A Software for Complete Calibration of Multicamera Systems

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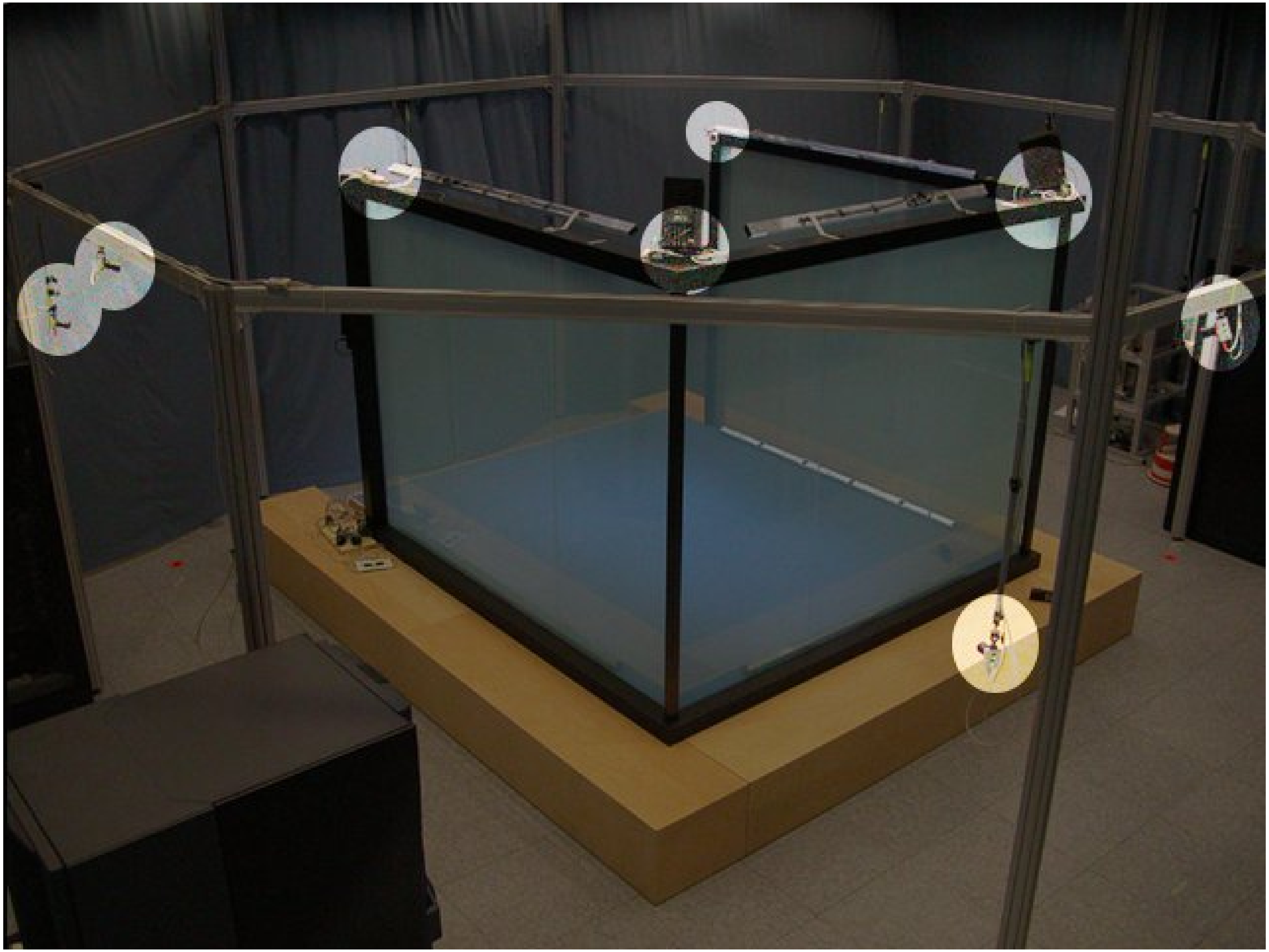
Outline

- Motivation
- Problem definition
- Proposed solution
- Results
- Applications

Motivation

- multiple cameras became common
- they can be found in . . .

Virtual reality room

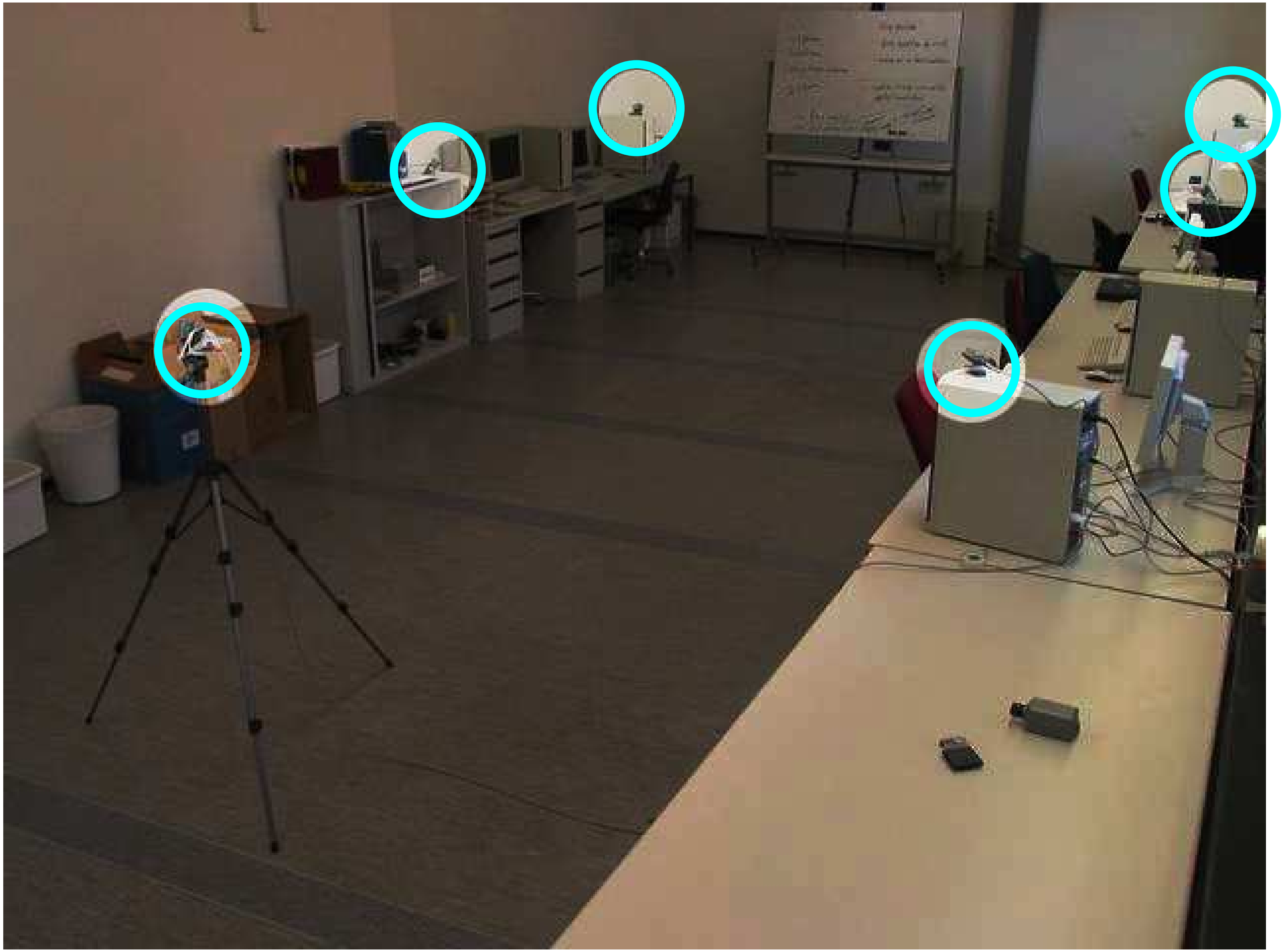


Telepresence setup

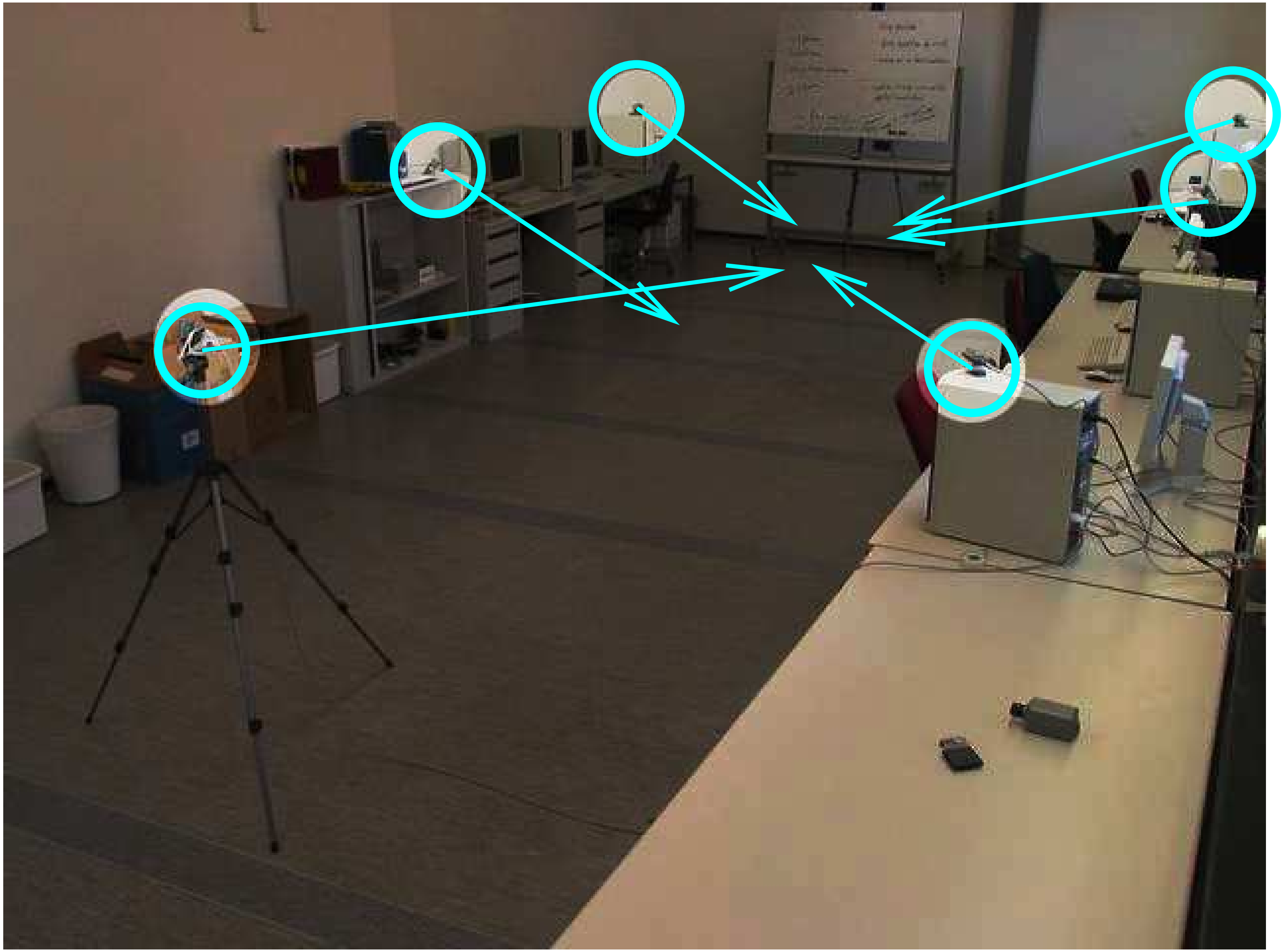
Calibration

- many tasks can be accomplished without knowing anything about the cameras
- however, many more when we know . . .

camera positions, and . . .

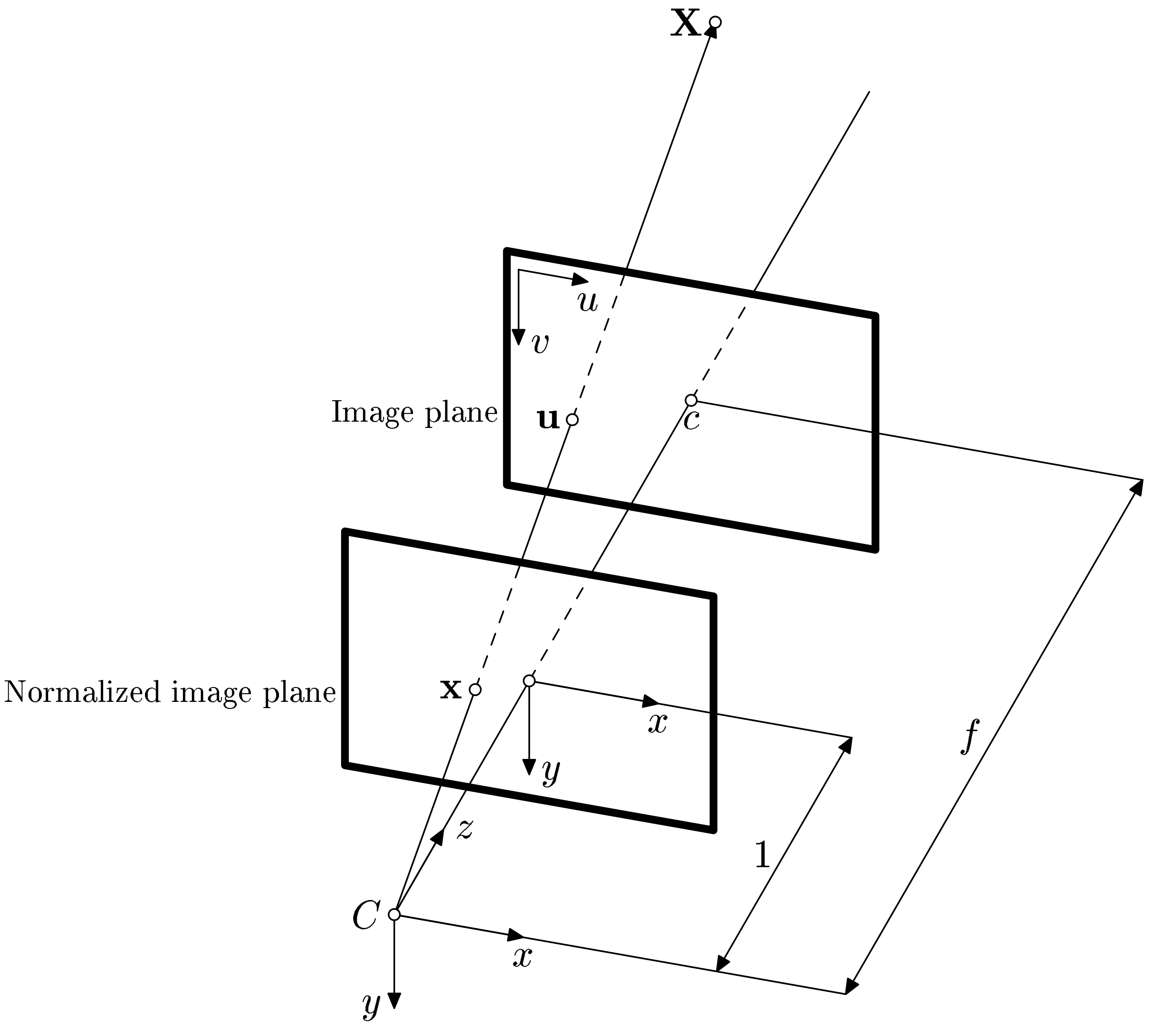


. . . camera orientations, and . . .



. . . camera internal parameters

from geometry to pixels

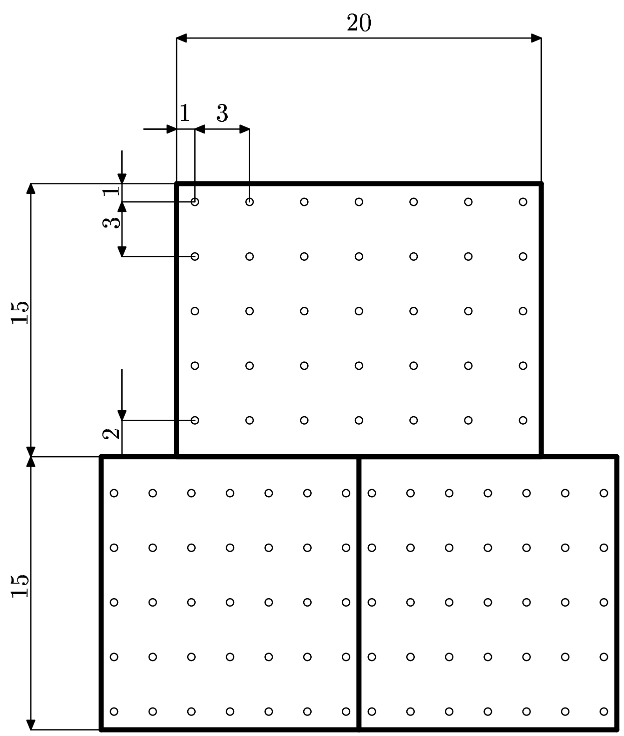


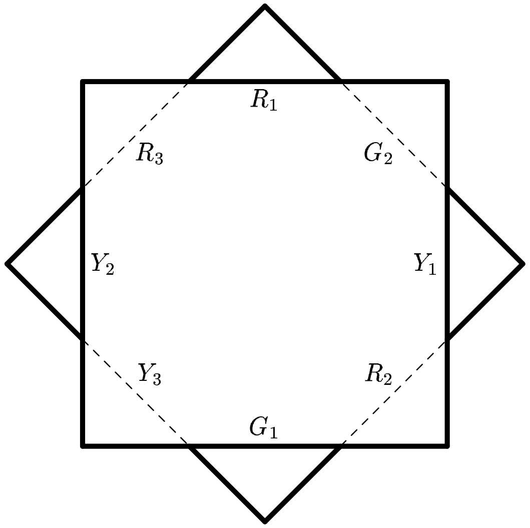
. . . nonlinear parameters included

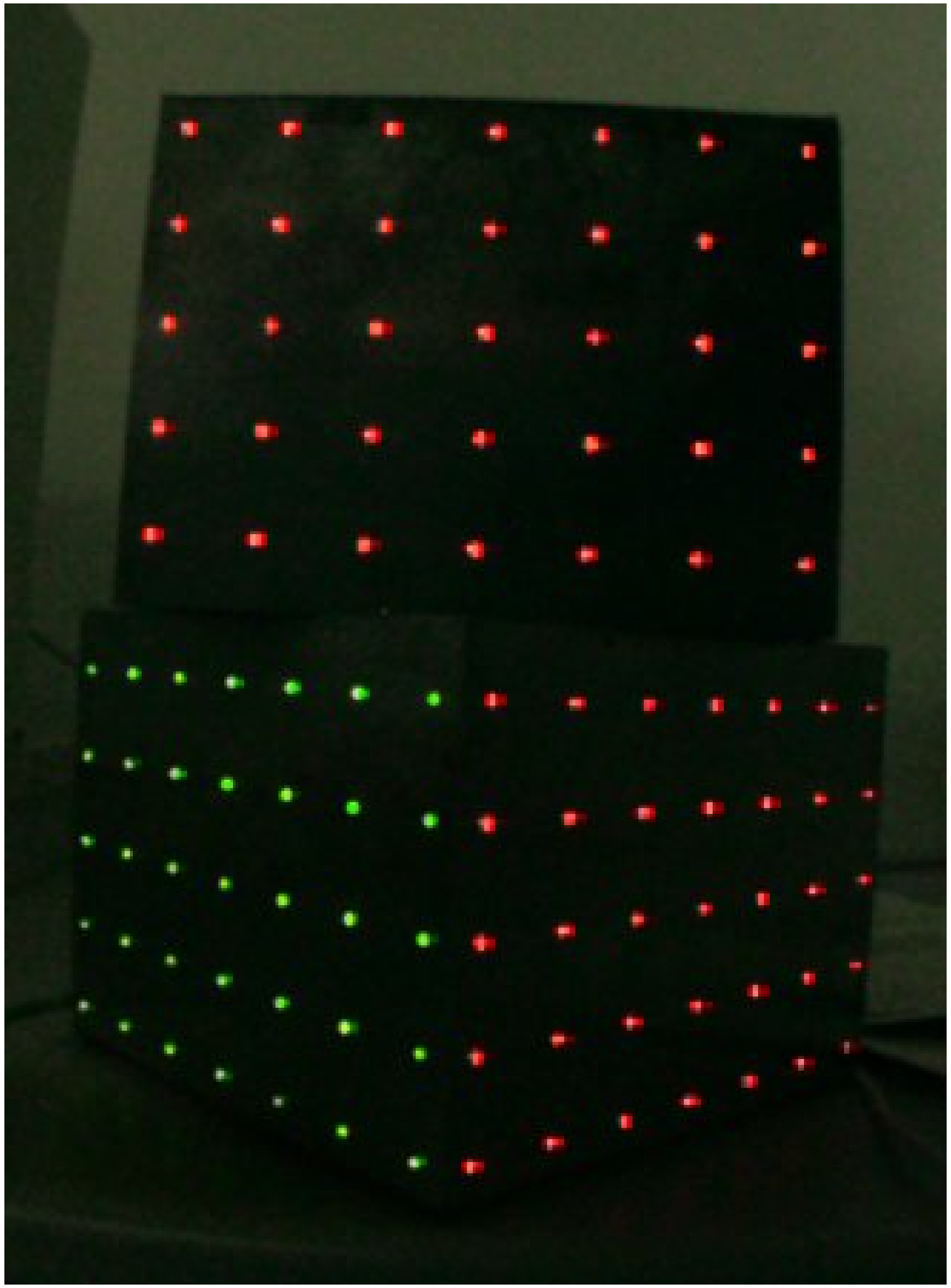
Camera calibration is an old problem

- for photogrammetrists (even older problem)
- in computer vision
- many methods exist

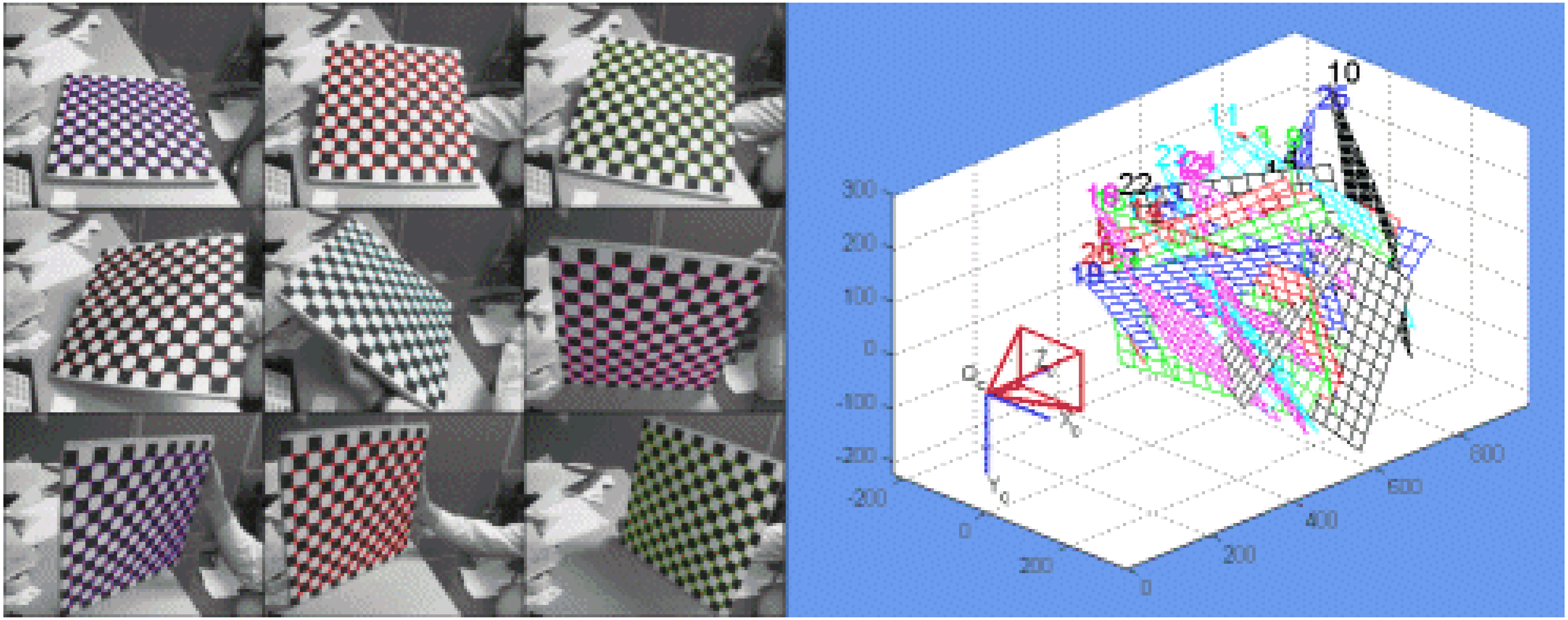
Classical approaches — known 3D points







Classical approaches — plate at several positions



http://www.vision.caltech.edu/bouguetj/calib_doc/

Classical methods — revisited

Pros:

