Noise in images
filtering in spatial and frequency domain

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- deterioration of analog signal
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How to suppress noise?

- digital only, ie. no A/D and D/A conversion. $\longrightarrow$ OK
- larger chips $\longrightarrow$ EXPENSIVE, EXPENSIVE LENSES
- cooled cameras (astronomy) $\longrightarrow$ SLOW, EXPENSIVE
- (local) image preprocessing

Example scene

image sequence

Statistical point of view

Suppose we can acquire N images of the same scene. For each pixels we obtain N results $x_i, i = 1 \ldots N$. Assume:

- observations independent
- each x_i has $\mathsf{E}\{x_i\} = \mu$ and $\mathrm{var}\{x_i\} = \sigma^2$

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Properties of the average value $s_N = \frac{1}{N}\sum_1^N x_i$

- Expectation: $\mathsf{E}\{s_N\} = \frac{1}{N}\sum_1^N \mathsf{E}\{x_i\} = \mu$
- Variance: We know that $\mathrm{var}\{x_i/N\} = \mathrm{var}\{x_i\}/N^2$, thus

$$\mathrm{var}\{s_N\} = \frac{\mathrm{var}\{x_1\}}{N^2} + \frac{\mathrm{var}\{x_2\}}{N^2} + \ldots + \frac{\mathrm{var}\{x_N\}}{N^2} = \frac{\sigma^2}{N}\,.$$

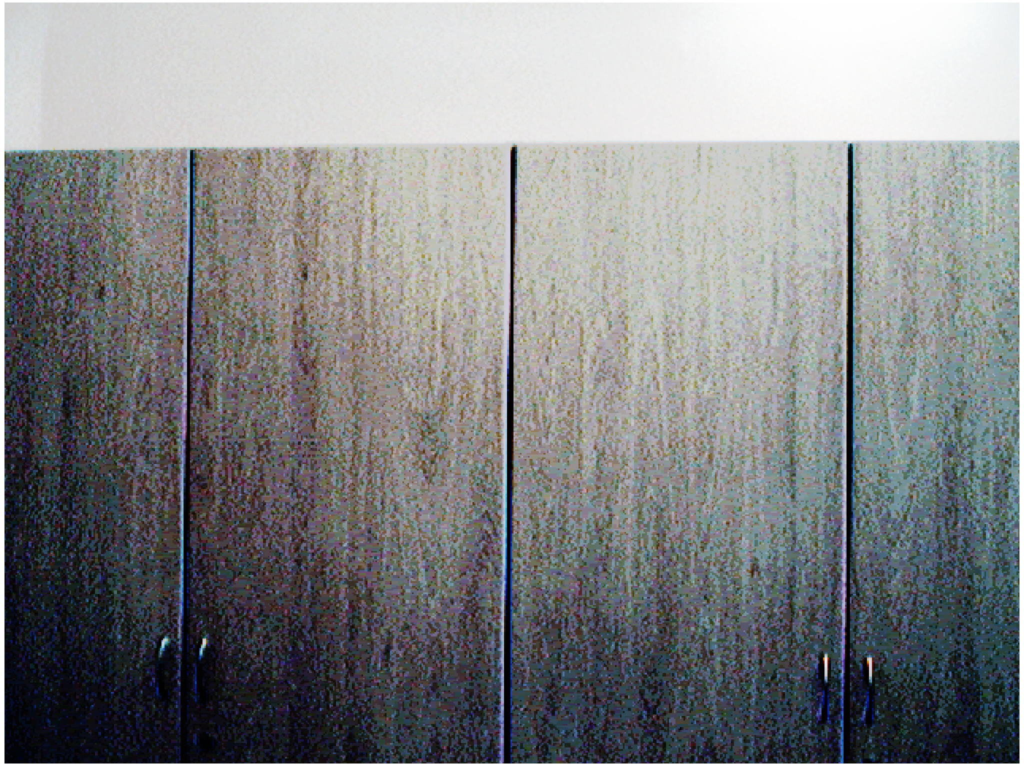
which means that standard deviation of s_N decreases as $\frac{1}{\sqrt{N}}$.

Example

a noisy image

average from $\approx$ 60 observations.

Example — equalized



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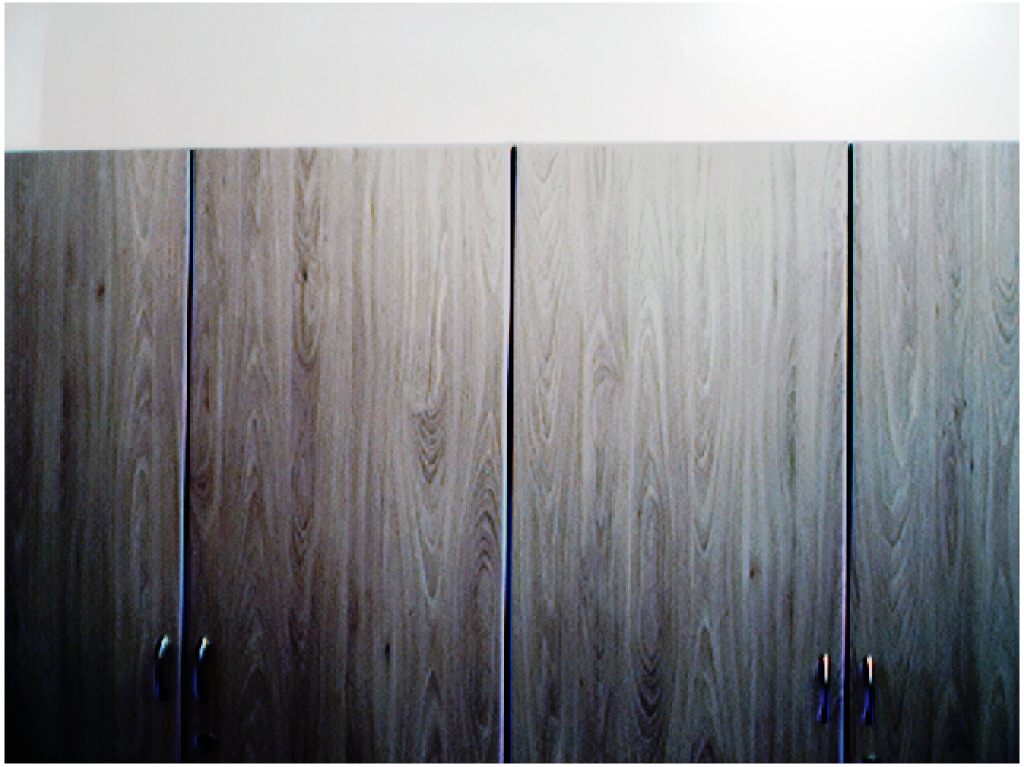
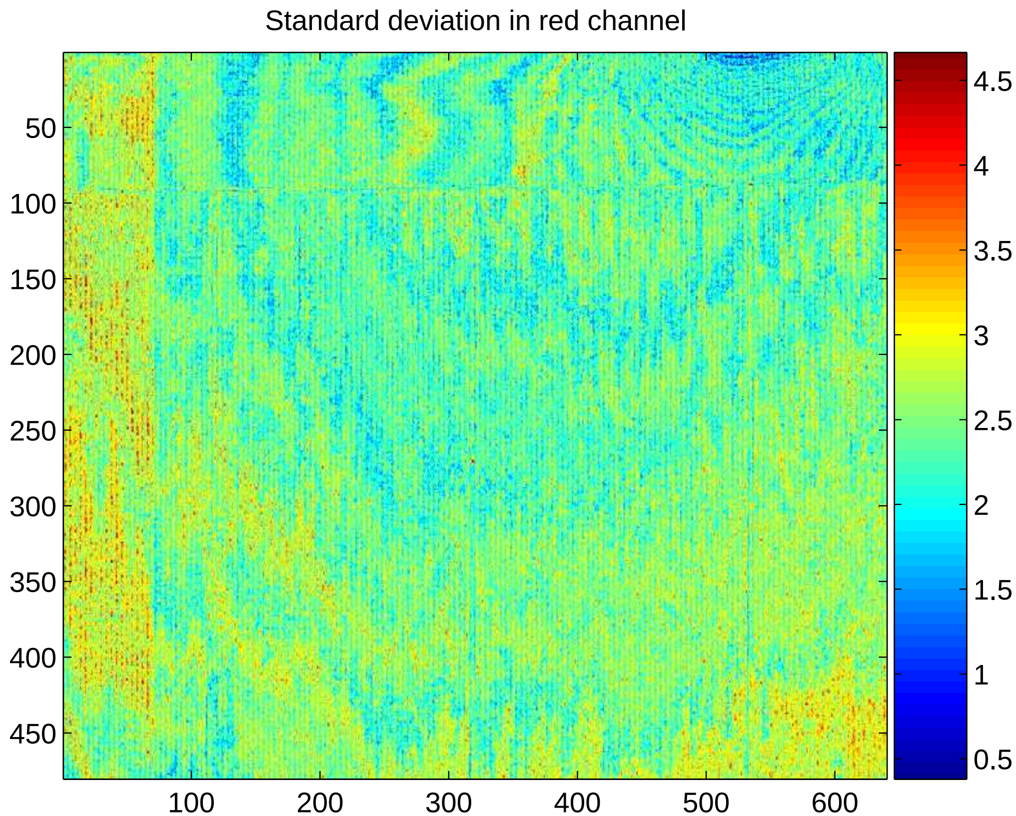


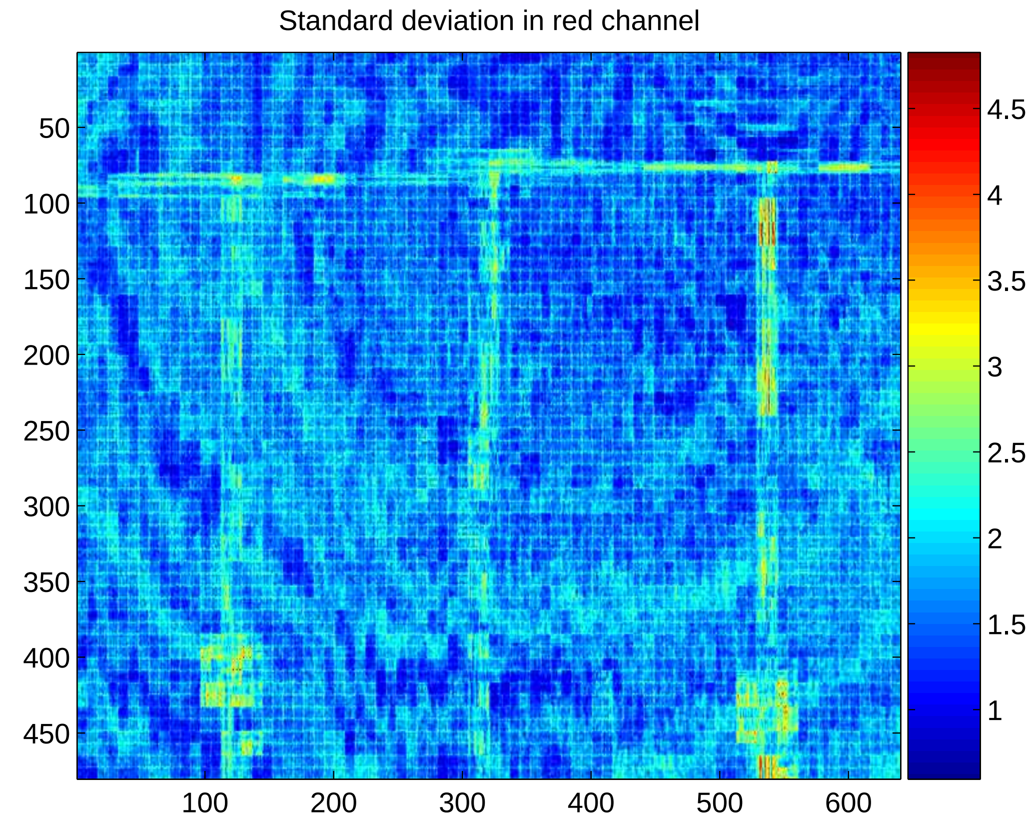
average from $\approx$ 60 observations.

Standard deviations in pixels

for images:



without compression



lossy compressed (jpg)

Lossy compression is generally not a good choice for machine vision!

Problem: noise suppression from just one image

- redundancy in images
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- correction of the pixel value based on an analysis of its neighbourhood
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spatial filtering

Spatial filtering — informally

Idea: Output is a function of a pixel value and those of its neighbours.

Example for $8-$connected region.

$$g(x,y) = \mathrm{Op}\begin{bmatrix} f(x-1,y-1) & f(x,y-1) & f(x+1,y-1) \\ f(x-1,y) & f(x,y) & f(x+1,y) \\ f(x-1,y+1) & f(x,y+1) & f(x+1,y+1) \end{bmatrix}$$

Possible operations: sum, average, weighted sum, min, max, median . . .

Spatial filtering by masks

- Very common neighbour operation is per-element multiplication with a set of weights and sum together.
- Set of weights is often called mask or kernel.

Local neighbourhood

f(x-1,y-1)	f(x,y-1)	f(x+1,y-1)
f(x-1,y)	f(x,y)	f(x+1,y)
f(x-1,y+1)	f(x,y+1)	f(x+1,y+1)

mask

w(-1,-1)	w(0,-1)	w(+1,-1)
w(-1,0)	w(0,0)	w(+1,0)
w(-1,+1)	w(0,+1)	w(+1,+1)

$$g(x,y) = \sum_{k=-1}^{1} \sum_{l=-1}^{1} w(k,l) f(x+k, y+l)$$

2D convolution

- Spatial filtering is often referred to as convolution.
- We say, we convolve the image by a kernel or mask.
- Though, it is not the same. Convolution uses a flipped kernel.

Local neighbourhood

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2D Convolution — Why is it important?

- Input and output signals need not to be related through convolution, but if they are (and only if) the system is linear and time invariant.

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- 2D convolution describes well the formation of images.
- Many image distortions made by imperfect acquisition may be modelled by 2D convolution, too.
- It is a powerful thinking tool.

2D convolution — definition

Convolution integral

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x-k, y-l)h(k,l)dkdl$$

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Symbolic abbreviation

$$g(x, y) = f(x, y) * h(x, y)$$

Discrete 2D convolution

$$g(x,y) = f(x,y) * h(x,y) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f(x-k, y-l) h(k,l)$$

What with missing values $f(x-k, y-l)$?

Zero-padding: add zeros where needed.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} =$$

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The result is zero elsewhere. The concept is somehow contra-intuitive, practice with a pencil and paper.

Thinking about convolution

$$g(x) = f(x) * h(x) = \sum_k f(k) h(x-k)$$

Blurring f:

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- break the f into each discrete sample
- send each one individually through h to produce blurred points

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$$g(x) = f(x) * h(x) = \sum_k f(k)h(x-k)$$

Blurring f:

- break the f into each discrete sample
- send each one individually through h to produce blurred points
- sum up the blurred points

Thinking about convolution II

$$g(x) = f(x) * h(x) = \sum_k f(x-k)h(k)$$

Mask filtering:

- flip the function h around zero

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- point-wise multiply for each position k value $f(x-k)$ and the shifted flipped copy of h.

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Mask filtering:

- flip the function h around zero
- shift to output position x
- point-wise multiply for each position k value $f(x-k)$ and the shifted flipped copy of h.
- sum for all k and write that value at position x

Motion blur modelled by convolution

Camera moves along x axis during acquisition.

$$g(x) = \sum_k f(x-k)h(k)$$

- $g(x)$ is the image we get
- $f(x)$ say to be the (true) 2D function
- g does not depend only on $f(x)$ but also on all k previous values of f
- $\#k$ measures the amount of the motion
- if the motion is steady then $h(k) = 1/(\#k)$

h is impulse response of the system (camera), image restoration

Spatial filtering vs. convolution — Flipping kernel

Why not $g(x) = \sum_k f(x+k)h(k)$ as in spatial filtering but $g(x) = \sum_k f(x-k)h(-k)$?

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In $g(x) = \sum_k f(x+k)h(k)$ we are asking for values of input function f that are yet to come!

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Solution: $h(-k)$

Convolution theorem

The Fourier transform of a convolution is the product of the Fourier transforms.

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The Fourier transform of a product is the convolution of the Fourier transforms.

$$\mathcal{F}\{f(x,y)h(x,y)\} = F(u,v) * H(u,v)$$

Convolution theorem — proof

$$\mathcal{F}\{f(x,y) * h(x,y)\} = F(u,v)H(u,v)$$

$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x) \exp\left(-i2\pi ux/M\right)$ and $g(x) = \sum_{k=0}^{M-1} f(k)h(x-k)$

$\mathcal{F}\{g(x)\} = \dots$

- $\frac{1}{M}\sum_{x=0}^{M-1}\sum_{k=0}^{M-1} f(k)h(x-k)e^{(-i2\pi ux/M)}$
- introduce new (dummy) variable $w = x - k$
- $\frac{1}{M}\sum_{k=0}^{M-1} f(k)\sum_{w=-k}^{(M-1)-k} h(w)e^{(-i2\pi u(w+k)/M)}$
- remember that all functions g, h, f are assumed to be periodic with period M
- $\frac{1}{M}\sum_{k=0}^{M-1} f(k)e^{(-i2\pi uk/M)}\sum_{w=0}^{M-1} h(w)e^{(-i2\pi uw/M)}$
- which is indeed $F(u)H(u)$

Convolution theorem — what is it good for?

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Enough theory for now. Go for examples . . .

Spatial filtering

What is it good for?

- smoothing
- sharpening
- noise removal
- edge detection
- pattern matching
- ...

Smoothing

Output value is computed as an average of the input value and its neighbourhood.

- Advantage: less noise

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Averaging:

$$g(x,y) = \frac{\sum_k \sum_l w(k,l) f(x+k, y+l)}{\sum_k \sum_l w(k,l)}$$

Smoothing kernels

Can be of any size, any shape

$$h = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad h = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix},$$

$$h = \frac{1}{25}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

Averaging ones($n \times n$) — increasing mask size

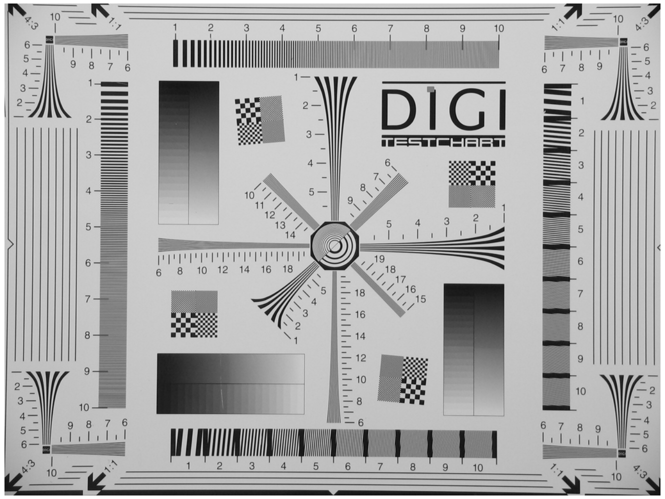
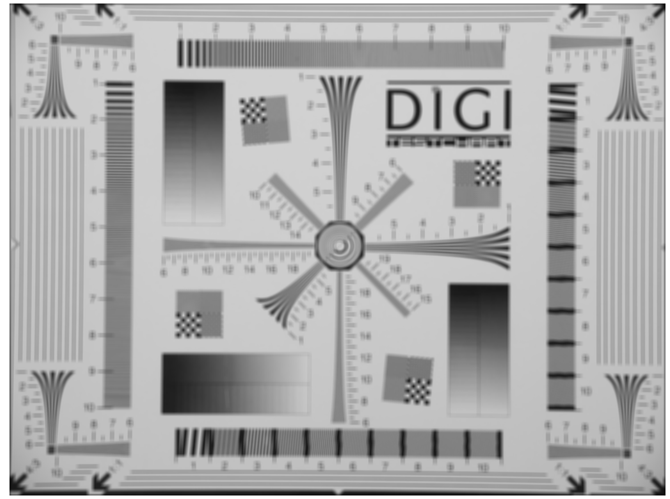
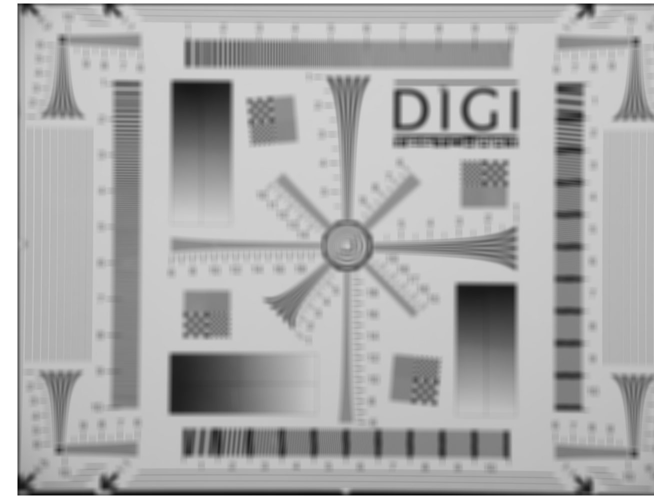


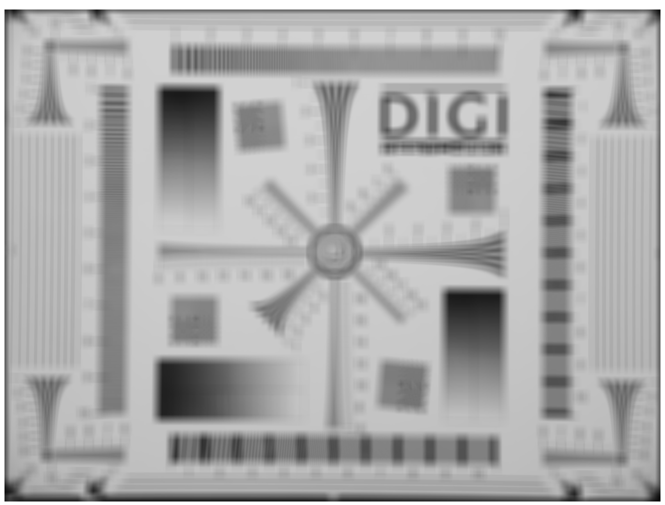
image 1024×768



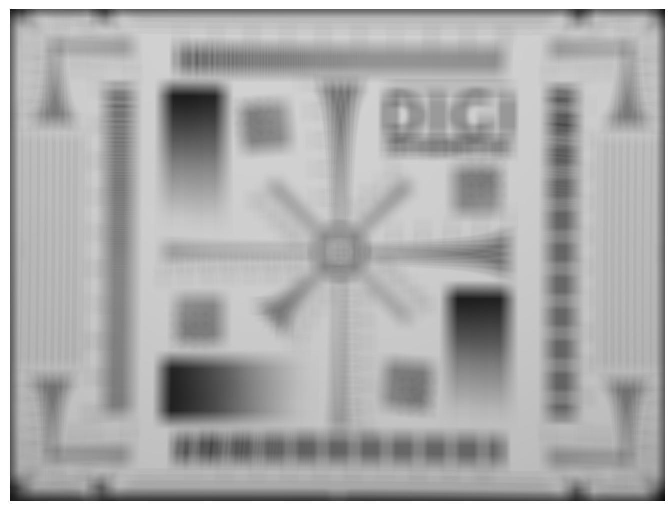
7×7



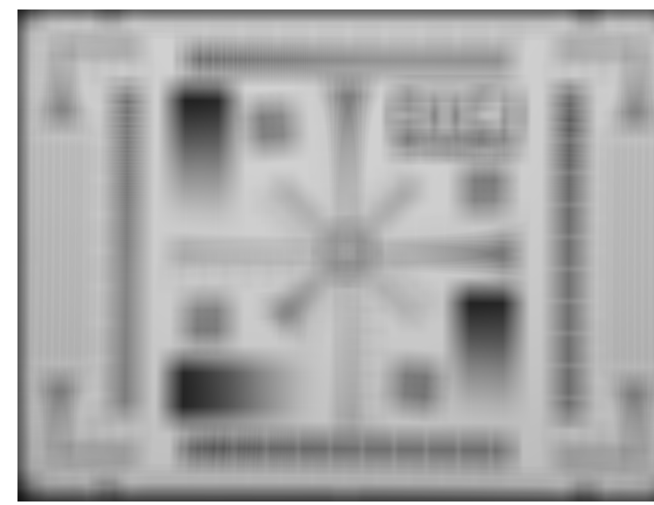
11×11



15×15



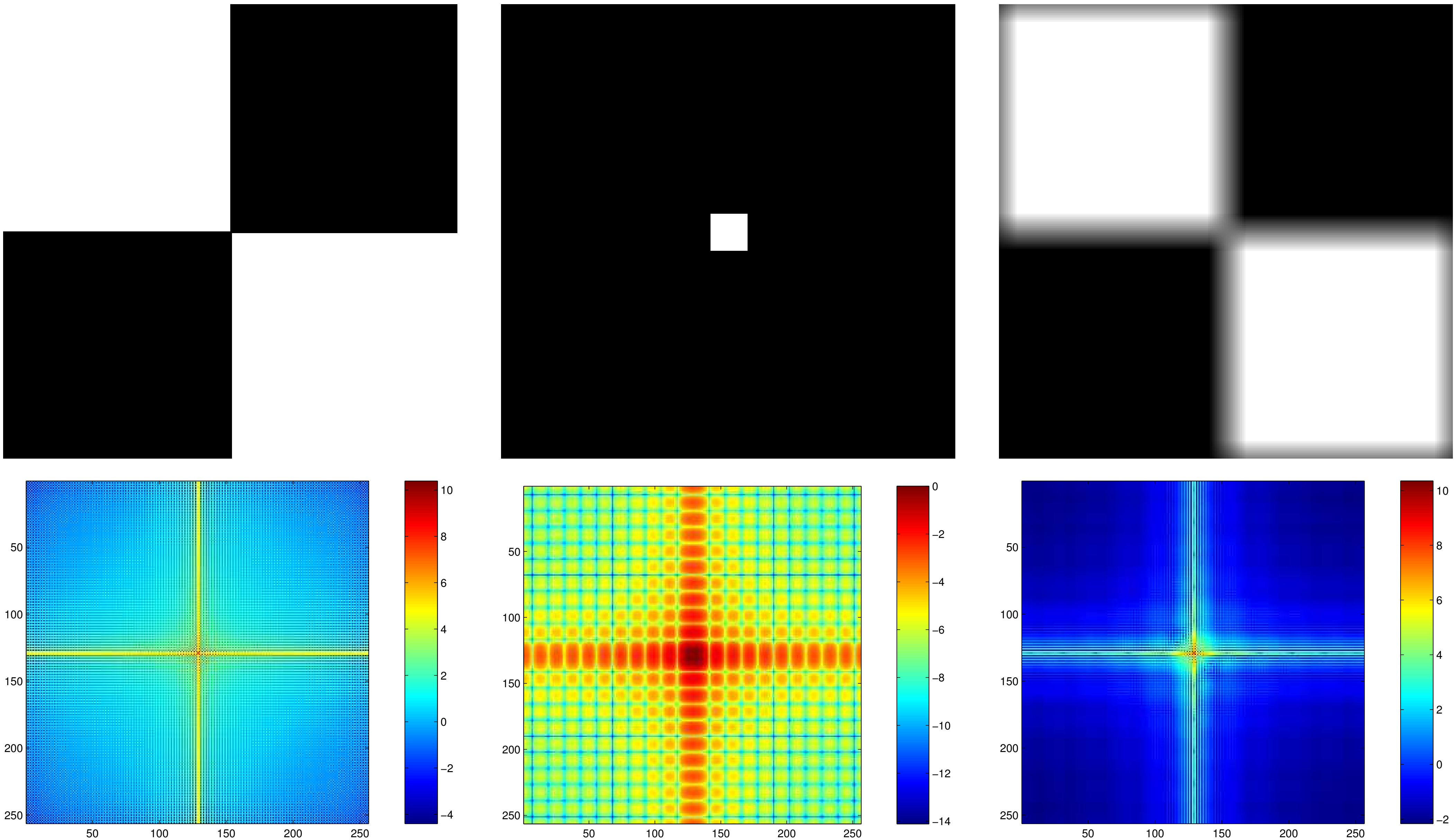
29×29



43×43

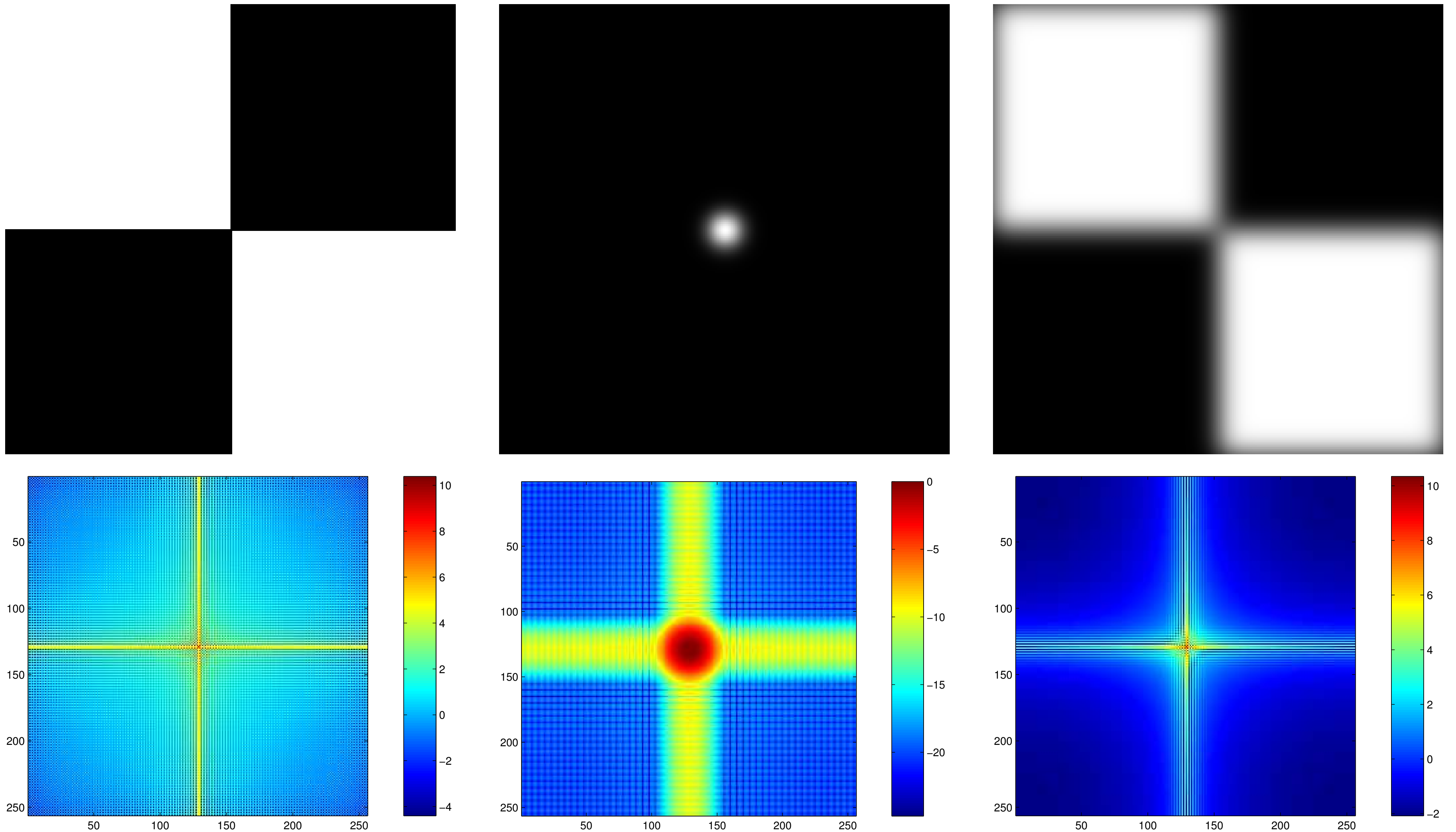
Frequency analysis of the spatial convolution — Simple averaging

