Mean Shift Segmentation

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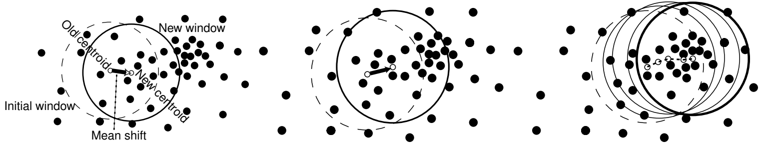
Mean shift segmentation overview

- No assumptions about probability distributions — rarely known
- Spatial-range domain $\big(x, y, f(x, y)\big)$ — normally $f(x, y)$
- Find maxima in the $\big(x, y, f\big)$ space — clusters close in space and range correspond to classes.

Mean shift procedure

Goal: Find local maxima of the probability density *(density modes)* given by samples.



1. Start with a random region of interest.
2. Determine a centroid of the data.
3. Move the region to the location of the new centroid.
4. Repeat until convergence.

Kernel estimation

$$K(\mathbf{x}) = c_k\, k\big(\|\mathbf{x}\|^2\big) \qquad \text{(radial symmetry)}$$

Epanechnikov kernel (other choices possible)

$$k(r) = \begin{cases} 1 - r & \text{for } r \leq 1 \\ 0 & \text{otherwise} \end{cases} \qquad \left(\text{profile, } r = \|\mathbf{x}\|^2\right)$$

Kernel density estimator

$$\widetilde{f}(\mathbf{x}) = \frac{1}{n\, h^d} \sum_{i=1}^{n} K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h}\right)$$

Mean shift procedure

At density maxima $\nabla \widetilde{f} = 0$

$$\widetilde{f}(\mathbf{x}) = \frac{1}{n\,h^d} \sum_{i=1}^{n} K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h}\right)$$

$$\nabla \widetilde{f}(\mathbf{x}) = \frac{2\,c_k}{n\,h^{(d+2)}} \sum_{i=1}^{n} (\mathbf{x} - \mathbf{x}_i)\, k'\left(\left\| \frac{\mathbf{x} - \mathbf{x}_i}{h} \right\|^2\right)$$

$$= \frac{2\,c_k}{n\,h^{(d+2)}} \left(\sum_{i=1}^{n} g_i\right) \left(\frac{\sum_{i=1}^{n} \mathbf{x}_i\, g_i}{\sum_{i=1}^{n} g_i} - \mathbf{x}\right)$$

for $g(r) = k'(r)$, $g_i = g\left(\|(\mathbf{x} - \mathbf{x}_i)/h\|^2 \right)$

Mean shift procedure

At density maxima $\nabla \widetilde{f} = 0$

$$\widetilde{f}(\mathbf{x}) = \frac{1}{n\,h^d} \sum_{i=1}^{n} \; K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h}\right)$$

$$0 = \nabla \widetilde{f}(\mathbf{x}) = \frac{2\,c_k}{n\,h^{(d+2)}} \sum_{i=1}^{n} (\mathbf{x} - \mathbf{x}_i)\,k'\left(\left\|\frac{\mathbf{x} - \mathbf{x}_i}{h}\right\|^2\right)$$

$$= \frac{2\,c_k}{n\,h^{(d+2)}} \left(\sum_{i=1}^{n} g_i\right) \underbrace{\left(\frac{\sum_{i=1}^{n} \mathbf{x}_i\,g_i}{\sum_{i=1}^{n} g_i} - \mathbf{x}\right)}_{\textit{mean shift vector} \text{ — must be 0 at optimum}}$$

for $g(r) = k'(r)$, $g_i = g\left(\,\|(\mathbf{x} - \mathbf{x}_i)/h\|^2\,\right)$

Mean shift procedure (2)

Mean shift vector

$$
\begin{aligned}
m(\mathbf{x}) &= \frac{\sum_{i=1}^{n} \mathbf{x}_i \, g_i}{\sum_{i=1}^{n} g_i} - \mathbf{x} \\
g_i &= g\left(\|(\mathbf{x} - \mathbf{x}_i)/h\|^2 \right) \\
g(r) &= k'(r)
\end{aligned}
$$

Successive locations $\mathbf{y}_j$ of the kernel:

$$
\mathbf{y}_{j+1} = \sum_{i=1}^{n} \mathbf{x}_i \, g\left(\left\| \frac{\mathbf{y}_j - \mathbf{x}_i}{h} \right\|^2 \right) \Big/ \sum_{i=1}^{n} g\left(\left\| \frac{\mathbf{y}_j - \mathbf{x}_i}{h} \right\|^2 \right)
$$

Mean shift procedure (2)