- many methods (and free codes)
- precise, even for complicated camera models

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- precise, even for complicated camera models

Cons (for multicamera systems):

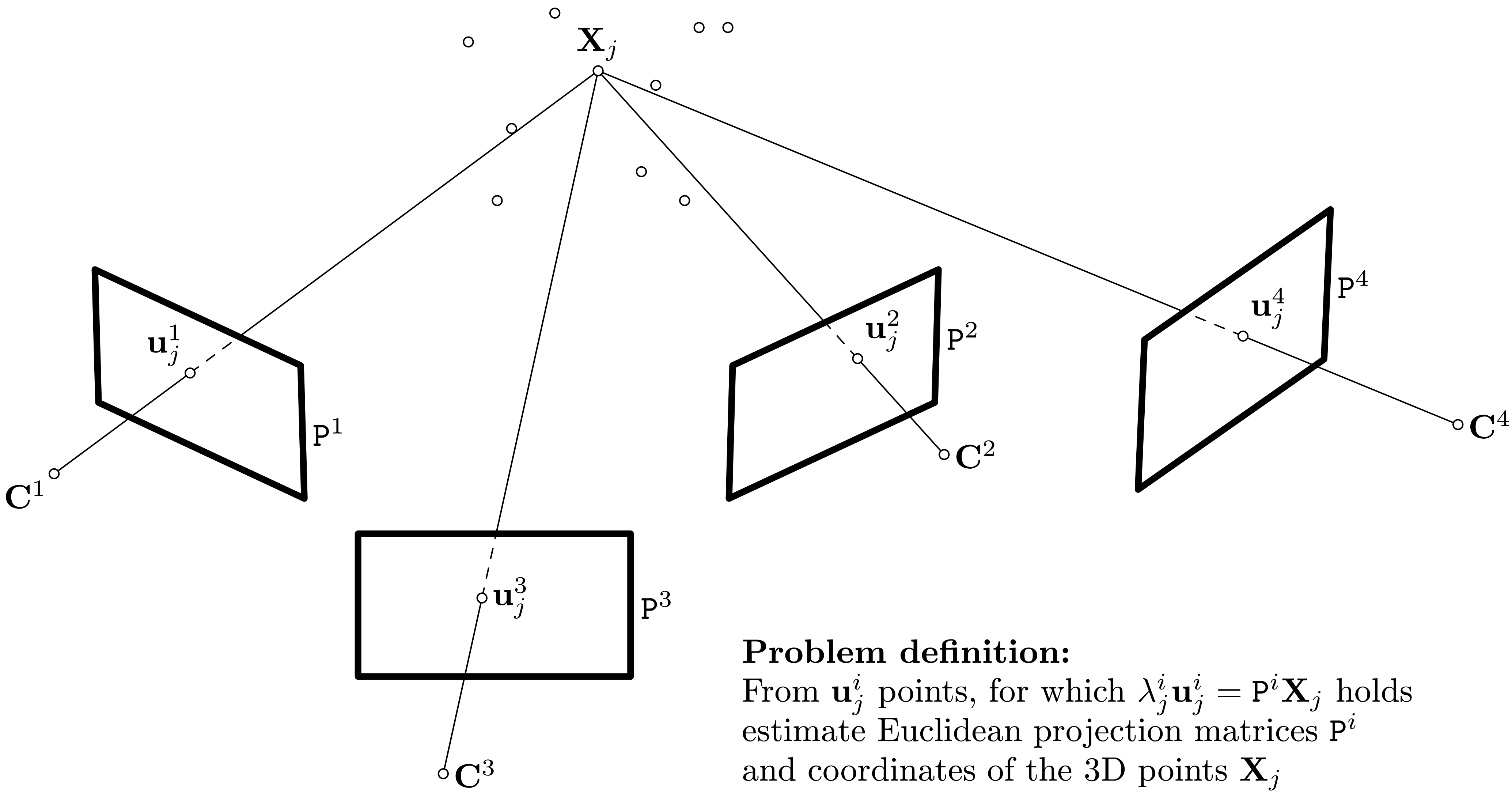
- many cameras $\longrightarrow$ hand work is not an option
- large working volume to fill $\longrightarrow$ big calibration objects/plates

Our solution — overview

We assume at least approximately synchronized multicamera ($N \geq 3$) setup.

- use 1-point calibration object easily detectable in images
- wave the calibration point through the working volume freely
- this will create a virtual calibration object (but the 3D position unknown!)
- apply theoretical results from self-calibration field
- estimate as complicated camera model as reasonable
- validate the results

Multiple cameras — Geometry



Problem definition:
From $\mathbf{u}_j^i$ points, for which $\lambda_j^i \mathbf{u}_j^i = \mathtt{P}^i \mathbf{X}_j$ holds
estimate Euclidean projection matrices $\mathtt{P}^i$
and coordinates of the 3D points $\mathbf{X}_j$

Pinhole camera model

$$\lambda_j^i \begin{bmatrix} u_j^i \\ v_j^i \\ 1 \end{bmatrix} = \lambda_j^i \mathbf{u}_j^i = \mathrm{P}^i \mathbf{X}_j \,, \qquad \lambda_j^i \in \mathcal{R}^+$$

- $_j$ index points
- ^{i} index camera
- λ_j^i projective depths
- $\mathbf{u}_j^i$ point projections (we find them in images)
- $\mathbf{X}_j$ 3D points (we do not know the positions!)
- P^i camera matrices

Multicamera linear model

$$
\mathtt{W}_s = \begin{bmatrix}
\lambda_1^1 \begin{bmatrix} u_1^1 \\ v_1^1 \\ 1 \end{bmatrix} & \cdots & \lambda_n^1 \begin{bmatrix} u_n^1 \\ v_n^1 \\ 1 \end{bmatrix} \\
\vdots & \vdots & \vdots \\
\lambda_1^m \begin{bmatrix} u_1^m \\ v_1^m \\ 1 \end{bmatrix} & \cdots & \lambda_n^m \begin{bmatrix} u_n^m \\ v_n^m \\ 1 \end{bmatrix}
\end{bmatrix}
= \underbrace{\begin{bmatrix} \mathtt{P}^1 \\ \vdots \\ \mathtt{P}^m \end{bmatrix}_{3m\times 4}}_{\mathtt{P}}
\underbrace{[\mathbf{X}_1 \cdots \mathbf{X}_n]_{4\times n}}_{\mathtt{X}}
$$

Self-calibration (Euclidean stratification)

$$
\mathtt{W}_s = \mathtt{P}\mathtt{X} = \underbrace{\mathtt{P}\mathtt{H}}\,\underbrace{\mathtt{H}^{-1}\mathtt{X}} = \hat{\mathtt{P}}\hat{\mathtt{X}}\,,
$$

What the software does:

1. Finds the projections $\mathbf{u}_j^i$ of the laser pointer in the images.

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5. Estimates the parameters of the non-linear distortion

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Many cross-validation steps inside.

Calibration object

A very standard laser pointer with a piece of transparent plastic attached.

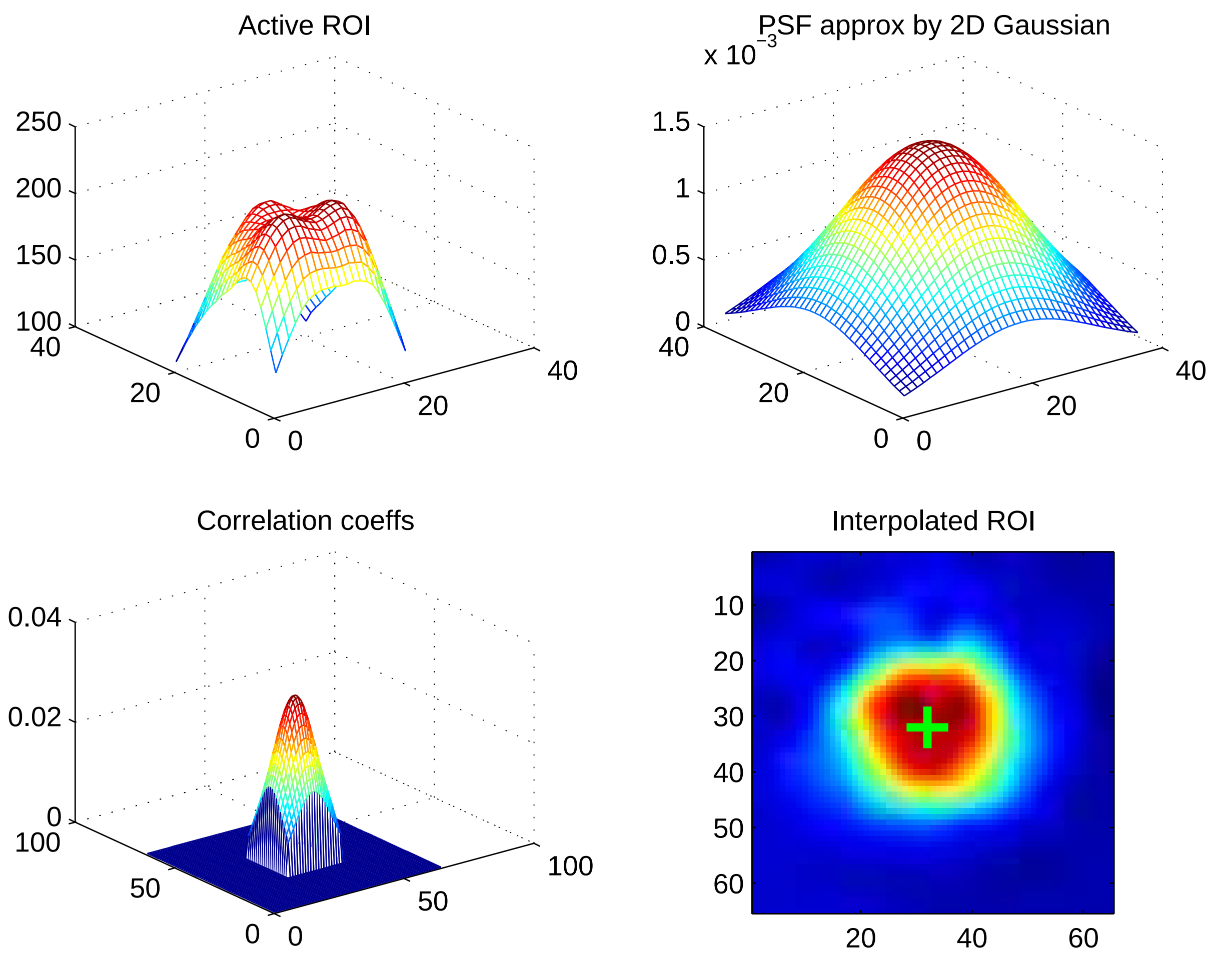
Finding points

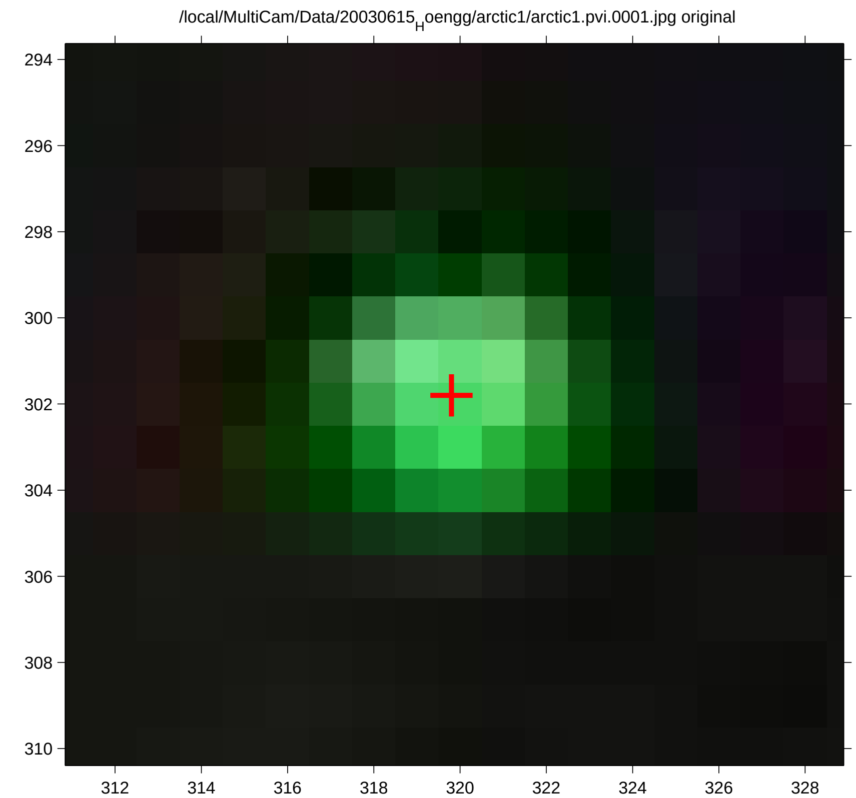
Needs to be a bit more clever than a simple thresholding

Statistical analysis of the images (almost) solves it.

Finding points

Sub-pixel accuracy is desirable





Around 100 ms per image.

Calibration input

Video

We know

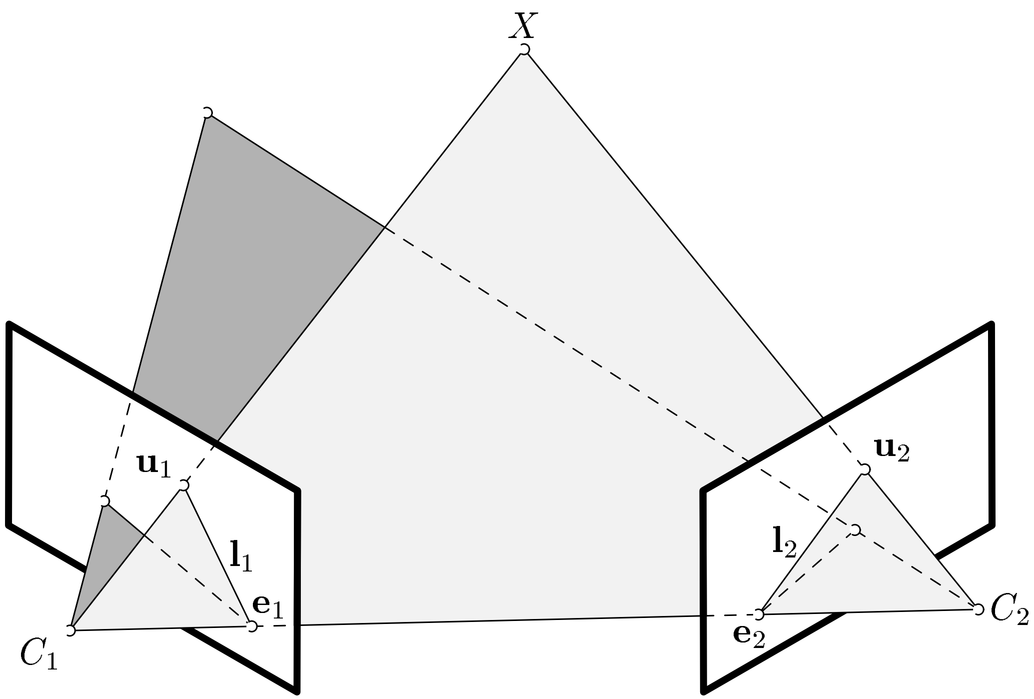
$$\mathtt{W}_s = \begin{bmatrix} \lambda_1^1 \begin{bmatrix} u_1^1 \\ v_1^1 \\ 1 \end{bmatrix} & \cdots & \lambda_n^1 \begin{bmatrix} u_n^1 \\ v_n^1 \\ 1 \end{bmatrix} \\ \vdots & \vdots & \vdots \\ \lambda_1^m \begin{bmatrix} u_1^m \\ v_1^m \\ 1 \end{bmatrix} & \cdots & \lambda_n^m \begin{bmatrix} u_n^m \\ v_n^m \\ 1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \mathtt{P}^1 \\ \vdots \\ \mathtt{P}^m \end{bmatrix}_{3m\times 4} [\mathbf{X}_1 \cdots \mathbf{X}_n]_{4\times n}$$

$$\mathtt{W}_s = \mathtt{P}\mathtt{X} = \mathtt{P}\mathtt{H}\mathtt{H}^{-1}\mathtt{X} = \hat{\mathtt{P}}\hat{\mathtt{X}} \ ,$$

However, some $[u_j^i, v_j^i]^\top$ may be missing!

Estimation of λ_j^i (Sturm & Triggs ECCV96)

uses the epipolar geometry



$$\lambda_j^i = \frac{(\mathbf{e}^{ik} \times \mathbf{u}_j^i) \cdot (\mathbf{F}^{ik}\mathbf{u}_j^k)}{\|\mathbf{e}^{ik} \times \mathbf{u}_j^i\|^2} \lambda_j^k$$

We know

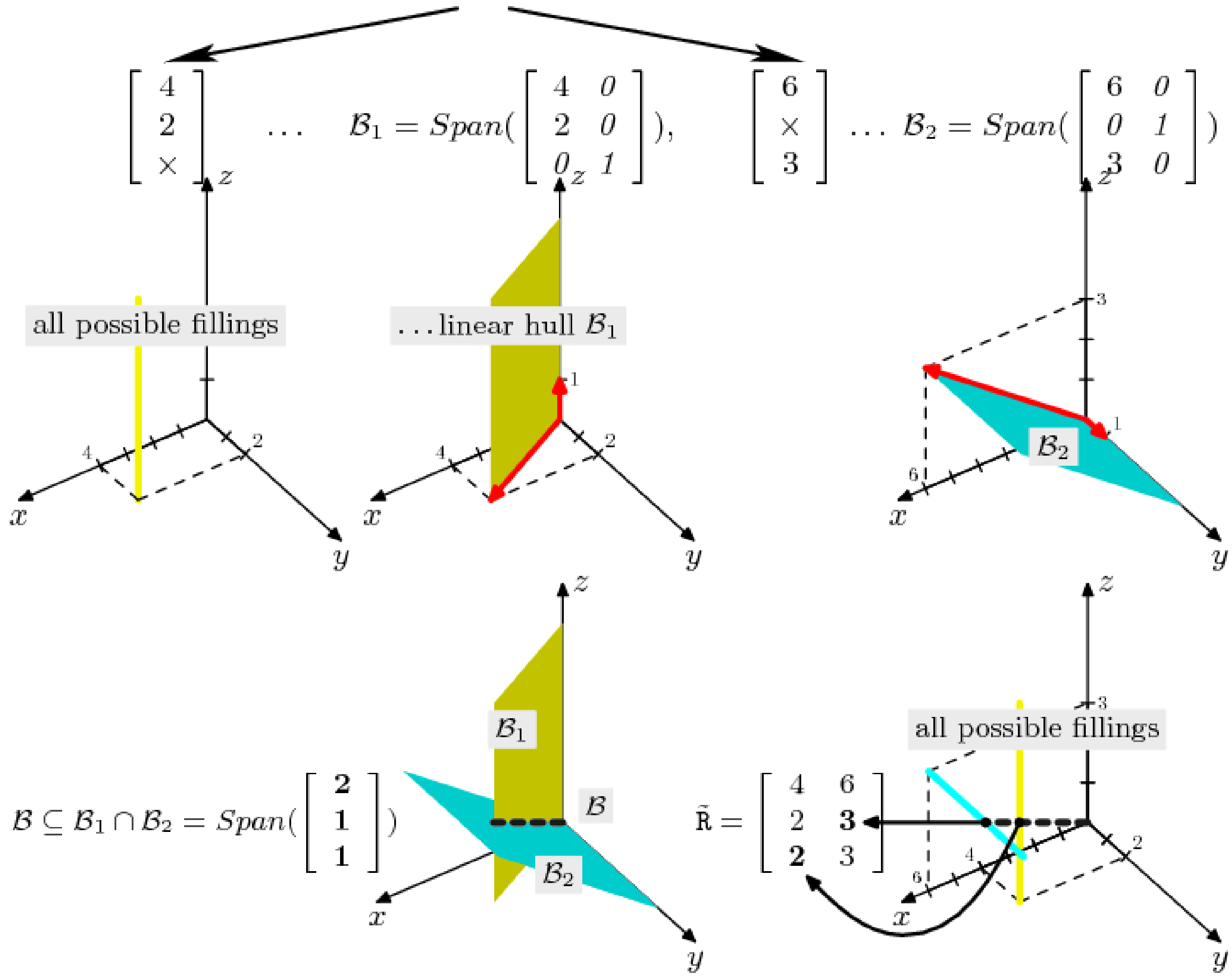
$$\mathtt{W}_s = \begin{bmatrix} \lambda_1^1 \begin{bmatrix} u_1^1 \\ v_1^1 \\ 1 \end{bmatrix} & \cdots & \lambda_n^1 \begin{bmatrix} u_n^1 \\ v_n^1 \\ 1 \end{bmatrix} \\ \vdots & \vdots & \vdots \\ \lambda_1^m \begin{bmatrix} u_1^m \\ v_1^m \\ 1 \end{bmatrix} & \cdots & \lambda_n^m \begin{bmatrix} u_n^m \\ v_n^m \\ 1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \mathtt{P}^1 \\ \vdots \\ \mathtt{P}^m \end{bmatrix}_{3m\times 4} [\mathbf{X}_1 \cdots \mathbf{X}_n]_{4\times n}$$

$$\mathtt{W}_s = \mathtt{P}\mathtt{X} = \mathtt{P}\mathtt{H}\mathtt{H}^{-1}\mathtt{X} = \hat{\mathtt{P}}\hat{\mathtt{X}} \,,$$

However, some $[u_j^i, v_j^i]^\top$ and λ_j^i may be missing!

Filling missing points (Martinec and Pajdla ECCV2002)

Example: $\mathtt{R} = \begin{bmatrix} 4 & 6 \\ 2 & \times \\ \times & 3 \end{bmatrix}$, for rank $\mathtt{R} = 1$ instead of rank $\mathtt{R} = 4$



We know

$$\mathtt{W}_s = \begin{bmatrix} \lambda_1^1 \begin{bmatrix} u_1^1 \\ v_1^1 \\ 1 \end{bmatrix} & \cdots & \lambda_n^1 \begin{bmatrix} u_n^1 \\ v_n^1 \\ 1 \end{bmatrix} \\ \vdots & \vdots & \vdots \\ \lambda_1^m \begin{bmatrix} u_1^m \\ v_1^m \\ 1 \end{bmatrix} & \cdots & \lambda_n^m \begin{bmatrix} u_n^m \\ v_n^m \\ 1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \mathtt{P}^1 \\ \vdots \\ \mathtt{P}^m \end{bmatrix}_{3m\times 4} \left[\mathbf{X}_1 \cdots \mathbf{X}_n\right]_{4\times n}$$

$$\mathtt{W}_s = \mathtt{P}\mathtt{X} = \mathtt{P}\mathtt{H}\mathtt{H}^{-1}\mathtt{X} = \hat{\mathtt{P}}\hat{\mathtt{X}}\ ,$$

Rank–4 factorization

$$\mathtt{W}_s = \begin{bmatrix} \mathtt{P}^1 \\ \vdots \\ \mathtt{P}^m \end{bmatrix}_{3m\times \mathbf{4}} [\mathbf{X}_1 \cdots \mathbf{X}_n]_{\mathbf{4}\times n}$$

So, matrix $\mathtt{W}_s$ should have rank at most 4

$$\mathtt{W}_s = \mathtt{U}\mathtt{S}\mathtt{V}^\top$$

$$\begin{bmatrix} \mathtt{P}^1 \\ \vdots \\ \mathtt{P}^m \end{bmatrix}_{3m\times 4} [\mathbf{X}_1 \cdots \mathbf{X}_n]_{4\times n} = (\mathtt{U}\sqrt{\mathtt{S}_4})(\sqrt{\mathtt{S}_4}\mathtt{V}^\top)$$

where $\mathtt{S}_4$ is the $\mathtt{S}$ with only 4 biggest diagonal values, rest is zeroed.

We know

$$\mathtt{W}_s = \mathtt{PX} = \mathtt{PHH}^{-1}\mathtt{X} = \hat{\mathtt{P}}\hat{\mathtt{X}}\ ,$$

We must find a 4×4 matrix $\mathtt{H}$ which upgrades the projective structures $\mathtt{P}, \mathtt{X}$ to metric ones, $\hat{\mathtt{P}}, \hat{\mathtt{X}}$.