Original image | 21×21 const. mask | filtered image



Frequency analysis of the spatial convolution — Gaussian smoothing

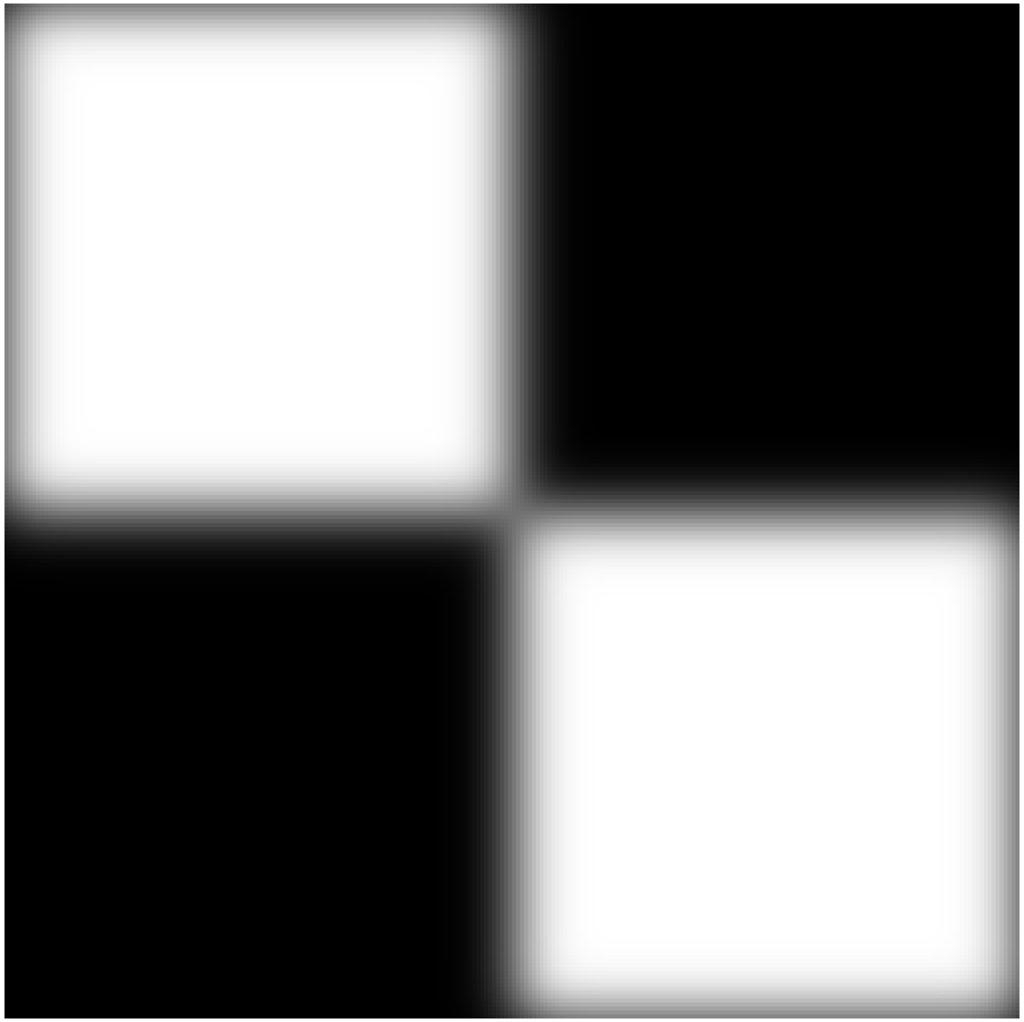
Original image | 21×21 Gauss. mask | filtered image



Simple averaging vs. Gaussian smoothing

simple averaging

Gaussian smoothing



Both images blurred but filtering by a constant mask still shows up some high frequencies!

Frequency analysis of the spatial convolution — Simple averaging

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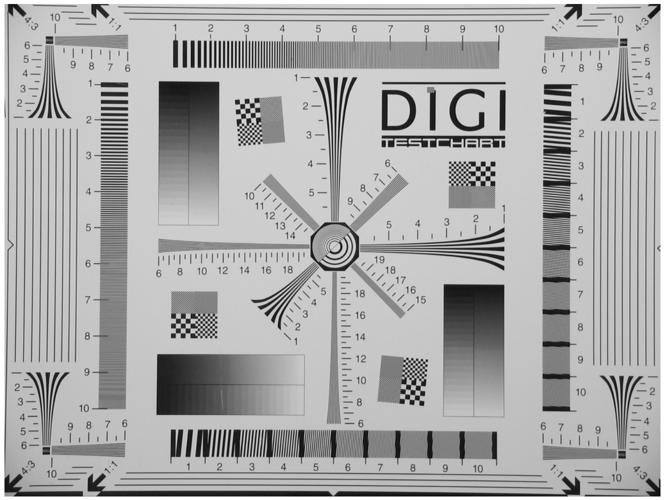

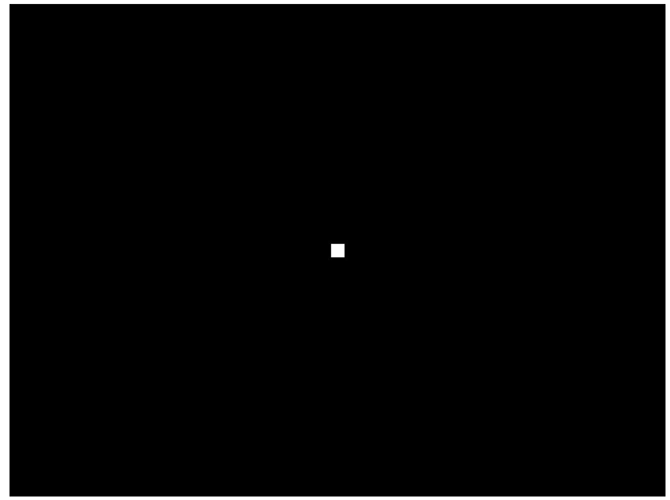

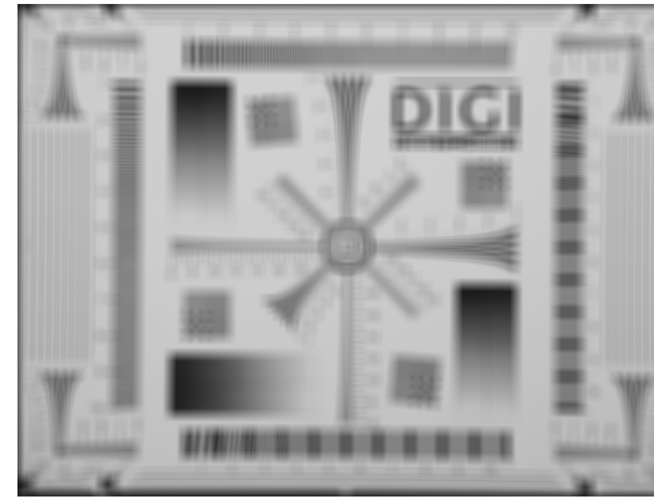

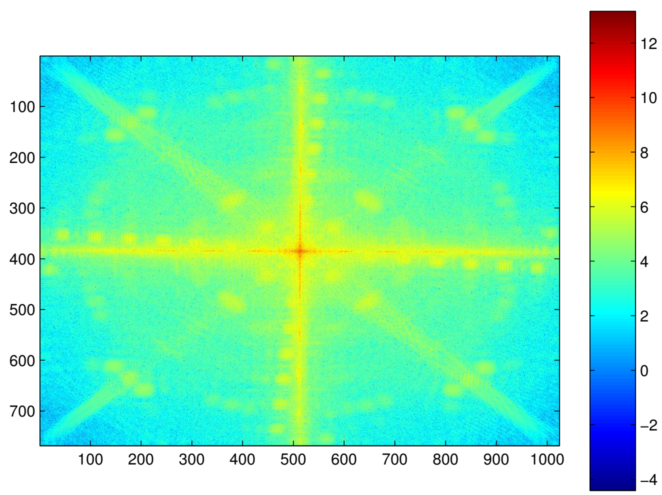

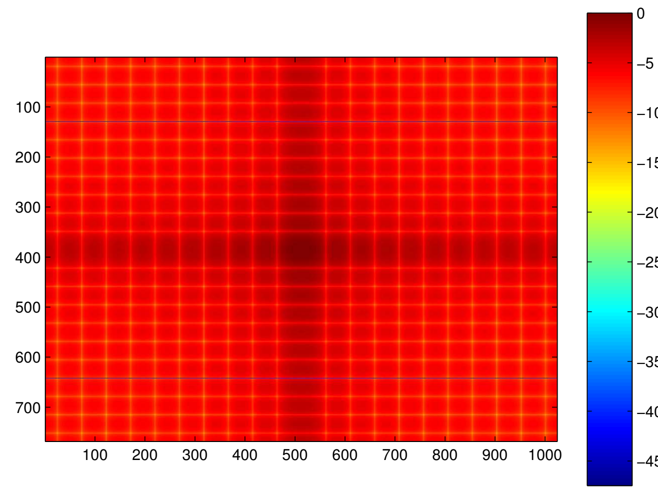

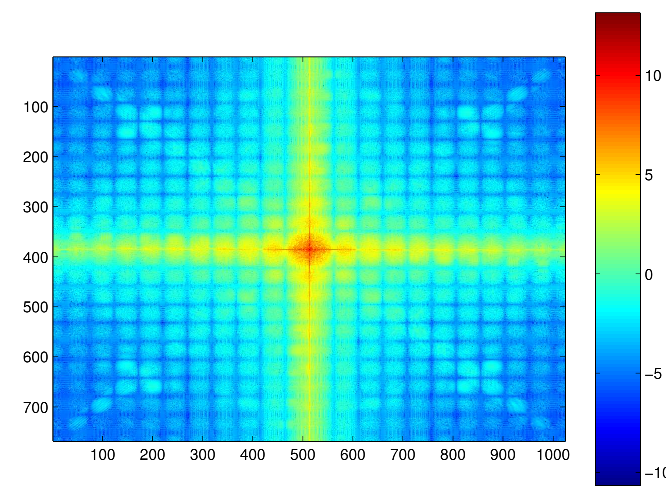

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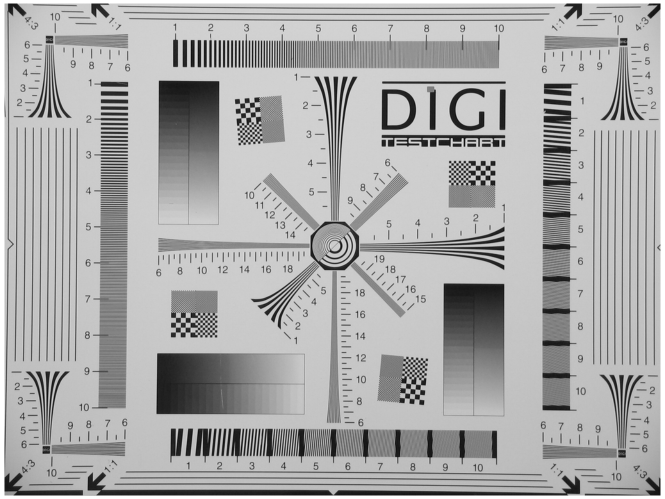

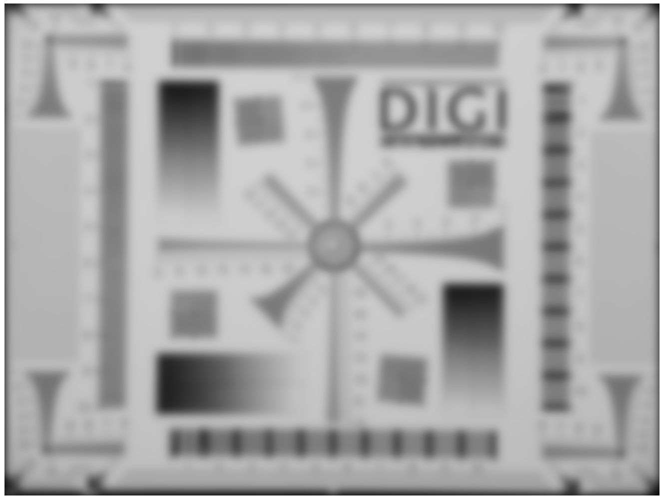

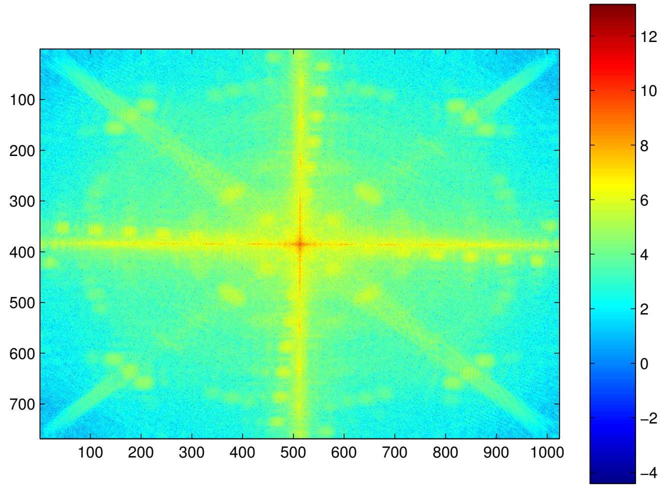

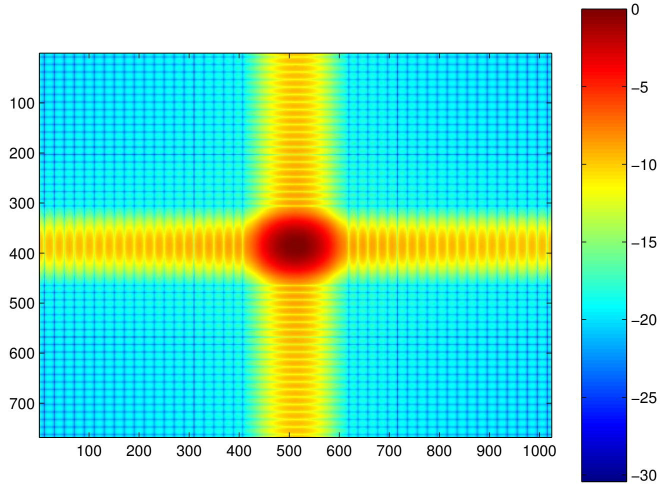

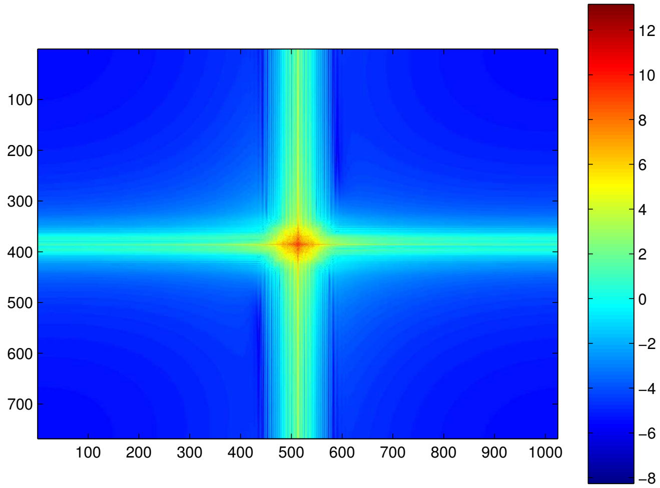

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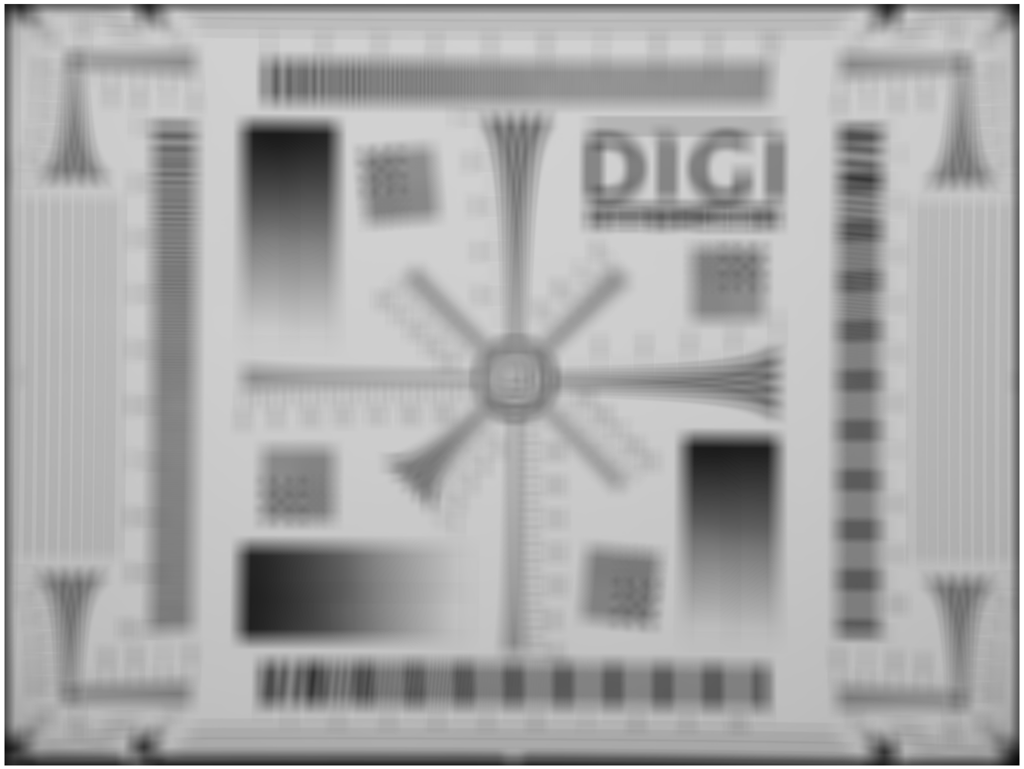
simple averaging



Gaussian smoothing

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Non-linear smoothing

Goal: reduce blurring of image edges during smoothing

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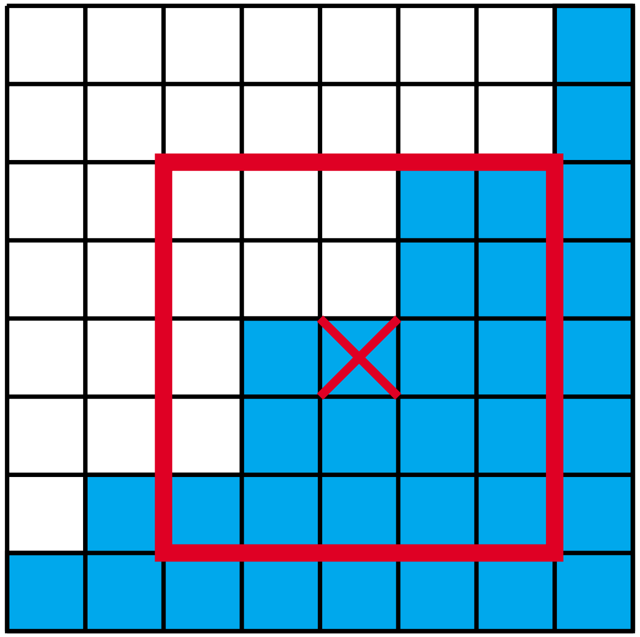
Homogeneous neighbourhood: find a proper neighbourhood where the values have minimal variance.



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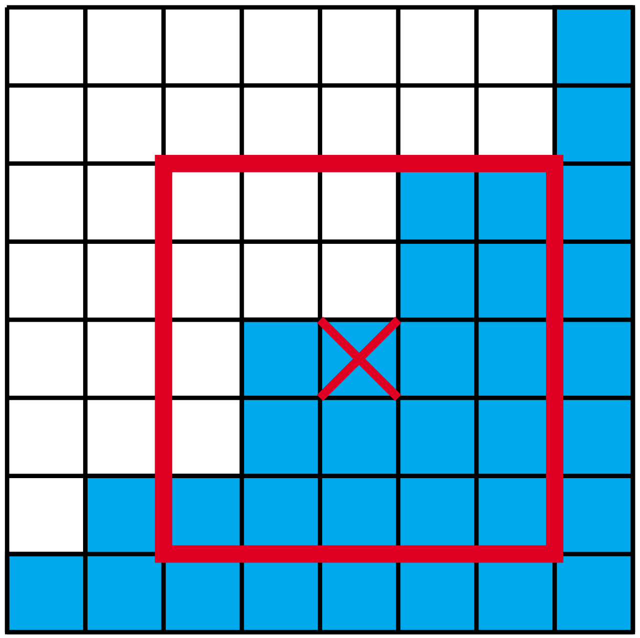
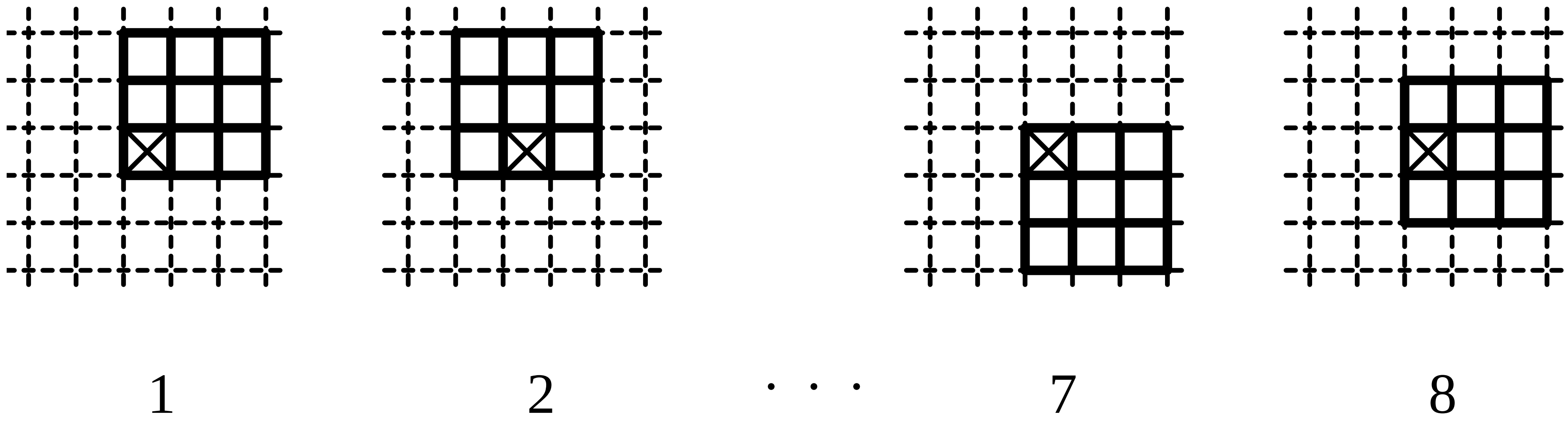


Robust statistics: something better than the mean.

Rotation mask

Rotation mask 3×3 seeks a homogeneous part at 5×5 neighbourhood.

Together 9 positions, 1 in the middle + 8 on the image



The mask with the lowest variance is selected as the proper neighbourhood.

Nonlinear smoothing — Robust statistics

Order-statistic filters

Nonlinear smoothing — Robust statistics

Order-statistic filters

- median

Nonlinear smoothing — Robust statistics

Order-statistic filters

- median
  - Sort values and select the middle one.

Nonlinear smoothing — Robust statistics

Order-statistic filters

- median
  - Sort values and select the middle one.
  - A method of edge-preserving smoothing.
  - Particularly useful for removing salt-and-pepper, or impulse noise.

Nonlinear smoothing — Robust statistics

Order-statistic filters

- median
  - Sort values and select the middle one.
  - A method of edge-preserving smoothing.
  - Particularly useful for removing salt-and-pepper, or impulse noise.
- trimmed mean
  - Throw away outliers and average the rest.
  - More robust to a non-Gaussian noise than a standard averaging.

Median filtering

100	98	102
99	105	101
95	100	255

Median filtering

100	98	102
99	105	101
95	100	255

Mean = 117.2

Median filtering

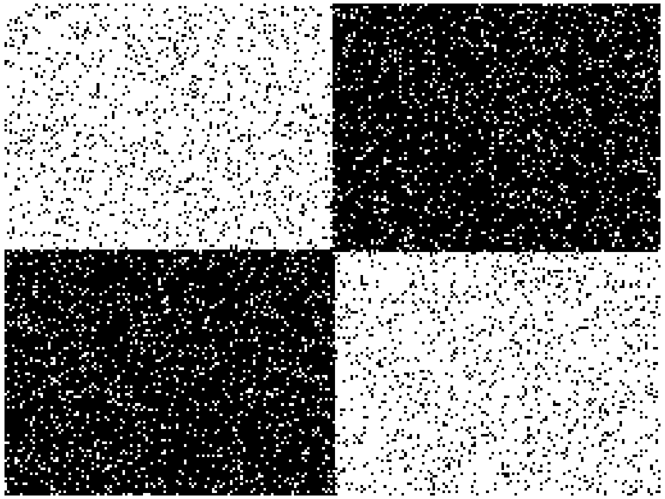
100	98	102
99	105	101
95	100	255

Mean = 117.2

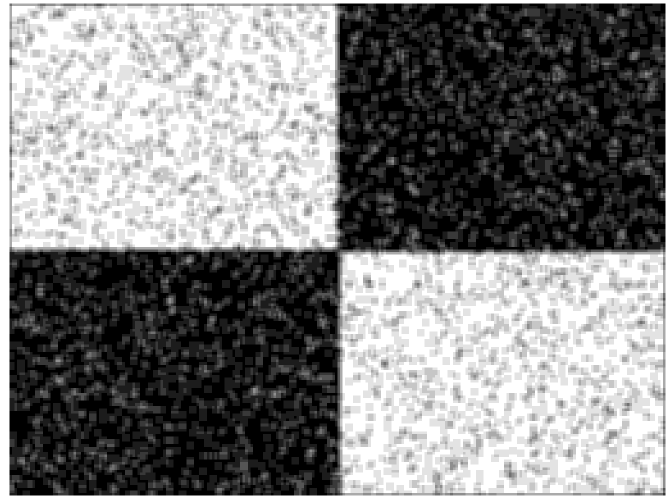
median: 95 98 99 100 **100** 101 102 105 255

Very robust, up to 50% of values may be outliers.

