Isophotes: the Key to Tractable Local Shading Analysis

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Abstract. When studying the formation of intensity image of an opaque surface, a natural question arises: how does the local surface structure project on the local image structure? We consider this question for the case of *a single and monocular intensity image.* In this paper, we discuss the the general problem of local shading analysis (LSA) of an arbitrary C^2 surface. We reformulate the problem so that it is well tractable without any restrictive assumptions about the surface. We also contribute to the discussion about the LSA ambiguity problem by identifying image structures where the local surface interpretation may only change.

1 Introduction

Local Shading Analysis (LSA) aims at looking for a *direct* relation between differential surface structure and the local structure of its intensity image. By surface it is meant *surface in the small*, i.e., defined in some open neighbourhood of one of its points. The structure is described by an m-th order local model.

The general advantage of LSA is that it can provide surface-related data for higher-level vision algorithms from single monocular 2-D intensity images *without reconstructing the surface* in an explicit depth form. This is possible, since intensity image is strongly related to local surface orientation. The change of intensity value in a particular direction in image is proportional to surface curvature in that direction. The surface normal vector and the shape operator ('curvature matrix') form thus a natural local shape model *recoverable from intensity image* by means of a local computational operation. The surface curvature (which is a rigid-motion invariant) is an important cue to local surface *quality* and plays thus a key role in surface description.

Formally, a set of independent n-th order local measurements ${\mathcal{F}_I}^n$ on image function is supposed to be related to surface differential parameters and to imaging geometry parameters by a mapping $\mathcal{P}$. The (ill-posed) inverse mapping $\mathcal{P}^{-1}$ is the subject of LSA aiming at obtaining local surface model from ${\mathcal{F}_I}^n$ with minimum ambiguity. The proper choice of ${\mathcal{F}_I}^n$ is crucial for the LSA *tractability.*

We assume known positions of infinitely distant illuminant and viewer, uniform surface albedo, and we exclude the presence of interreflections among surface patches. The latter two are the necessary conditions for the problem to remain local. The shape recovery in the presence of surface texture has already received a lot of attention and we will not cope with it here. The illumination direction can be estimated by the shading analysis near the occlusion contour [2]. If a non-local method is applied, the effect of interreflections can be undone [9].

1.1 Previous Work on LSA

In his work on LSA, Pentland makes two more restrictive assumptions: (i) the surface must be *spherical*, and (ii) the reflectance function must be *Lambertian* [11]. He then shows that the surface normal up to convex/concave case, the surface albedo, and the incident light direction can be estimated *without knowledge of the illuminant position.* If the qualitative illuminant position is known, the ambiguity can be resolved. When the assumptions are violated, the estimates degrade very fast. The solution need not exist in the case of parabolic surfaces.

Pentland uses image Hessian eigendirections as surface normal tilt estimator. Lee and Rosenfeld [8] show that image gradient does *for spheres* as well, since, in this case, the image gradient vector is parallel to the projection of surface normal onto the plane perpendicular to light direction vector $\mathbf{L}$. This projected vector is then related to surface normal vector in the viewer-centered coordinate system by a simple 3-D space transformation, whose parameters are known if the components of $\mathbf{L}$ are known in this system.

The general drawback of the above methods is that the recovered normals need not model a smooth (integrable) surface when the heuristic assumptions about the surface are violated.

Oliensis [10] argues that a surface point neighbourhood can be approximated by the *osculating paraboloid* at that point. Then, for a Lambertian surface, the set ${\mathcal{F}_I}^2$ and *known* $\mathbf{L}$ suffice to estimate the surface normal, principal curvatures, and the surface albedo. The formulation makes the problem poorly tractable because of the mathematical description complexity.

The shading at singular image points was studied in [7, 13]. Due to lack of space, we will not cope with this problem here. We refer reader to [13].