Euclidean stratification (Pollefeys et al, Hartley, . . .)

based on the idea of absolute quadric (conic)

$$\hat{\mathrm{P}}^i = \mu_i \left[\; \mathrm{K}^i \mathrm{R}^i \quad \mathrm{K}^i \mathbf{t}^i \; \right]$$

$$\hat{\mathrm{P}}^i \hat{\Omega}_\infty \hat{\mathrm{P}}^{i\top} \sim \mathrm{K}^i \mathrm{K}^{i\top}$$

where

$$\hat{\Omega}_\infty = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Euclidean stratification cont.

absolute conic exists also in the projective world!

$$\begin{aligned} \mathtt{K}^i\mathtt{K}^{i\top} &\sim (\hat{\mathtt{P}}^i\mathtt{H}^{-1})(\mathtt{H}\hat{\Omega}_\infty\mathtt{H}^\top)(\mathtt{H}^{-\top}\hat{\mathtt{P}}^{i\top}) \\ \mathtt{K}^i\mathtt{K}^{i\top} &\sim \mathtt{P}^i\Omega_\infty\mathtt{P}^{i\top} \end{aligned}$$

We know the projective $\mathtt{P}^i$. The projective Ω_∞ is 4×4 symmetric.

Once Ω_∞ is known, then we can compute $\mathtt{H}$ from

$$\Omega_\infty = \mathtt{H}\hat{\Omega}_\infty\mathtt{H}^\top$$

by eigenvalue decomposition and get the sought Euclidean structures $\hat{\mathtt{P}}^i = \mathtt{P}^i\mathtt{H}$ and $\hat{\mathbf{X}}_j = \mathtt{H}^{-1}\mathbf{X}_j$.

Euclidean stratification — Example of solution

assume everything is known except focal lenghts

$$\mathtt{K}^i = \begin{bmatrix} f^i & 0 & u_0^i \\ 0 & \alpha^i f^i & v_0^i \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \mathtt{K}^i \mathtt{K}^{i\top} = \begin{bmatrix} f^{i2} & 0 & 0 \\ 0 & f^{i2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Remember that $\mathtt{K}^i \mathtt{K}^{i\top} \sim \mathtt{P}^i \Omega_\infty \mathtt{P}^{i\top}$

$$\begin{aligned} (\mathtt{P}^i \Omega_\infty \mathtt{P}^{i\top})_{11} - (\mathtt{P}^i \Omega_\infty \mathtt{P}^{i\top})_{22} &= 0 \\ (\mathtt{P}^i \Omega_\infty \mathtt{P}^{i\top})_{12} &= 0 \\ (\mathtt{P}^i \Omega_\infty \mathtt{P}^{i\top})_{13} &= 0 \\ (\mathtt{P}^i \Omega_\infty \mathtt{P}^{i\top})_{23} &= 0 \end{aligned}$$

Each camera contributes by 4 contraints.

We have the metric linear model

$$\mathtt{W}_s = \hat{\mathtt{P}}\hat{\mathtt{X}}$$

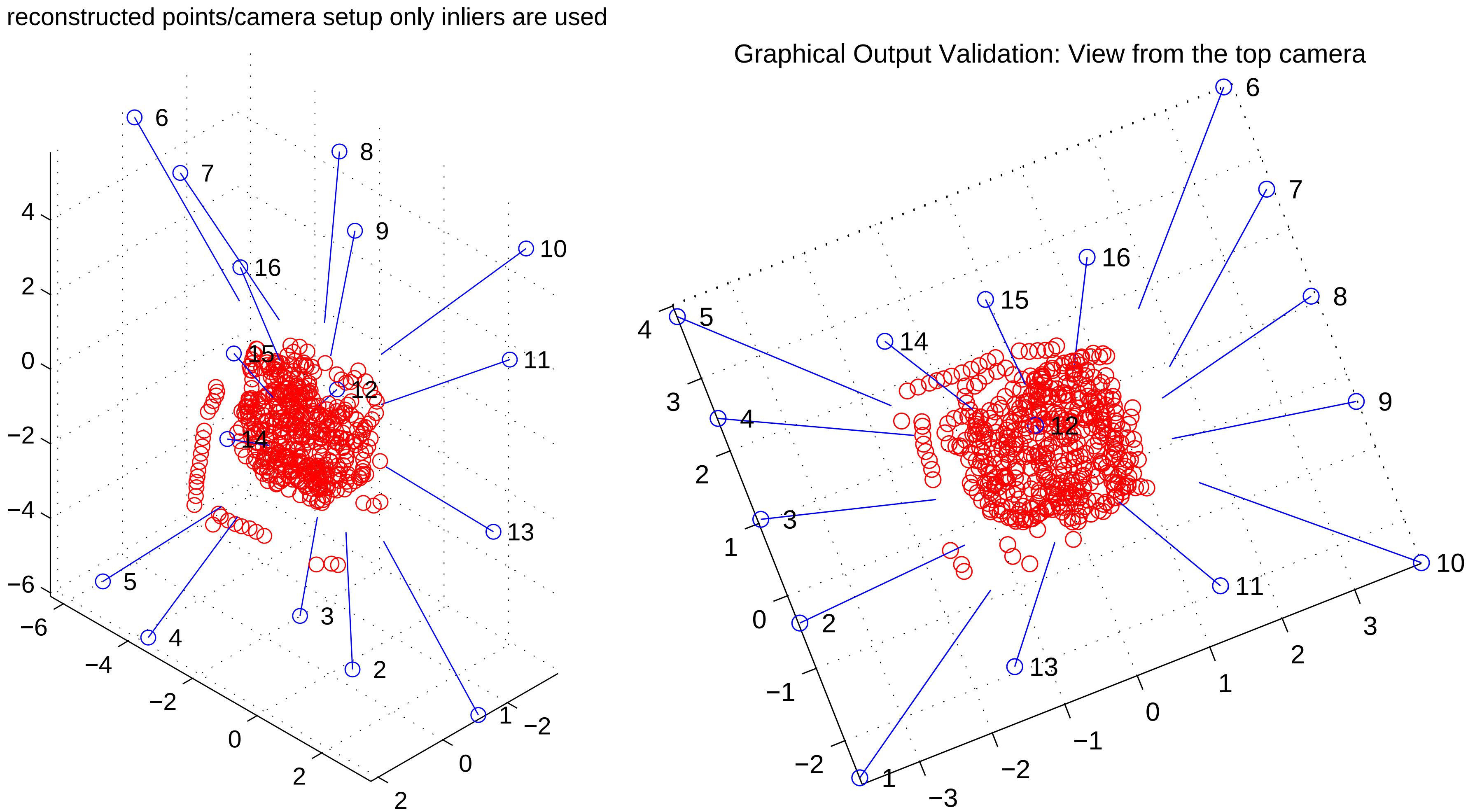
Estimation of non-linear distortion starts from

$$\hat{\mathtt{X}}_j \leftrightarrow \mathbf{u}_j^i$$

correspondences. We use the CalTech package
http://www.vision.caltech.edu/bouguetj/calib_doc/

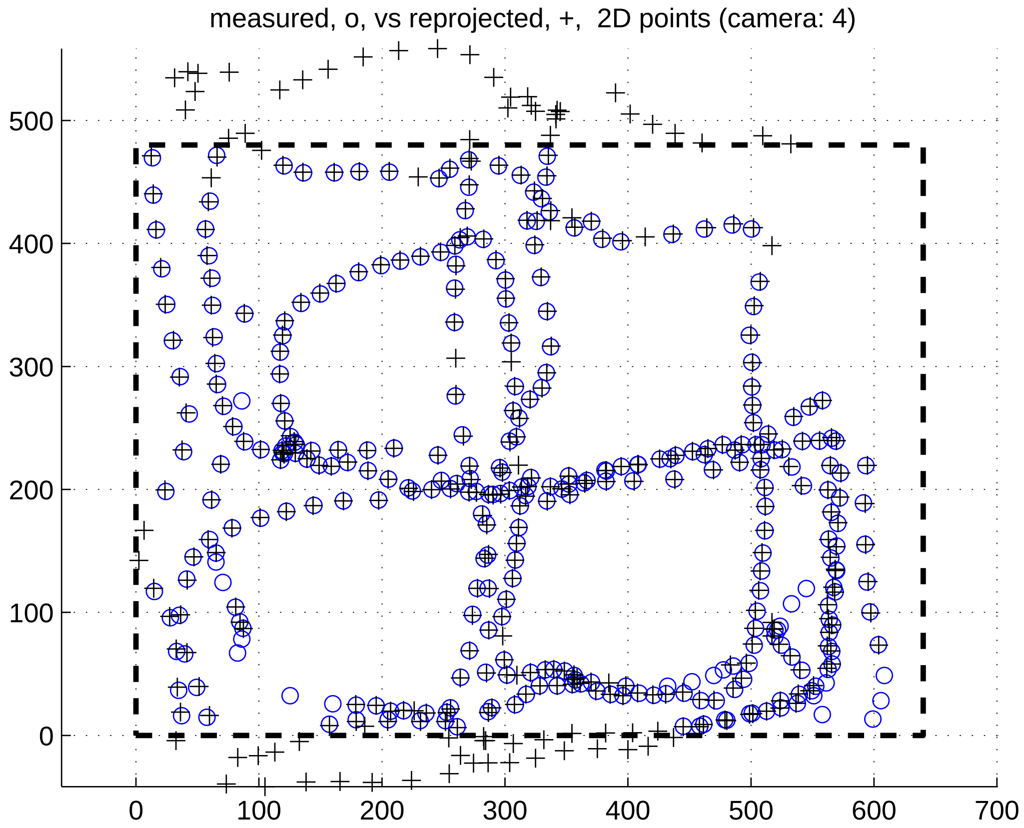
Then it goes back, adapt parameters and . . .

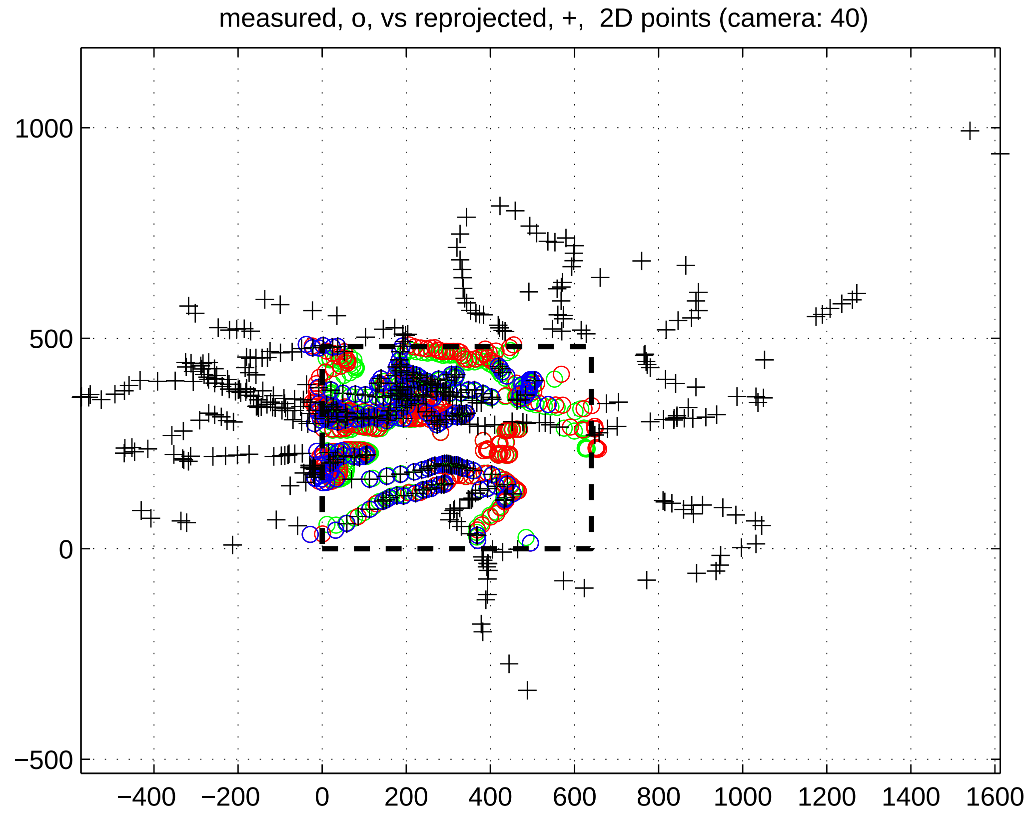
Aligning the results with the world



User provides some 3D information. Example: “Cameras No. 11,13,15 define the xy plane”.

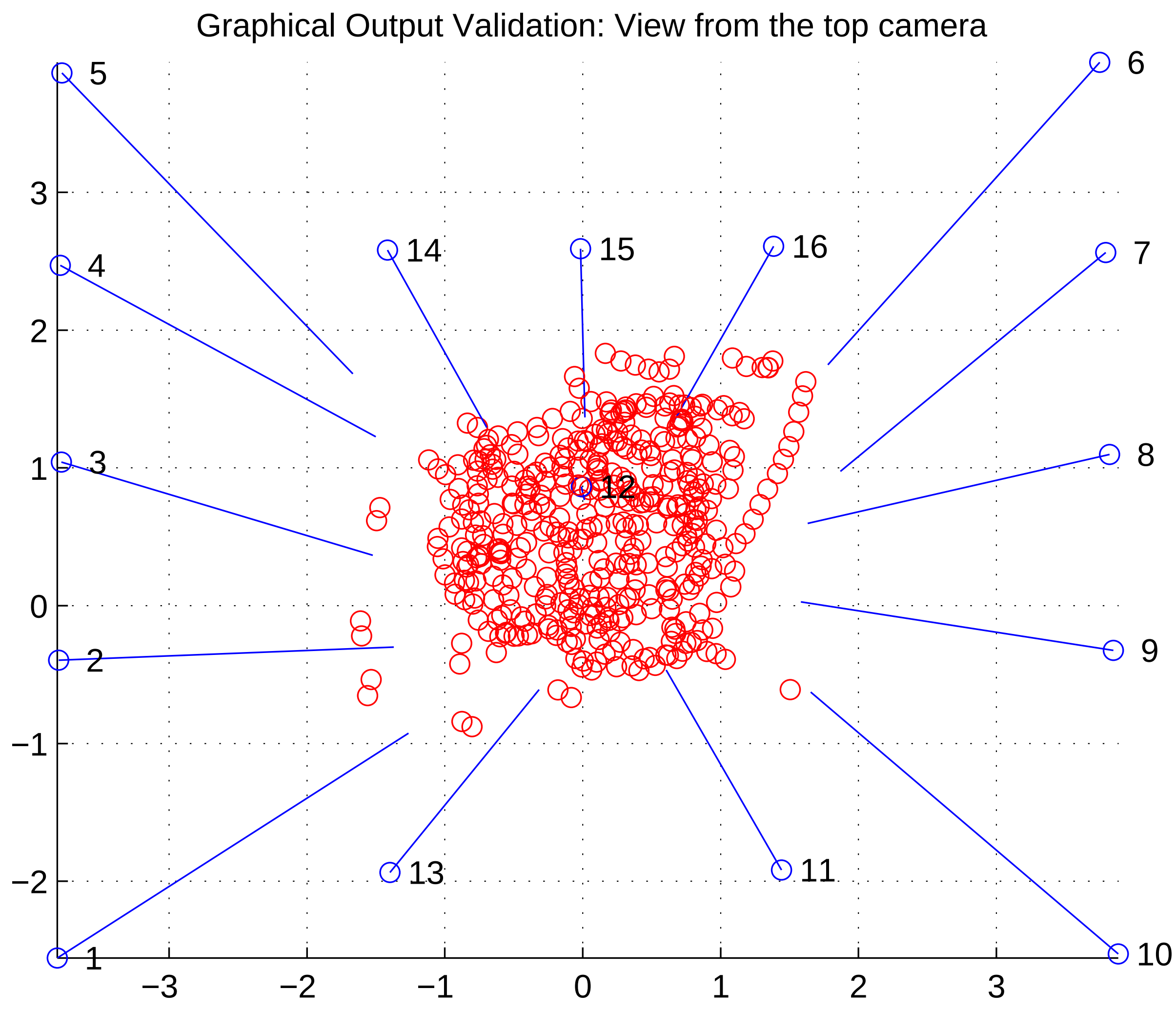
Results — Filling points

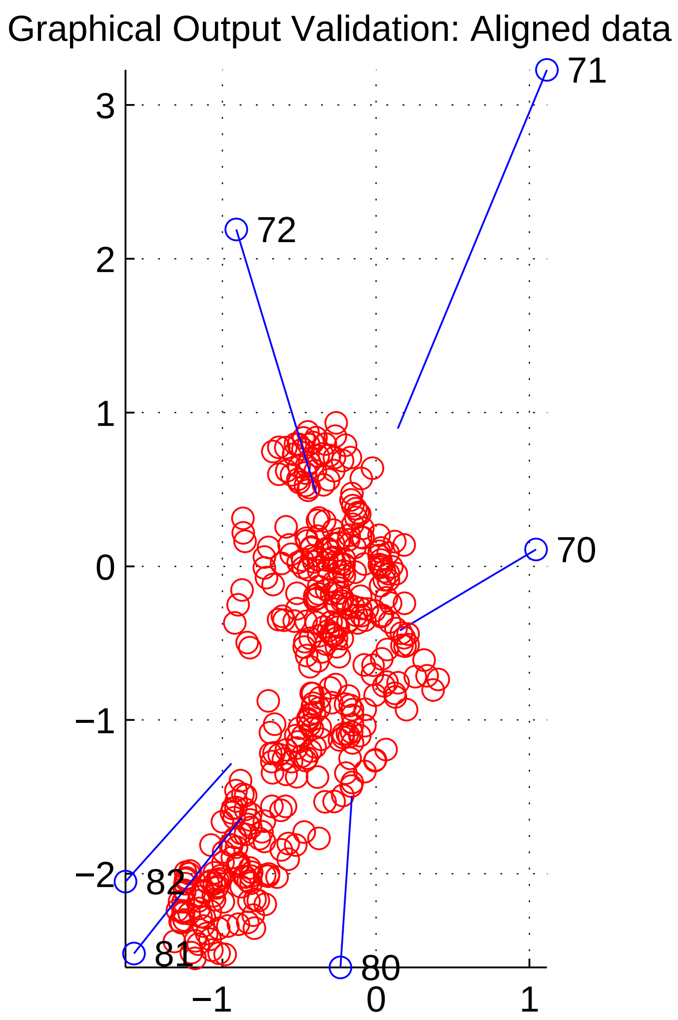




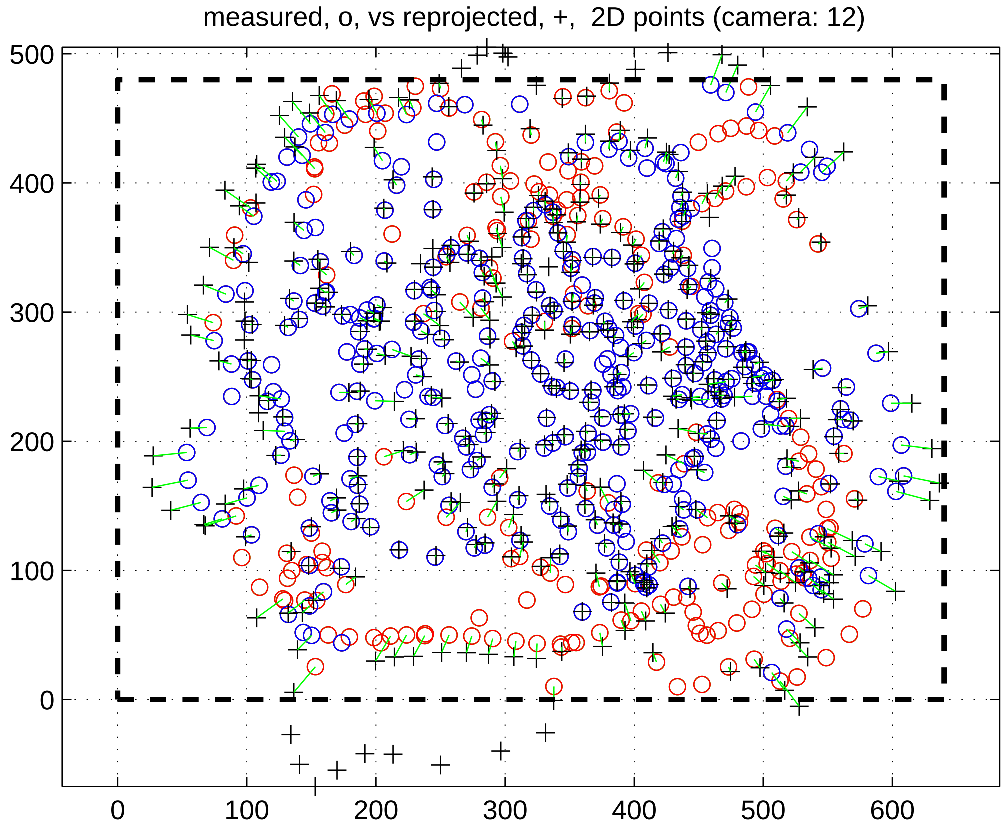
The calibration “point” needs not to be visible in all cameras!