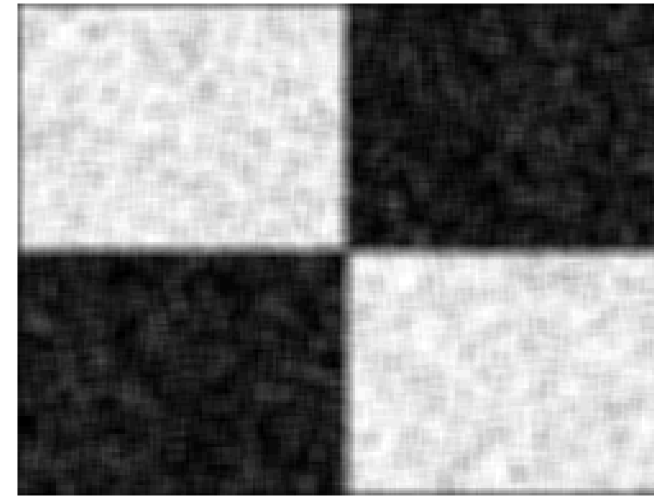
Nonlinear smoothing examples



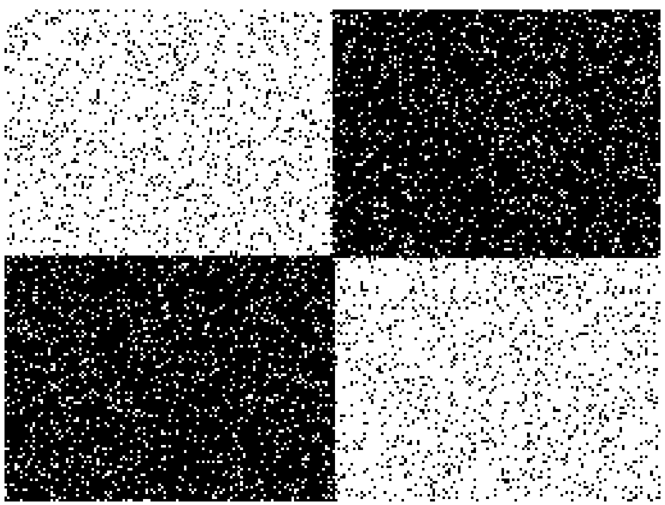
noisy image



averaging 3×3



averaging 7×7



noisy image

median 3×3

median 7×7

The median filtering damage corners and thin edges.

Filtering in frequency domain

Filtering in frequency domain

1. $F(u,v) = \mathcal{F}\{f(x,y)\}$

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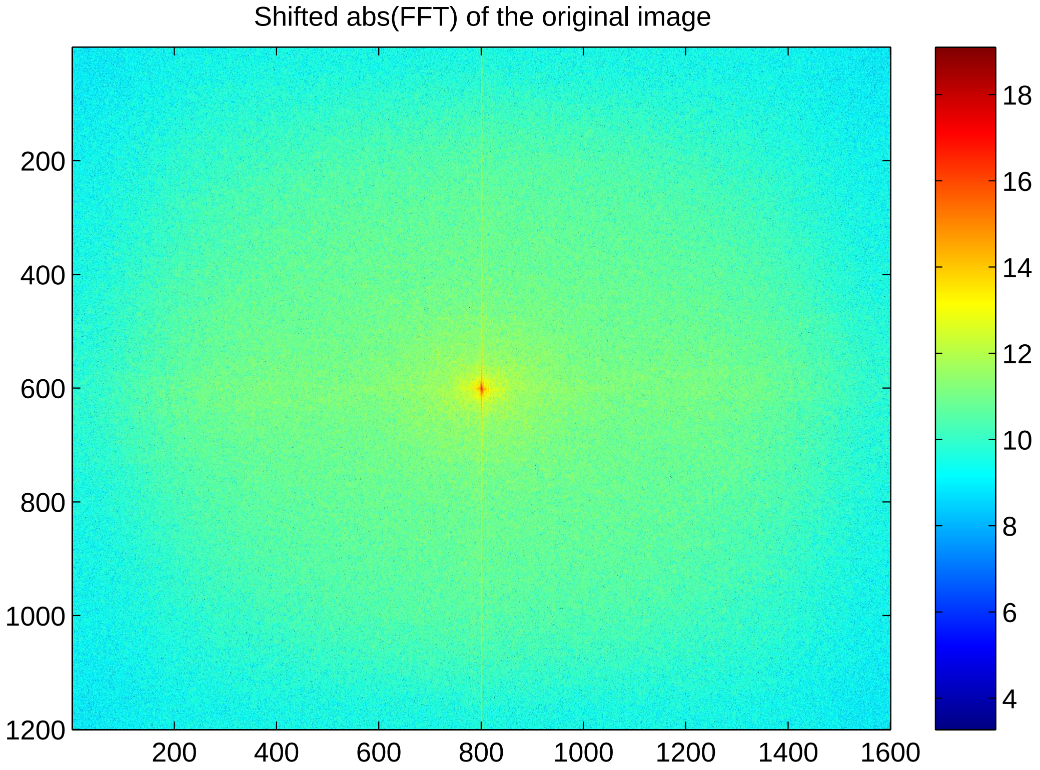
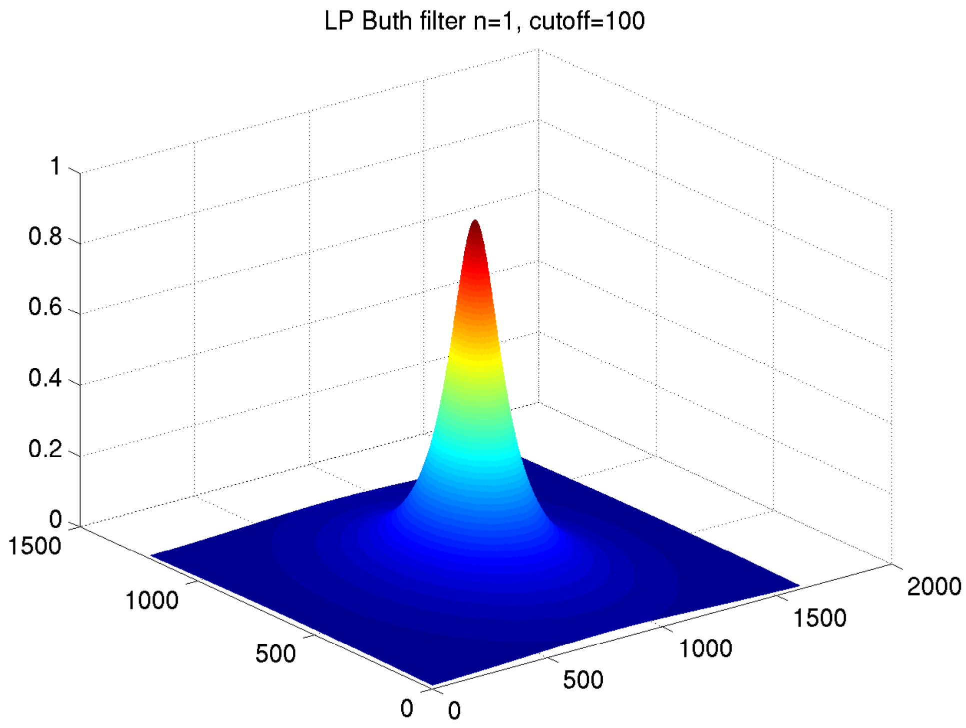
Do not forget: We display $\ln \|F(u,v)\|$. The filter must be applied to the $F(u,v)$.

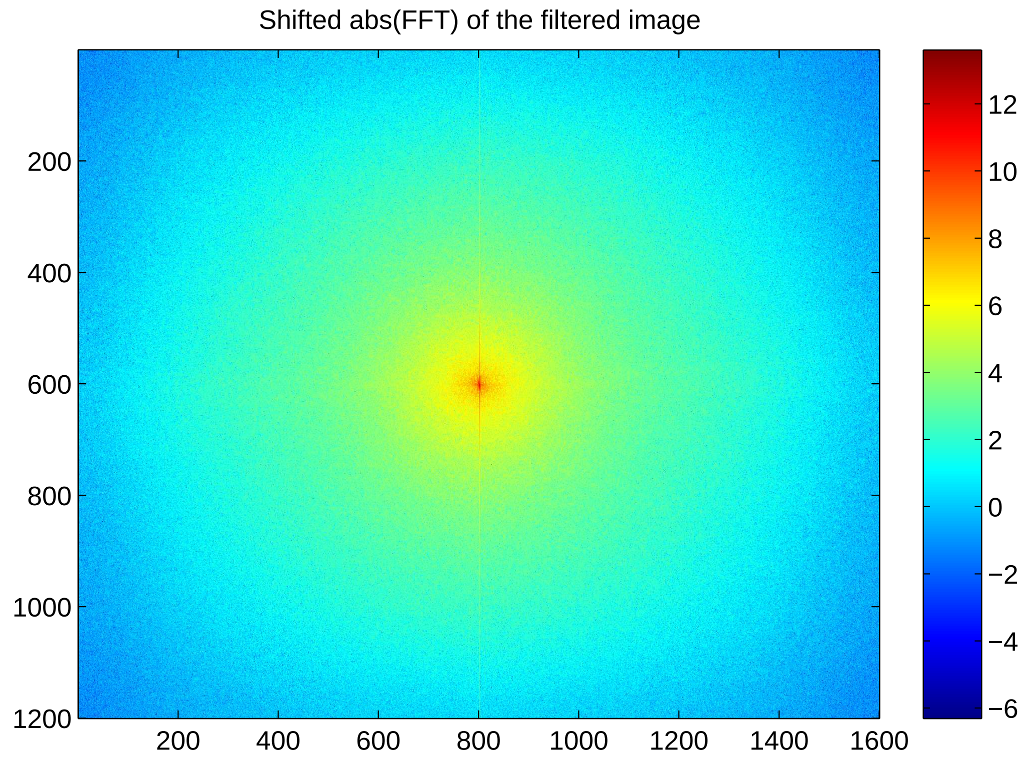
Lowpass filtering — Buttherworth filter I



Lowpass filtering — Buttherworth filter II



Buttherworth lowpass filter



FFT of the filtered image

$H(u,v) = \frac{1}{1+(D(u,v)/D_0)^{2/n}}$, where $D(u,v) = \sqrt{u^2+v^2}$

Lowpass filtering — Buttherworth filter III

Original image

Filtered image

More advanced filtering — Homomorphic filtering

Idea: simultaneously normalize the brightness across an image and increase contrast.

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Image is a product of illumination and reflectance components:
$f(x,y) = i(x,y)r(x,y)$

More advanced filtering — Homomorphic filtering

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Illumination i — slow spatial variations (low frequency)

More advanced filtering — Homomorphic filtering

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Reflectance r — fast varitations (dissimilar objects)

More advanced filtering — Homomorphic filtering

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Image is a product of illumination and reflectance components:
$f(x, y) = i(x, y)r(x, y)$

Illumination i — slow spatial variations (low frequency)

Reflectance r — fast varitations (dissimilar objects)

Use logarithm to separate the components and filter the logarithms!

Homomorphic filtering — cont.

$$z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$$

Homomorphic filtering — cont.

$$z(x, y) = \ln f(x, y) = \ln i(x, y) + \ln r(x, y)$$

Fourier pair

$$Z(u, v) = I(u, v) + R(u, v)$$

Homomorphic filtering — cont.

$$z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$$

Fourier pair

$$Z(u,v) = I(u,v) + R(u,v)$$

Filtering

$$S(u,v) = H(u,v)Z(u,v) = H(u,v)I(u,v) + H(u,v)R(u,v)$$

Homomorphic filtering — cont.

$$z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$$

Fourier pair

$$Z(u,v) = I(u,v) + R(u,v)$$

Filtering

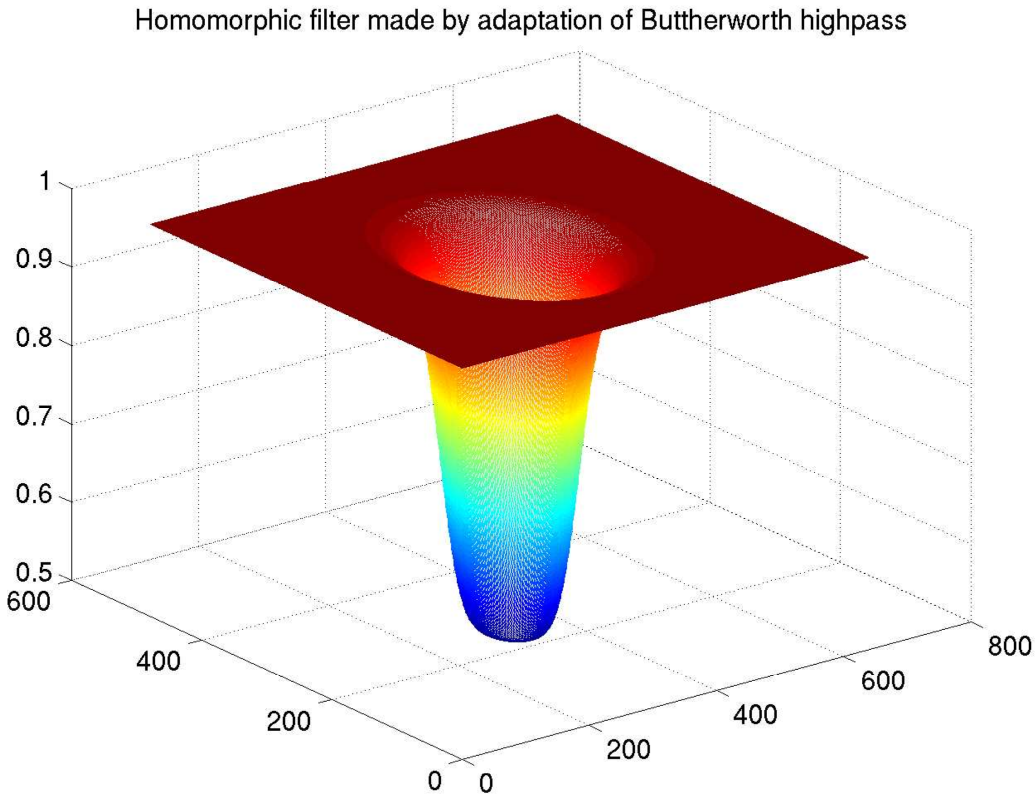
$$S(u,v) = H(u,v)Z(u,v) = H(u,v)I(u,v) + H(u,v)R(u,v)$$

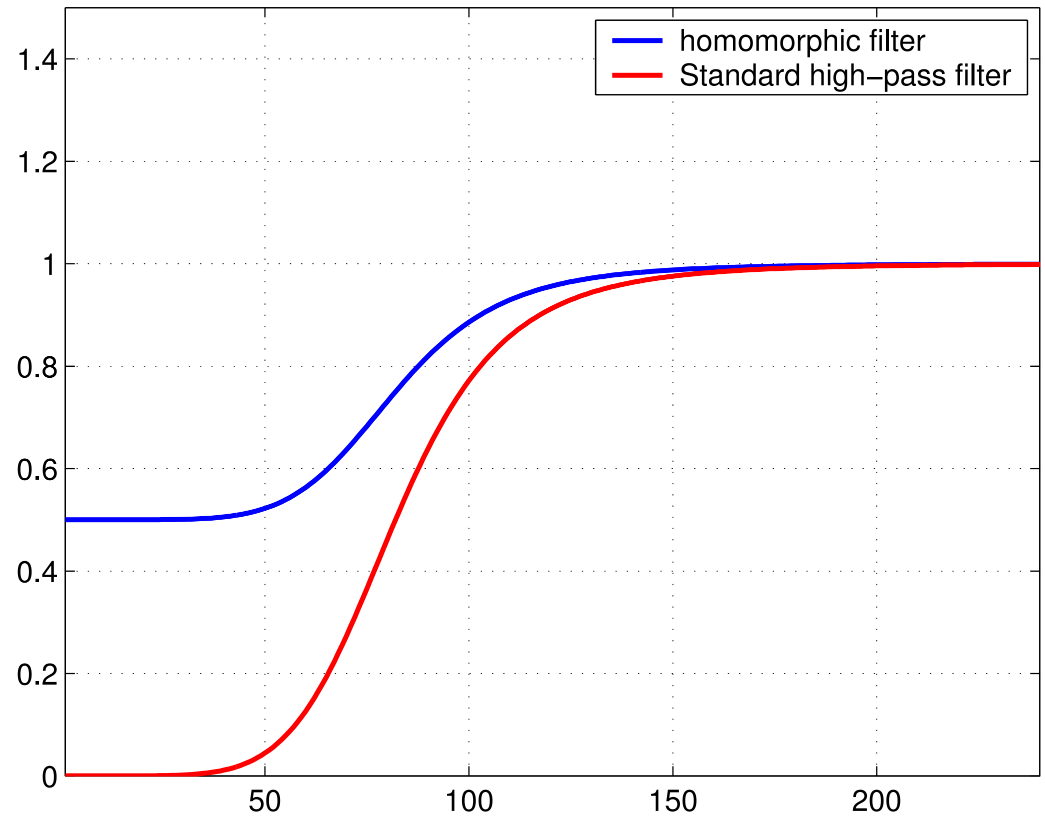
back to space $s(x,y) = \mathcal{F}^{-1}\{S(u,v)\}$ and back from $\ln$

$$g(x,y) = \exp\left(s(x,y)\right)$$

So, we can suppress variations in illumination and enhance reflectance component.

Homomorphic filtering — filters





Remember: The filter is applied to $Z(u,v)$. Not to $F(u,v)$!

Homomorphic filtering — results

Original image.

Homomorphic filtering — results

Original image.

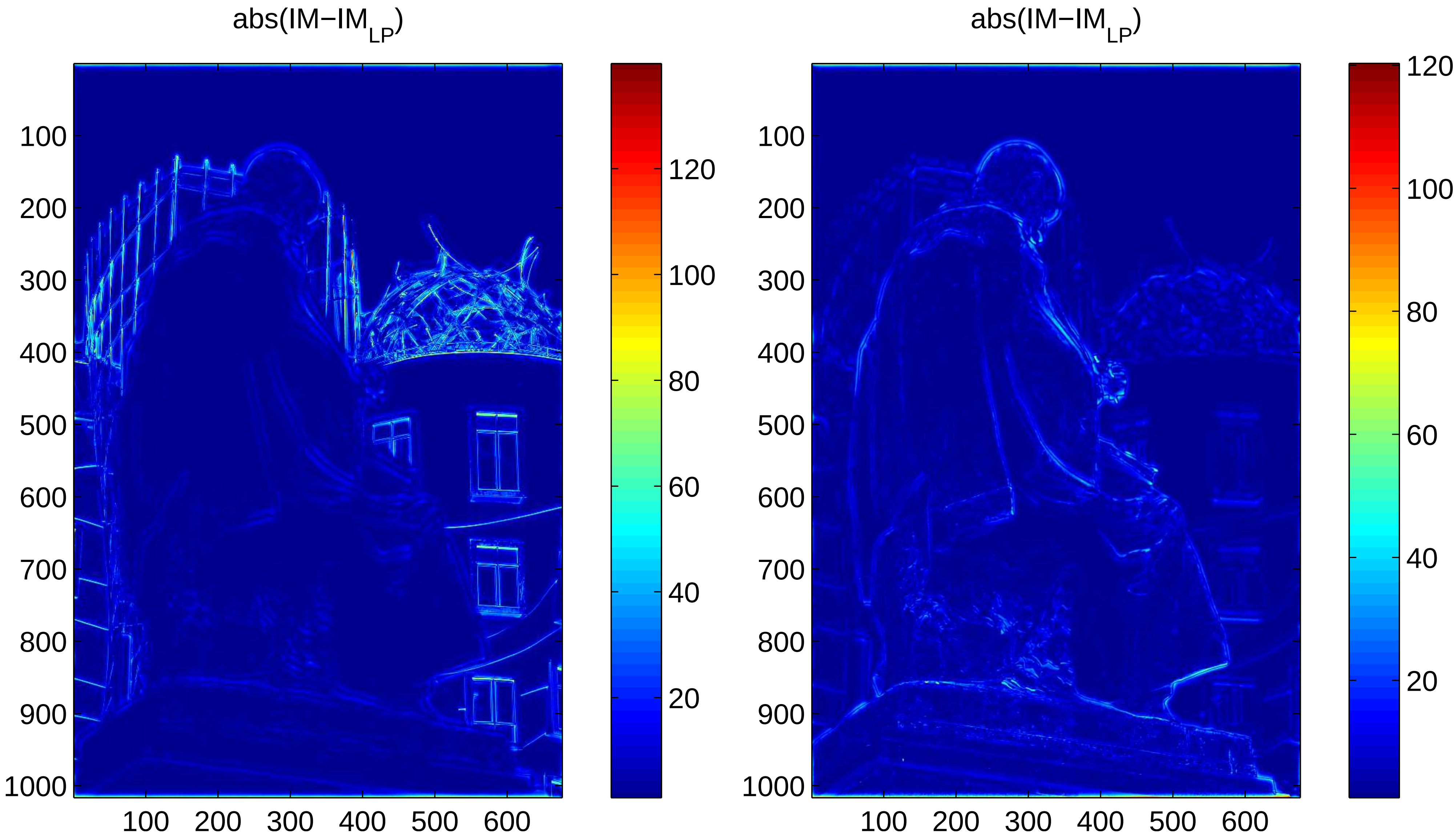
Filtered image.

Where are the frequencies in image?

Both image low-pass filtered

$\|IM_{orig} - IM_{lp}\|$


abs(IM−IM_LP)
abs(IM−IM_LP)
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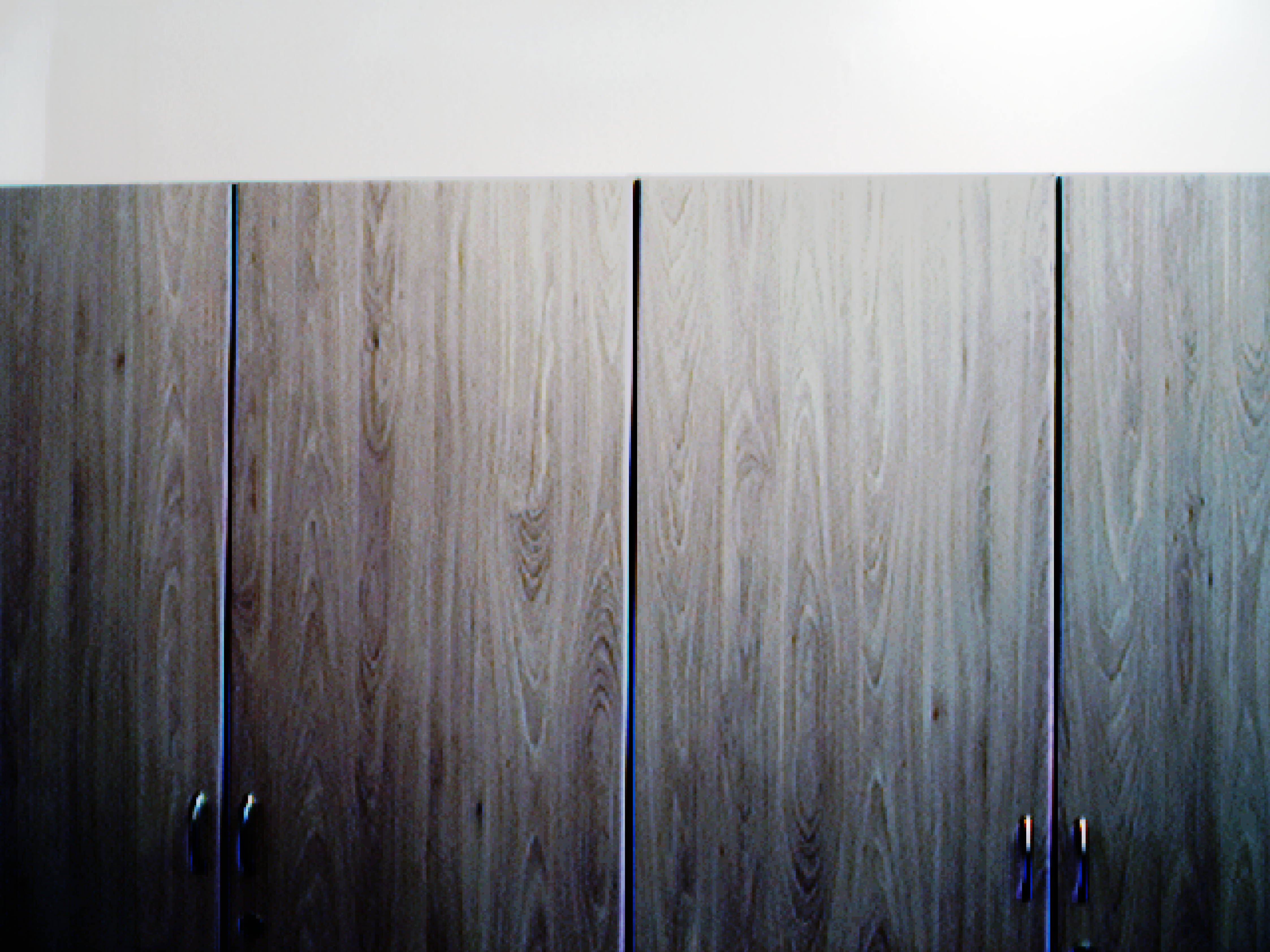
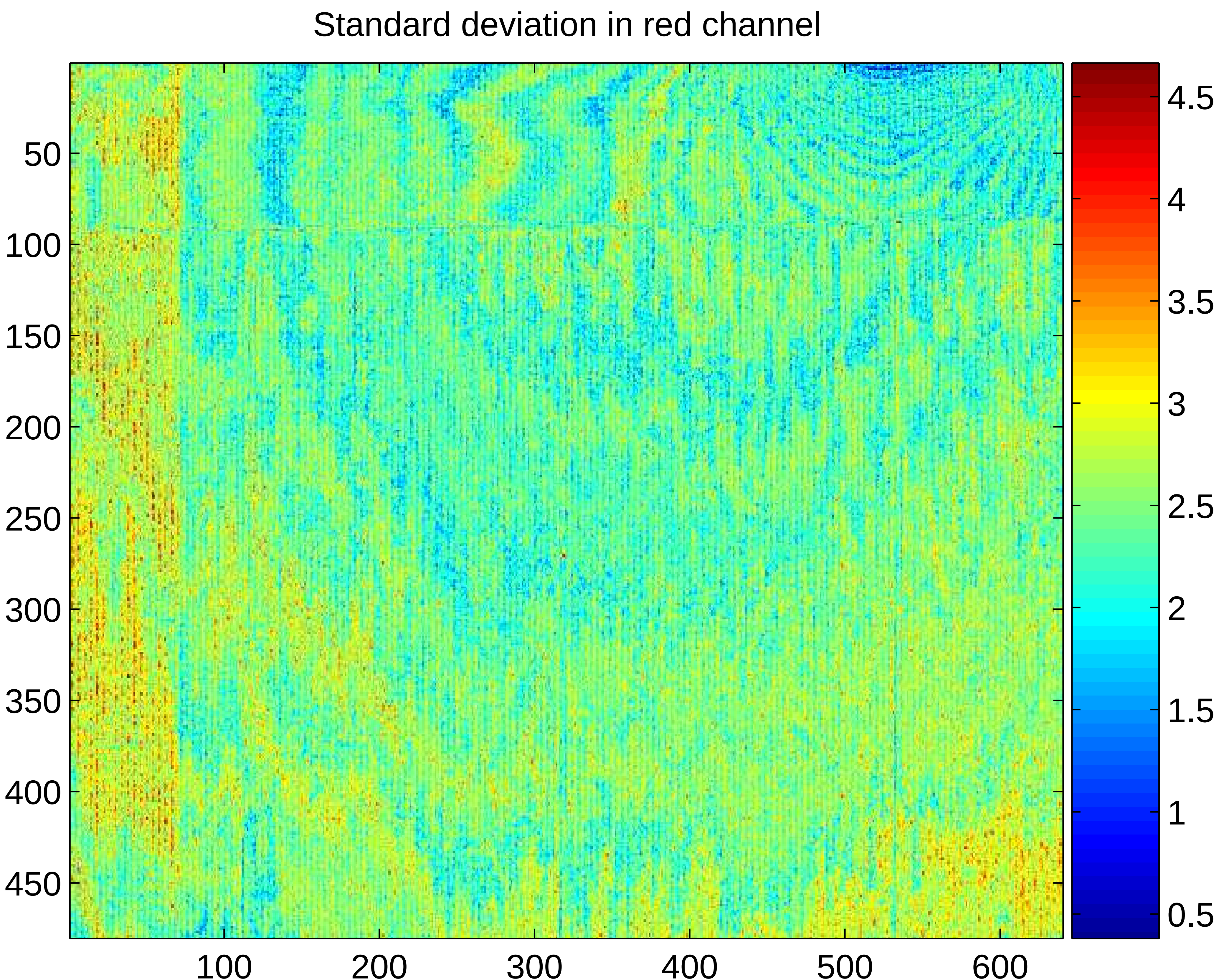
20