1.2 The Existence of Second-Order LSA Algorithm

At a *regular* image point, we construct the set ${\mathcal{F}_I}^n$ from partial image function derivatives:

$${\mathcal{F}_I}^n = \{I,\, I_x,\, I_y,\, I_{xx},\, I_{xy},\, I_{yy},\, \ldots\}.$$

For practical reasons, the highest derivative order n should not be high. There is a natural question if the set is sufficient to recover the m-th order surface model.

The Existence of Second-Order LSA Assuming the reflectance function and the illuminant position are known and the norm of the shape operator derivative is small, the ${\mathcal{F}_I}^2$ are sufficient to *estimate* the second order surface model and the surface albedo. There are six elements of ${\mathcal{F}_I}^2$ related to six unknown parameters[1]. Later it will be shown why ${\mathcal{F}_I}^1$ suffice for estimating normal vectors on spheres.

Fundamental Limitation of LSA On developable surfaces, the shape operator is not of full rank and the components of image gradient thus do not represent two independent shape-related measurements. The surface isoreflectance curves are then always parallel to the vanishing-curvature principal direction—independently on illumination geometry and surface reflectance model [7]. This explains why isophotes are *always* parallel to the rulings of developable surfaces and why shading is not sufficient for recovering the local surface shape of a developable by *any* shape-from-shading algorithm.

2 Isophotes

In this section, we show how to transform the LSA problem so that the relation between the local measurements ${\mathcal{F}_I}^2$ and the local surface shape is made explicit and algebraically well tractable.

An image $I(x,y)$ is a system of brightness isocontours, *image isophotes*, constituting a family of non-intersecting curves in the image plane. In the large, they are curves along which $I(x, y) = I_0$, and, in the small, their tangent direction $\mathbf{v}$ is the direction of vanishing image function directional derivative, $I_{\mathbf{v}} = 0$.

Image isophotes are projections of surface isoreflectance curves. These *surface isophotes* are structures *uniquely induced on the surface* by viewing and illumination geometry. The local structure of the field of surface isophotes is then related to the local structure of the surface.

Closely related is the system of image *flow-lines* (gradient curves). Flow-lines are perpendicular to isophotes at each point: their unit tangent vector $-\mathbf{u}$ is parallel to image gradient. Flow-lines can be back-projected on surface forming thus another system of curves defined up to a finite ambiguity. The surface is then doubly covered by the two systems of curves. Let us recall from differential geometry that any two independent systems of regular parametric curves define a regular surface, which was also a motivation for our choice.

2.1 A Set Equivalent to ${\mathcal{F}_I}^2$

There are local features of the image isophote field that form an irreducible set equivalent to ${\mathcal{F}_I}^2$. They are: the isophote 'label' I_0, the isophote density ρ, the direction of image gradient γ, the isophote curvature κ_{I}, the flow-line curvature κ_{F}, and the relative variation of isophote density along the flow-line ν. They are related to scaled directional image derivatives [4]:

$$\rho^* = -\frac{\rho}{I_0} = -\frac{I_{\mathbf{u}}}{I_0}, \quad \kappa_{\mathrm{F}} = -\frac{I_{\mathbf{uv}}}{I_{\mathbf{u}}}, \quad \kappa_{\mathrm{I}} = -\frac{I_{\mathbf{vv}}}{I_{\mathbf{u}}}, \quad \nu = \frac{I_{\mathbf{uu}}}{I_{\mathbf{u}}} . \qquad (1)$$

[1] Two of the four coefficients forming the shape operator are related by a known function if the surface parameterization is Gaussian [13].

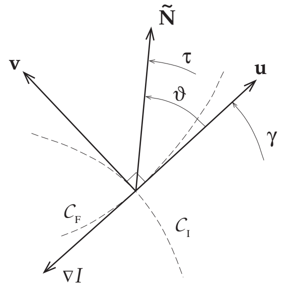

Fig. 1. The relation between the surface normal vector $\mathbf{N}$ projected on image plane and the vectors $\mathbf{u}$ and $\mathbf{v}$ that are tangent to image flow-line $\mathcal{C}_{\mathrm{F}}$ and isophote $\mathcal{C}_{\mathrm{I}}$, respectively. The angles τ and γ are measured from a fixed direction.