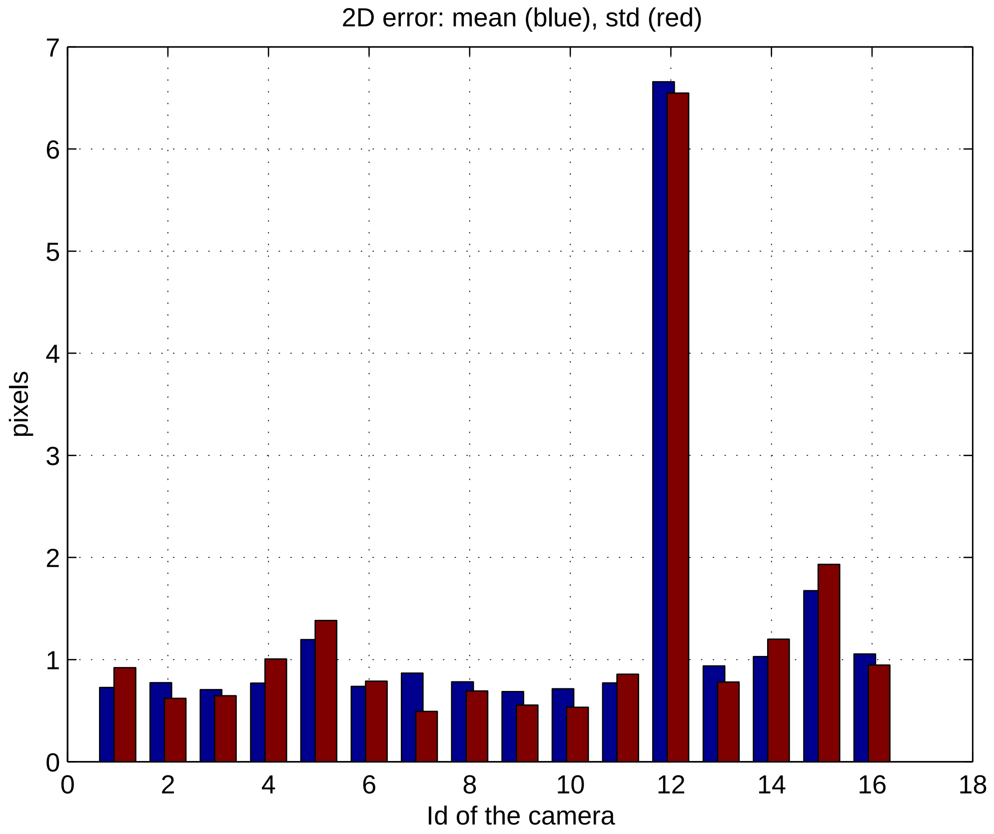
Results — Calibrated setups



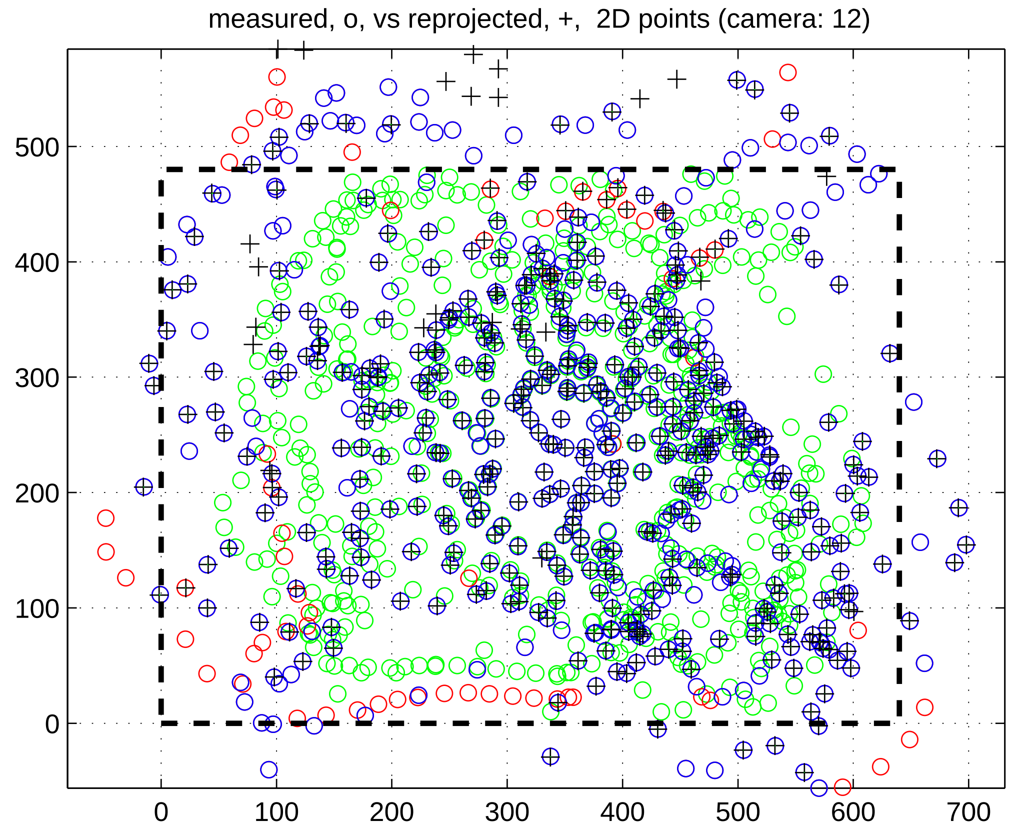


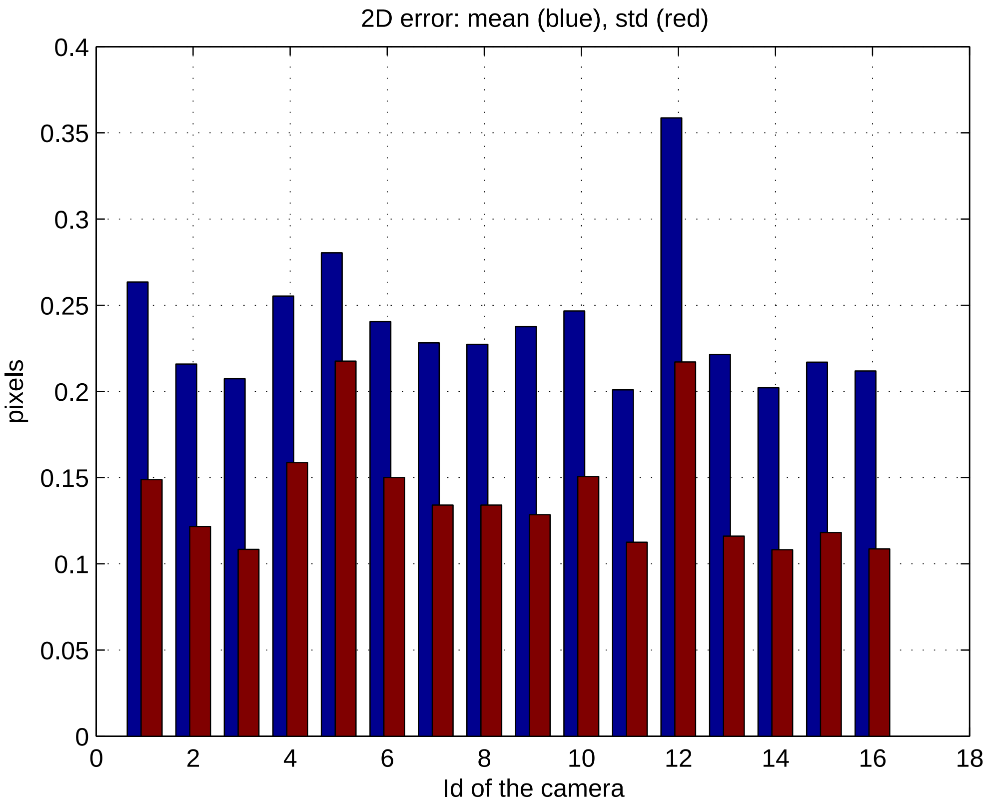
Results — Linear model





Results — Complete model

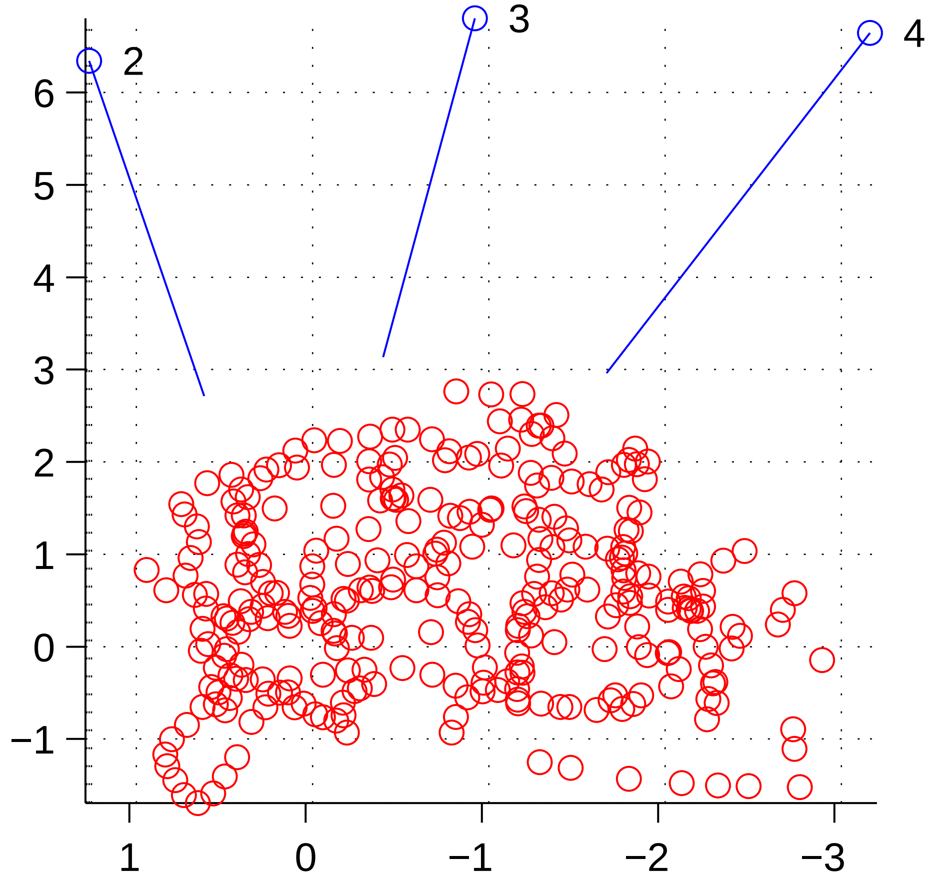


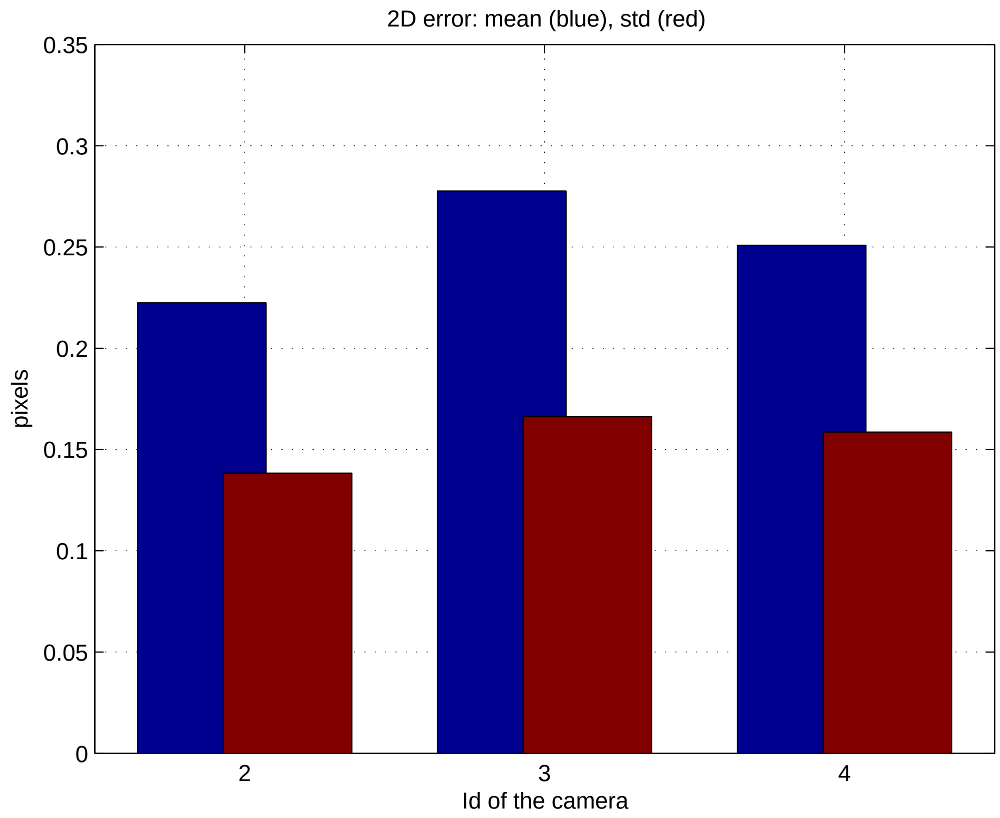


Very fine results from (almost) nothing!

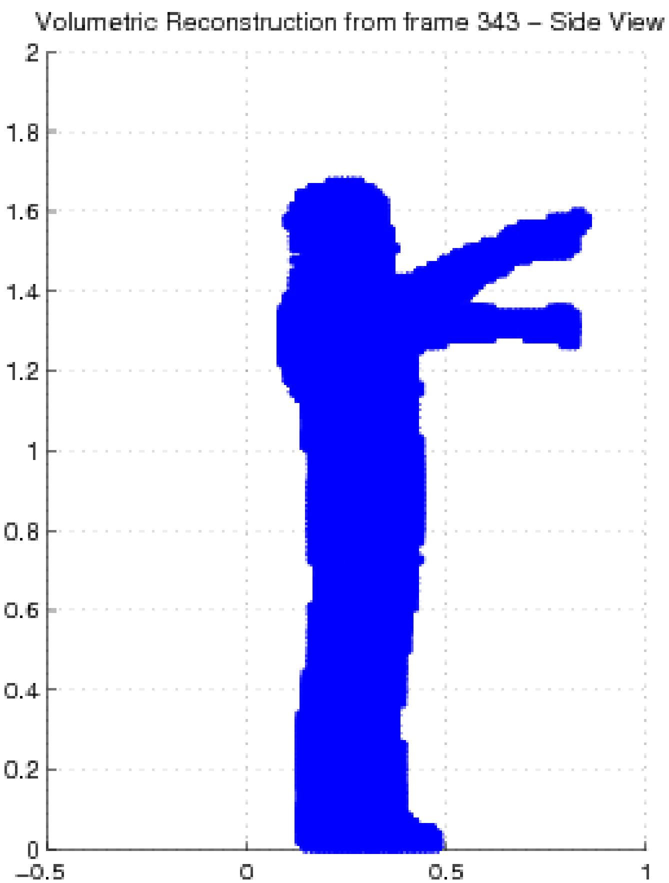
Results — Simple setup

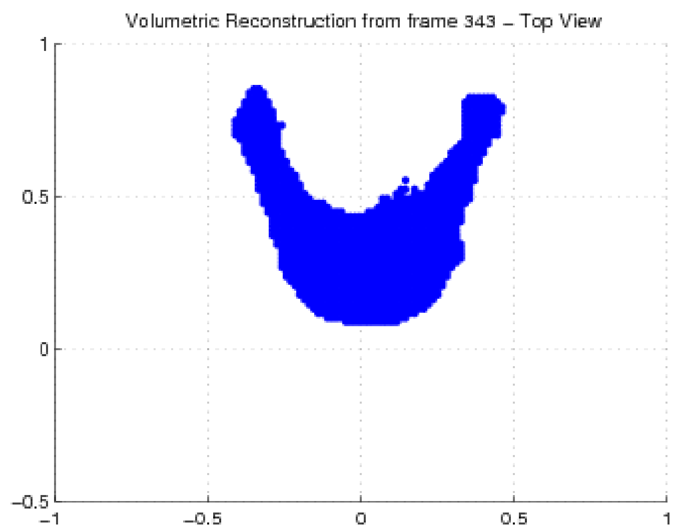
reconstructed points/camera setup only inliers are used

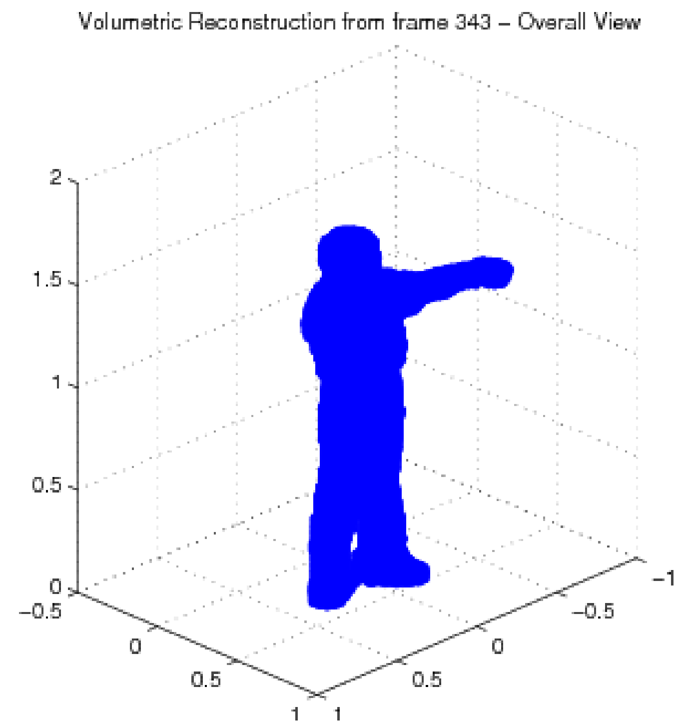




Application example — volumetric reconstruction







I know, it is just toy example. Still, it shows that the metric is OK.

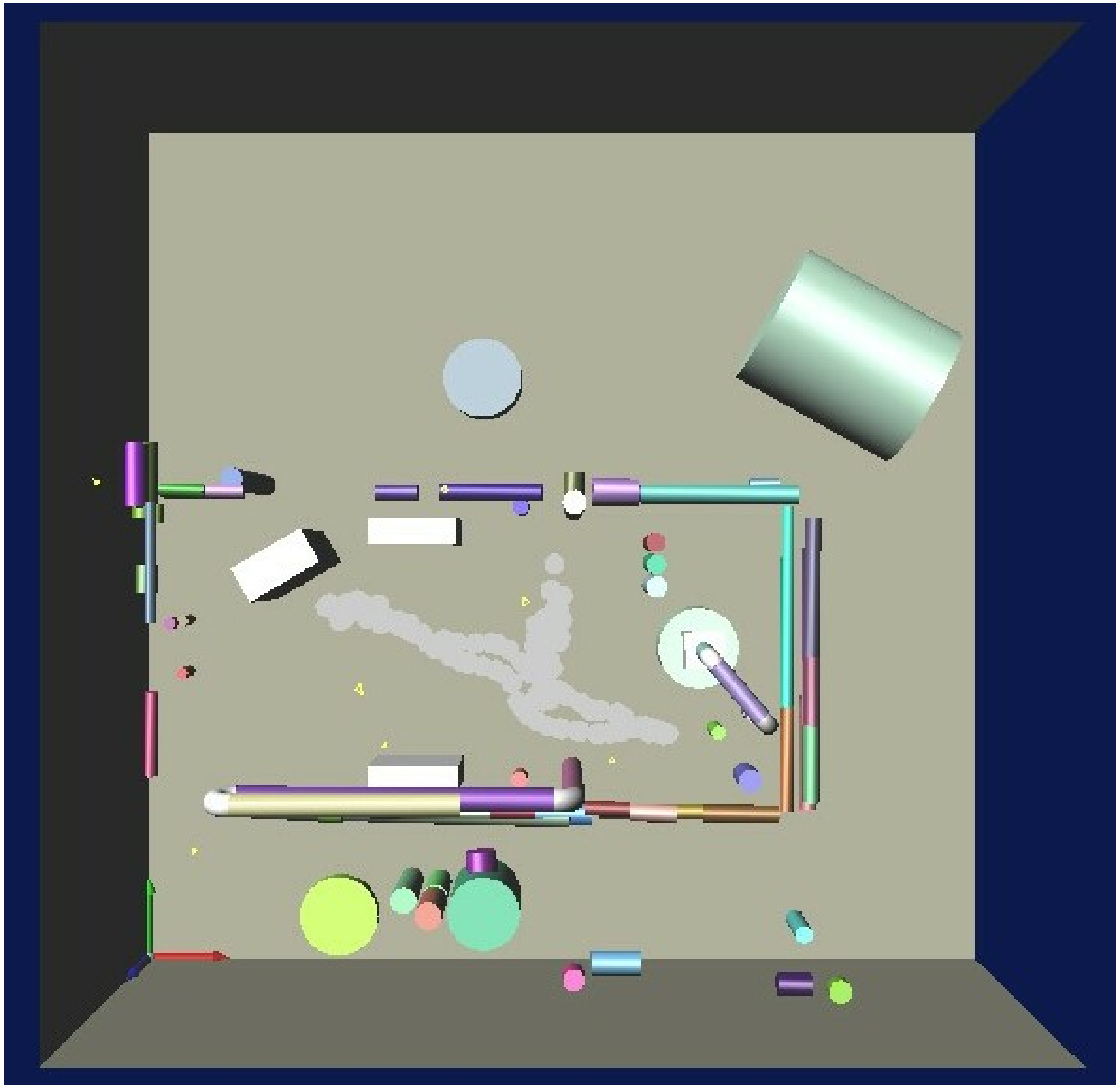
Application example — mobile multicamera setup

Video

Mobile multicamera setup - worker 3D tracking

Video

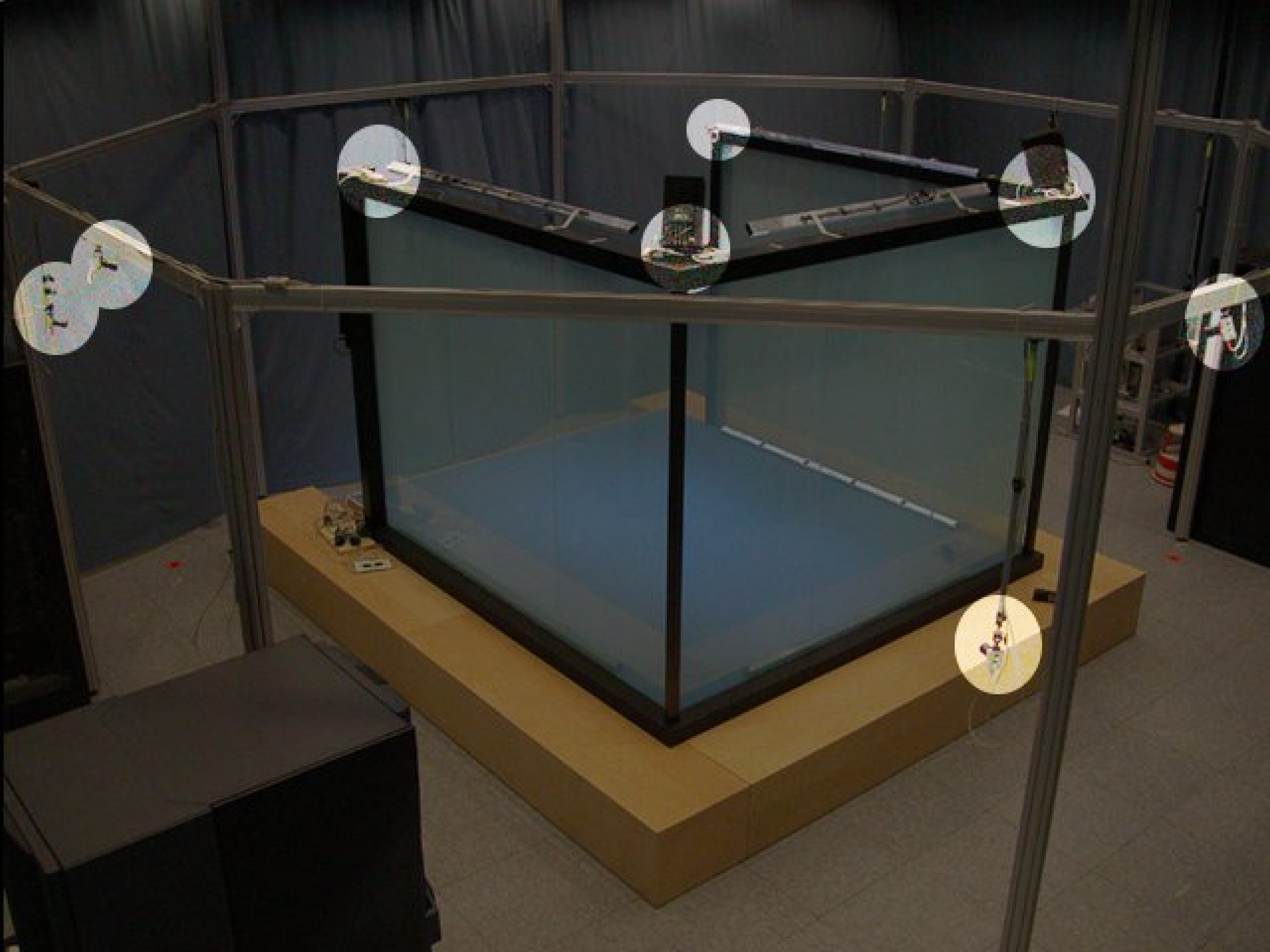
Mobile multicamera setup - worker 3D tracking