Make one focused image

Make one focused image

Make one focused image




Standard deviation in red channel
50
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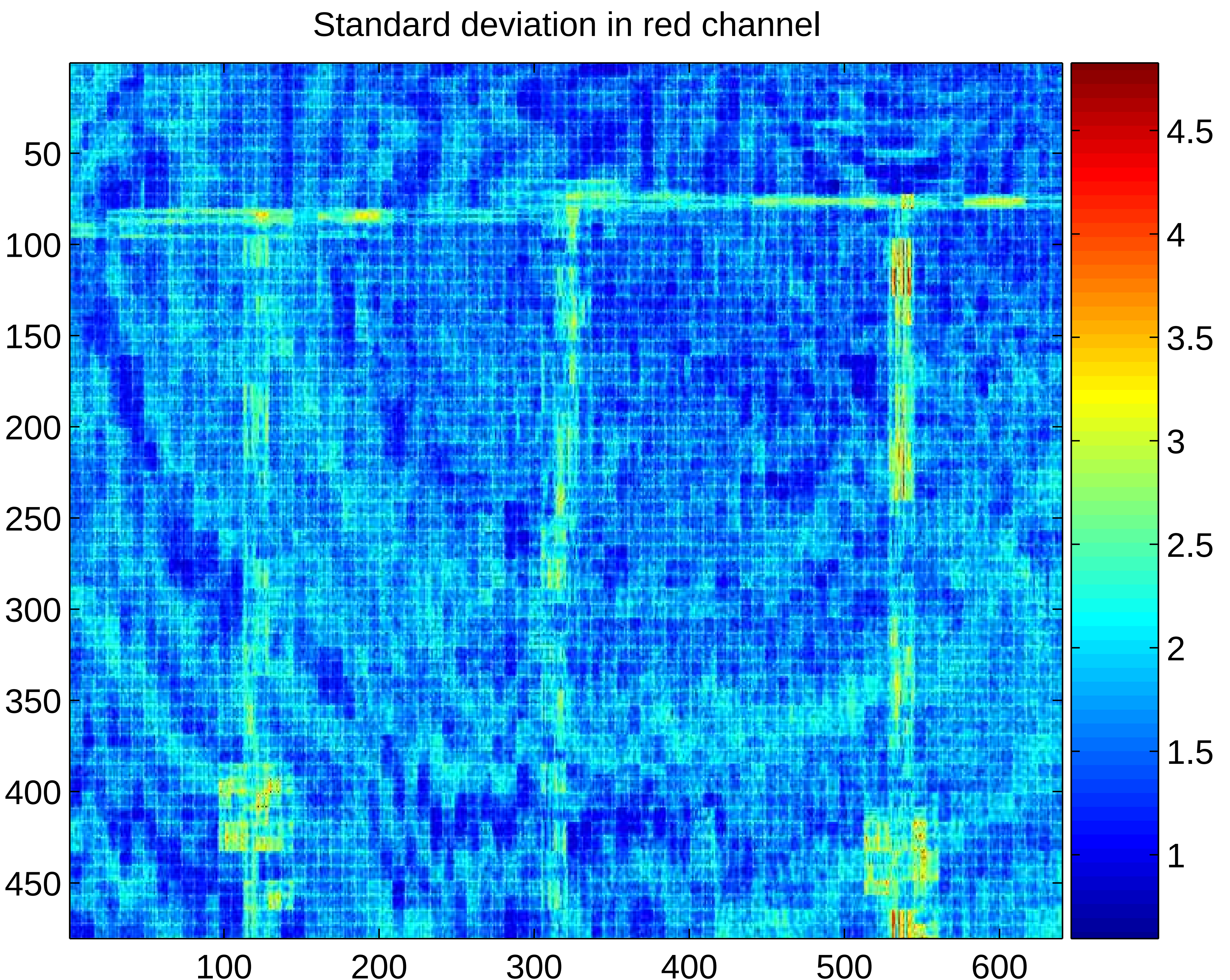
Standard deviation in red channel
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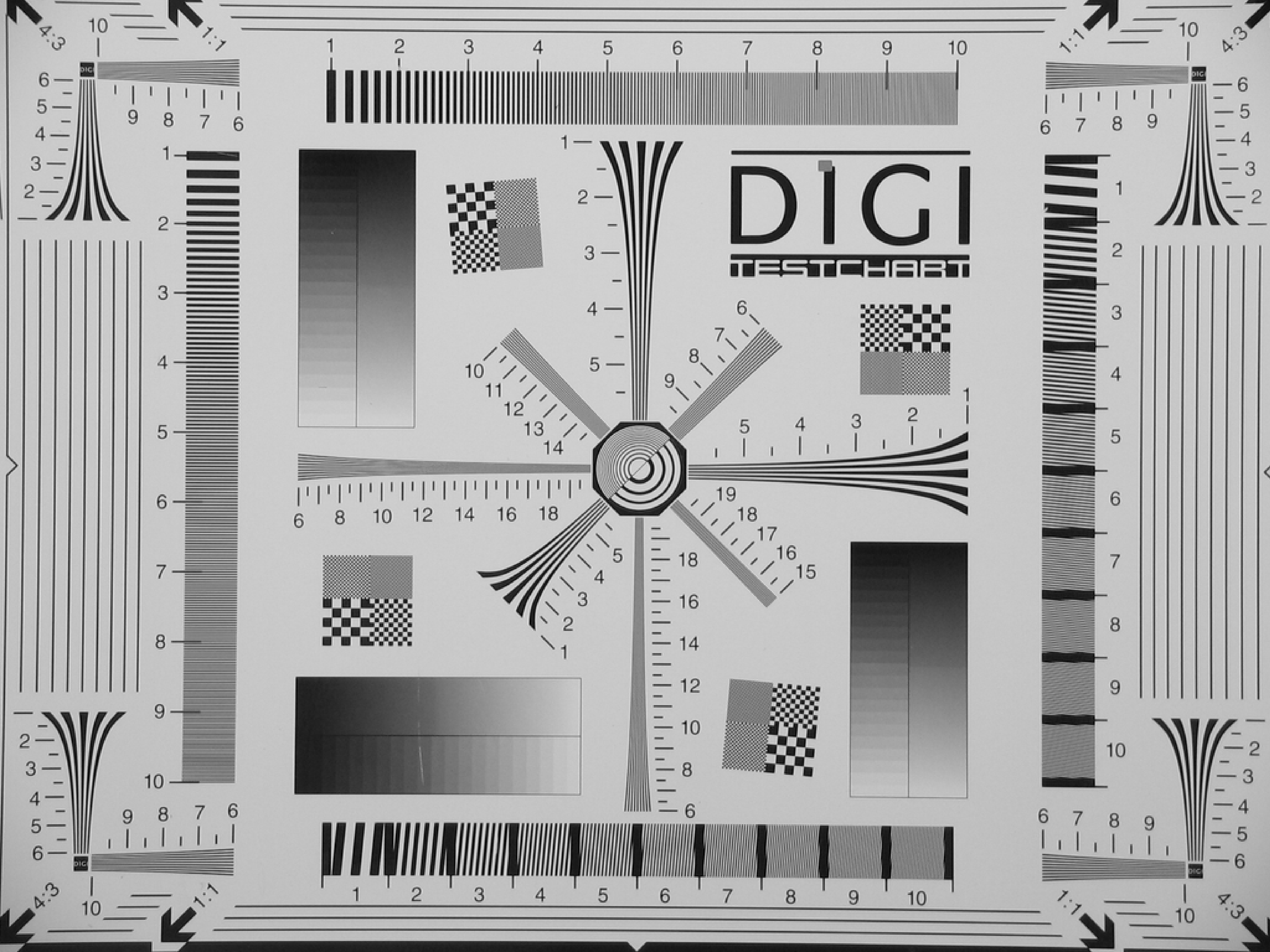
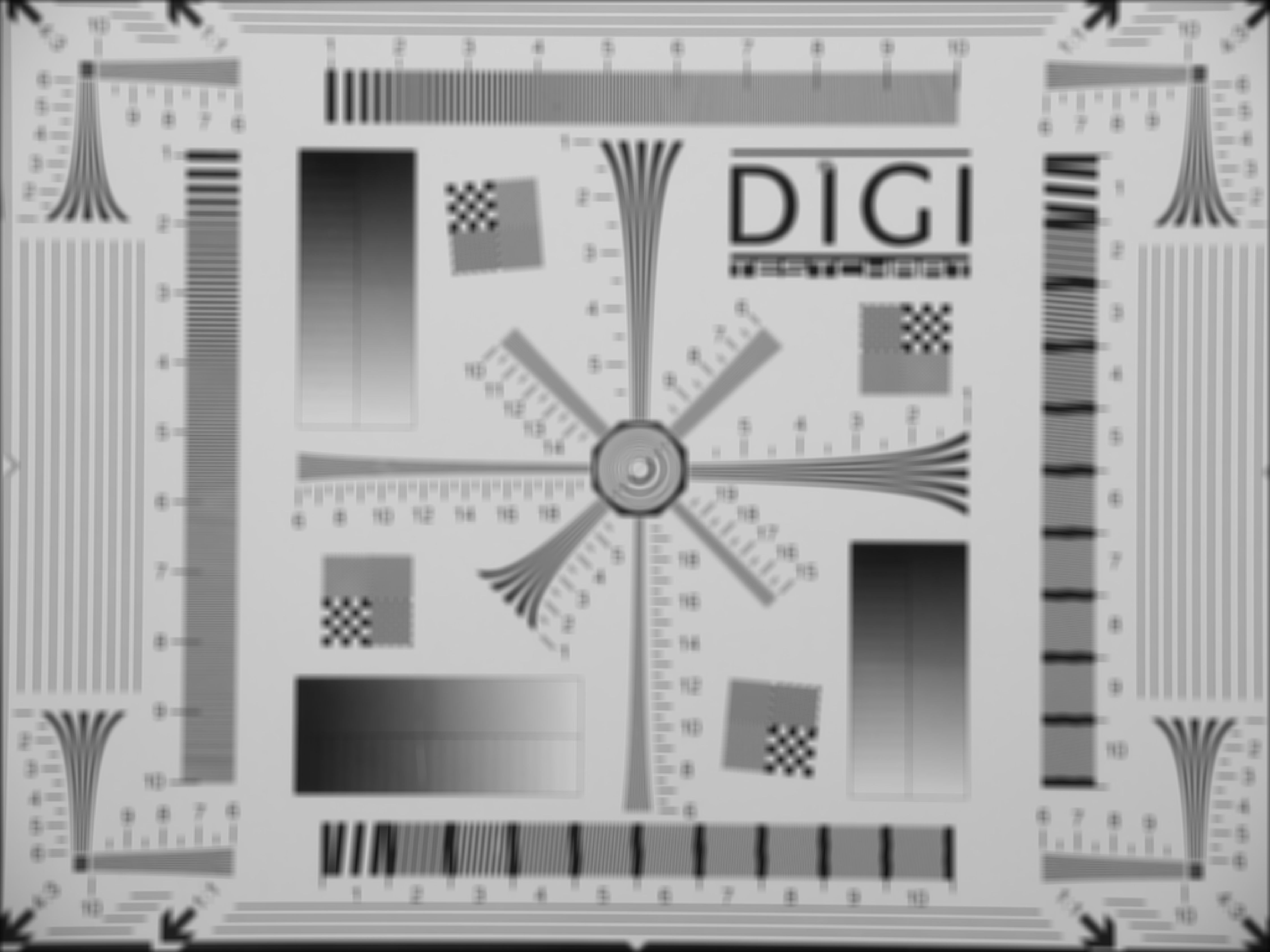
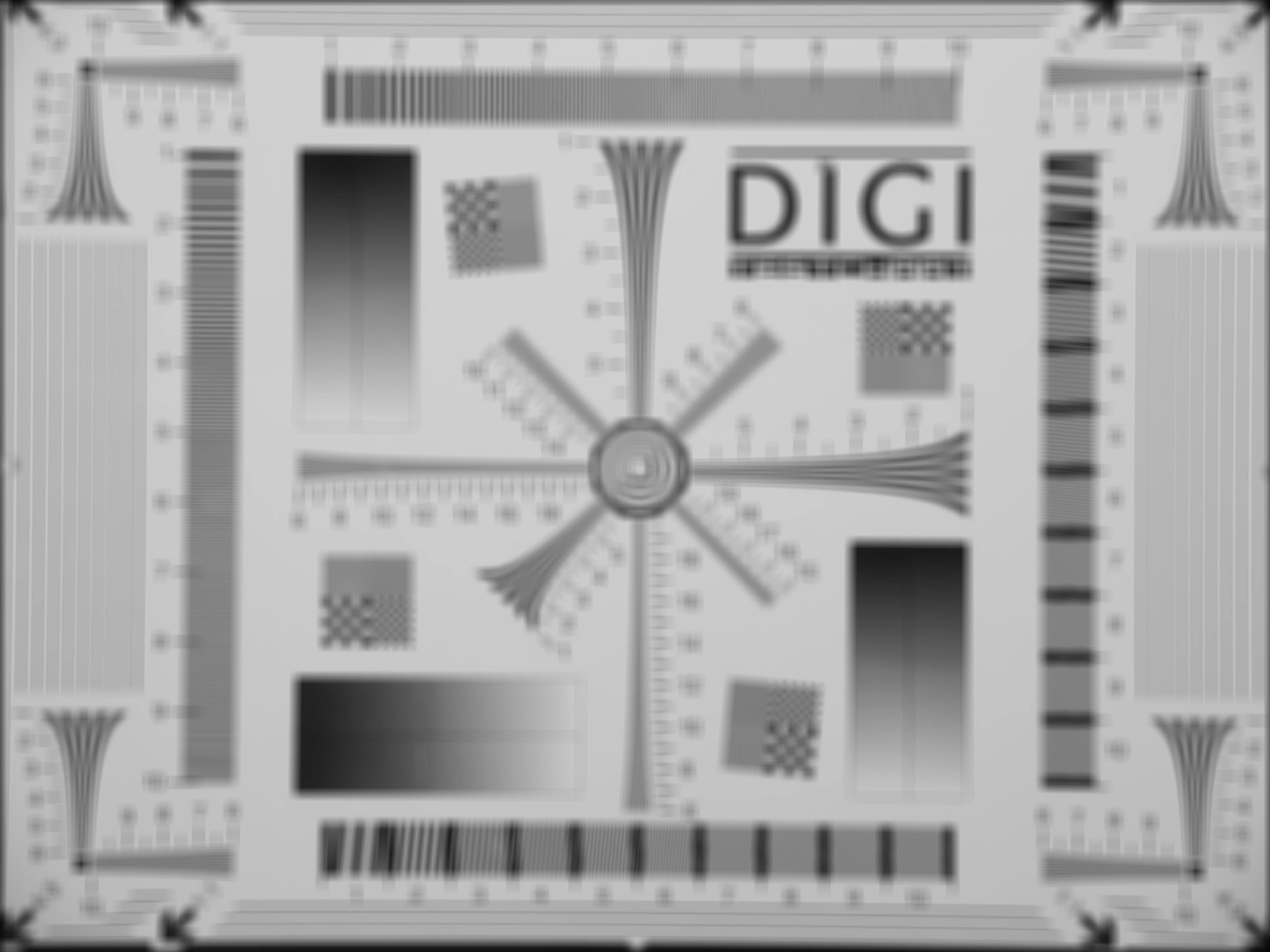
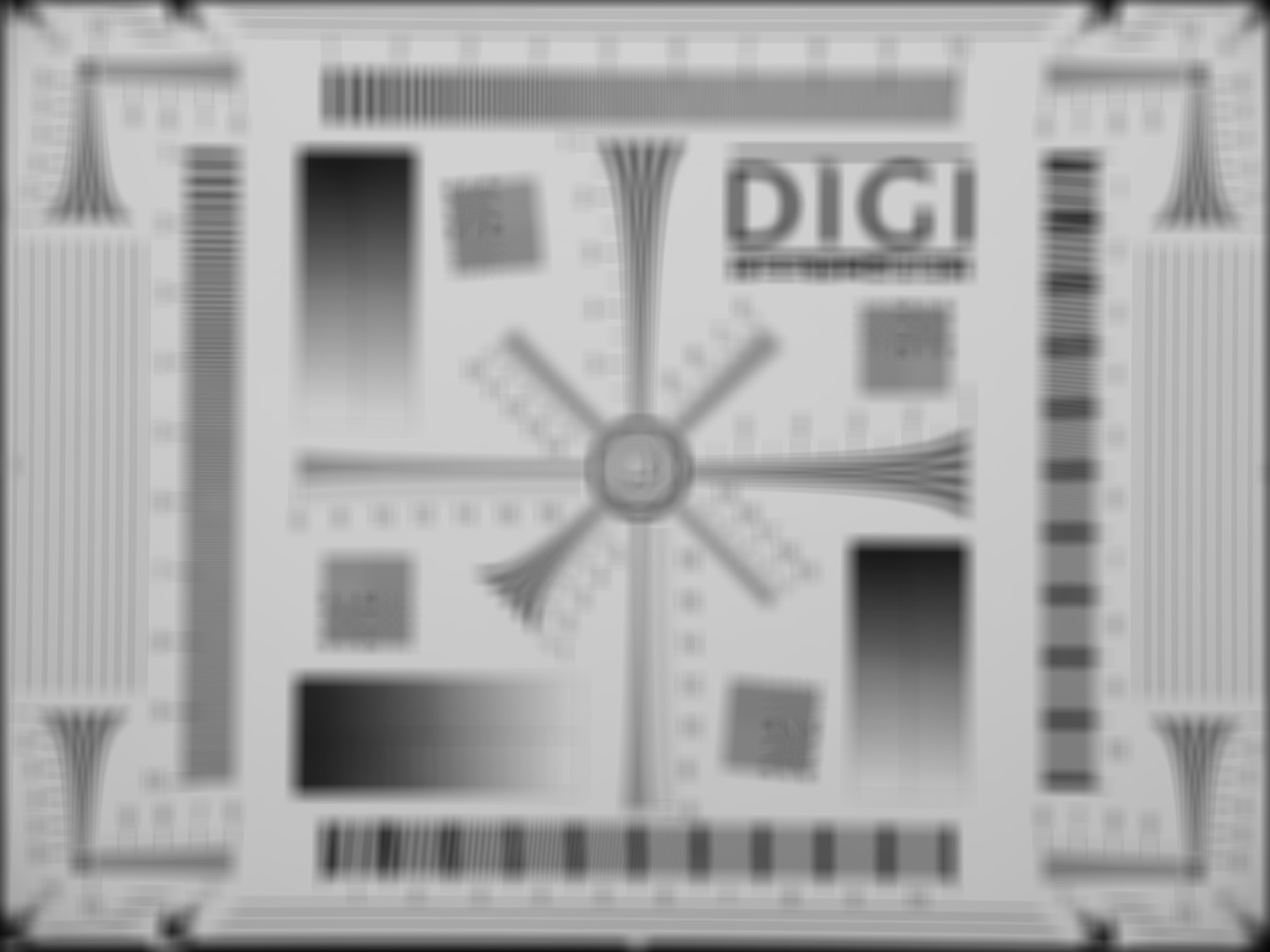
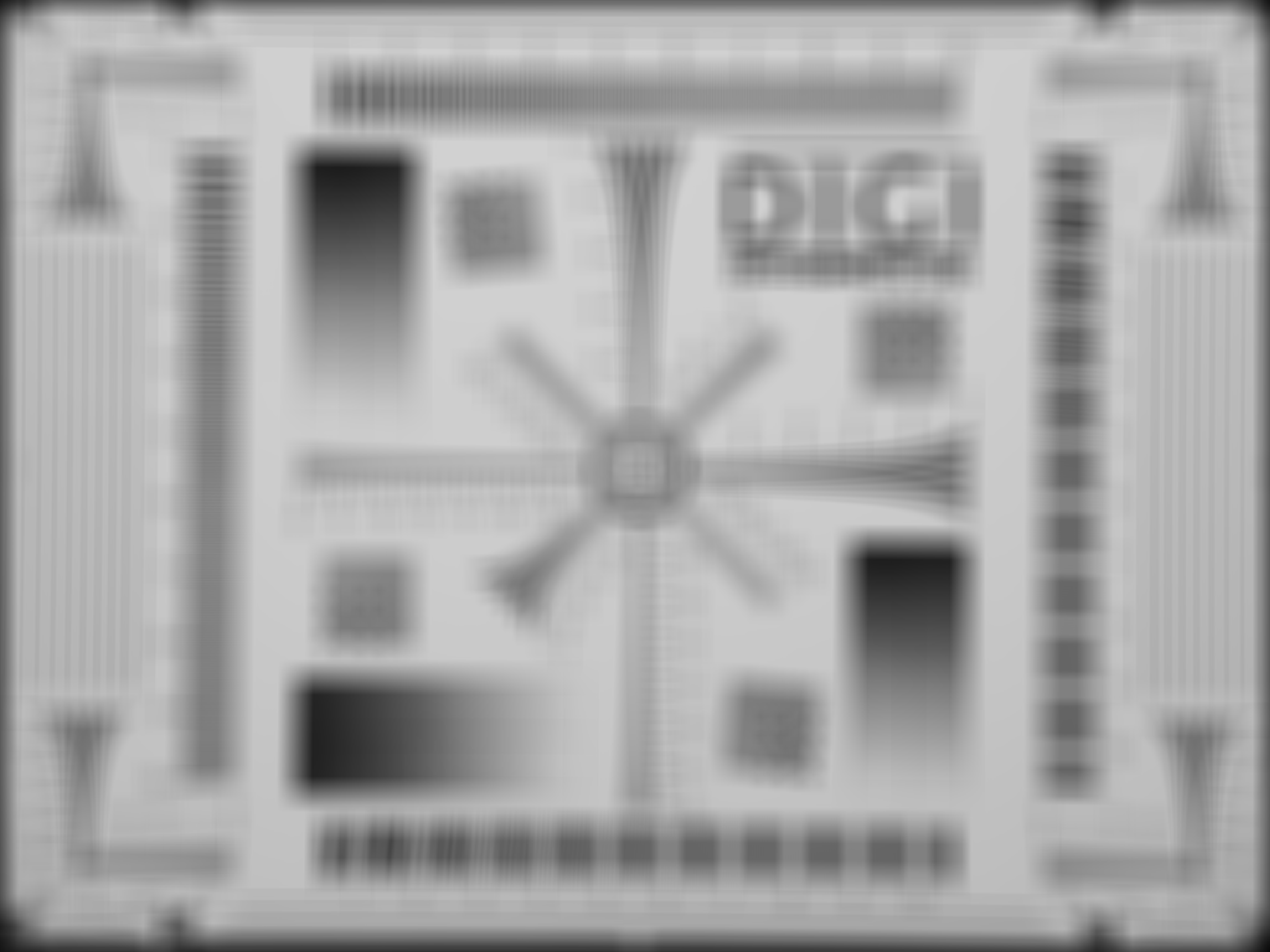
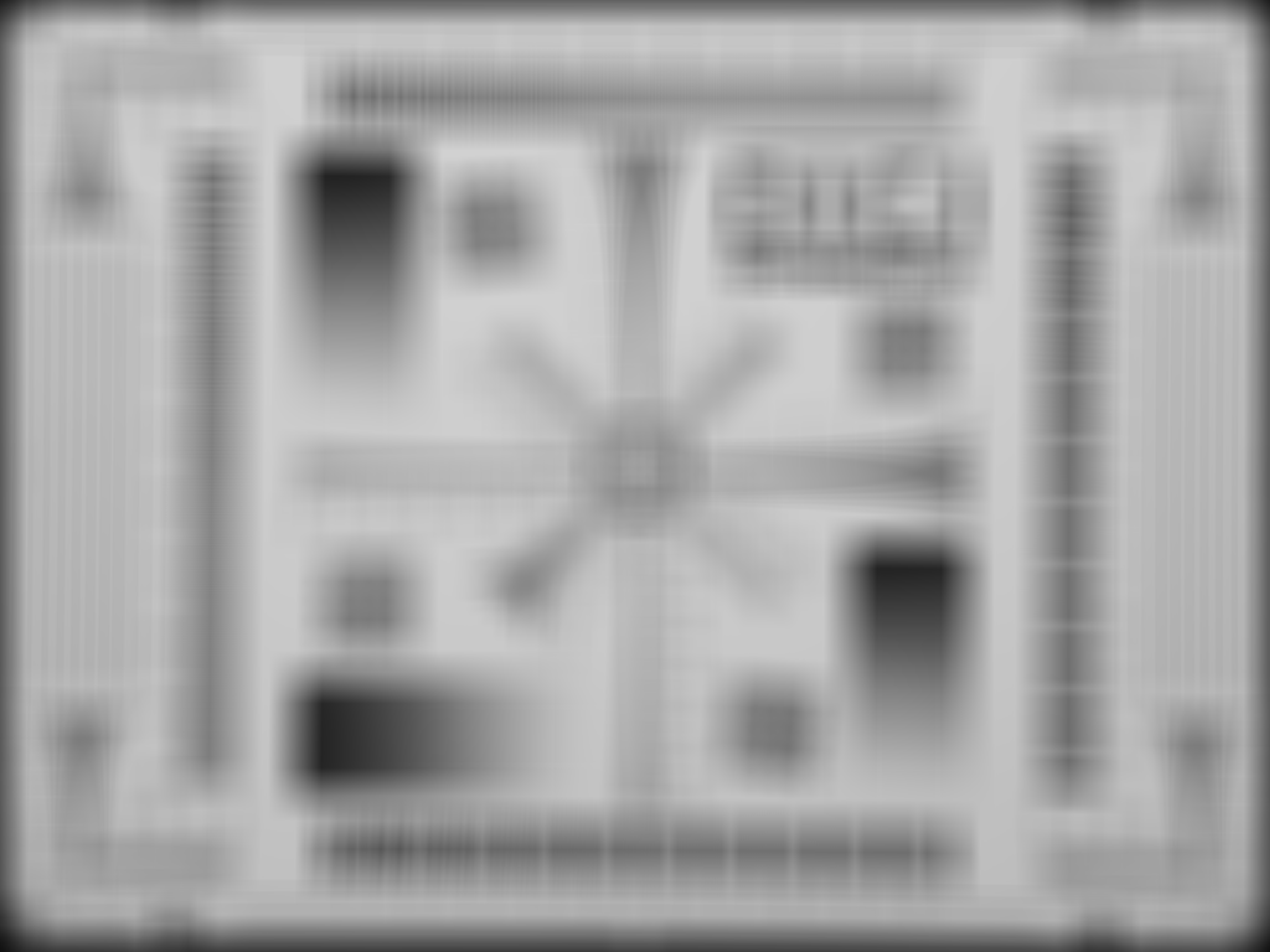