Some of the members of $\mathcal{F}_I^{*2} = \{I_0, \rho^*, \gamma, \kappa_{\mathrm{F}}, \kappa_{\mathrm{I}}, \nu\}$ have interesting photometric properties: the subset $\{\gamma, \kappa_{\mathrm{F}}, \kappa_{\mathrm{I}}\}$ is invariant to reflectance function if either (i) the illuminant is located at the vantage point or (ii) the reflectance function is space-invariant (it is invariant to the viewer position)[2]. The proof is given in [13]. The advantage of ρ^* over ρ is that it is invariant to (constant) surface albedo.

The tilt estimator proposed in Section 3.3 which is robust under the change of reflectance function benefits from the fact that it is partially based on the above introduced invariants.

3 Local Shading Analysis

3.1 Fundamental Equation of LSA

Let the normal vector be represented by its slant σ and its tilt $\tau = \gamma + \vartheta$, where ϑ is the angle between the image gradient in the direction γ and the orthographic projection of the surface normal onto image plane (see Fig.1). The normal vector is then

$$\mathbf{N}(\sigma, \tau) = [\cos\tau\ \sin\sigma,\ \sin\tau\ \sin\sigma,\ \cos\sigma]^{\mathrm{T}}\,,\quad \sigma \neq k\,\pi\,. \tag{2}$$

The parameters σ and τ can be used to parameterize a non-developable surface $\mathbf{x} = \mathbf{x}(\sigma, \tau)$. We will say such a surface has Gaussian parameterization. The σ-parameter curves coincide with the surface isophotes if either the surface reflectance function is space-invariant or the surface is illuminated from the vantage point. For a Gaussian parameterized Lambertian surface, the following fundamental partial differential equation was derived [13]:

$$\rho^*\ \sin\vartheta + \sin^2\sigma\ \Big[(\vartheta_{\mathbf{v}} \pm \kappa_{\mathrm{I}})\sin\vartheta + (\vartheta_{\mathbf{u}} \pm \kappa_{\mathrm{F}})\cos\vartheta\Big] = 0\,. \tag{3}$$

This equation relates the local image measurements ρ^*, κ_{I}, and κ_{F} to the unknown angles ϑ and σ. The equation is derived from Weingarten equations for surfaces using the explicit expression for the normal vector (2), see [13]. It holds

[2] This does not mean, however, the reflectance function may *vary* over the surface!

for the case of illuminant located at the vantage point and the orthogonal projection (a generalization is possible). The unknown signs in (3) give four local interpretations of the surface; another interpretations are due to the angle ϑ, since both ϑ and $\vartheta + \pi$ satisfy (3).

A necessary condition for ϑ to vanish at a regular image point p is one of the following [13]: (i) the point p lies on the occlusion contour, or, (ii) the parametric curves of the Gaussian parameterization coincide with the lines of curvature. The second case is important, since it follows that on *spheres*, the gradient vector of image projected on the light plane and the normal vector projected on the same plane become parallel. This is confirmed by the analysis given in [8]. In this case, ${\mathcal{F}_I}^1$ suffice for estimating normal vector and shape operator.

Besides the normal vector related to the solution of (3), the expression for the shape operator of the surface can be derived:

$$-\mathbf{S} = \begin{bmatrix} \rho^* \cot\sigma \cos\sigma \cos\vartheta & \cos\sigma\left[(\vartheta_{\mathbf{V}} \pm \kappa_{\mathrm{I}}) \sin\vartheta + (\vartheta_{\mathbf{U}} \pm \kappa_{\mathrm{F}}) \cos\vartheta\right] \\ -\rho^* \cos\sigma \sin\vartheta & \sin\sigma\left[(\vartheta_{\mathbf{V}} \pm \kappa_{\mathrm{I}}) \cos\vartheta - (\vartheta_{\mathbf{U}} \pm \kappa_{\mathrm{F}}) \sin\vartheta\right] \end{bmatrix}. \quad (4)$$

Gaussian curvature corresponds to the determinant of $\mathbf{S}$ and the mean curvature is a half of the trace of $\mathbf{S}$. Note that $\mathbf{S}(\vartheta + \pi) = -\mathbf{S}(\vartheta)$. This means that, e.g., if ϑ is the solution of (3) locally corresponding to a convex interpretation, the solution $\vartheta + \pi$ corresponds to a concave interpretation.