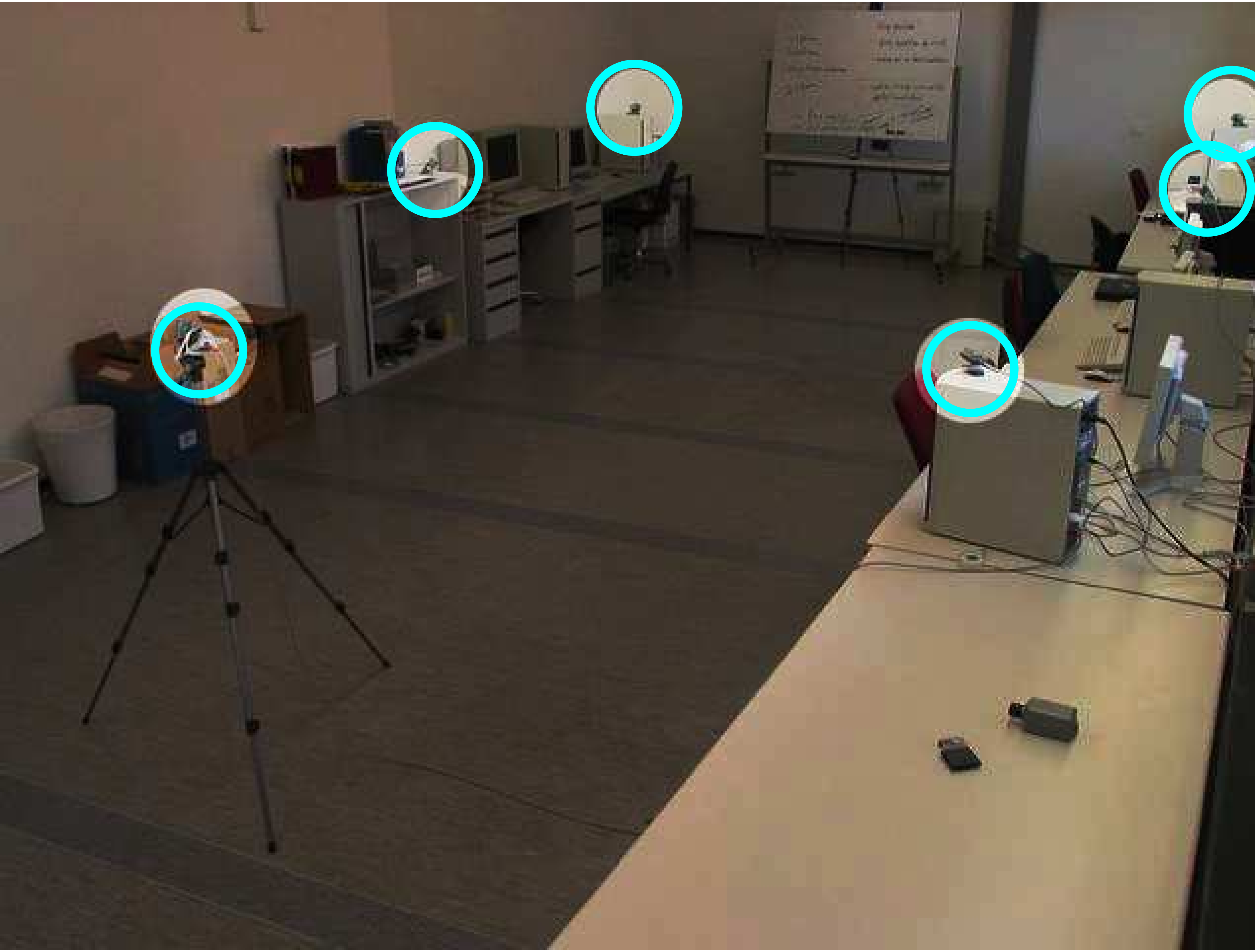


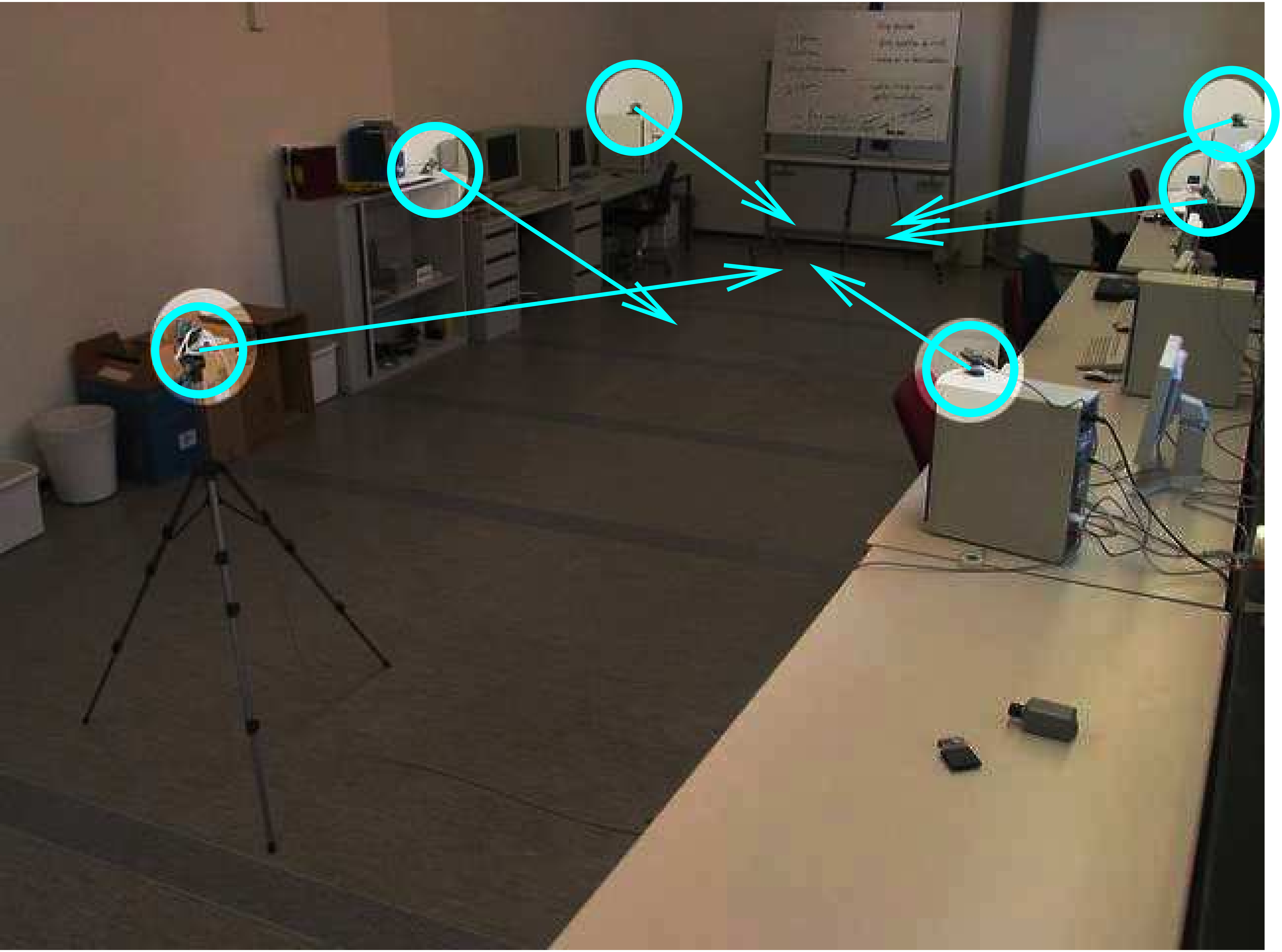
Summary

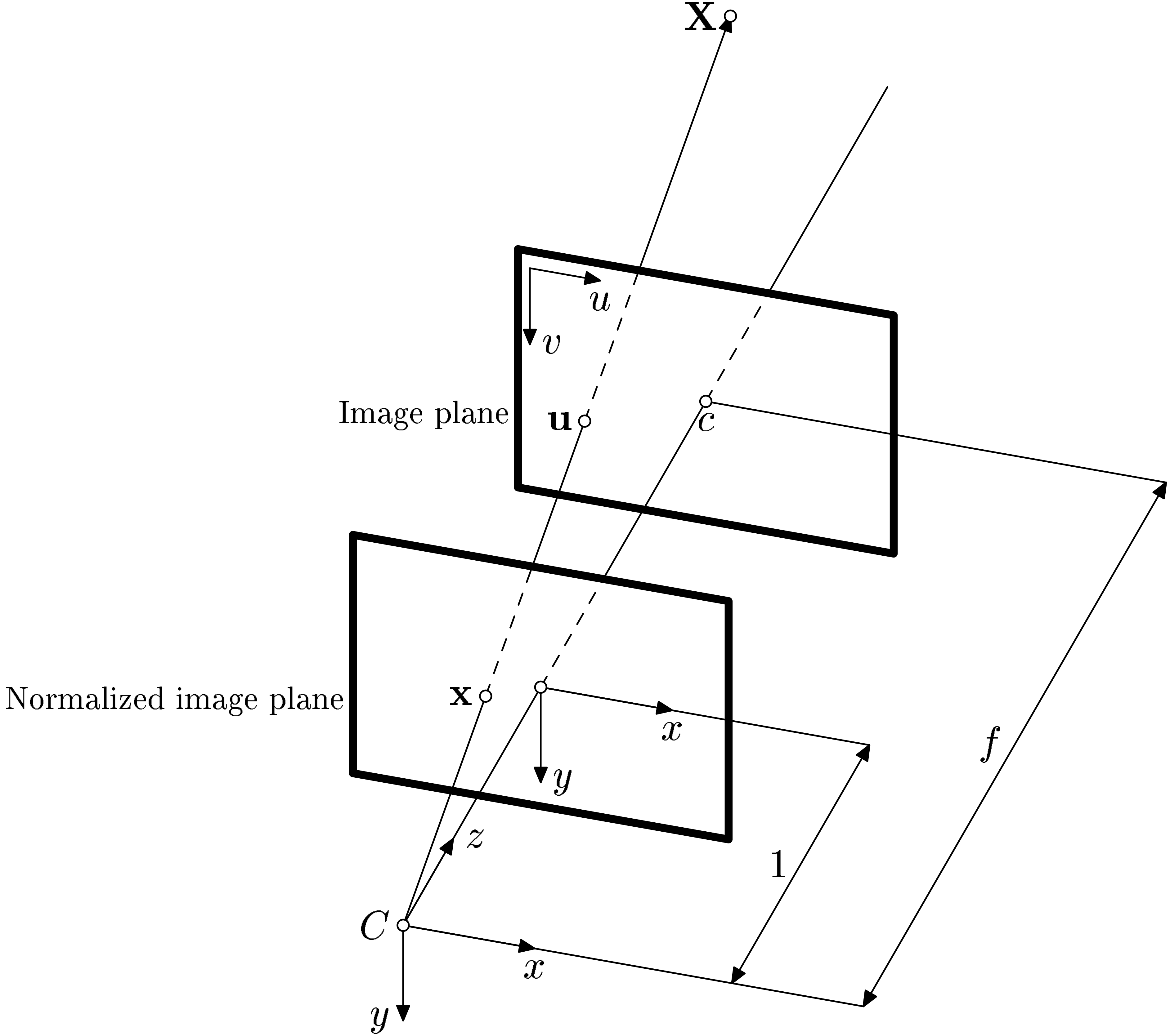
- waving the point object is the only hand work required
- no user interaction
- complete calibration of 16 camera setup may be done in 60-90 minutes (95% computation)

Codes, sample data, papers, etc. downloadable from
http://cmp.felk.cvut.cz/~svoboda/SelfCal









X
u
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Image plane
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Normalized image plane
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1
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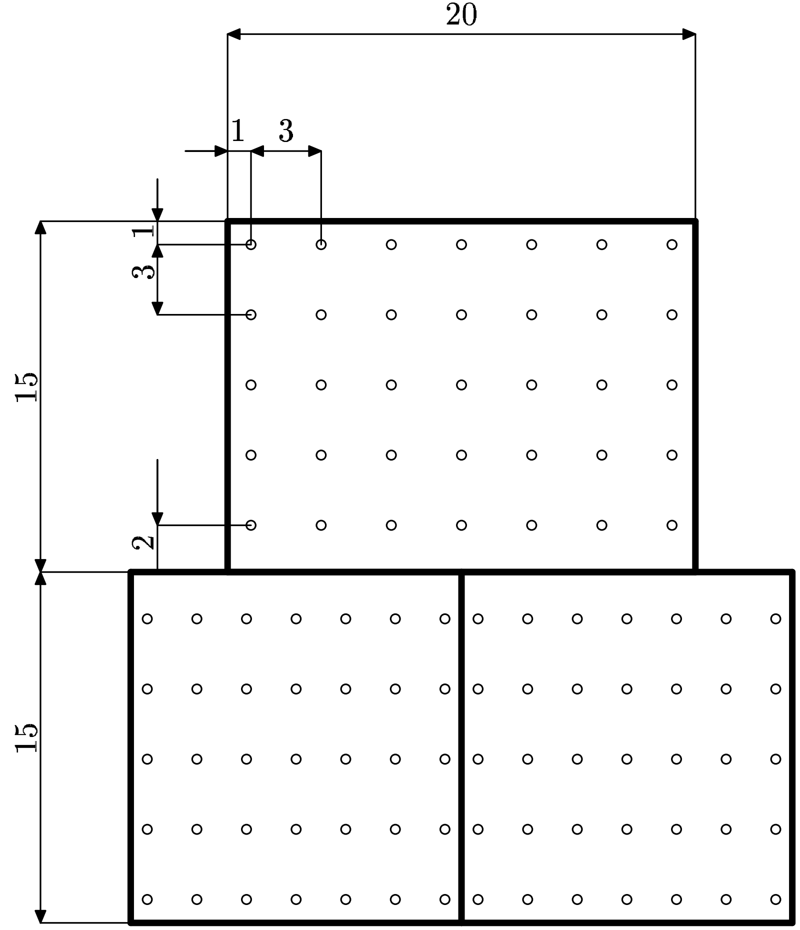
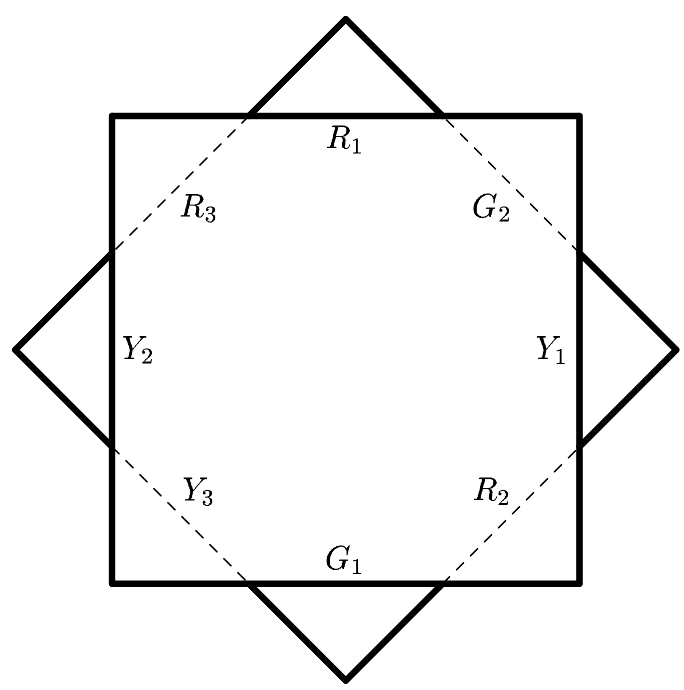
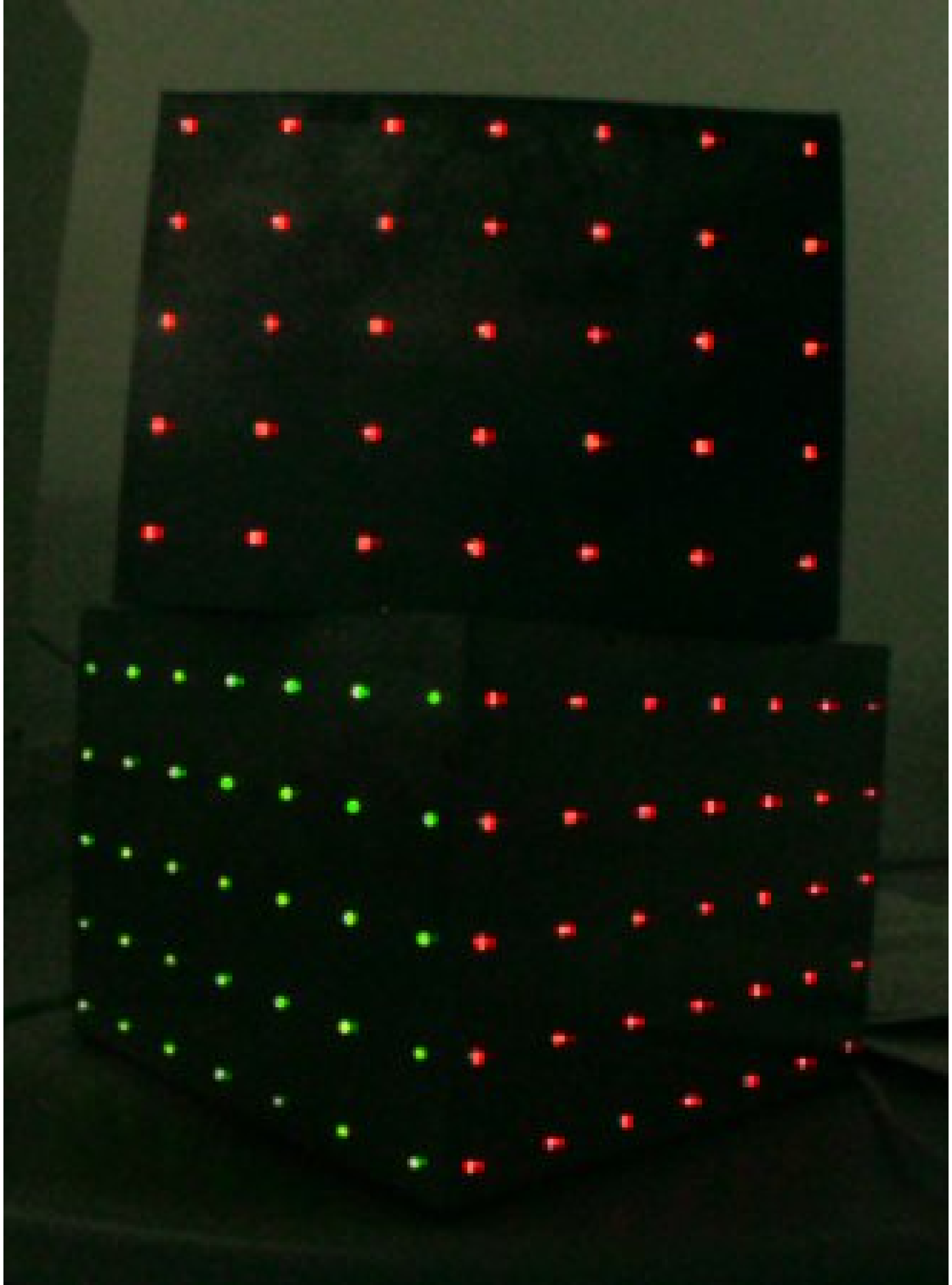
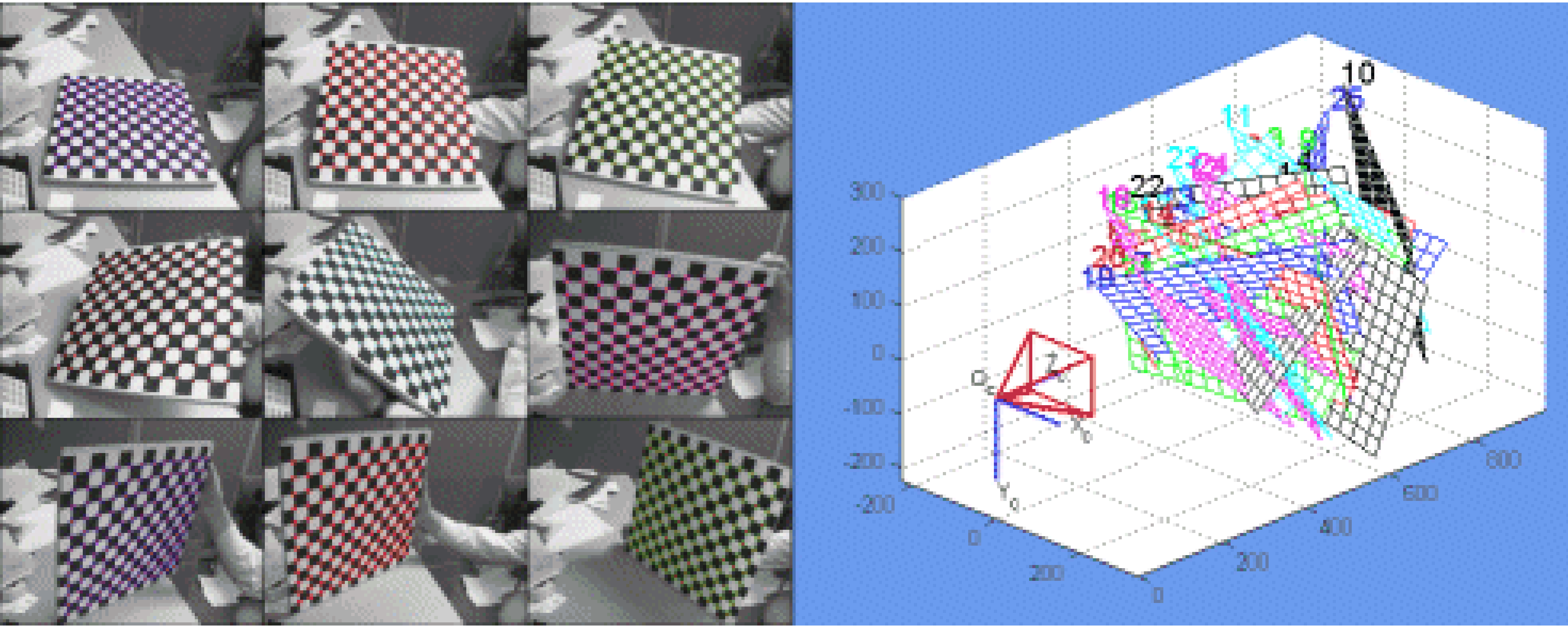


20
1
3
1
3
15
2
15



R_1
R_3
G_2
Y_2
Y_1
Y_3
R_2
G_1





300
200
100
0
-100
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-200
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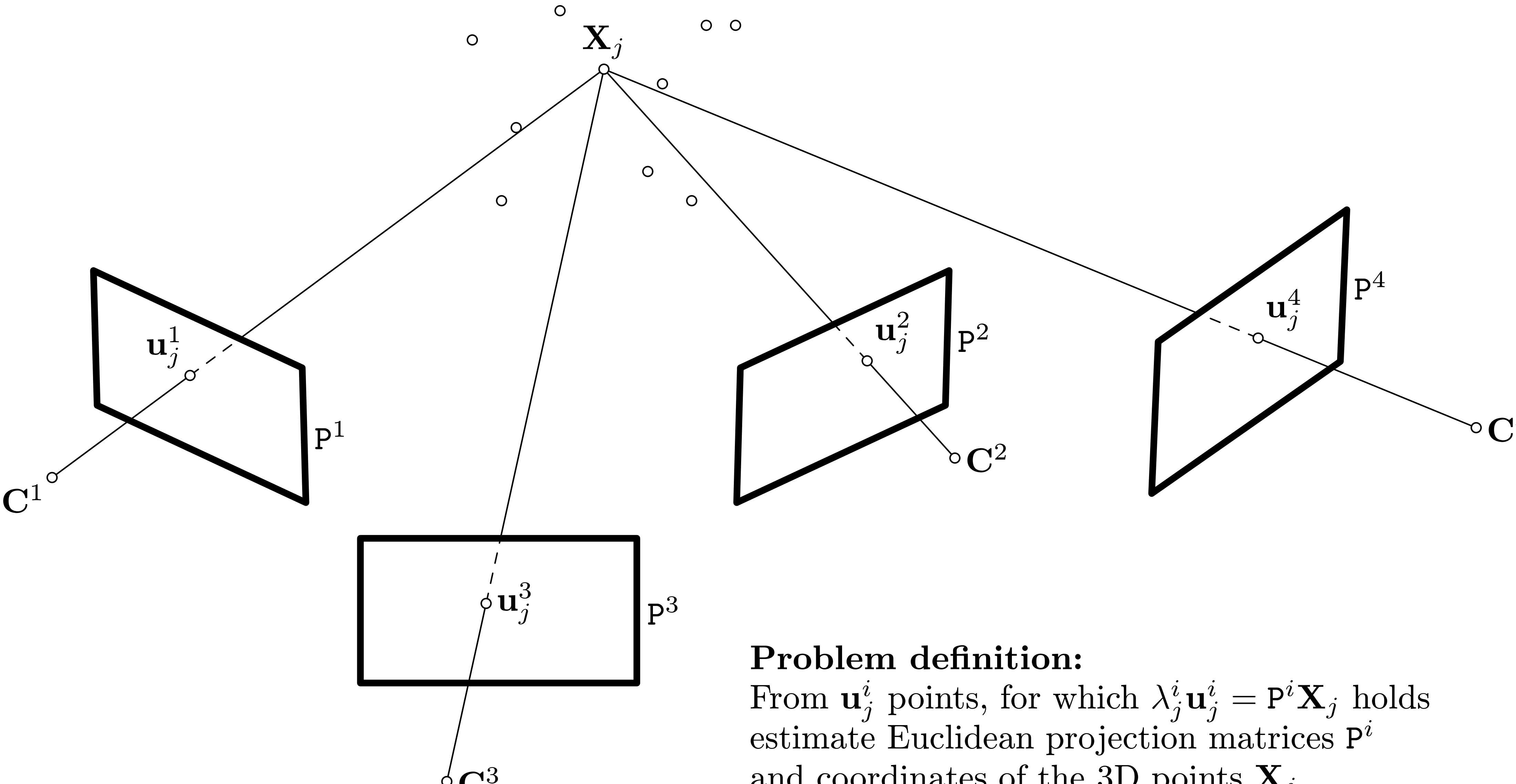
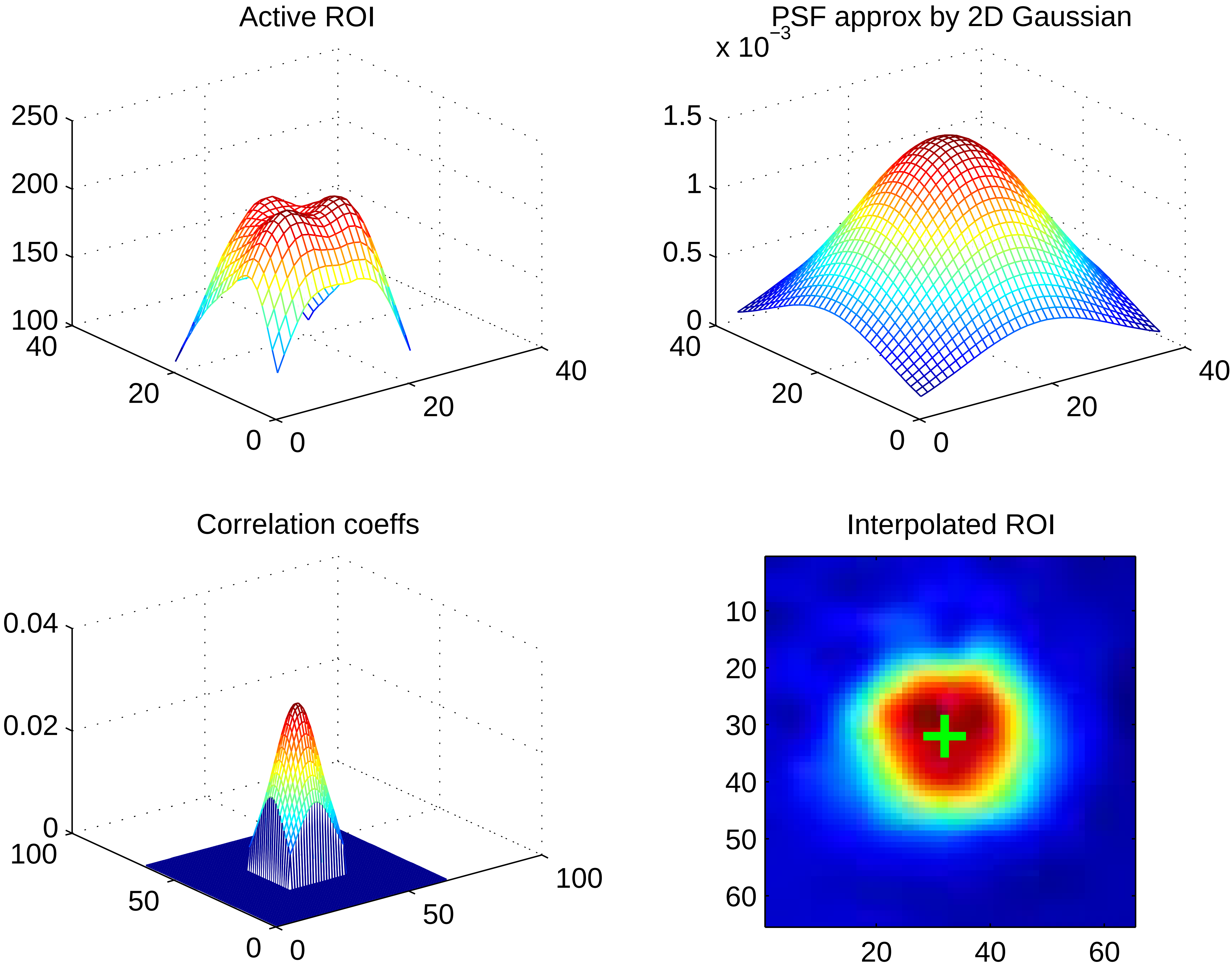
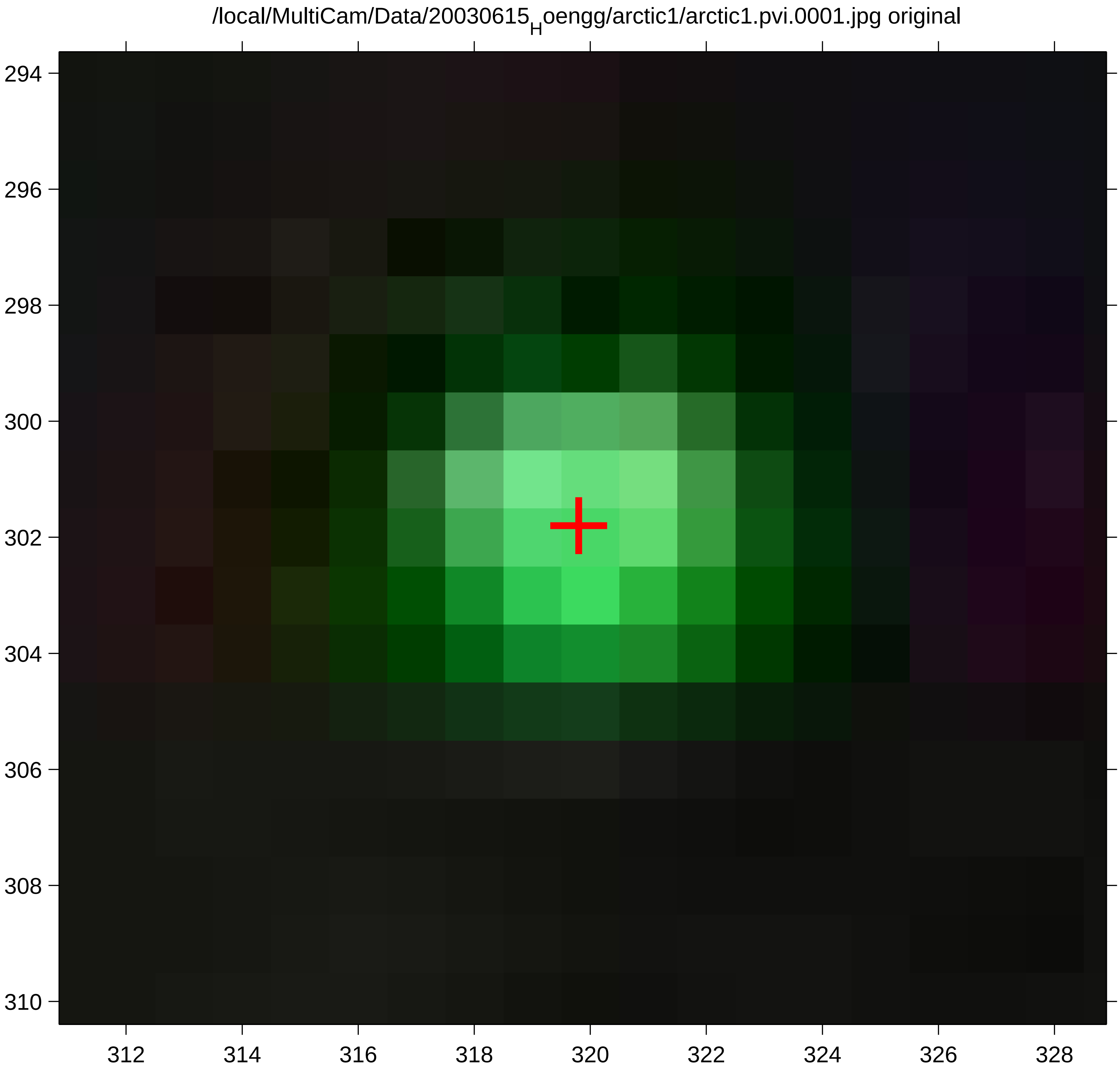
200
0
200
400
600
800
10
11
22