DIGI
TESTCHART

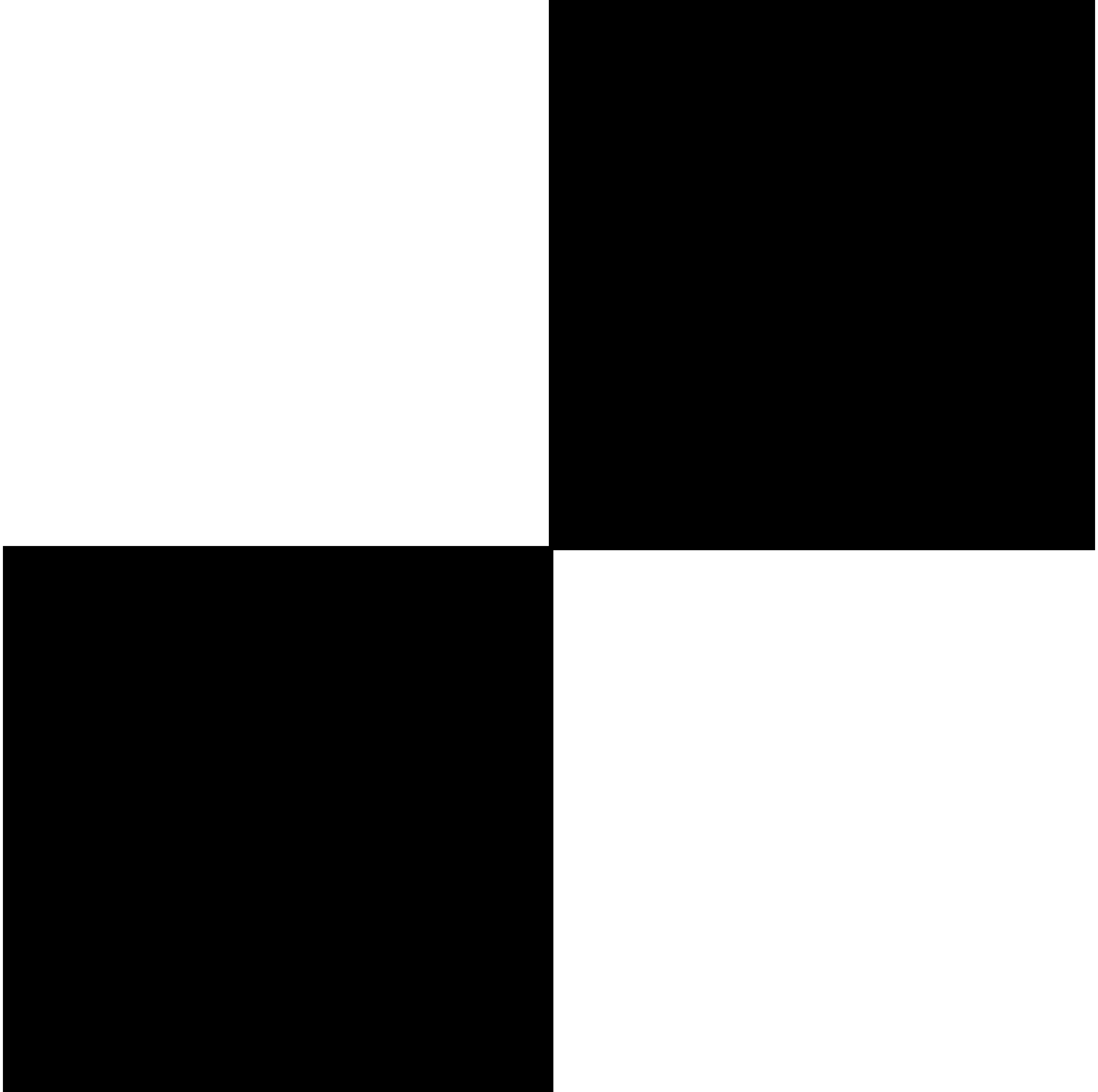

DIGI

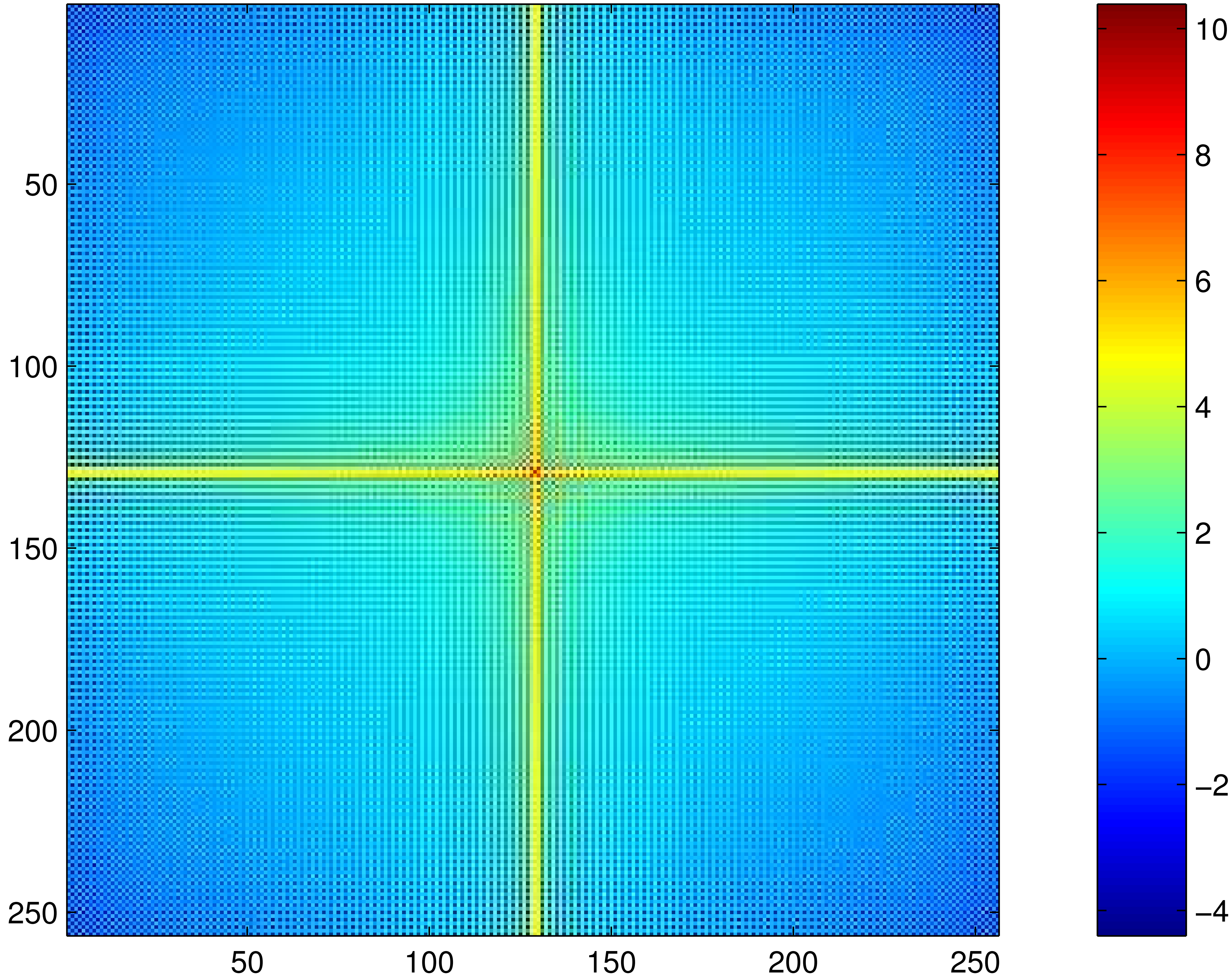

DIGI


DIGI


DIGI



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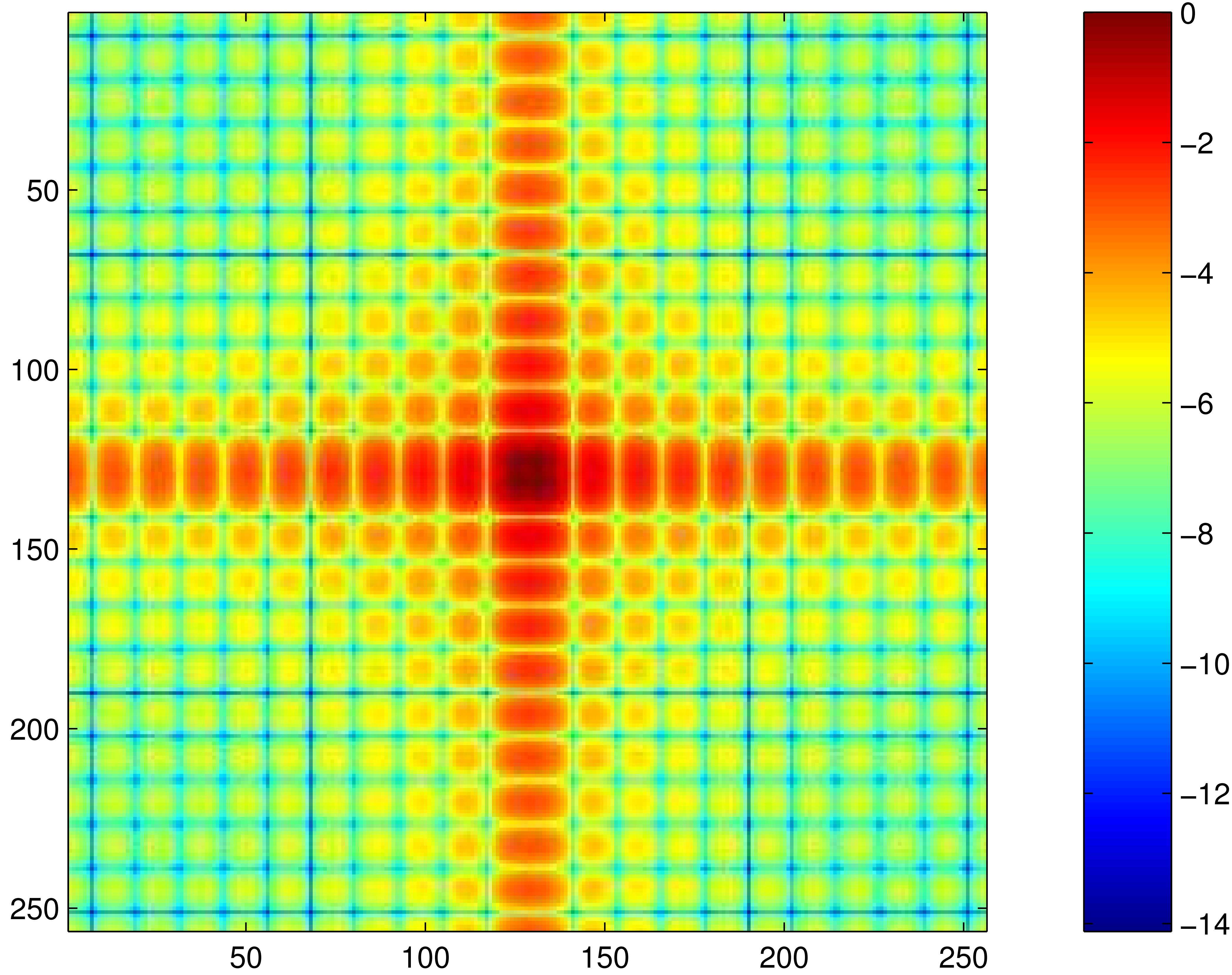

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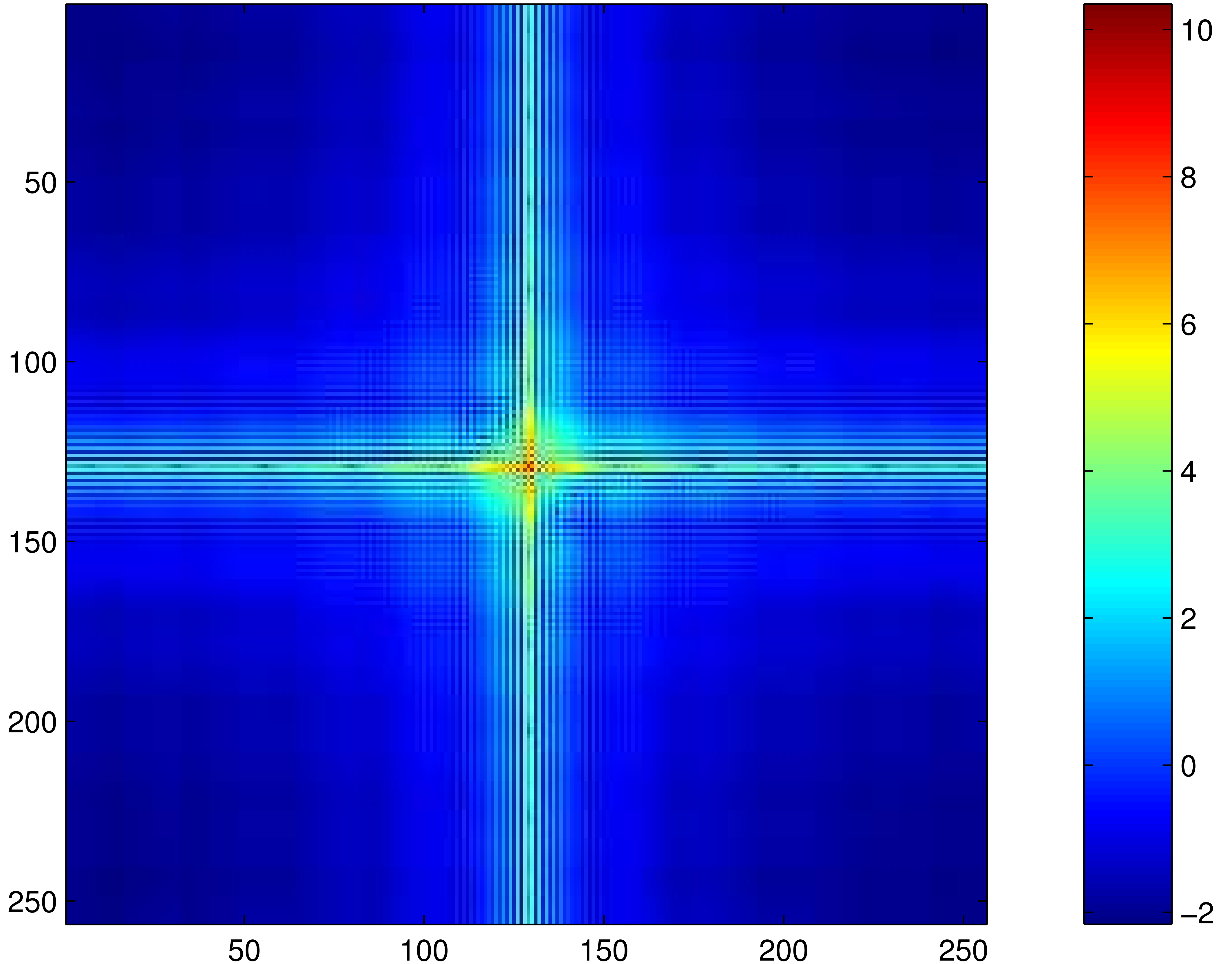

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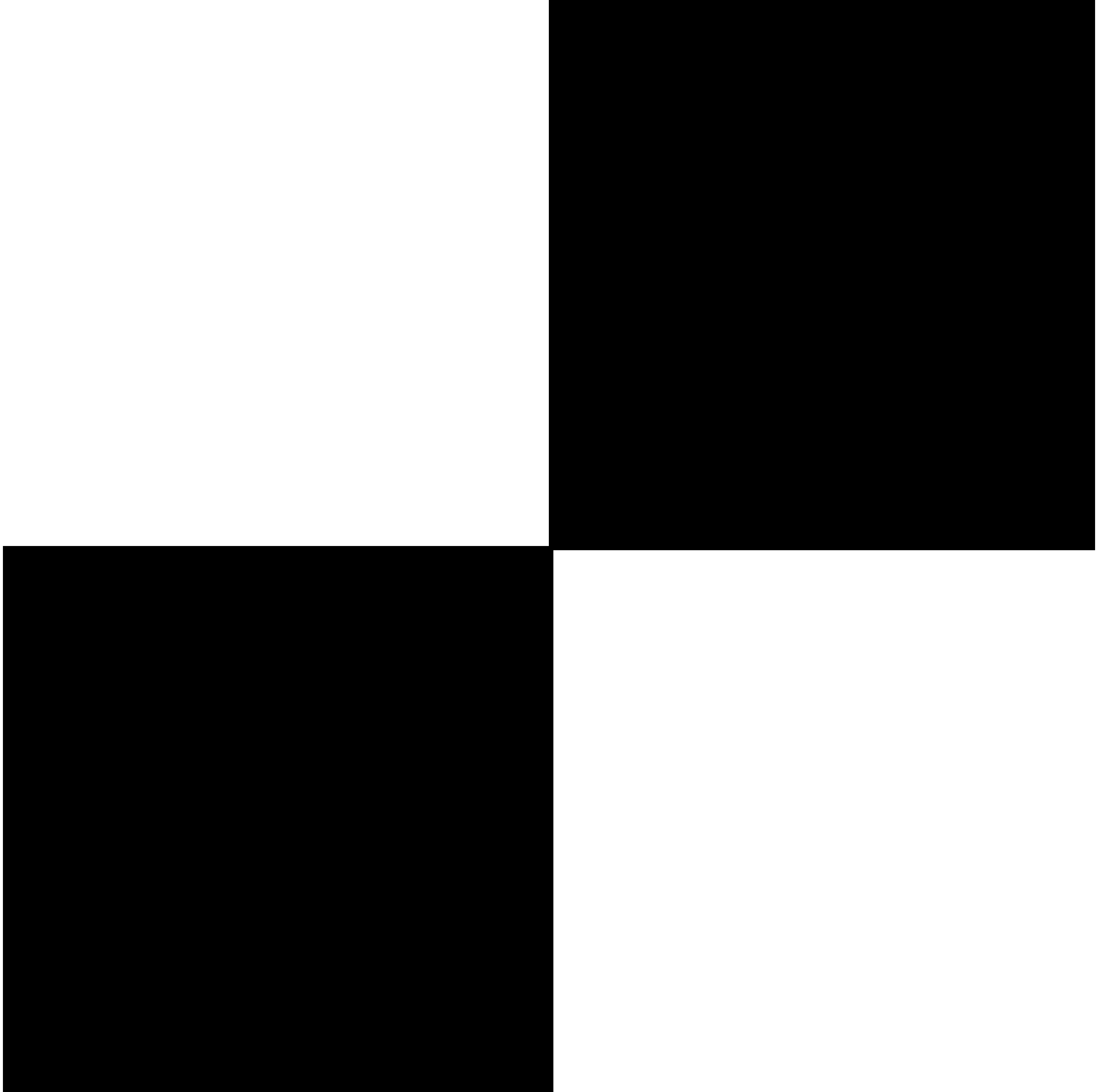

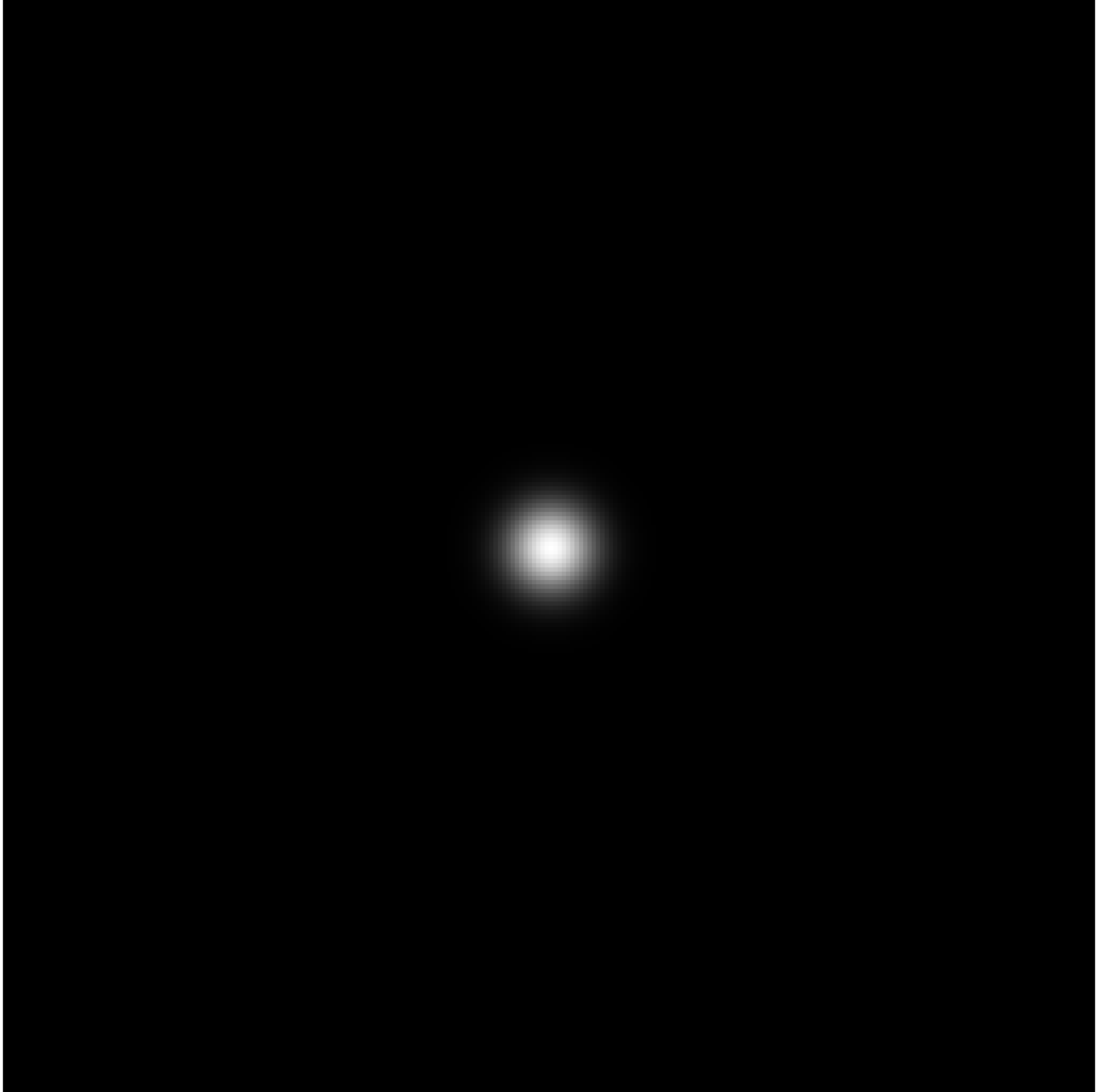

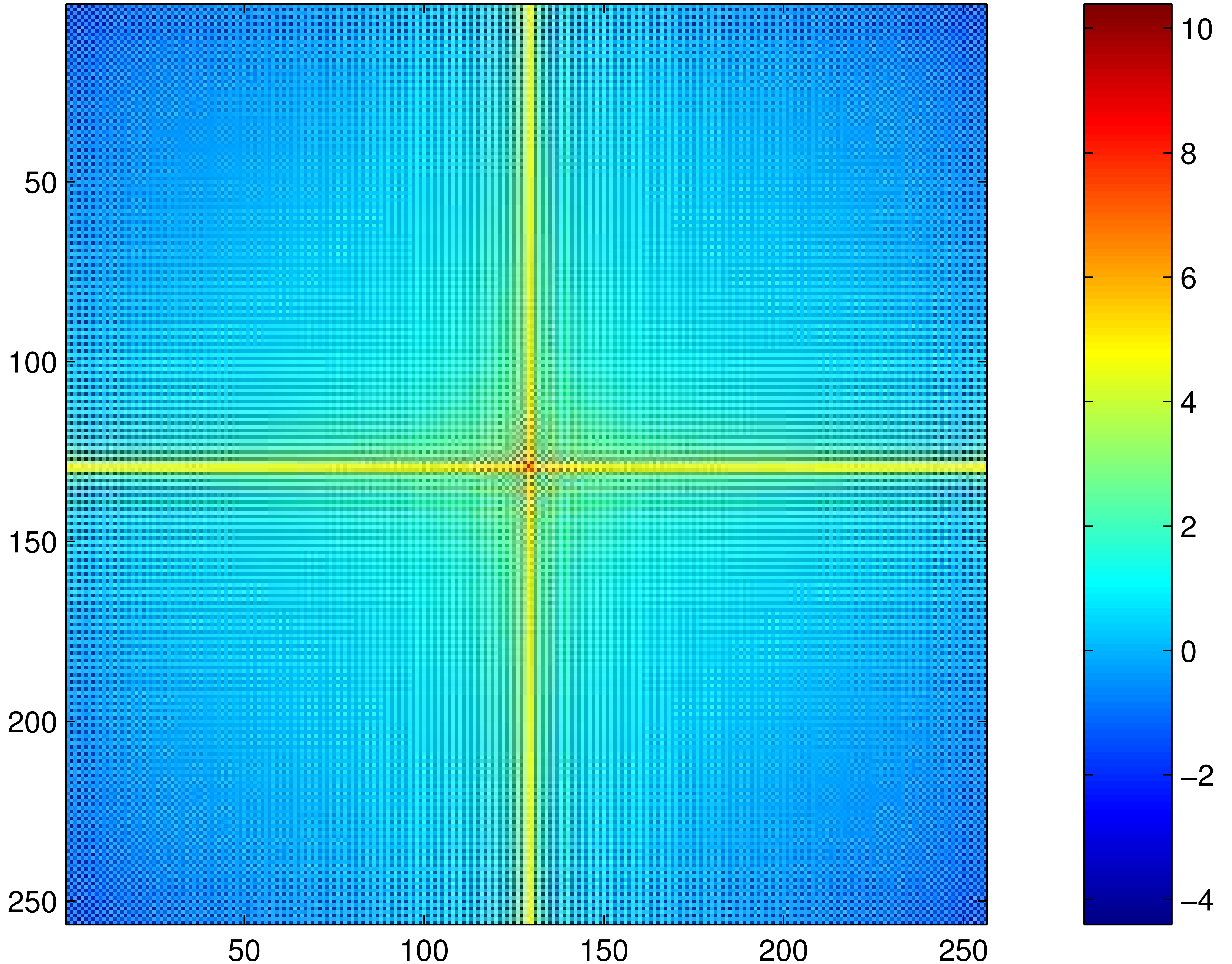

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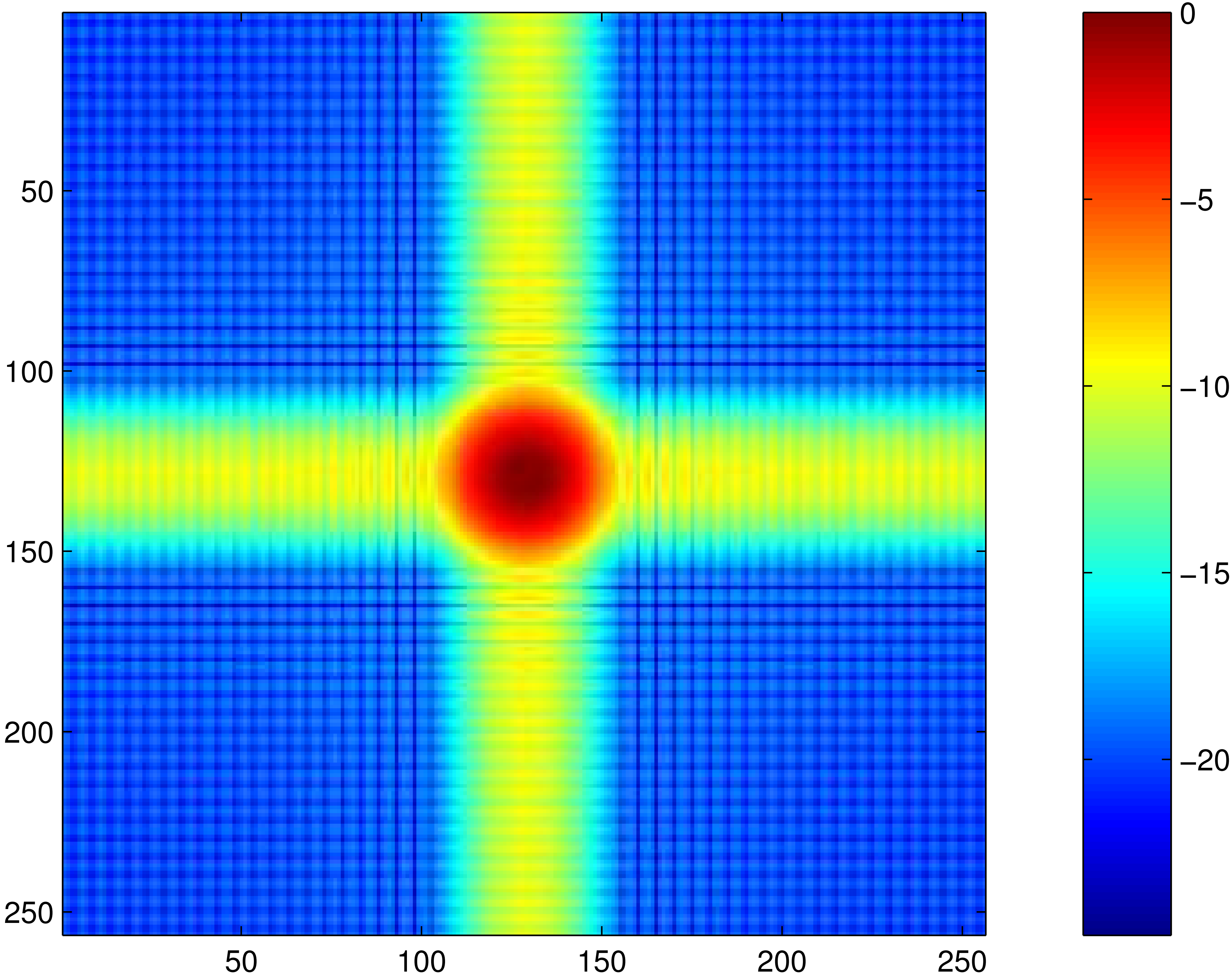
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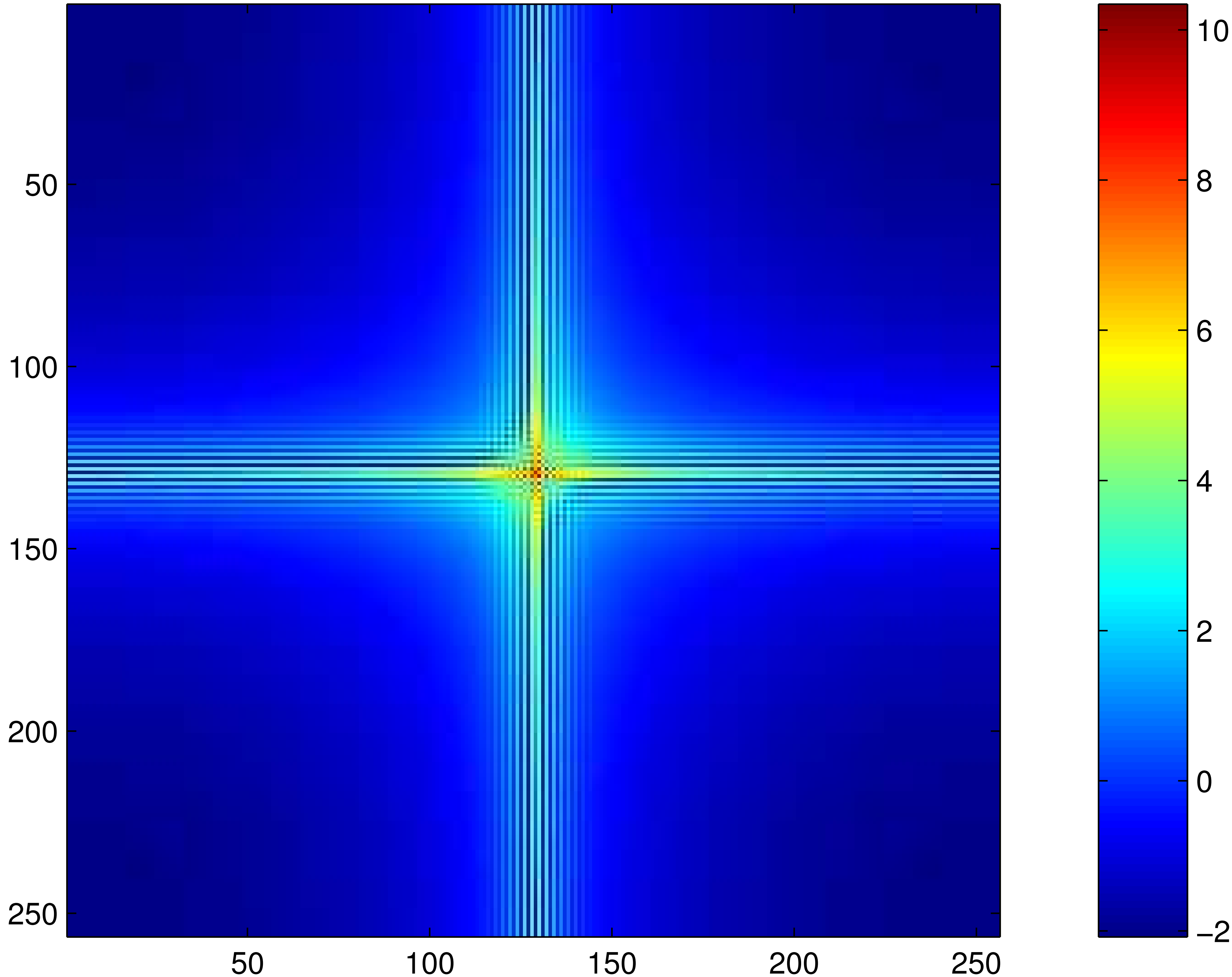