3.2 Occlusion Contours: the Curves of Unique Local Surface Interpretation

Since isophotes, like all surface curves projected on image plane, have a common tangent vector with the occlusion contour, the angle ϑ must vanish identically along the contour. It follows from (4) that, at the contour,

$$K = \det \mathbf{S} = \pm\kappa_{\mathrm{I}} \lim_{\sigma \to \left(\frac{\pi}{2}\right)_-} \rho^* \cos^2\sigma \,.$$

Since the limit exists and it is positive, the unknown sign of the isophote curvature κ_{I} will be equal to the sign of Gaussian curvature K. But it is known that the sign of Gaussian curvature is equal to the sign of occlusion contour curvature, which is equal to κ_{I} (isophote is parallel to the contour). The sign is therefore uniquely given and *the surface can unambiguously be interpreted along the occlusion contour in image.*

3.3 Single-Step Algorithm

The terms $\vartheta_{\mathbf{U}}$ and $\vartheta_{\mathbf{V}}$ in (3) capture the changes of surface curvature along the flow-line and the isophote, respectively. Assuming this change is small, we can neglect the terms. Then, by rearranging (3) and evaluating Gaussian curvature, we get

$$\vartheta \approx \arctan \frac{\pm\kappa_{\mathrm{F}} \sin^2\sigma}{\rho^* \pm \kappa_{\mathrm{I}} \sin^2\sigma} + k\,\pi, \qquad k = 0, 1 \quad (5)$$

$$K \approx \pm\kappa_{\mathrm{I}}\, \rho^* \cos^2\sigma. \quad (6)$$

It can be shown that a reasonable *slant estimator* is

$$\sigma \approx \arccos \frac{I}{I_{\max}}, \tag{7}$$

where $I_{\max}$ is the image function value at a singularity corresponding to the critical point of the reflectance function. If the reflectance function is space-invariant or if the illuminant is located at the vantage point, this singularity corresponds to the global image function maximum. The estimator (7) has already been suggested by several authors [8, 3]. The influence of slant misestimation on the resulting tilt estimate (3) is *surprisingly small* [13].

The qualitative local surface interpretation is ambiguous because of the unknown signs in (5). There are thus four convex and four concave interpretations possible. But, by continuity, the interpretation cannot change from one to another at loci where both κ_{I} and κ_{F} are nonzero. *This means that only along zero-crossings of isophote and flow-line curvature, the local surface interpretation may change.* The image can thus be segmented into a set of non-overlapping regions, each corresponding to a *constant* (although unknown) qualitative surface interpretation.

The single step LSA algorithm is based on (5) and (7) and it requires computing the image gradient and the second-order image derivatives, see formulas (1). This limits the robustness of the algorithm. If we use *blurred derivatives kernels* to compute the members of $\mathcal{F}_I{}^2$, the noise sensitivity of the algorithm is reduced [12].

The tilt estimator (5) was tested together with a modification of the Pentland's tilt estimator[3] and the Lee and Rosenfeld's estimator to compare their behaviour on non-trivial surfaces. We used slant estimator (7) in all three LSA algorithms.

The difference between the tested algorithms is nicely visible if the surface does not conform to the sphericity assumption. A synthetic image of a hyperboloid illuminated from the vantage point is used to show the difference. The needle maps resulting from the three LSA algorithms were integrated to get the explicit depth representation of the surface. The results are shown in Fig. 2.