Problem definition:
From $\mathbf{u}_j^i$ points, for which $\lambda_j^i \mathbf{u}_j^i = \mathtt{P}^i \mathbf{X}_j$ holds estimate Euclidean projection matrices $\mathtt{P}^i$ and coordinates of the 3D points $\mathbf{X}_j$

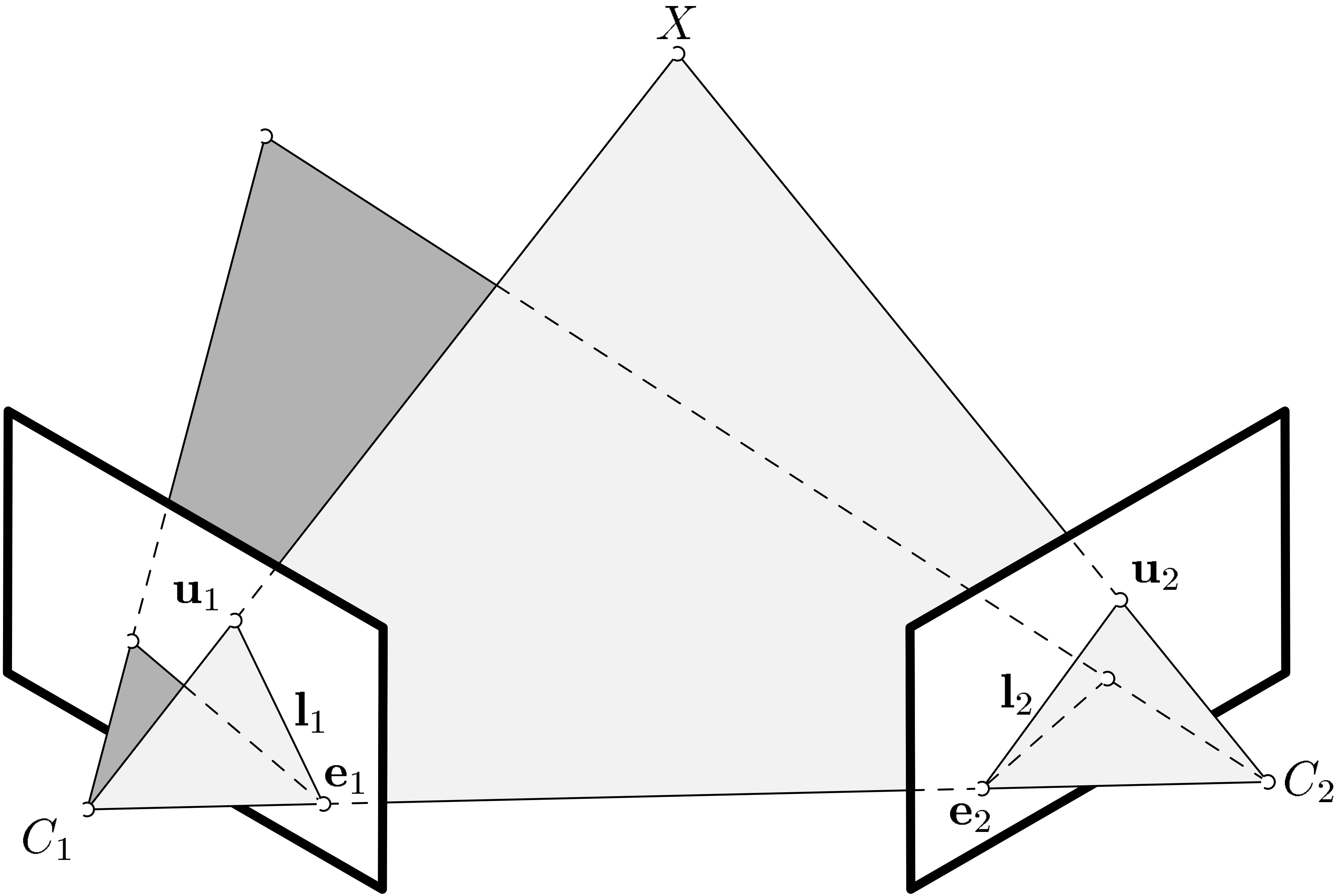


Active ROI
250
200
150
100
40
20
0
0
20
40
PSF approx by 2D Gaussian
x 10^-3
1.5
1
0.5
0
40
20
0
0
20
40
Correlation coeffs
0.04
0.02
0
100
50
0
0
50
100
Interpolated ROI
10
20
30
40
50
60
20
40
60



/local/MultiCam/Data/20030615_Hoengg/arctic1/arctic1.pvi.0001.jpg original
294
296
298
300
302
304
306
308
310
312
314
316
318
320
322
324
326
328

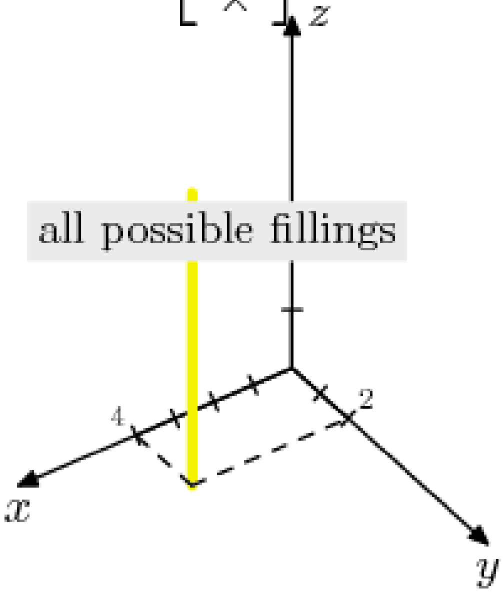

No point found
No point found
No point found

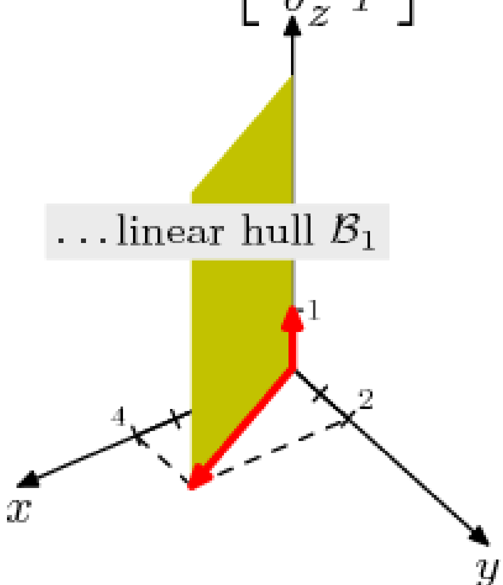


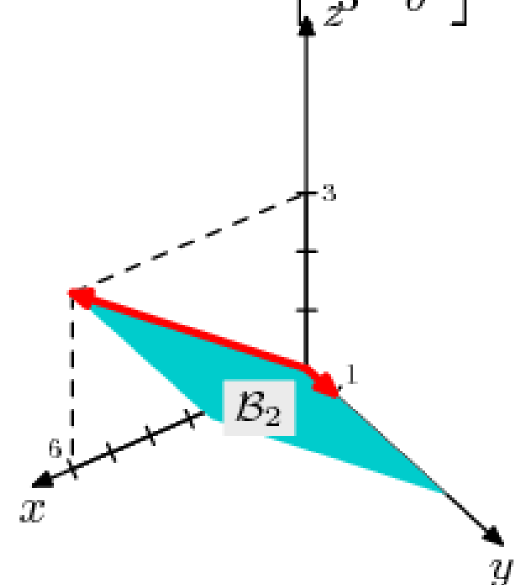
X
$\mathbf{u}_1$
$\mathbf{l}_1$
$\mathbf{e}_1$
C_1
$\mathbf{u}_2$
$\mathbf{l}_2$
$\mathbf{e}_2$
C_2

Example: $\mathtt{R} = \begin{bmatrix} 4 & 6 \\ 2 & \times \\ \times & 3 \end{bmatrix}$, for rank $\mathtt{R} = 1$ instead of rank $\mathtt{R} = 4$

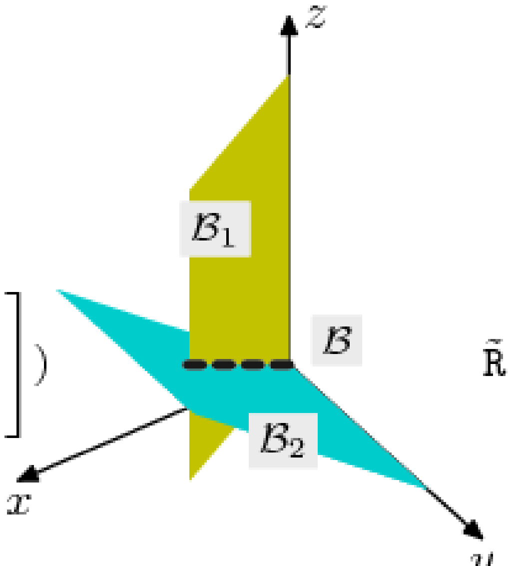
$\begin{bmatrix} 4 \\ 2 \\ \times \end{bmatrix}$ ... $\mathcal{B}_1 = Span(\begin{bmatrix} 4 & \mathit{0} \\ 2 & \mathit{0} \\ \mathit{0} & \mathit{1} \end{bmatrix})$, $\begin{bmatrix} 6 \\ \times \\ 3 \end{bmatrix}$ ... $\mathcal{B}_2 = Span(\begin{bmatrix} 6 & \mathit{0} \\ \mathit{0} & \mathit{1} \\ 3 & \mathit{0} \end{bmatrix})$



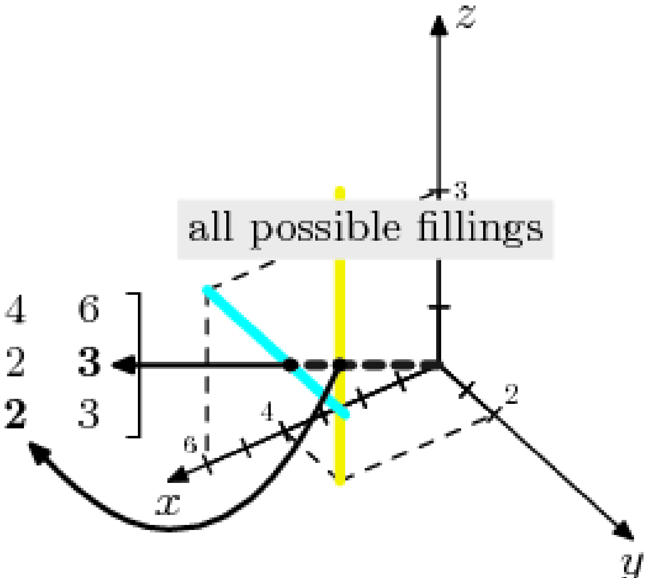


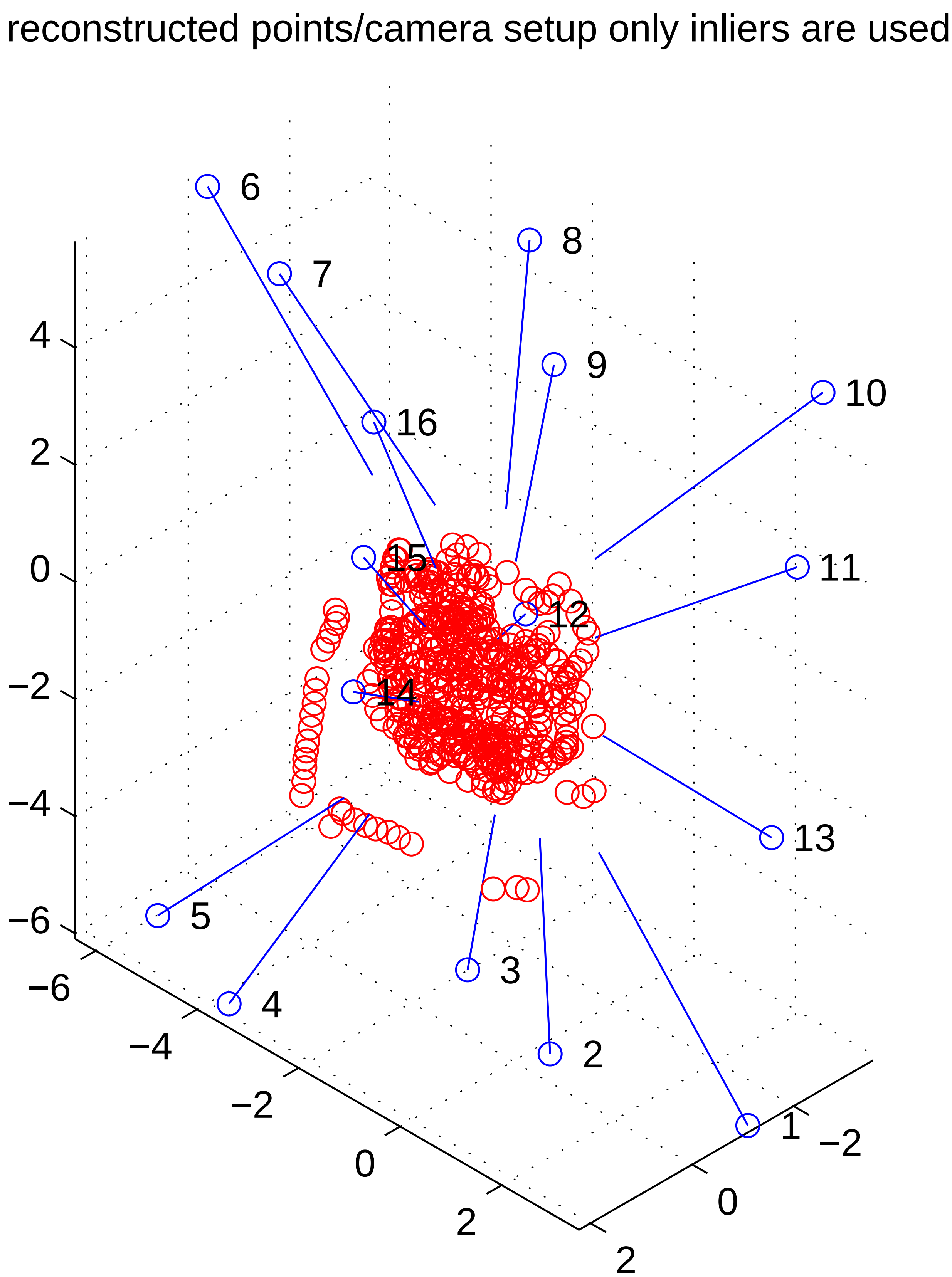


$\mathcal{B} \subseteq \mathcal{B}_1 \cap \mathcal{B}_2 = Span(\begin{bmatrix} \mathbf{2} \\ \mathbf{1} \\ \mathbf{1} \end{bmatrix})$

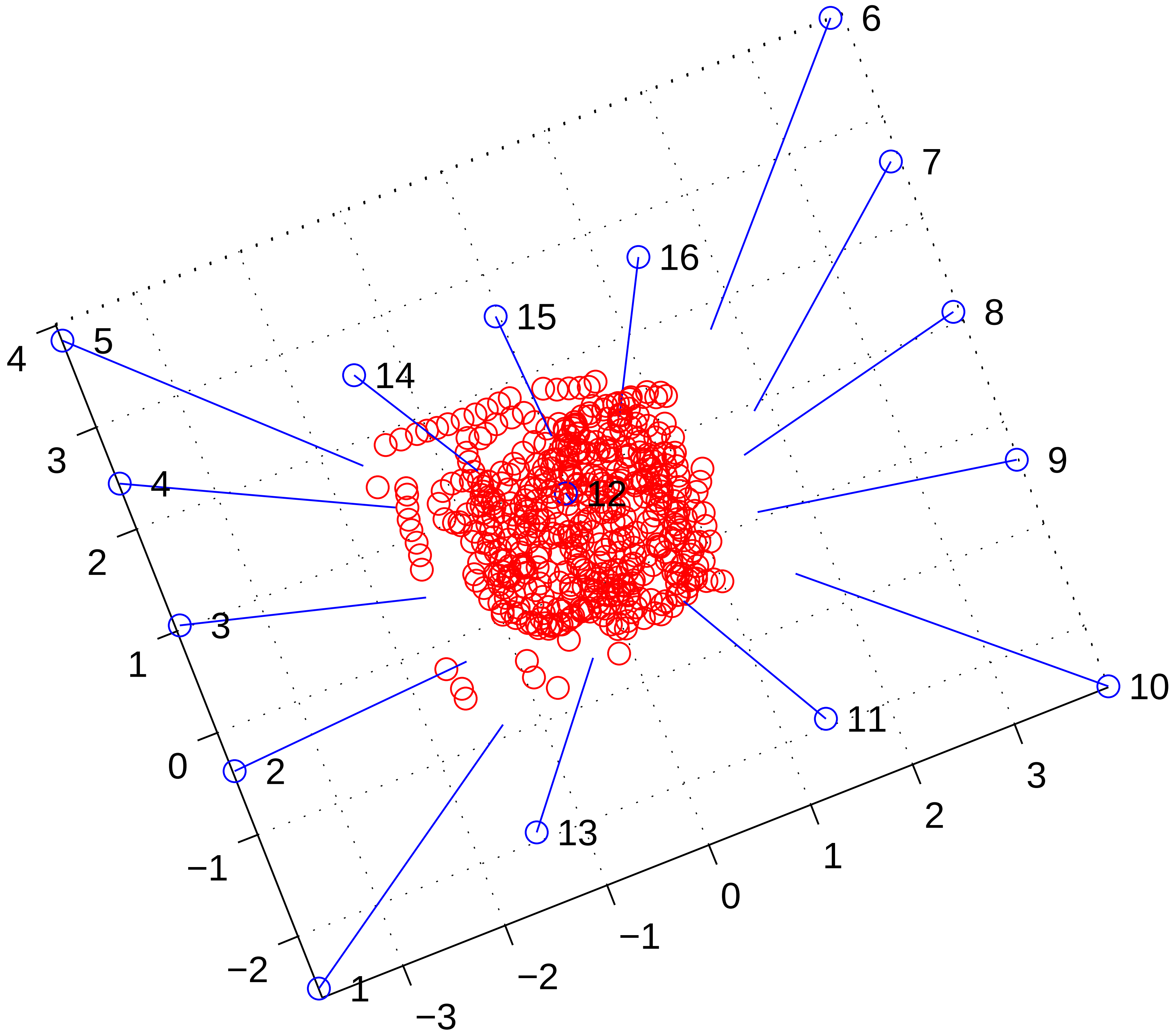


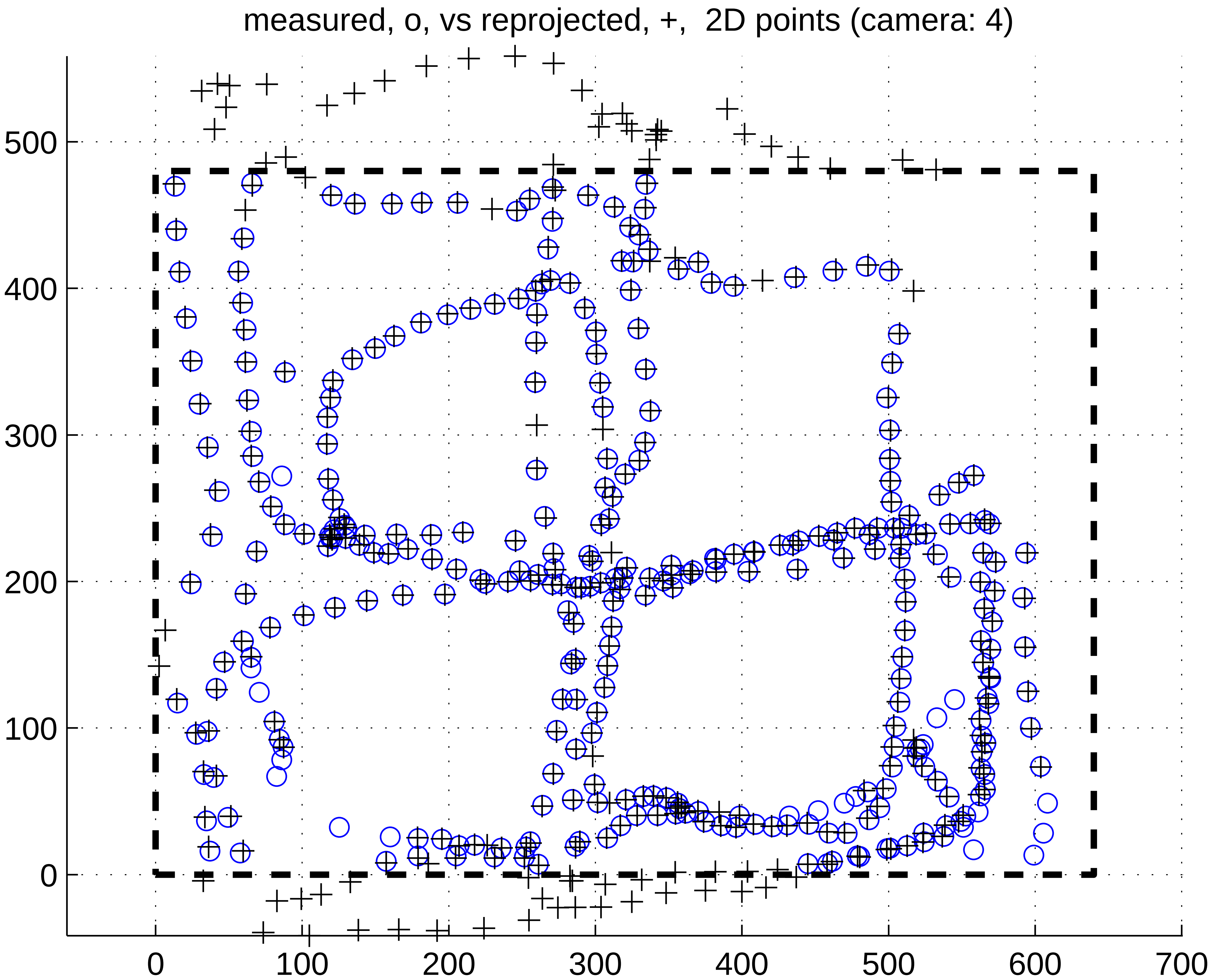
$\tilde{\mathtt{R}} = \begin{bmatrix} 4 & 6 \\ 2 & \mathbf{3} \\ \mathbf{2} & 3 \end{bmatrix}$