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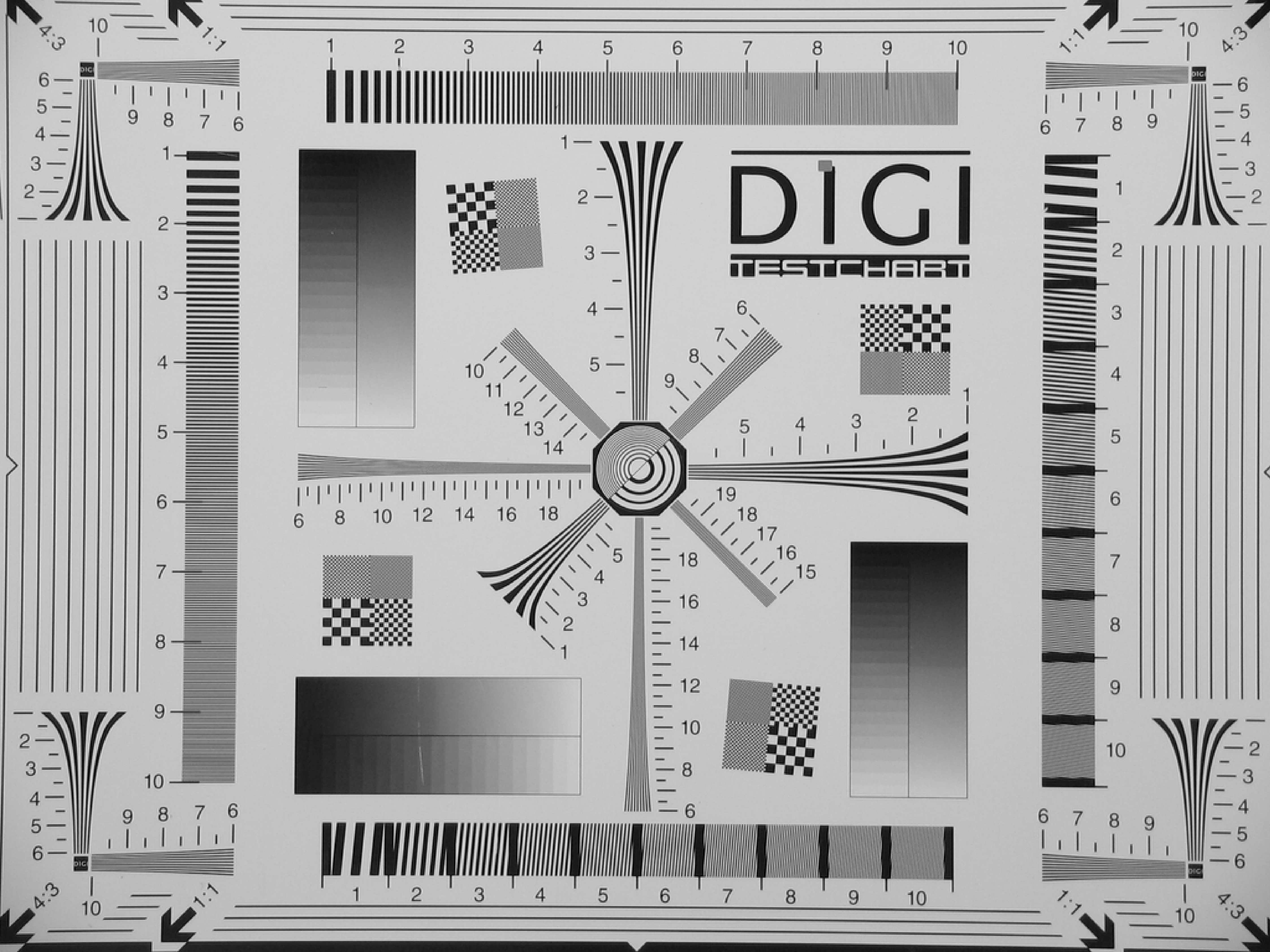
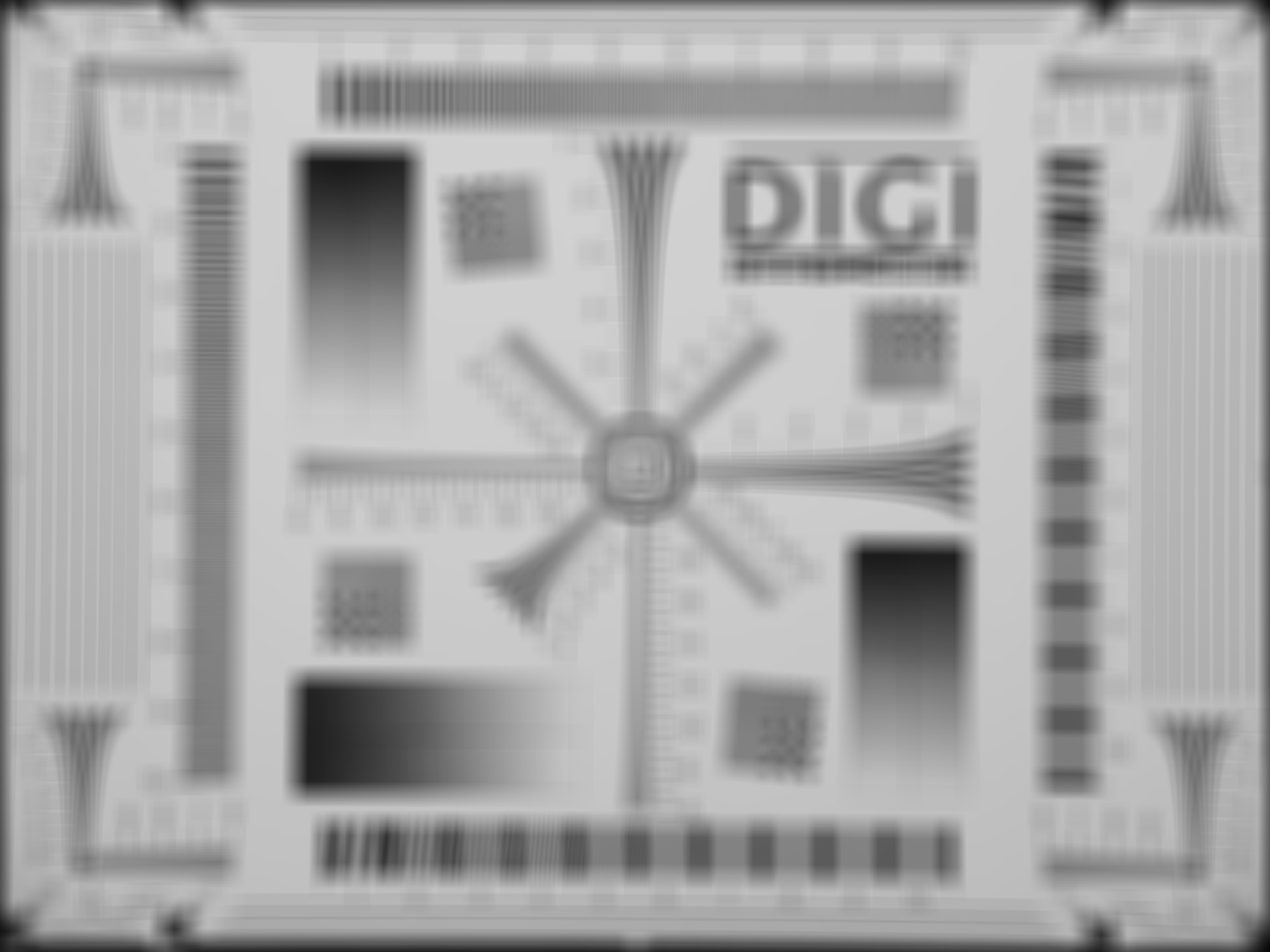
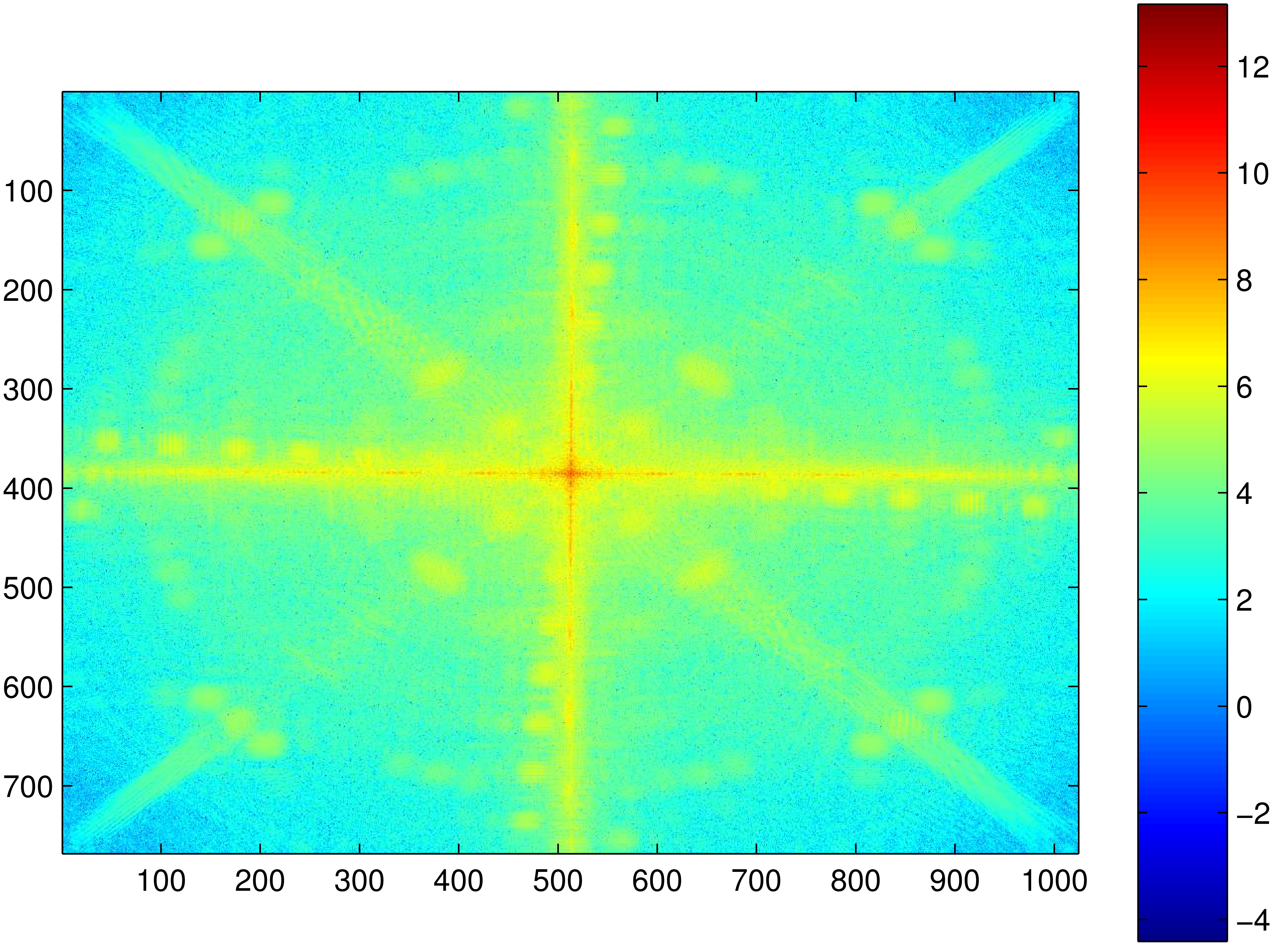


DIGI
TESTCHART


DIGI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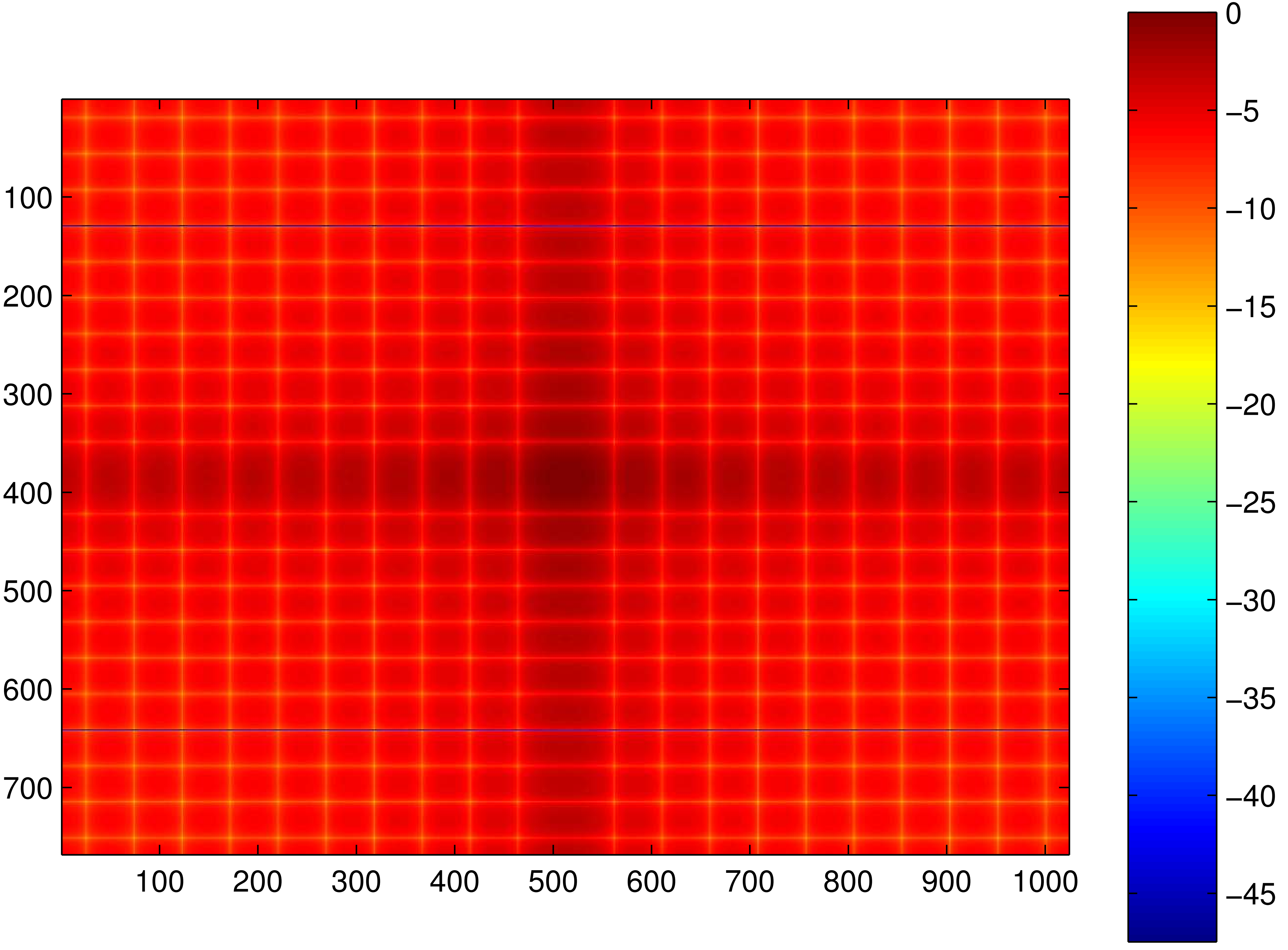

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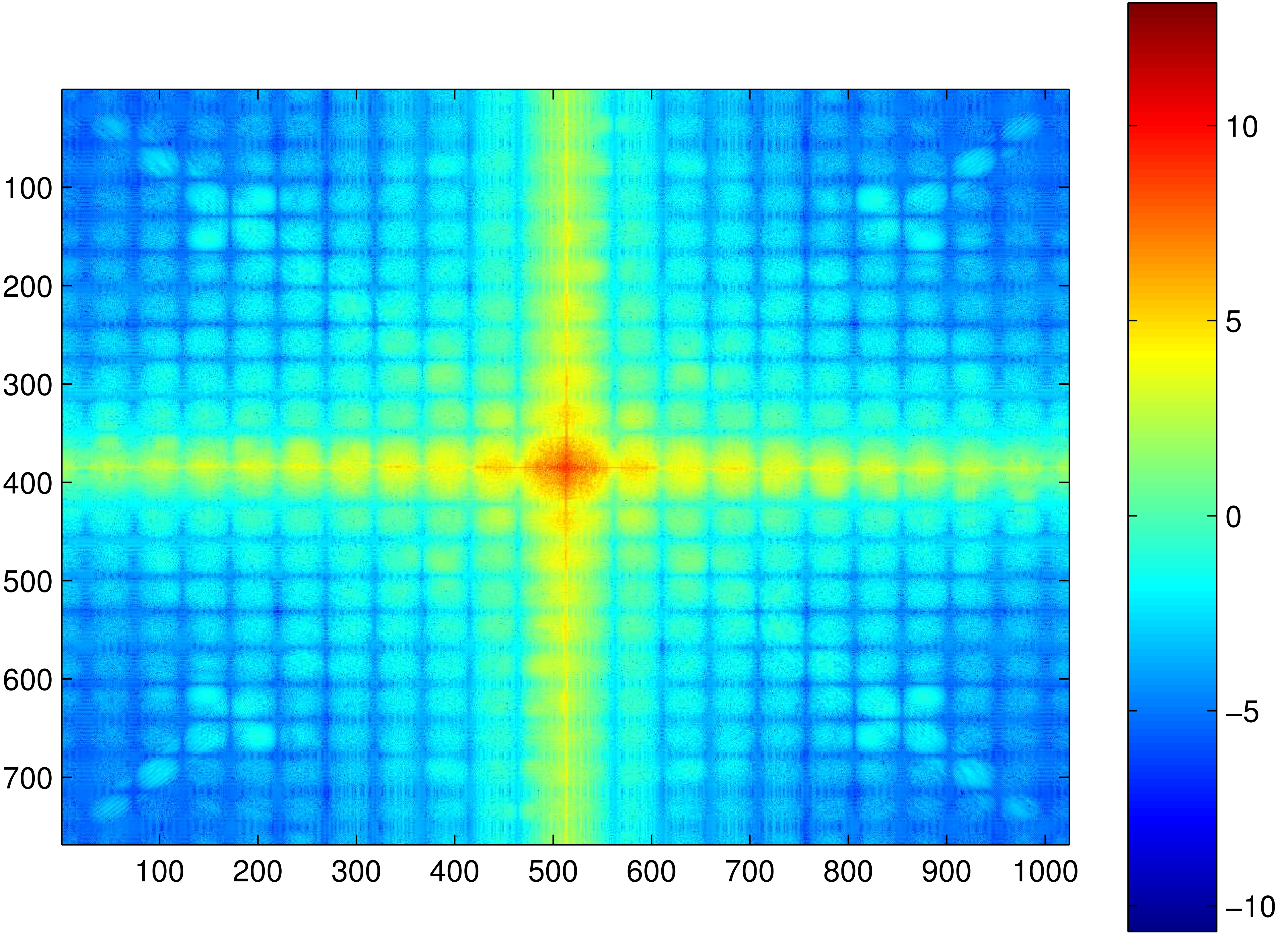

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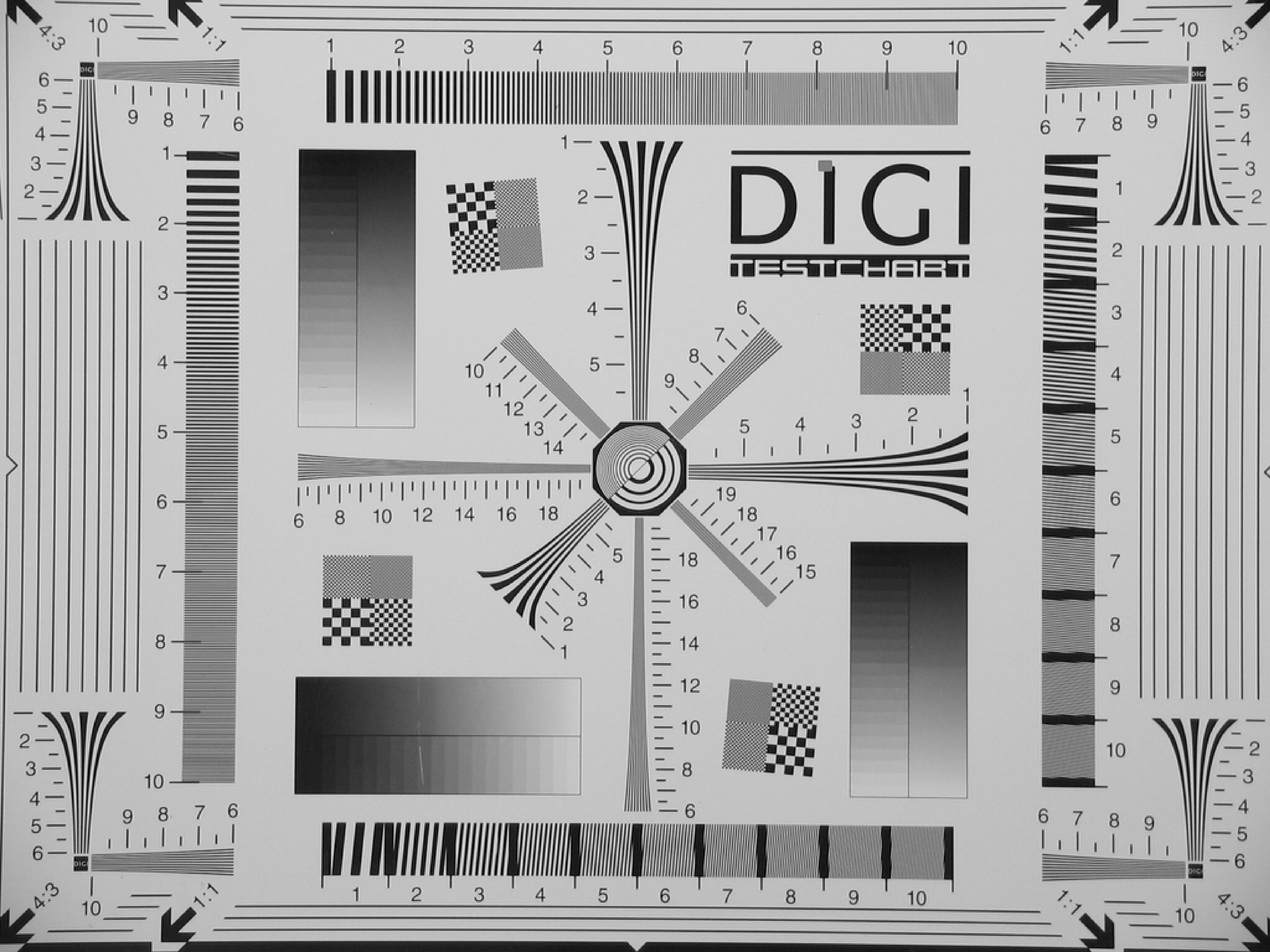
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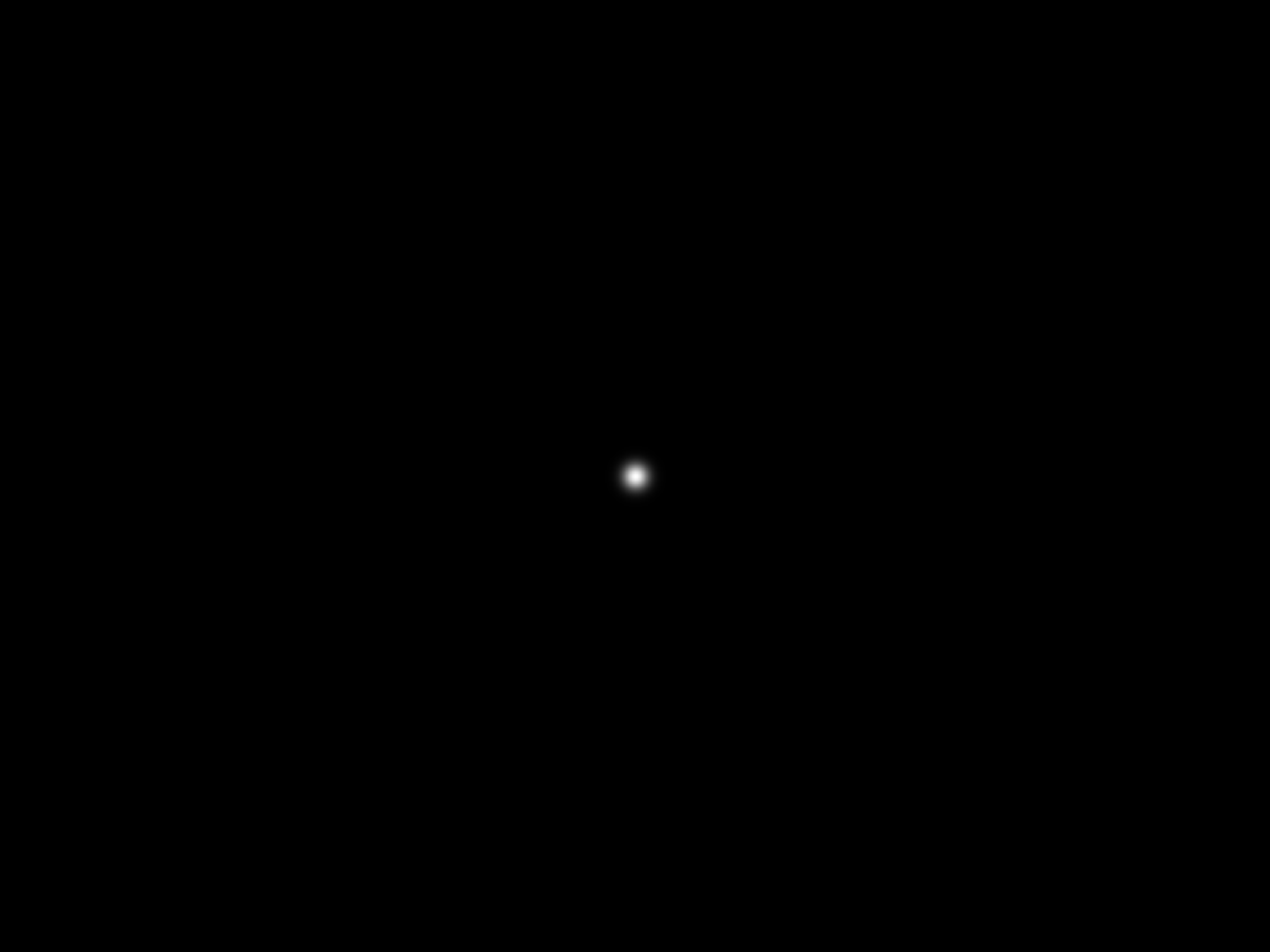
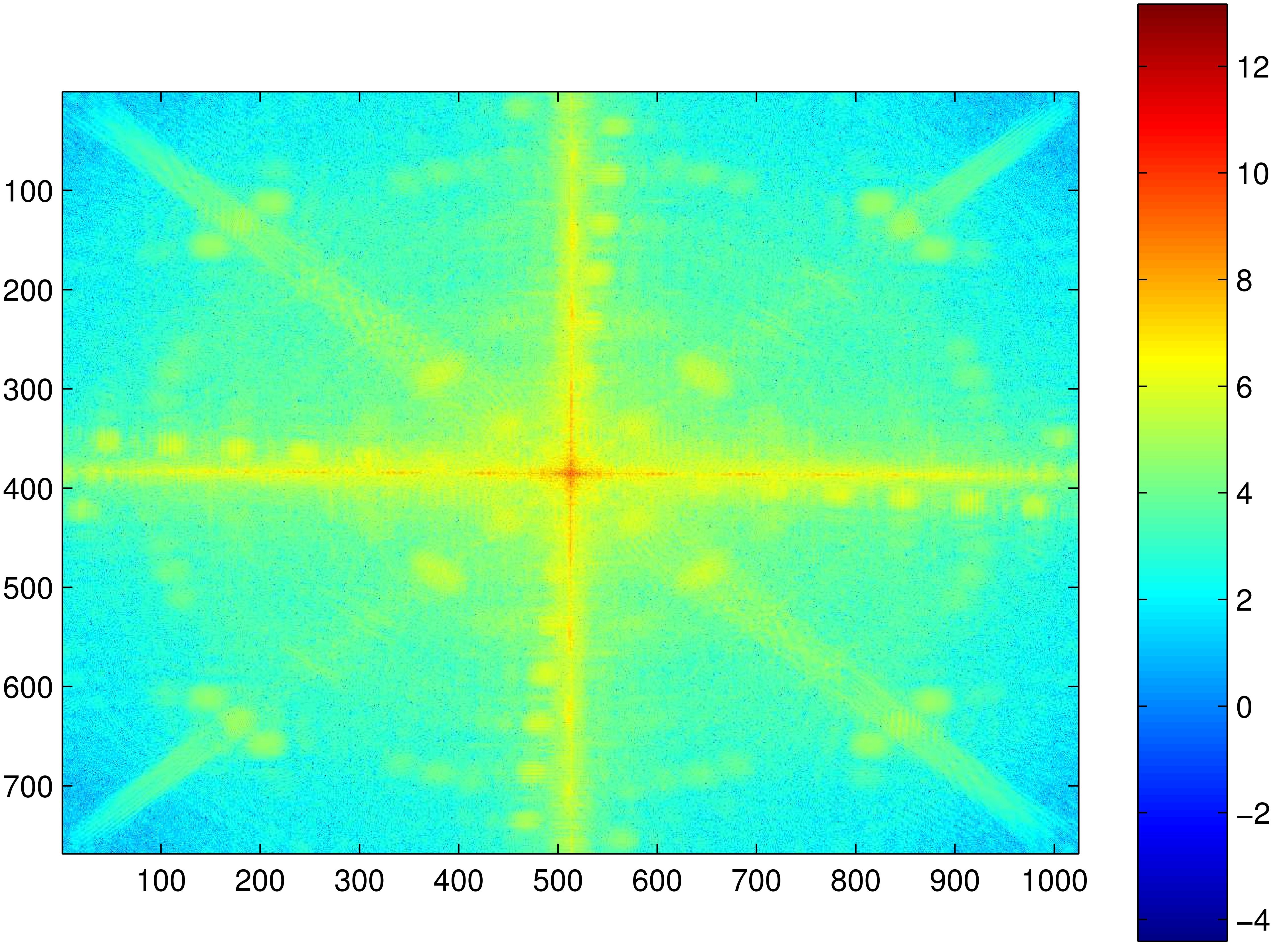
TESTCHART