4 Conclusions and Future Research

The theory briefly overviewed here is presented in [13] in full detail. We related the local image structure expressed by the structure of the image isophotes field to the differential surface structure. The generality of the approach allows further analysis of this relation. The advantage of choosing isophotes as the surface parameterization generators is that the local shading analysis problem becomes *well-tractable* with *no a priori restriction on the surface form* in the sense of restricting the surface type like Pentland in [11], Lee & Rosenfeld in [8], or Ferrie & Levine in [3], or the surface order like Arnspang in [1], or assuming an

[3] The modification ensures a unique solution is found such that it is closest to image gradient [13].

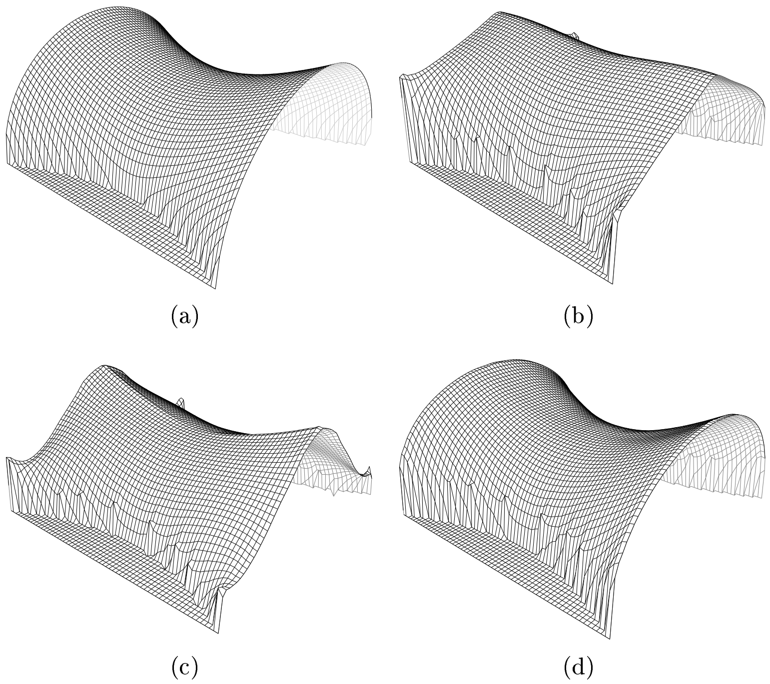

Fig. 2. Comparison of the reconstructed results of different LSA algorithms. The true surface (a), the result of Lee and Rosenfeld's algorithm (b), the result of Pentland's algorithm (c), and the result of the proposed algorithm (d).

explicit local surface approximation like Oliensis in [10]. The drawback of the proposed algorithm (and all image gradient based LSA algorithms) is its ill-posedness at extremal image points, since it is based on geometric properties of image gradient. The shading analysis at these singular points can be performed independently, however [13]. It was argued that no LSA algorithm can be used for recovering local surface model of developable (and planar) surfaces (see Section 1.2).

From the experiments on synthetic data it follows that: (i) the proposed algorithm does not miss any of the possible local solutions, (ii) the tilt estimates are very accurate, (iii) the recovered curvature-maps are qualitatively correct, (iv) there is no bad behavior of the algorithm on the occlusion contour, (v) the influence of non-Lambertianity of the surface is very small, (vi) the expected error of the recovered normal orientation due to misestimated illuminant position increases linearly as the illuminant moves from its presupposed position [13].

The main issue in local shading analysis is the ambiguity of the qualitative surface interpretation. It was shown that this ambiguity is minimal in the neighbourhood of certain image structures (occlusion contour). We have found unique image structures along which the interpretation may change. These structures segment the image into a set of non-overlapping regions. The overall image interpretation is then possible by propagating the information from less-ambiguous regions. If other cues to surface quality are available (motion, stereo), the structure induced by region adjacency relation gives a constraint that may enforce the robustness of the interpretation. It might also be used for distinguishing various types of surface contours and discontinuities. This issue needs further research. We are currently testing a *semi-global shading analysis*, where there are smoothness constraints imposed on the "seems" between the interpreted regions.

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