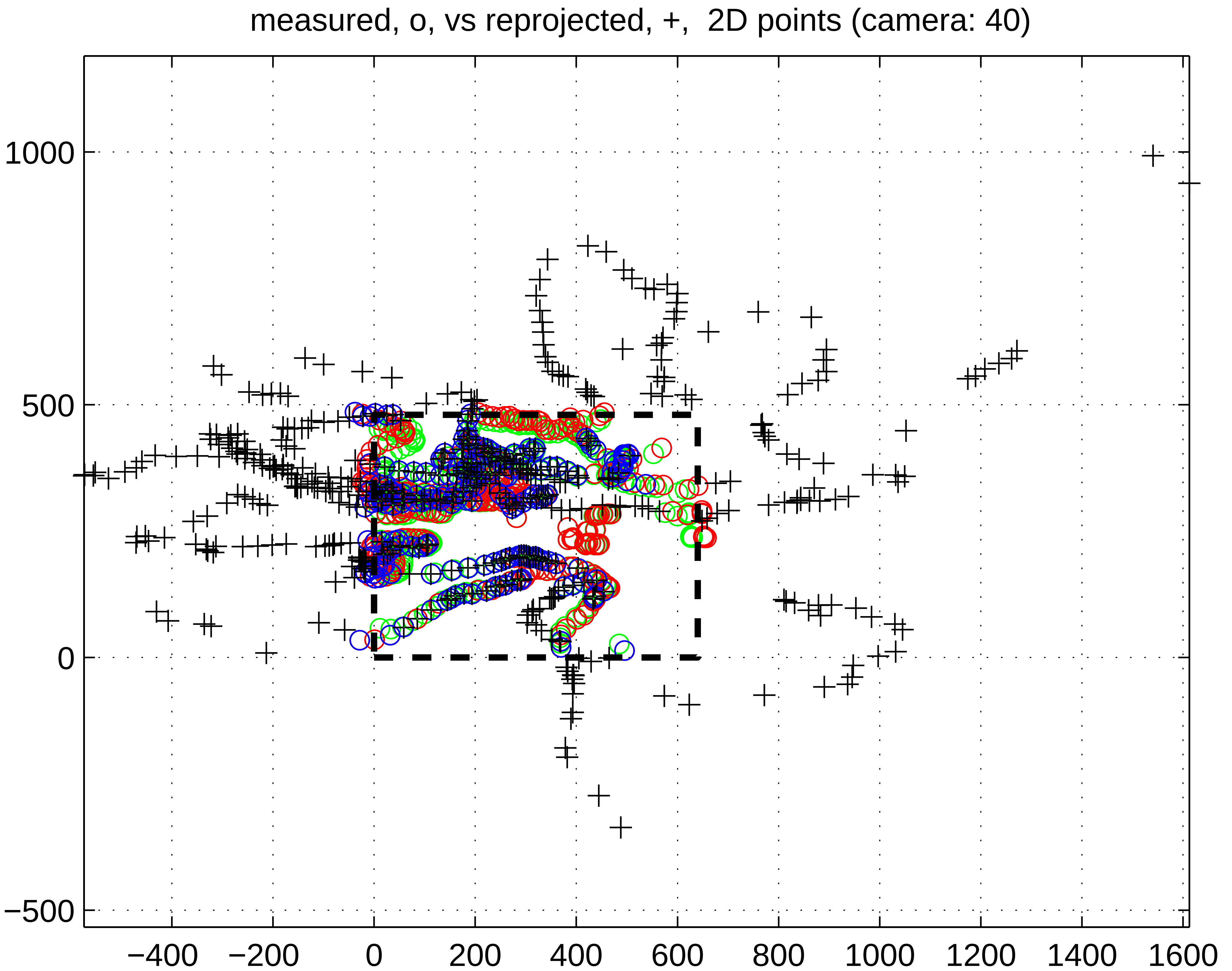

reconstructed points/camera setup only inliers are used
6
7
8
9
10
16
15
11
12
14
13
5
4
3
2
1
4
2
0
−2
−4
−6
−6
−4
−2
0
2
−2
0
2

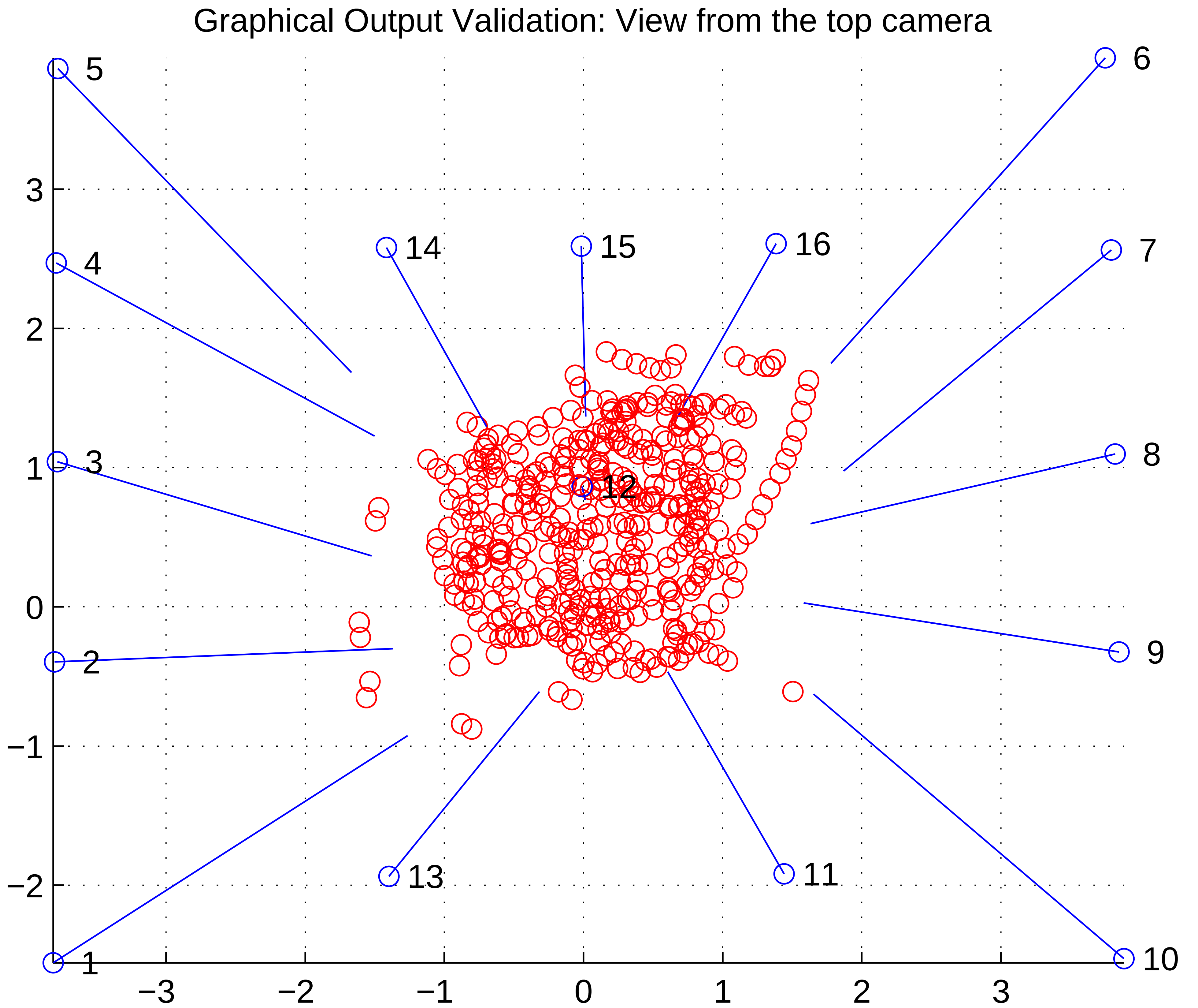
Graphical Output Validation: View from the top camera

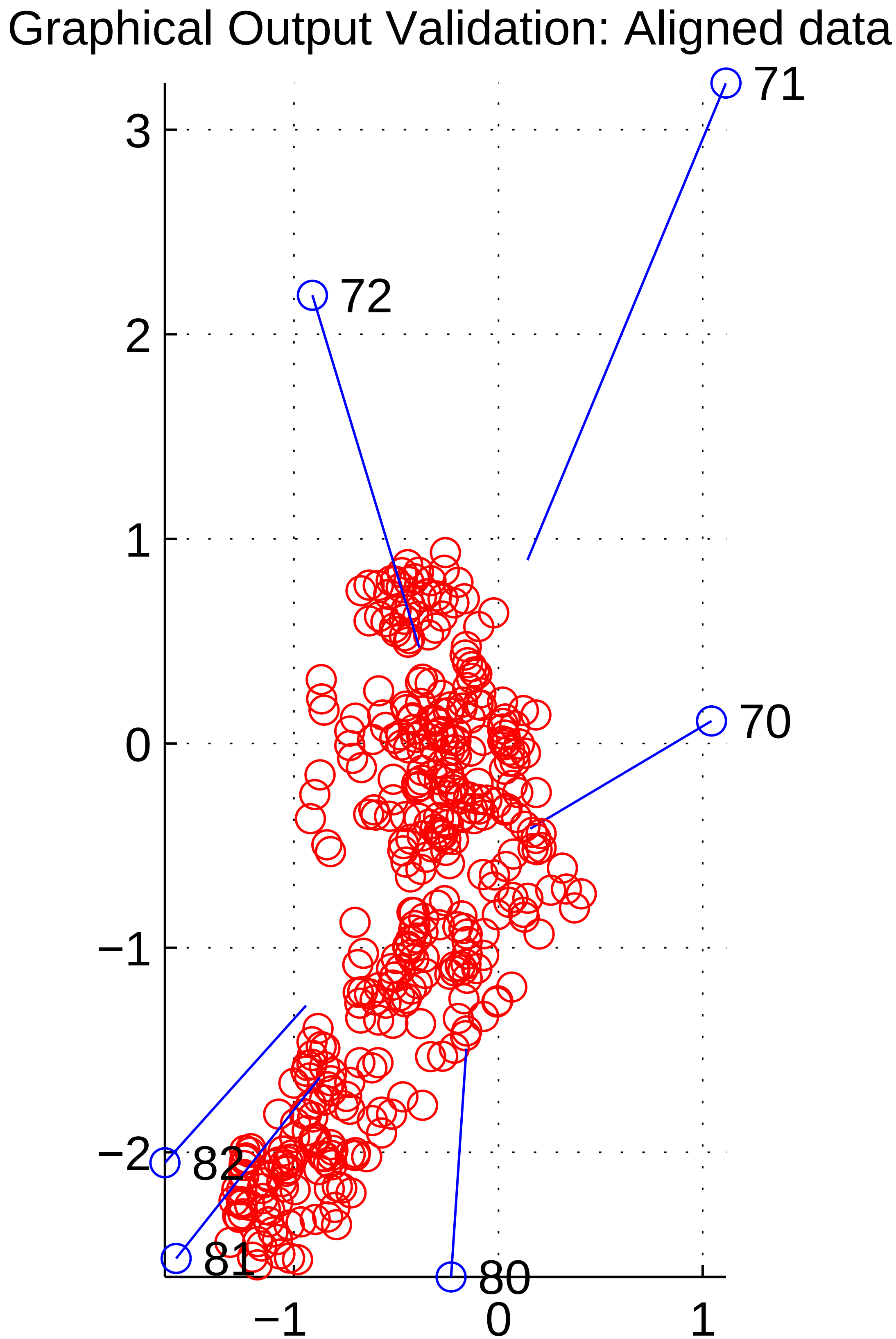




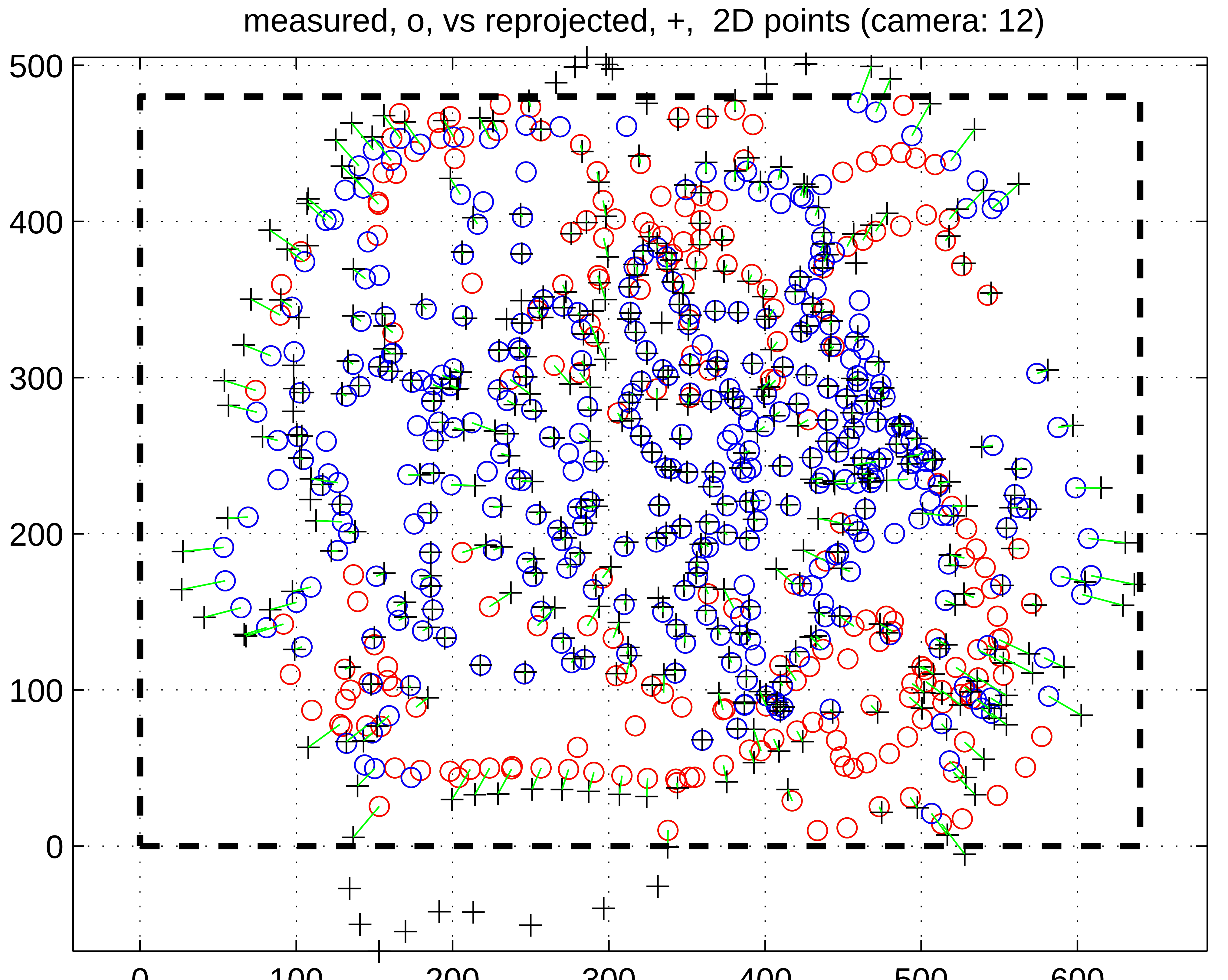
measured, o, vs reprojected, +, 2D points (camera: 4)
0
100
200
300
400
500
0
100
200
300
400
500
600
700

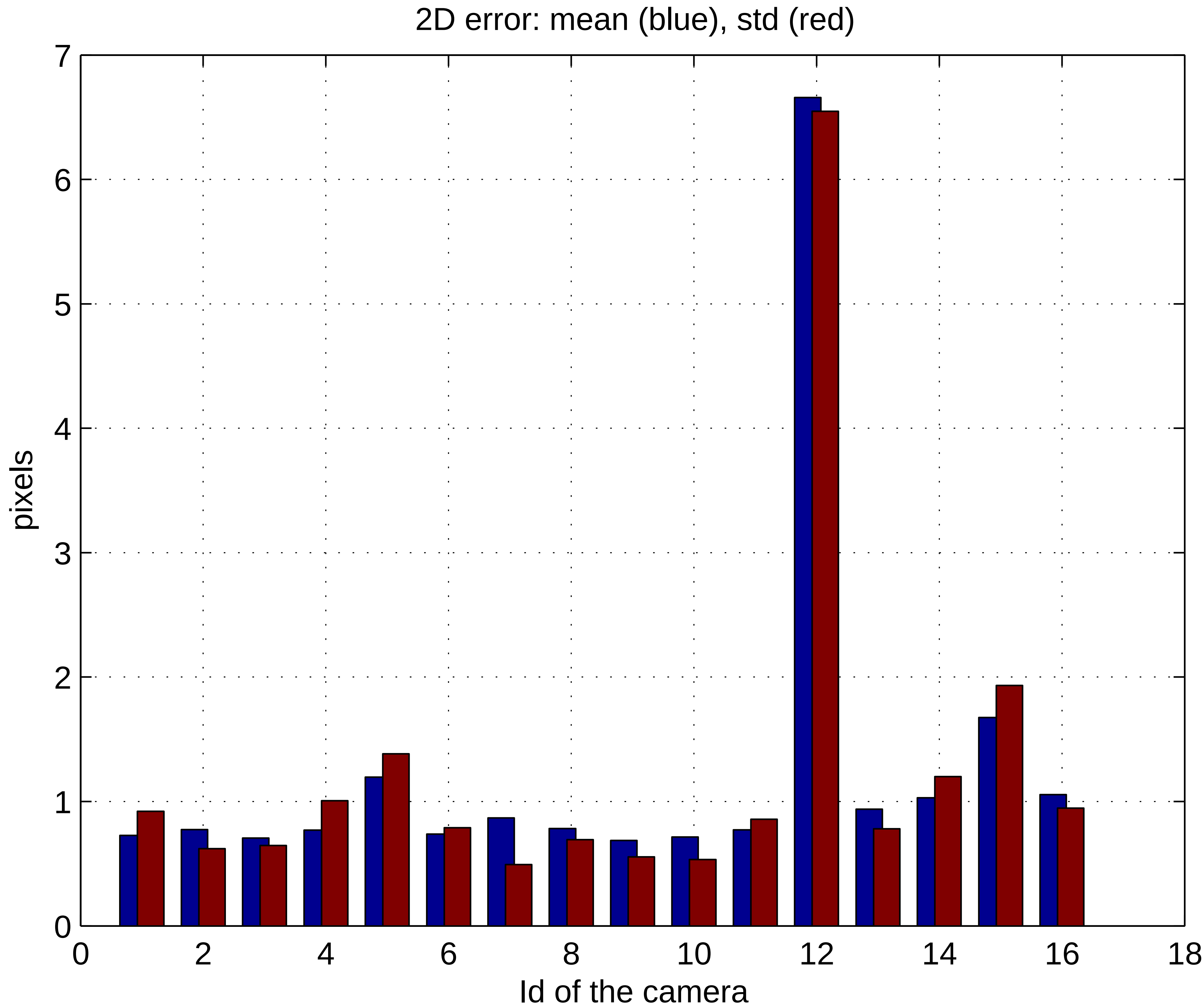

measured, o, vs reprojected, +, 2D points (camera: 40)
1000
500
0
−500
−400
−200
0
200
400
600
800
1000
1200
1400
1600


Graphical Output Validation: View from the top camera
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
−2
−1
0
1
2
3
−3
−2
−1
0
1
2
3