DIGI



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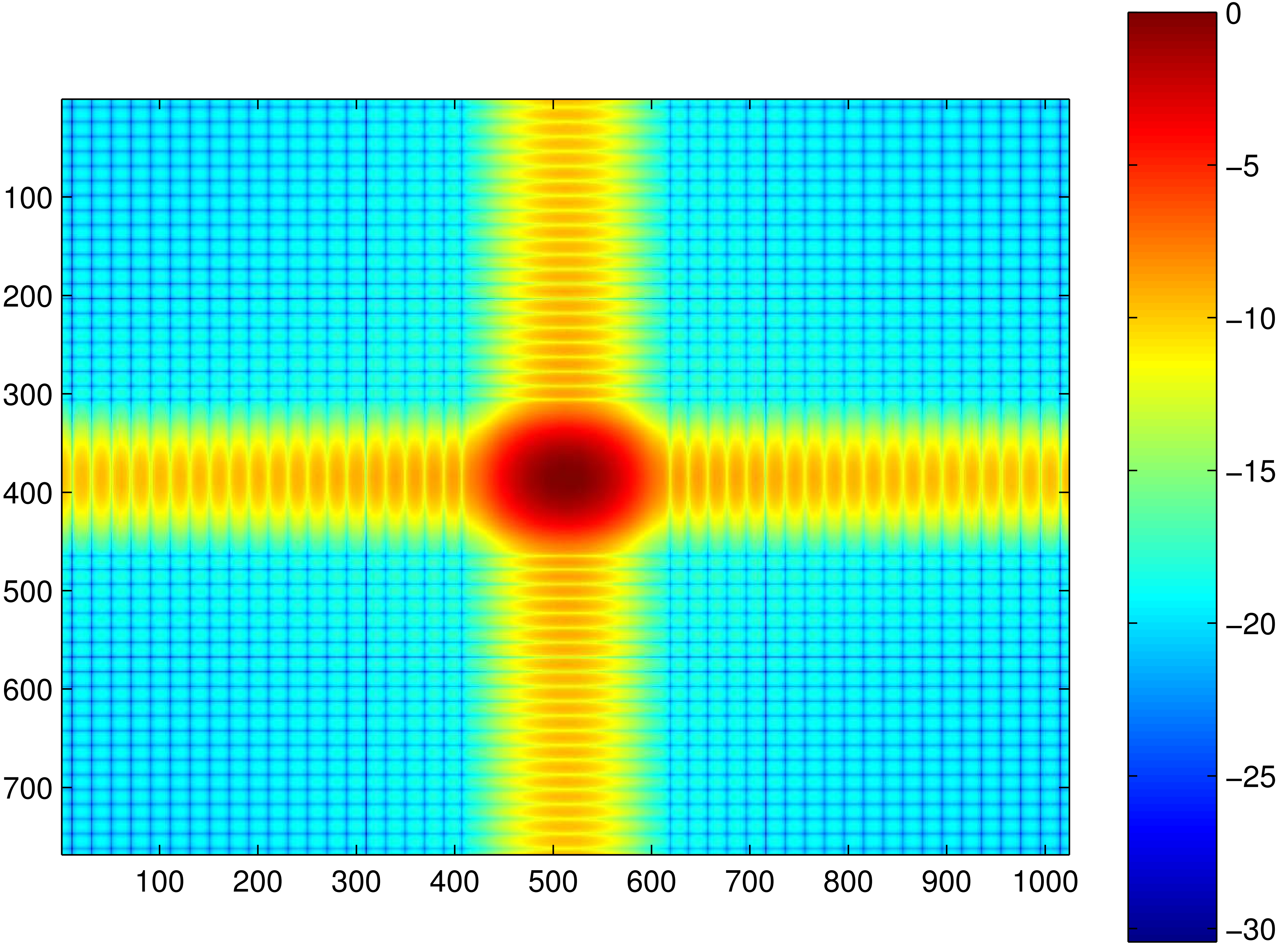

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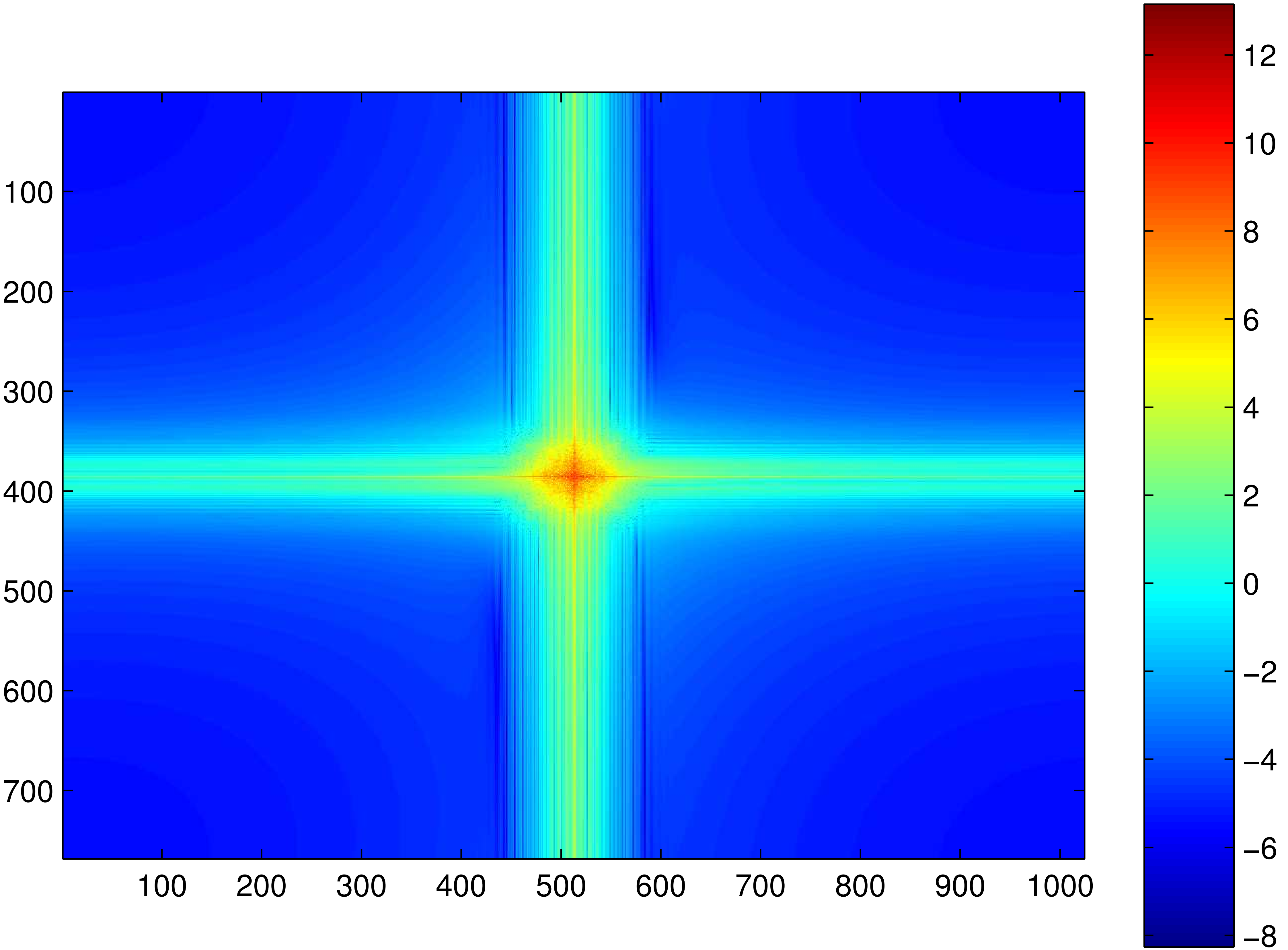

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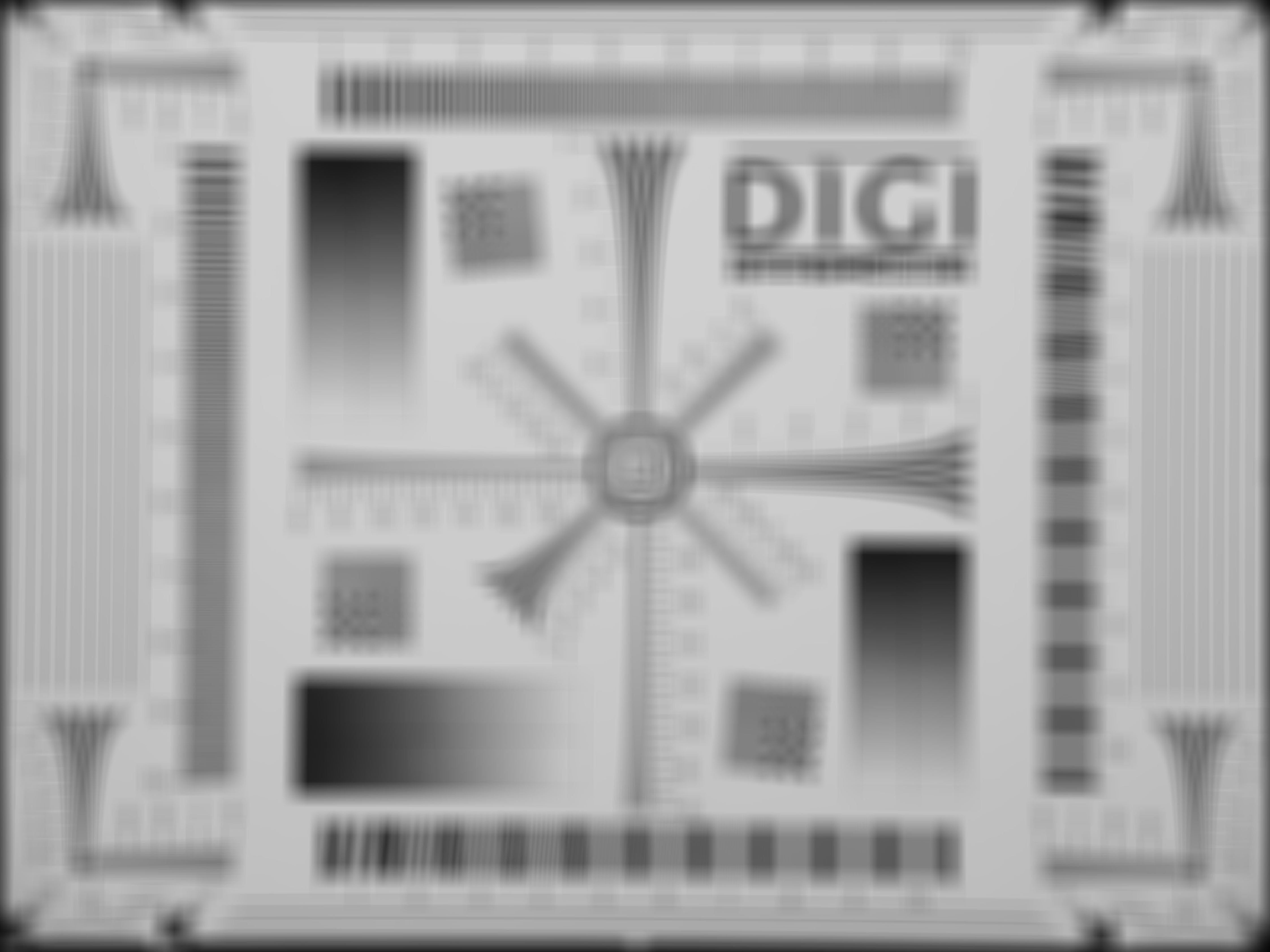
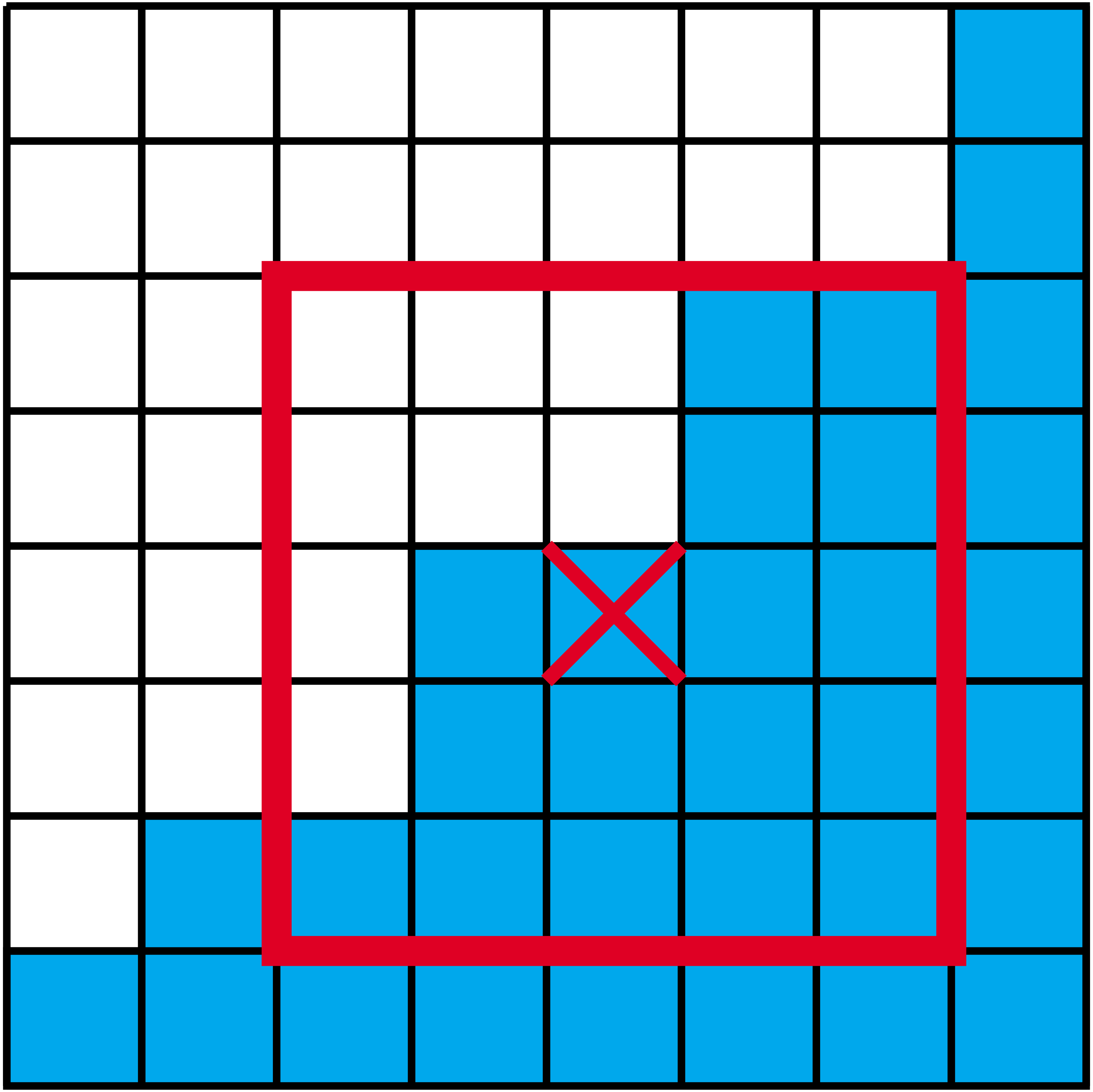
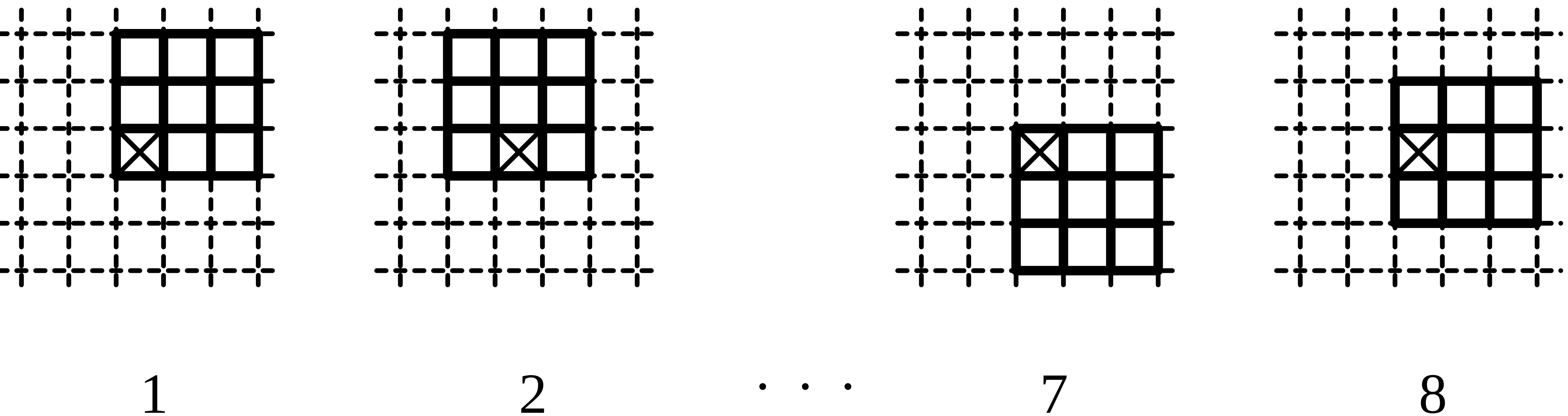
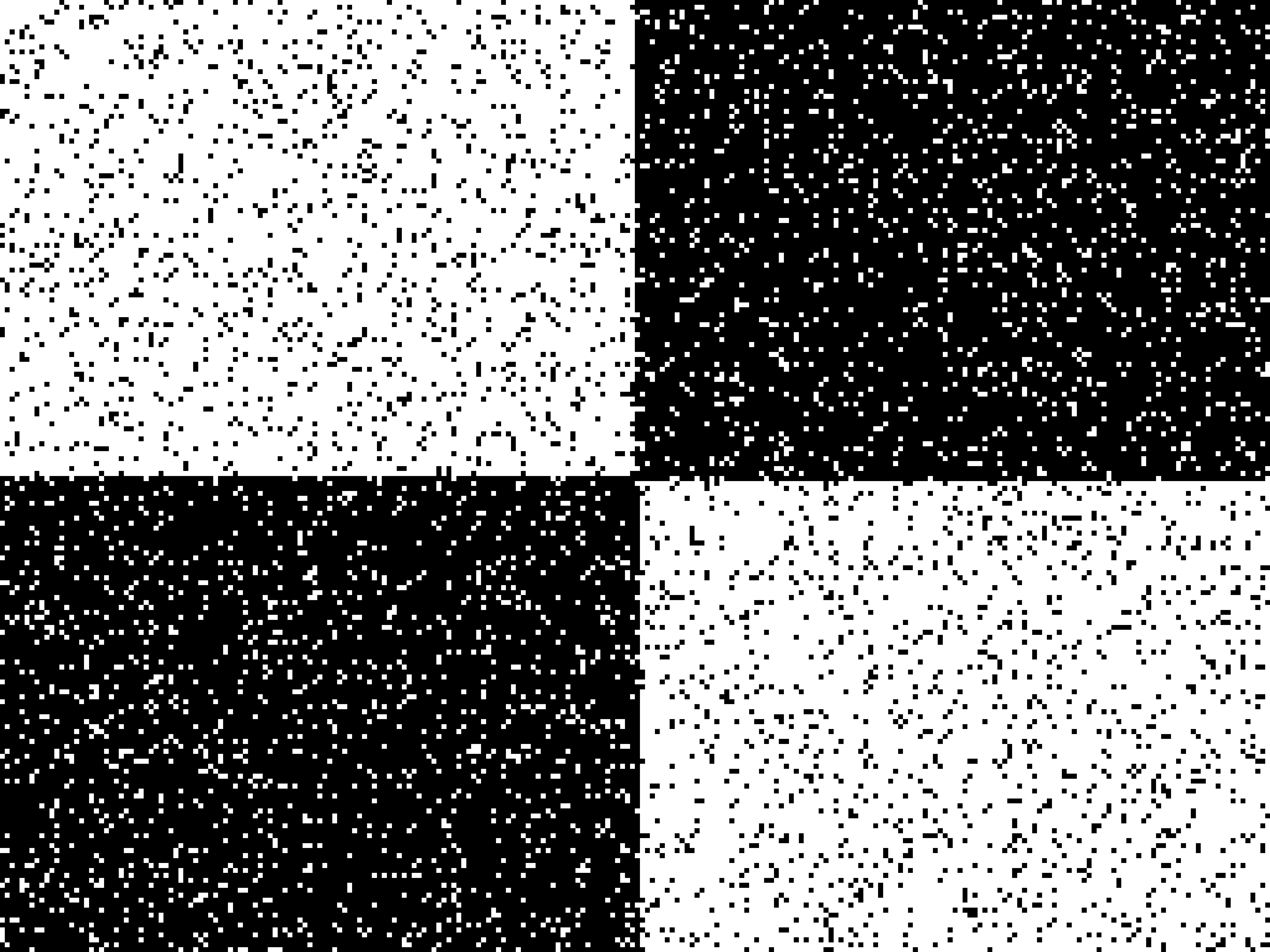
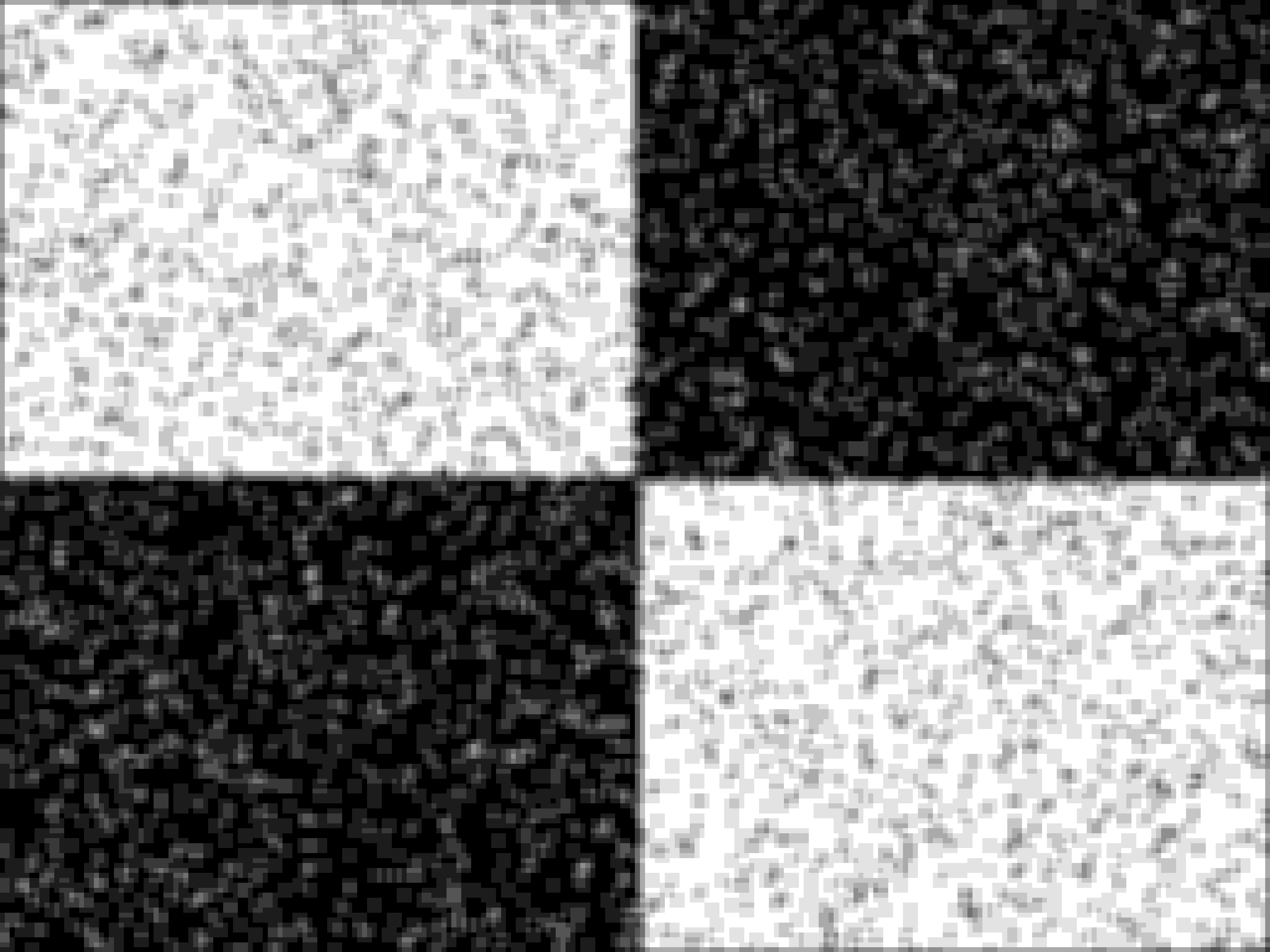