Mean shift vector

$$m(\mathbf{x}) = \frac{\sum_{i=1}^{n} \mathbf{x}_i \, g_i}{\sum_{i=1}^{n} g_i} - \mathbf{x}$$

$$g_i = g\left(\|(\mathbf{x} - \mathbf{x}_i)/h\|^2 \right)$$

$$g(r) = k'(r)$$

Successive locations $\mathbf{y}_j$ of the kernel:

$$\mathbf{y}_{j+1} = \sum_{i=1}^{n} \mathbf{x}_i \, g\left(\left\| \frac{\mathbf{y}_j - \mathbf{x}_i}{h} \right\|^2 \right) \Bigg/ \sum_{i=1}^{n} g\left(\left\| \frac{\mathbf{y}_j - \mathbf{x}_i}{h} \right\|^2 \right)$$

Theorem: If k is convex and monotonically decreasing, the sequence $\left\{ y_j \right\}_{j=1,2,\ldots}$ converge and $\left\{ \widetilde{f}(\mathbf{y}_j) \right\}_{j=1,2,\ldots}$ increases monotonically.

For Epanechnikov kernel $\rightarrow$ convergence in finite number of steps.

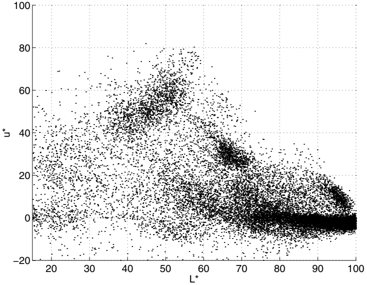
Mean shift mode detection

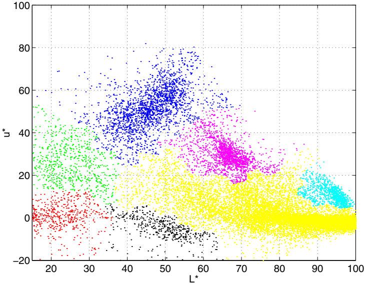
Points from a *basin of attraction* converge to the same mode.

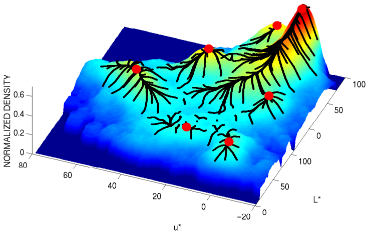
Algorithm:

1. Using multiple initializations covering the entire feature space, identify modes (stationary points).
2. Using small random perturbation, retain only local maxima.

Mean shift mode detection example







Mean shift discontinuity preserving filtering

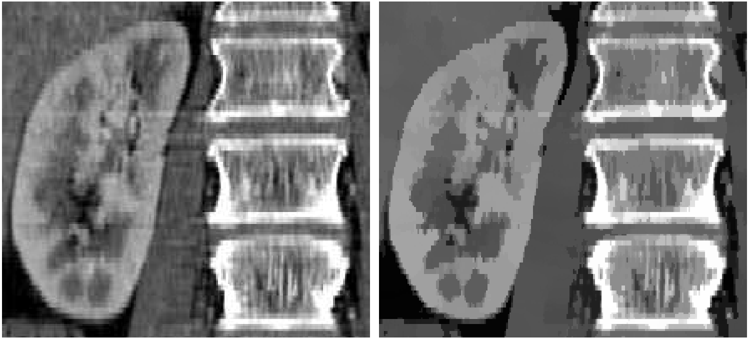
Combine spatial and range values

$$K(\mathbf{x}) = ([\mathbf{x}^s \ \mathbf{x}^r]) = \frac{c}{h_s^d \ h_r^p} \, k \left(\left\| \frac{\mathbf{x}^s}{h_s} \right\|^2 \right) k \left(\left\| \frac{\mathbf{x}^r}{h_r} \right\|^2 \right) ,$$

Algorithm:

1. For each image pixel $\mathbf{x}_i$, initialize $\mathbf{y}_{i,1} = \mathbf{x}_i$.
2. Iterate the mean shift procedure until convergence.
3. The filtered pixel values are defined as $\mathbf{z}_i = (\mathbf{x}_i^s, \mathbf{y}_{i,\mathrm{con}}^r)$; the value of the filtered pixel at the location $\mathbf{x}_i^s$ is assigned the image value of the pixel of convergence $\mathbf{y}_{i,\infty}^r$.

Mean shift discontinuity preserving filtering



Mean shift segmentation

1. Mean shift discontinuity preserving filtering
2. Determine the clusters $\{C_p\}_{p=1,\dots,m}$ by grouping all $\mathbf{z}_i$, which are closer than h_s in the spatial domain and h_r in the range domain, i.e. merge the basins of attractions.
3. Assign class labels to clusters
4. If desired, eliminate regions smaller than P pixels.

Mean shift segmentation examples

Mean shift segmentation examples