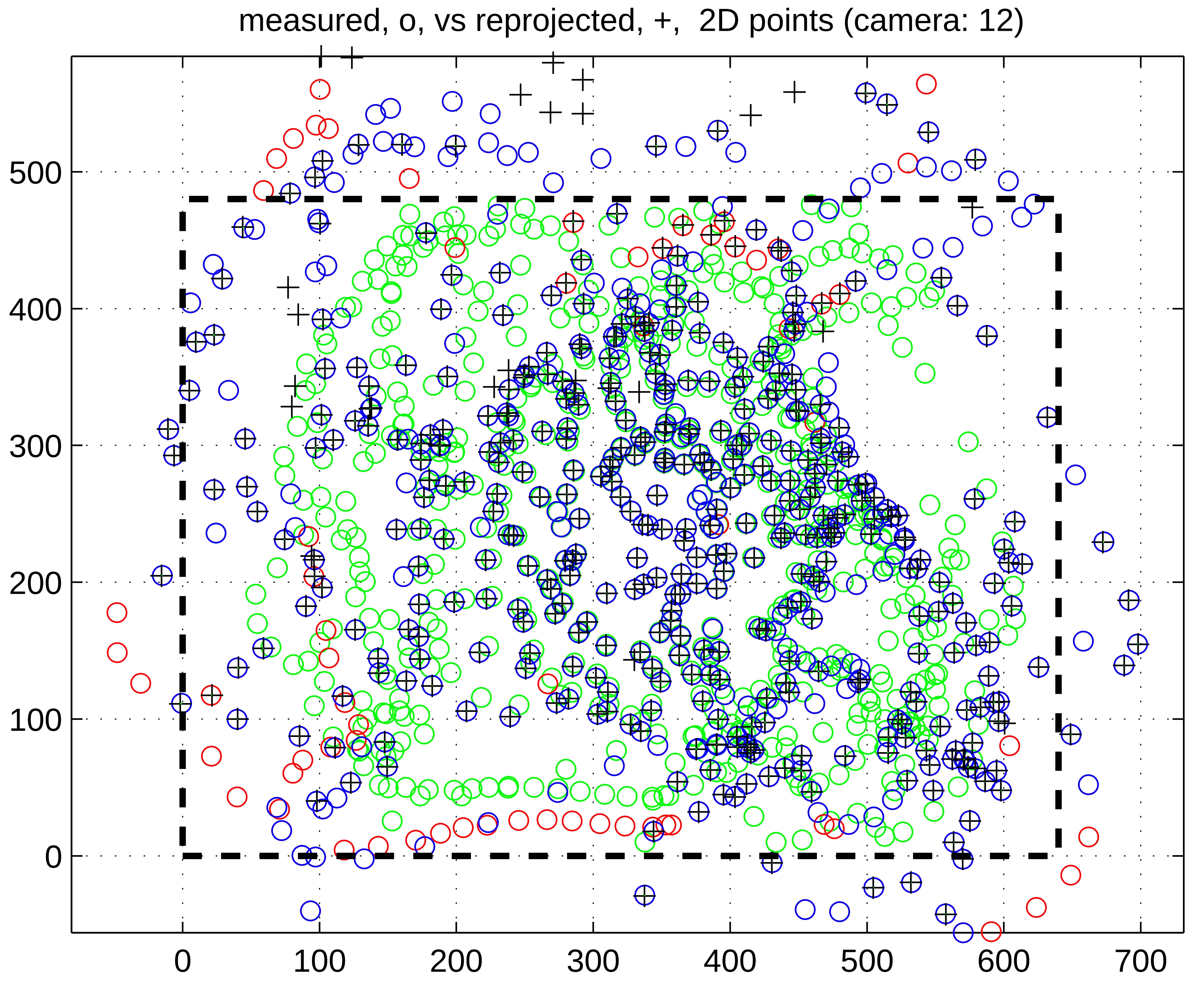


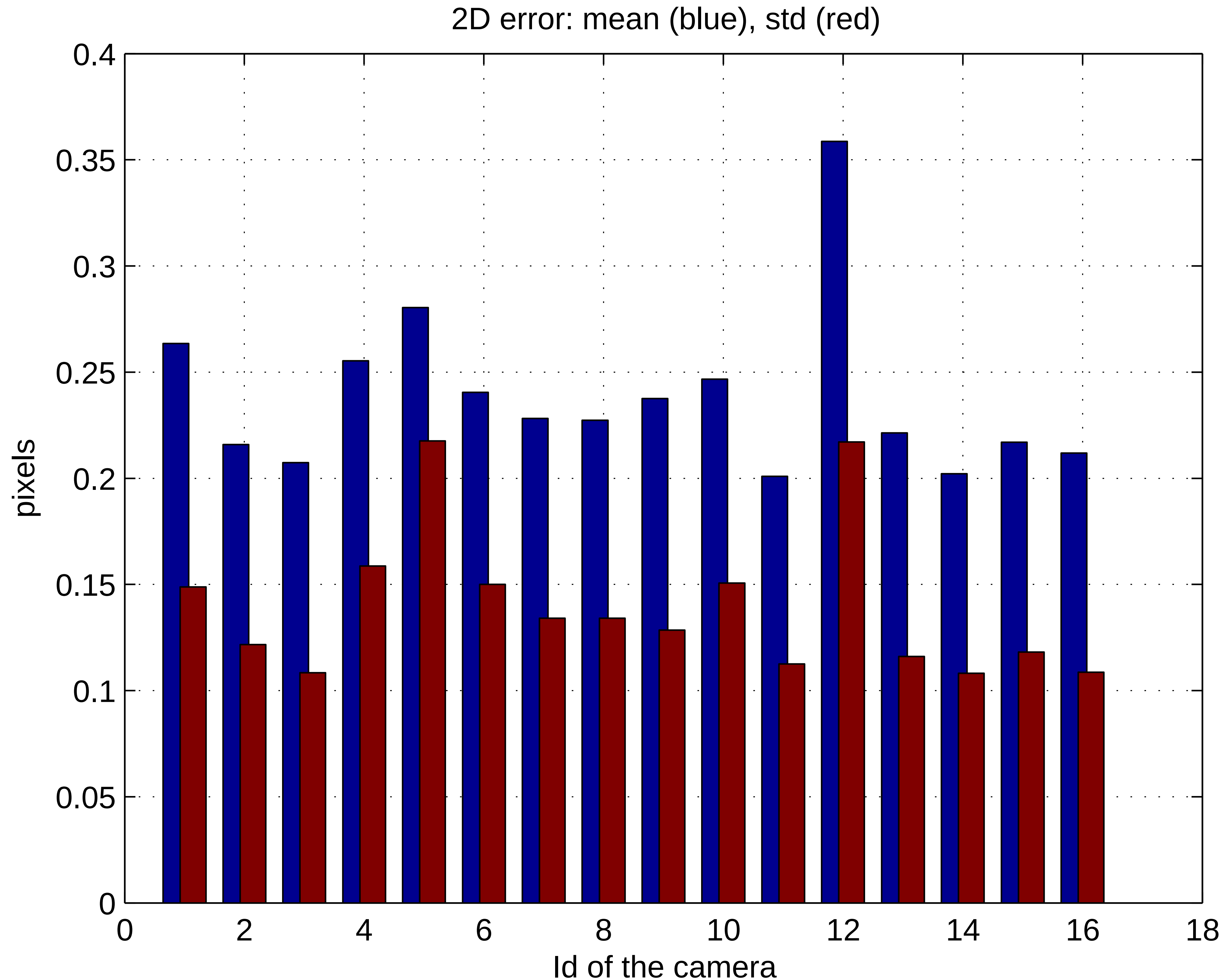
Graphical Output Validation: Aligned data
3
2
1
0
−1
−2
−1
0
1
71
72
70
82
81
80

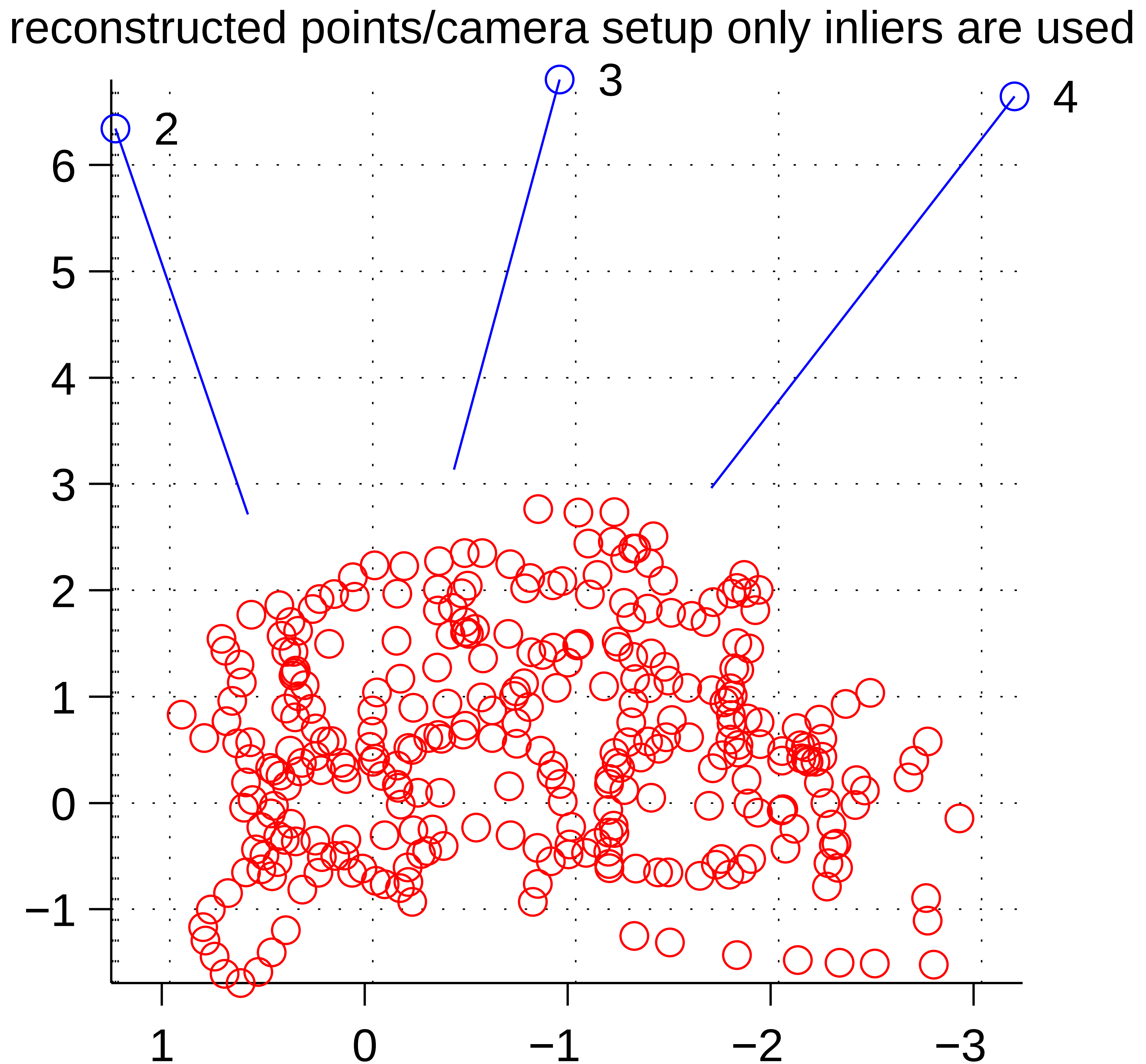

measured, o, vs reprojected, +, 2D points (camera: 12)
500
400
300
200
100
0
0
100
200
300
400
500
600

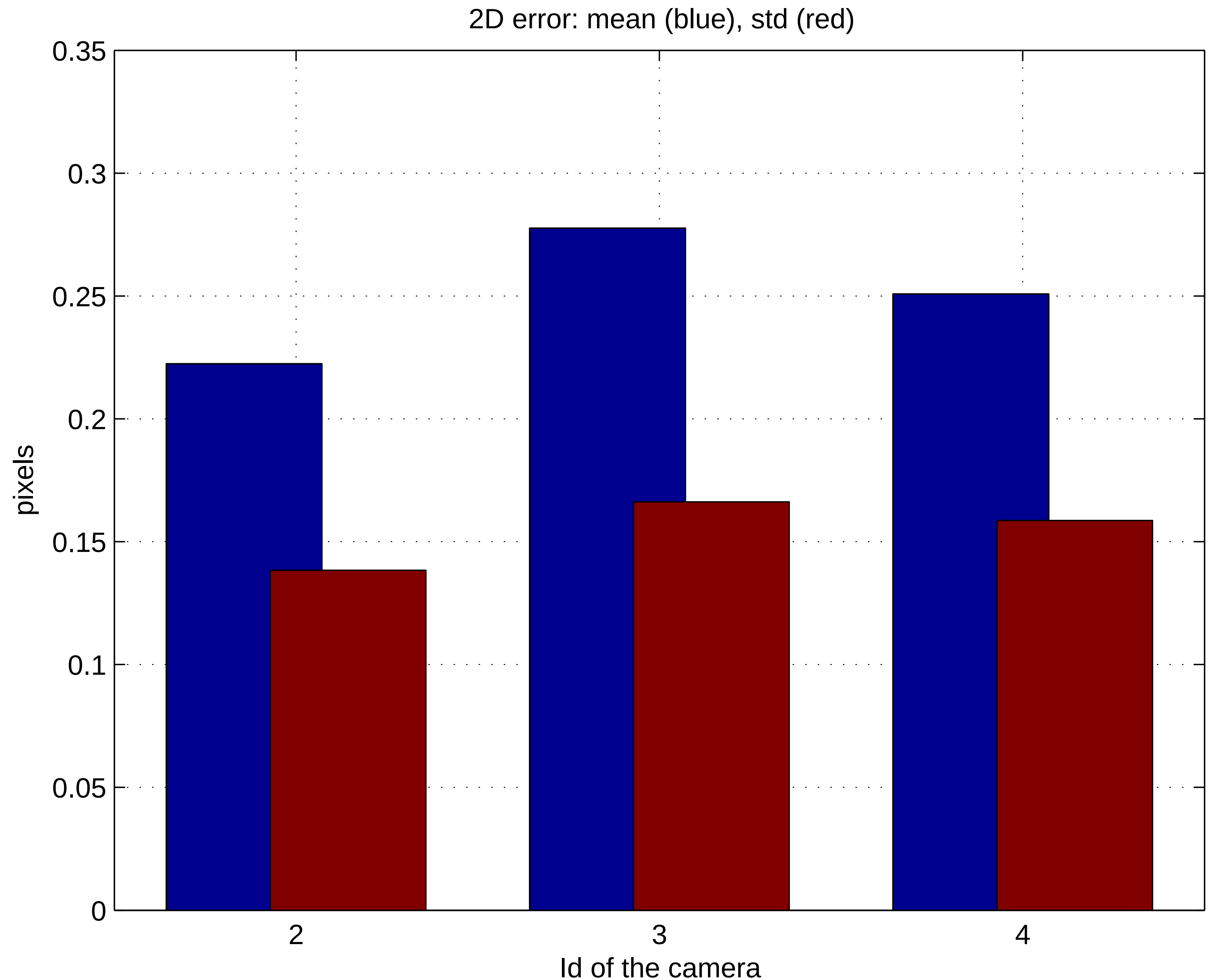


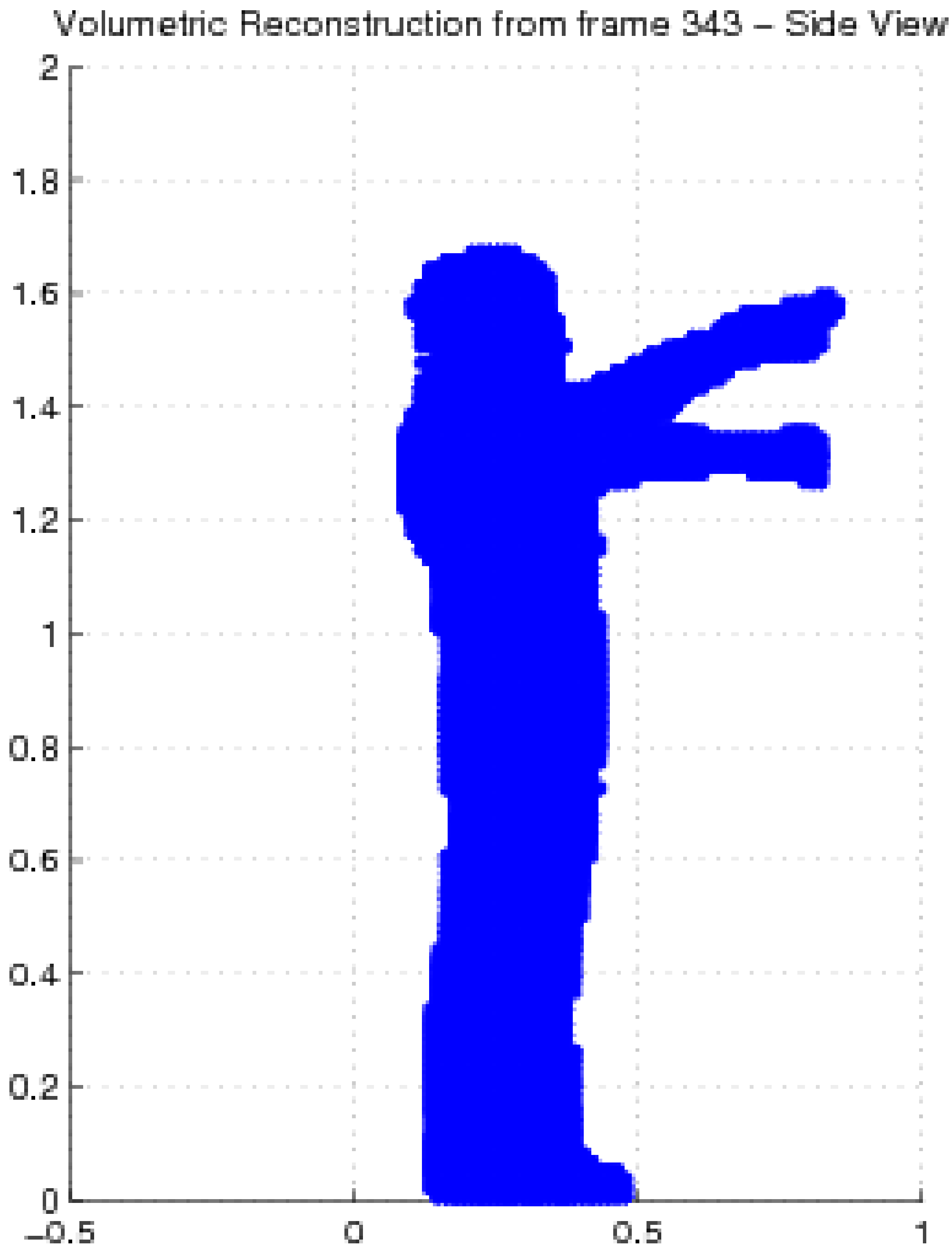
2D error: mean (blue), std (red)
pixels
Id of the camera
0
1
2
3
4
5
6
7
0
2
4
6
8
10
12
14
16
18

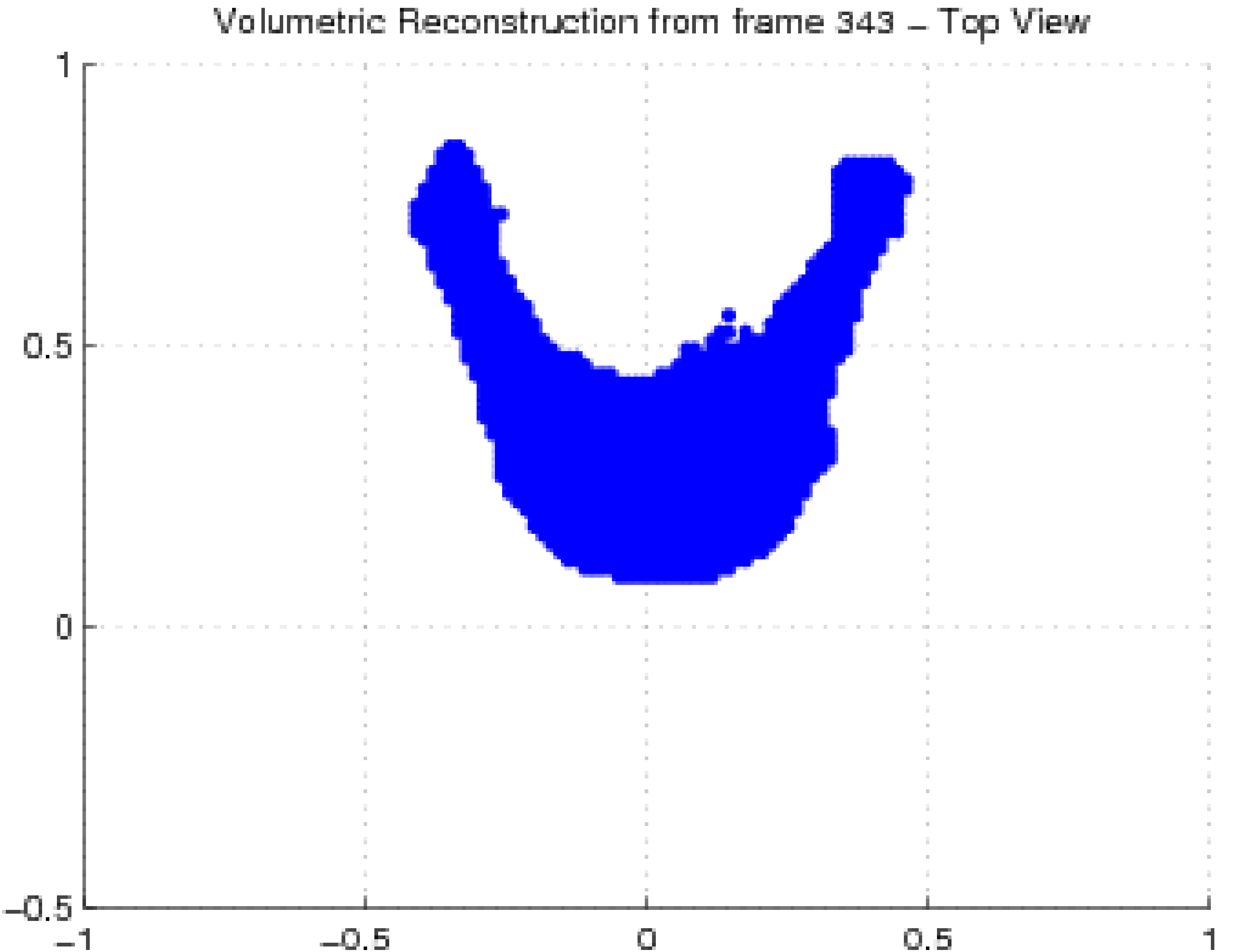

measured, o, vs reprojected, +, 2D points (camera: 12)
500
400
300
200
100
0
0
100
200
300
400
500
600
700


2D error: mean (blue), std (red)
pixels
Id of the camera
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0
2
4
6
8
10
12
14
16
18

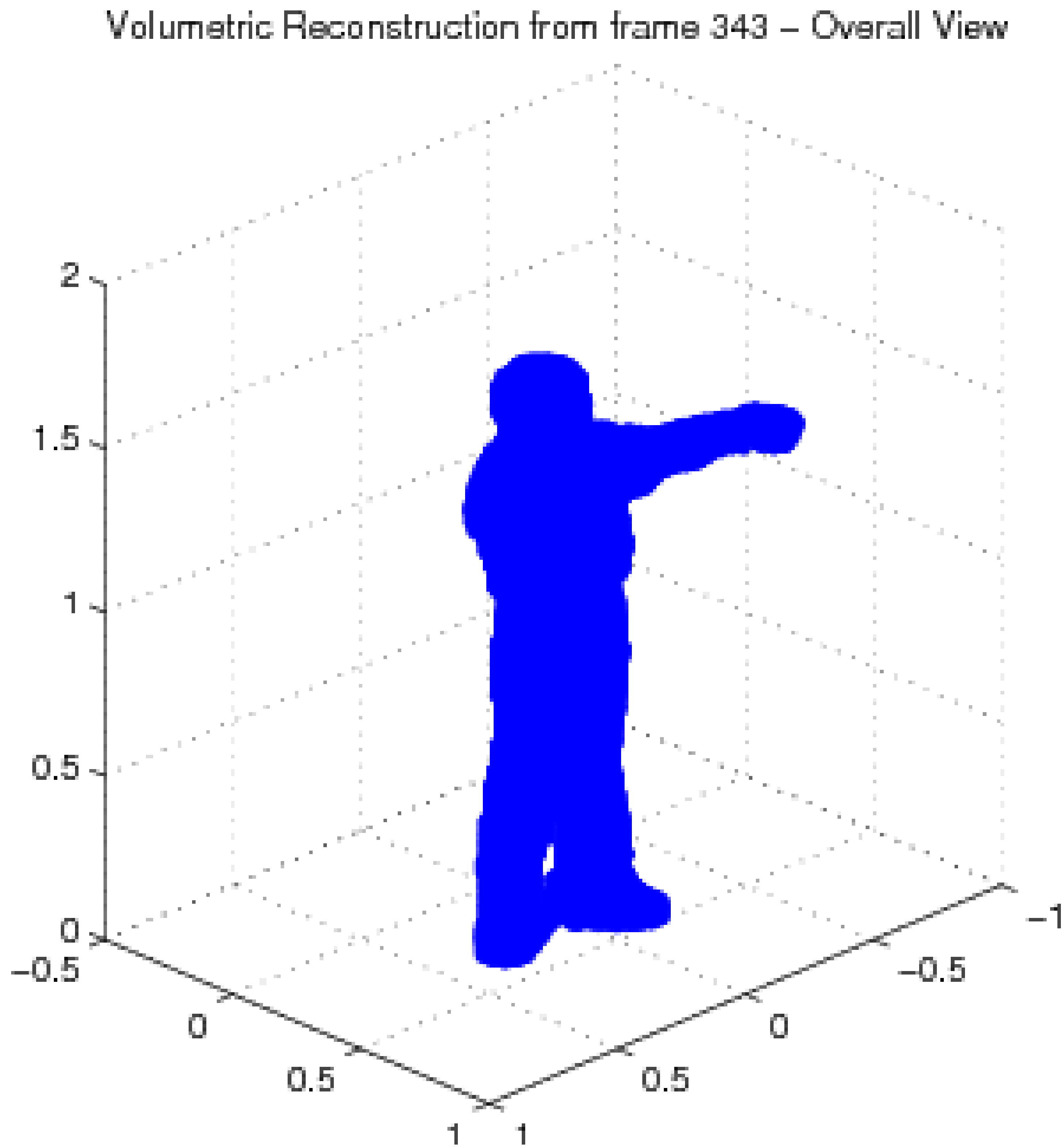

reconstructed points/camera setup only inliers are used
2
3
4
6
5
4
3
2
1
0
−1
1
0
−1
−2
−3


2D error: mean (blue), std (red)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
pixels
2
3
4
Id of the camera


Volumetric Reconstruction from frame 343 – Side View
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
–0.5
0
0.5
1



Volumetric Reconstruction from frame 343 – Top View
1
0.5
0
−0.5
−1
−0.5
0
0.5
1


Volumetric Reconstruction from frame 343 – Overall View
2
1.5
1
0.5
0
−0.5
0
0.5
1
1
0.5
0
−0.5
−1