DIGI

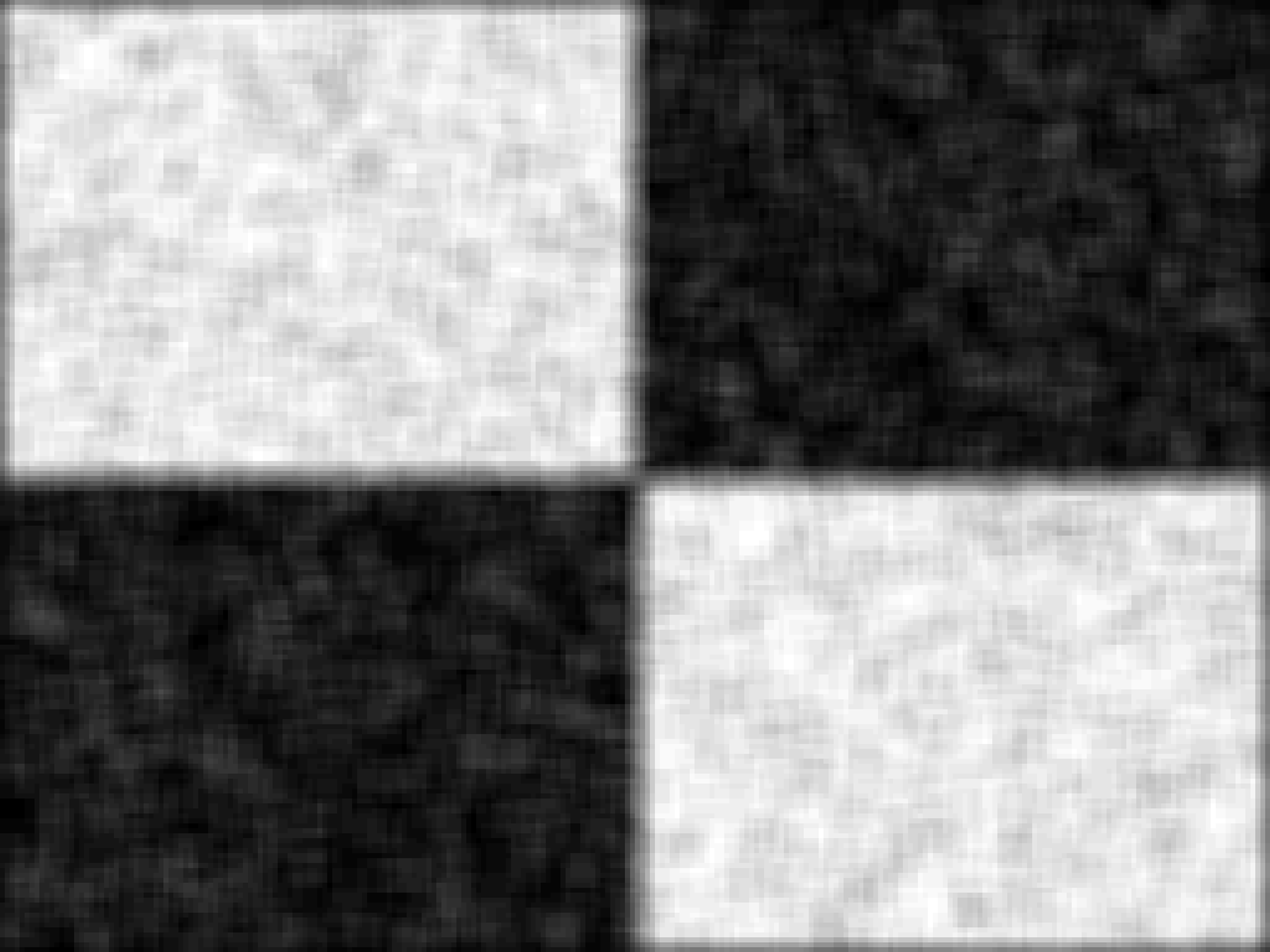

DIGI

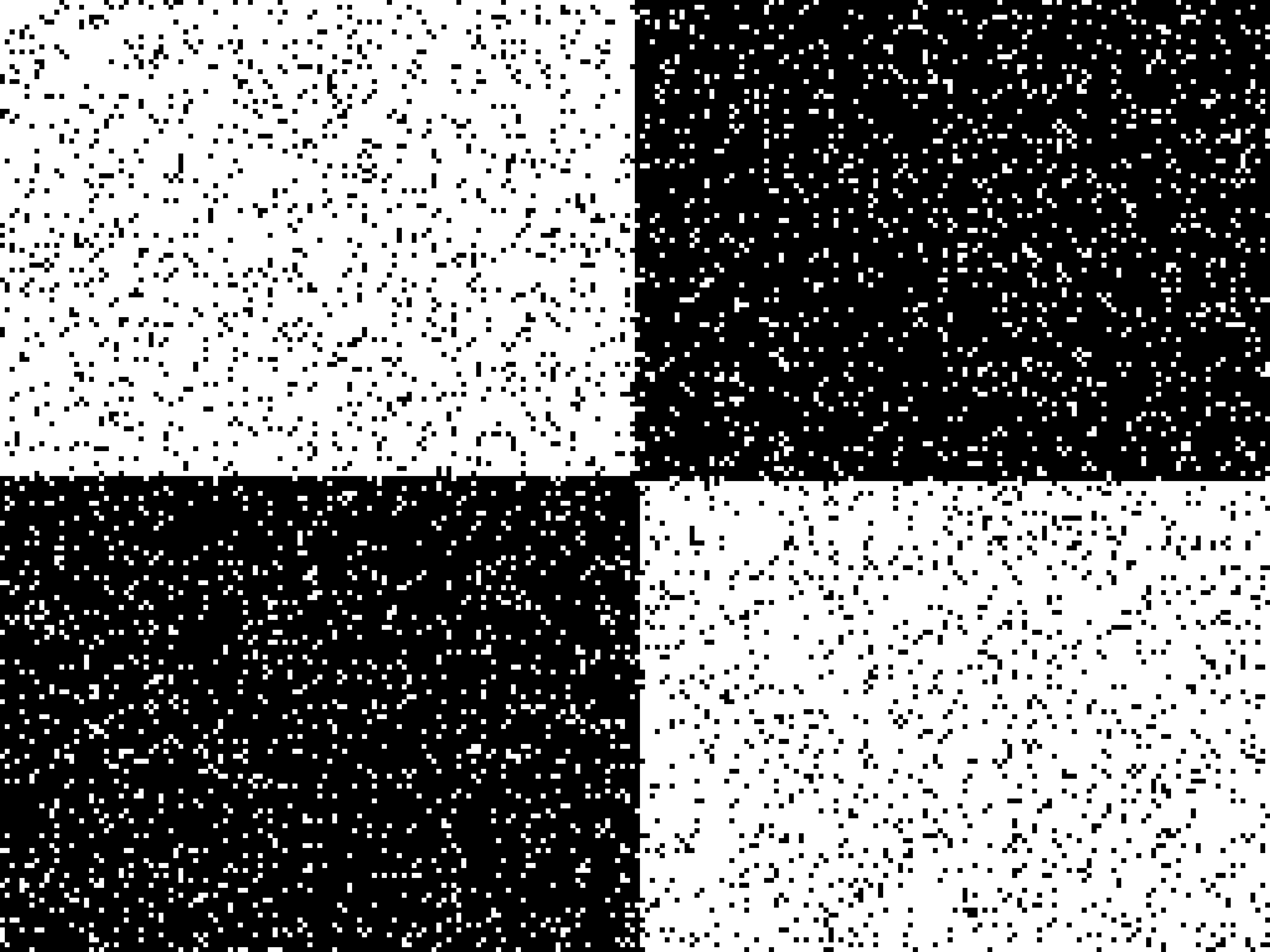


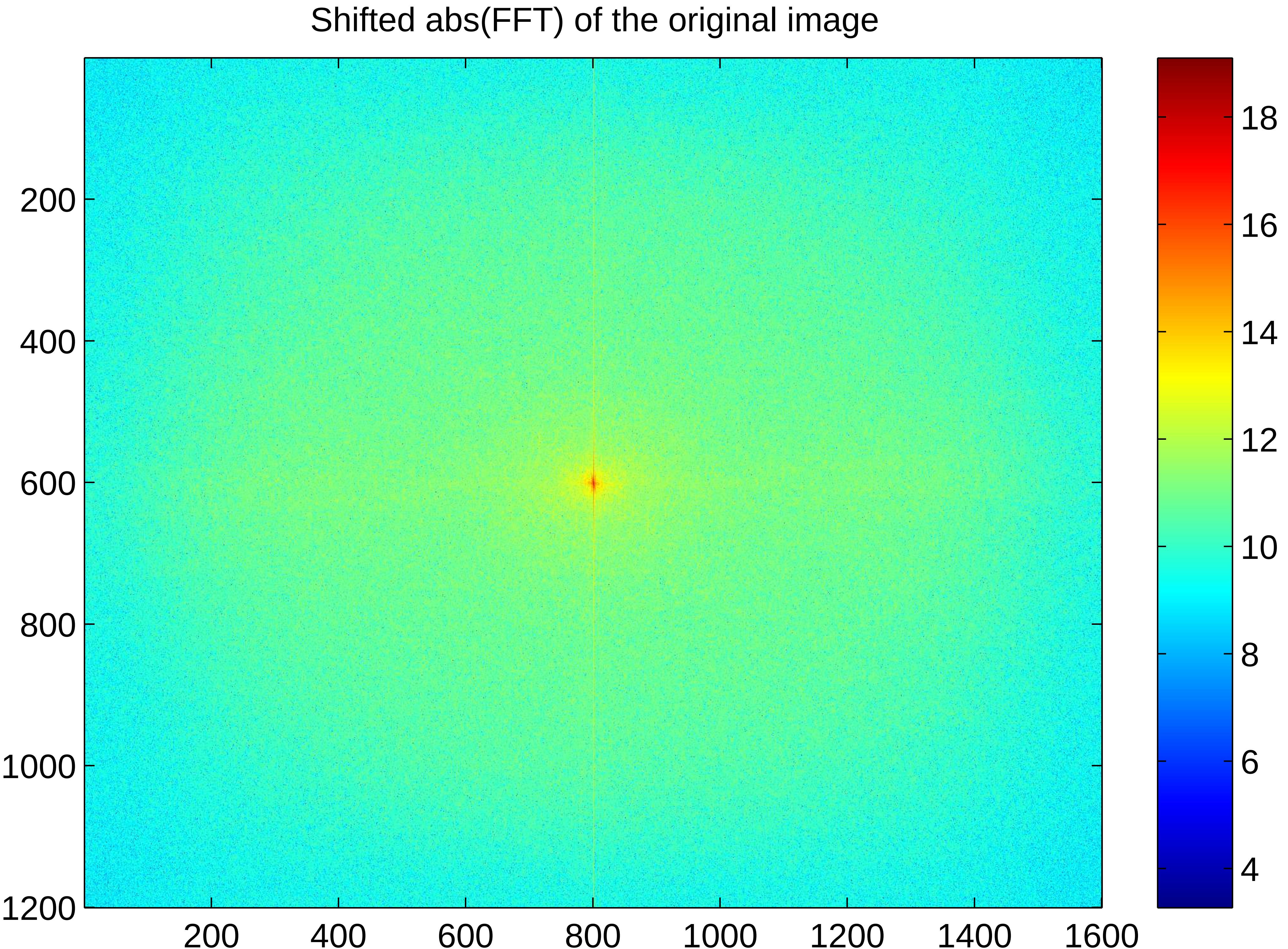

1
2
. . .
7
8










Shifted abs(FFT) of the original image
200
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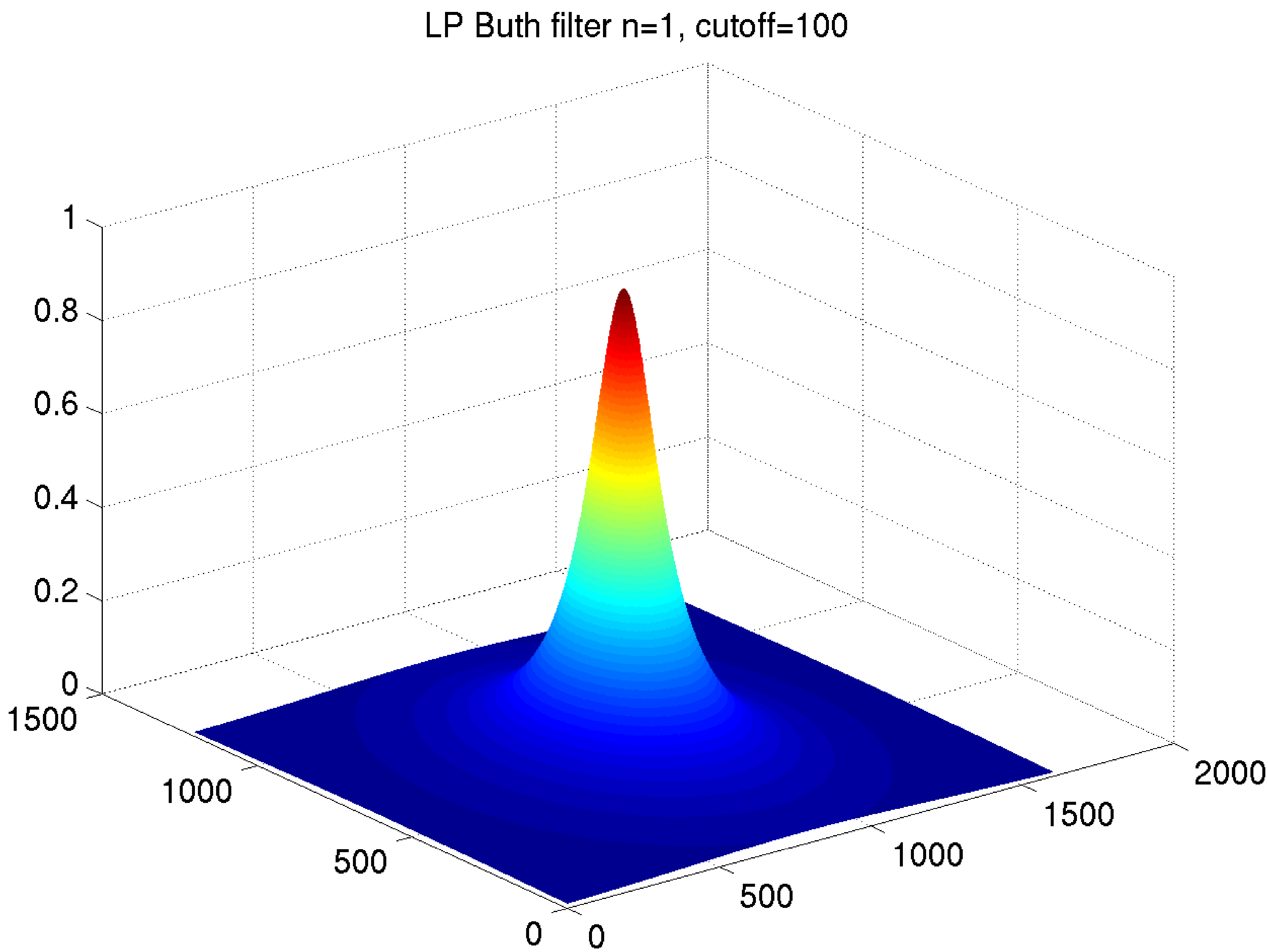
LP Buth filter n=1, cutoff=100
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0.6
0.4
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500
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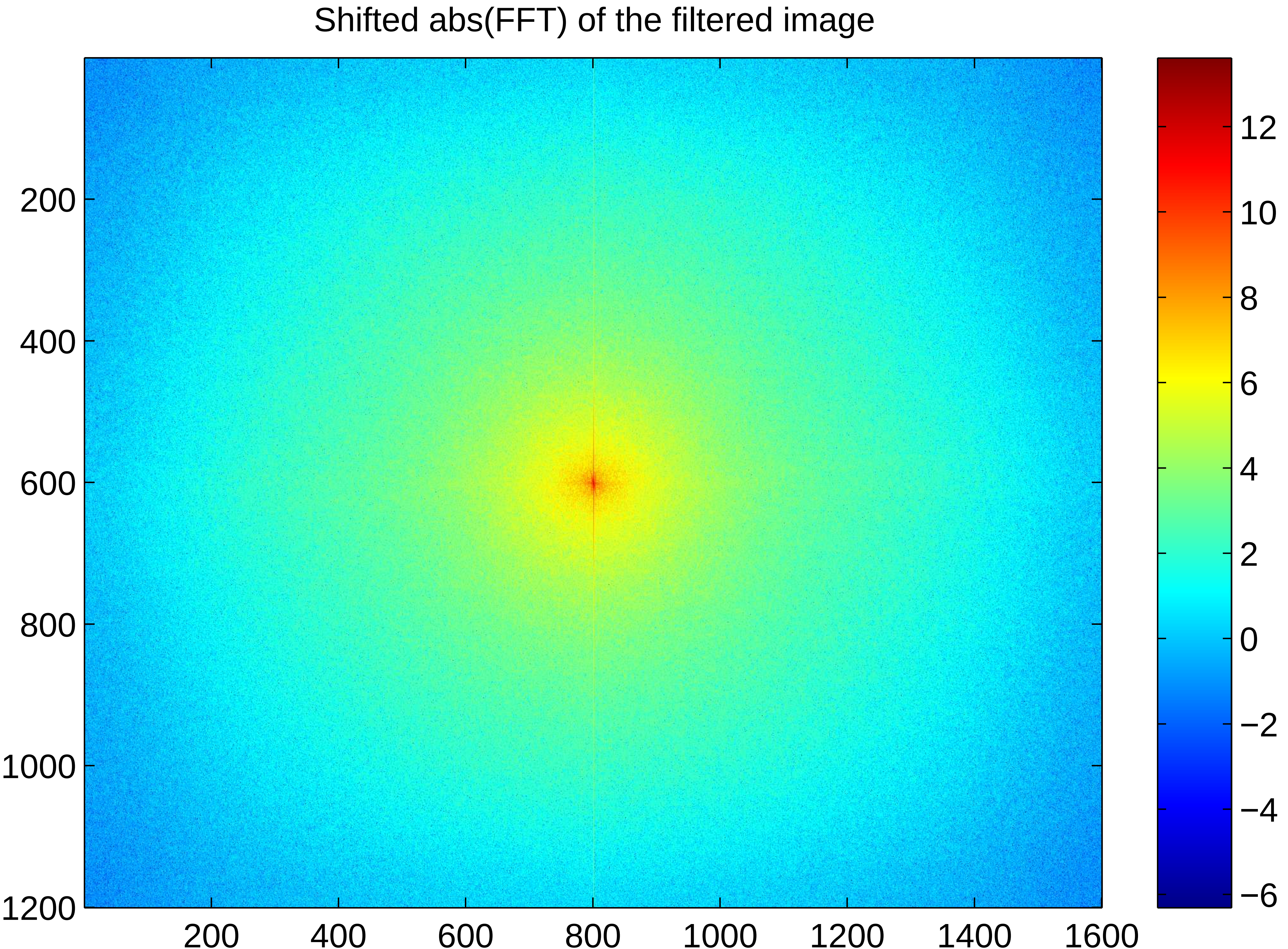
1500
2000



Shifted abs(FFT) of the filtered image
200
400
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8
6
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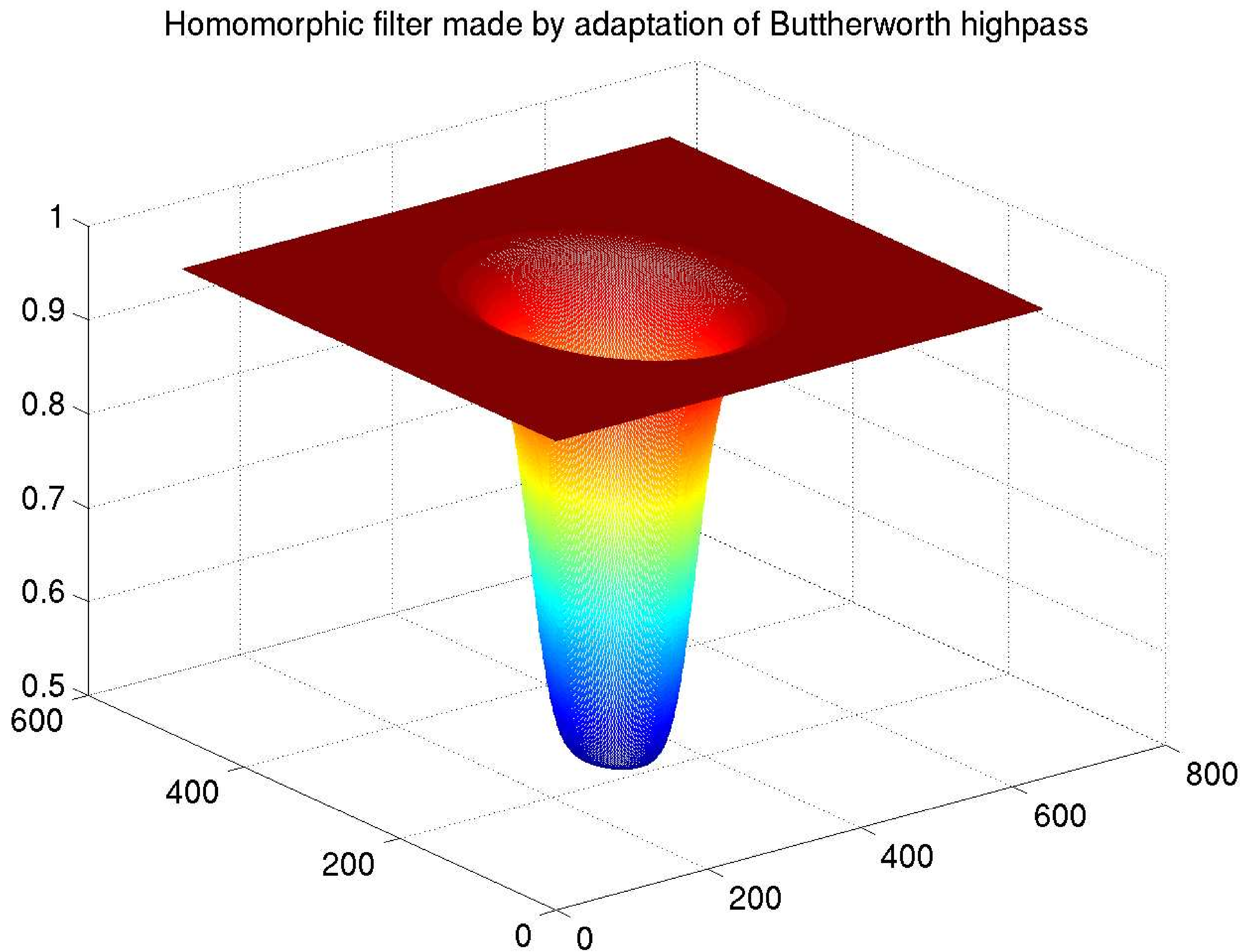
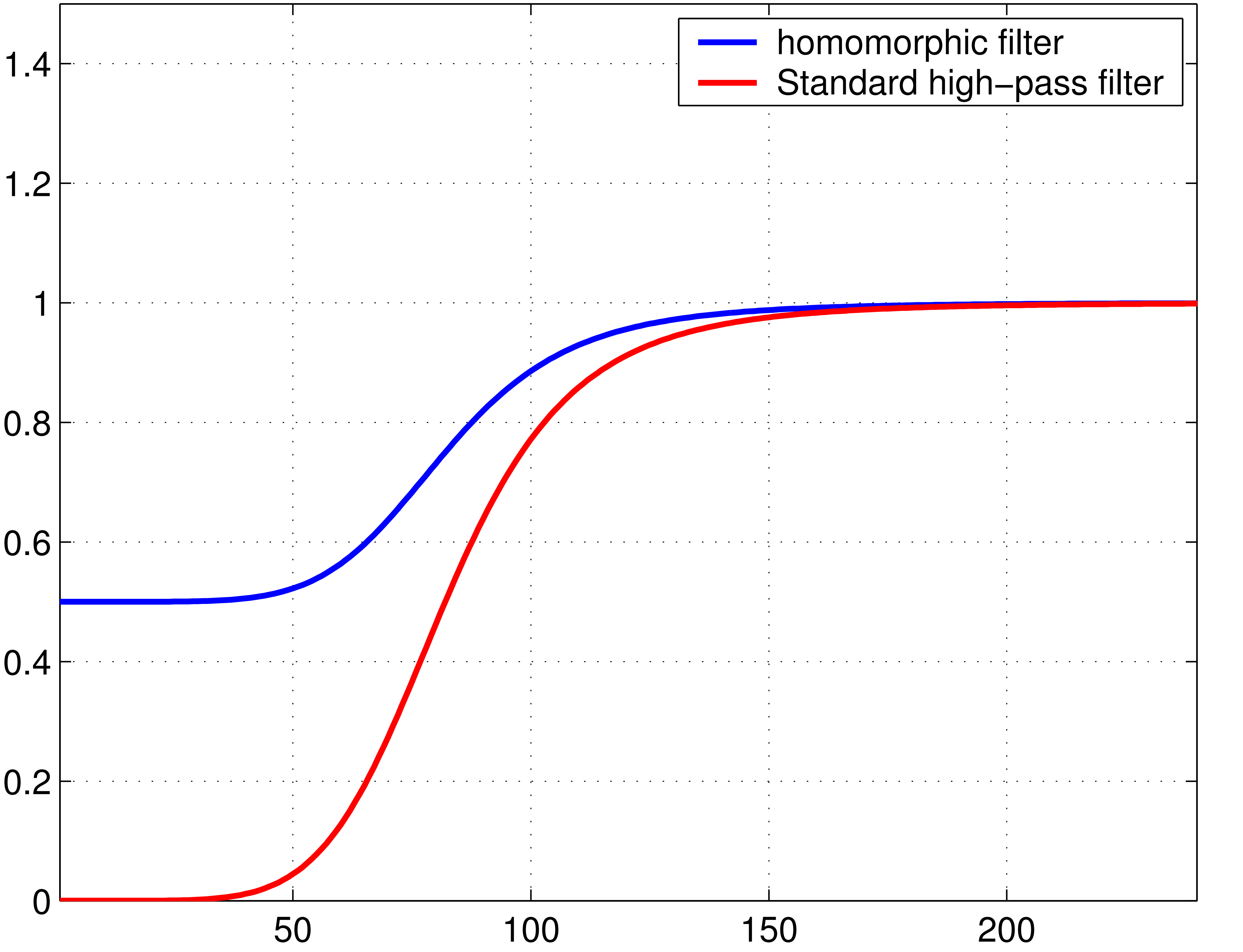
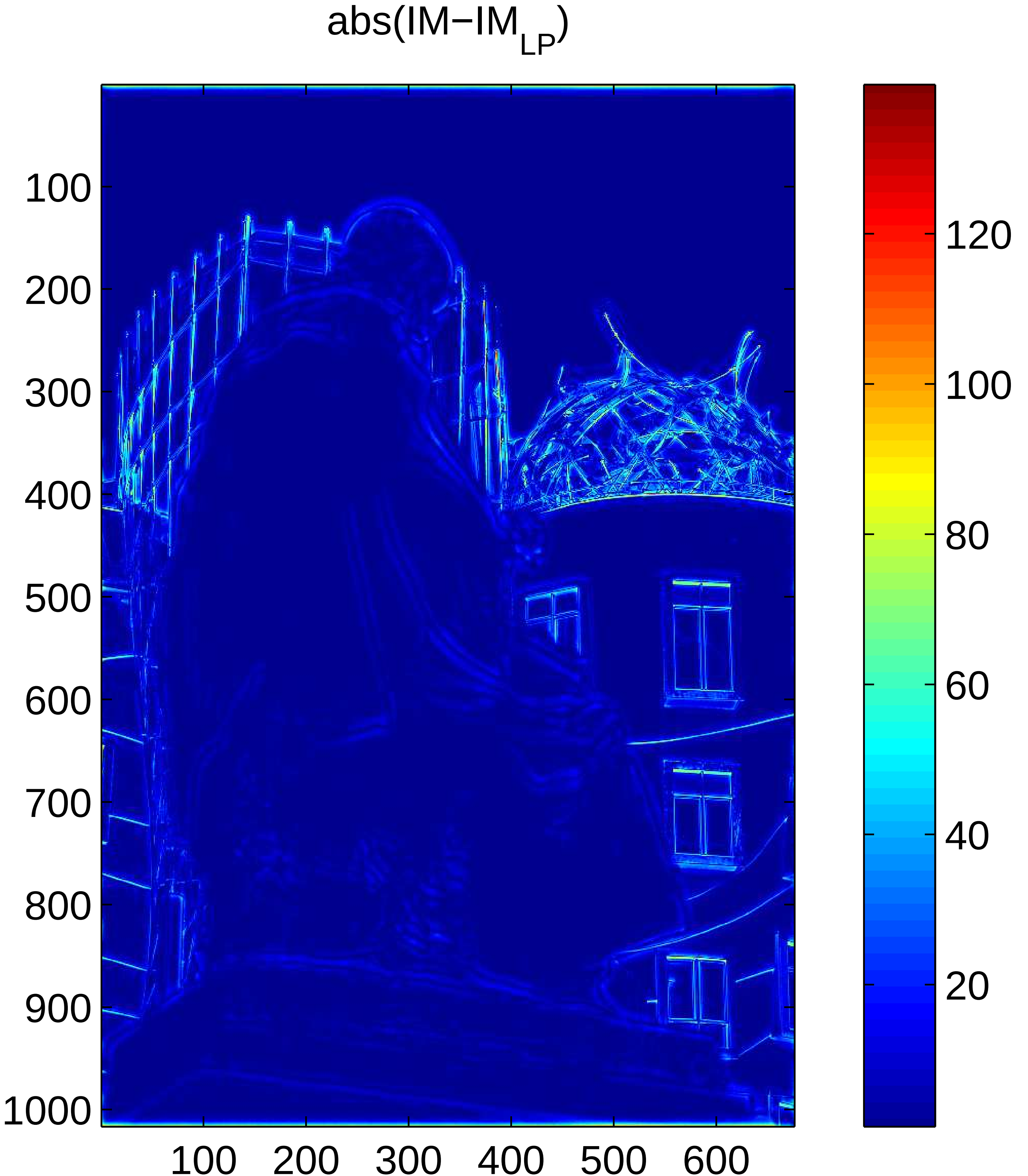
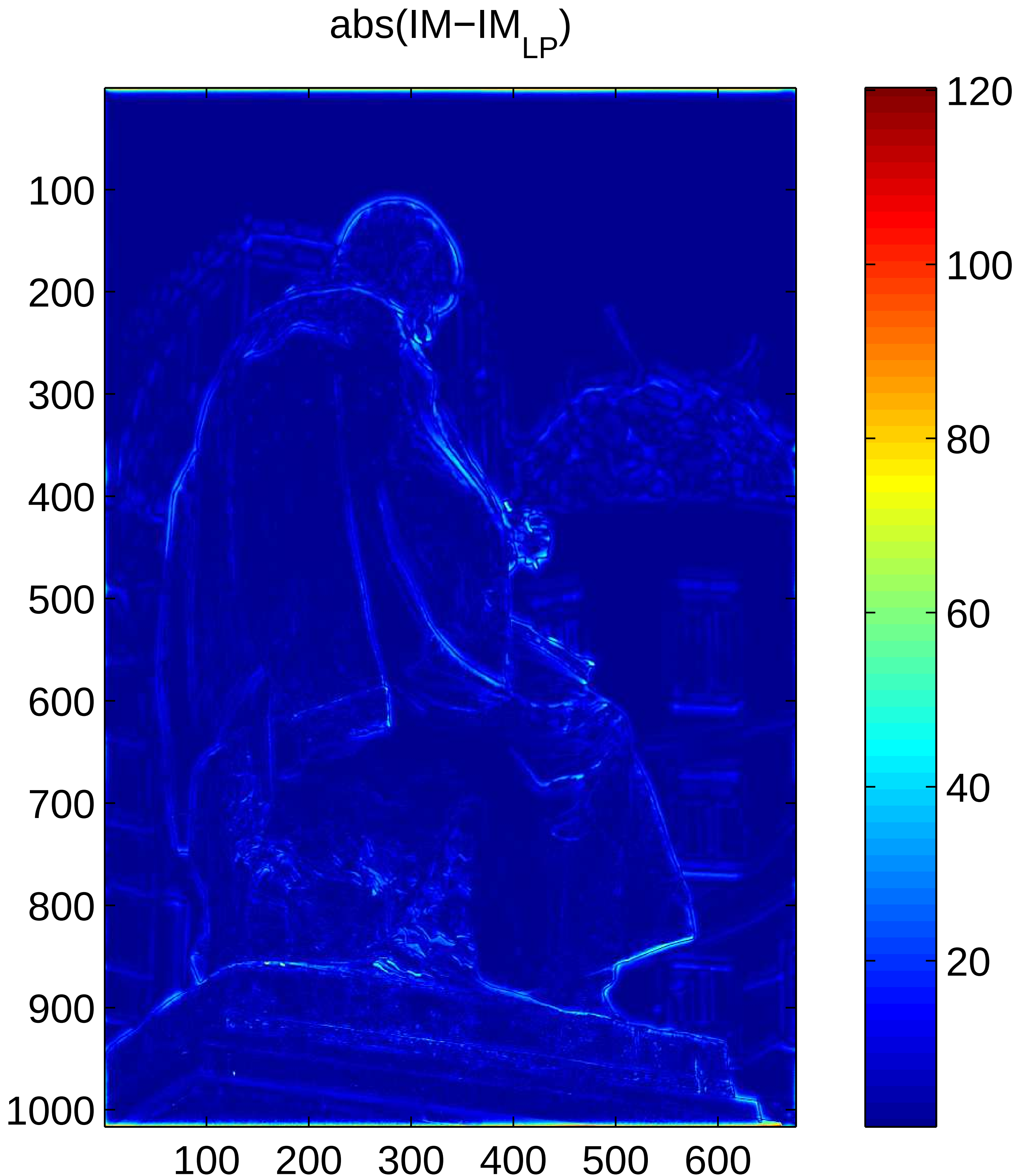
−4
−6


Homomorphic filter made by adaptation of Buttherworth highpass
1
0.9
0.8
0.7
0.6
0.5
600
400
200
0
0
200
400
600
800


homomorphic filter
Standard high−pass filter
1.4
1.2
1
0.8
0.6
0.4
0.2
0
50
100
150
200



abs(IM−IM_LP)
100
200
300
400
500
600
700
800
900
1000
100
200
300
400
500
600
120
100
80
60
40
20


abs(IM−IM_LP)
100
200
300
400
500
600
700
800
900
1000
100
200
300
400
500
600
120
100
80
60
40
20