Image Registration – Introduction

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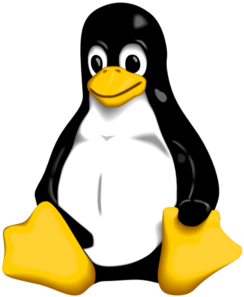
Outline

- What is image registration
- The purpose of image registration
- Warping and interpolation
- Classification of registration methods
- Manual landmark registration
- Automatic elastic B-spline registration
- Automatic elastic dense PDE based registration
- ITK Registration Toolkit

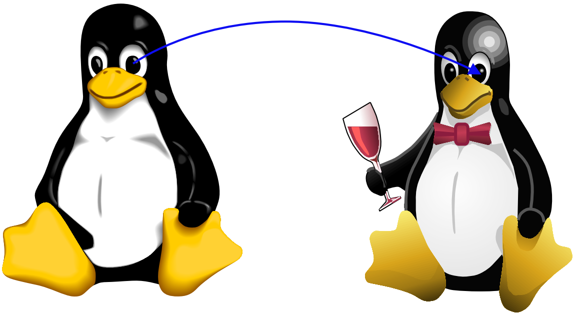
Find corresponding points



American Tux

Tux bordelais

Find corresponding points



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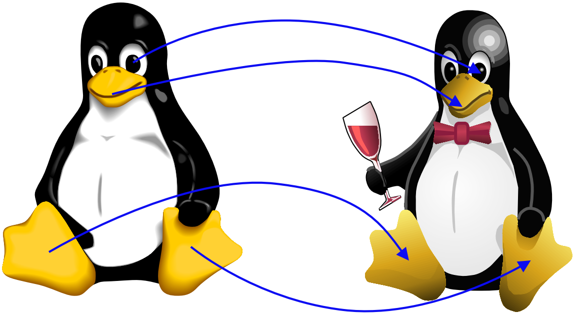


American Tux

Tux bordelais

Registration example 2

Michael Unser

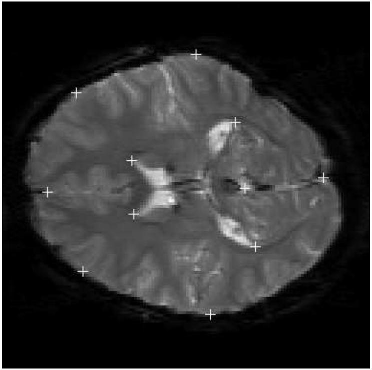
Philippe Thévenaz

Registration example 3

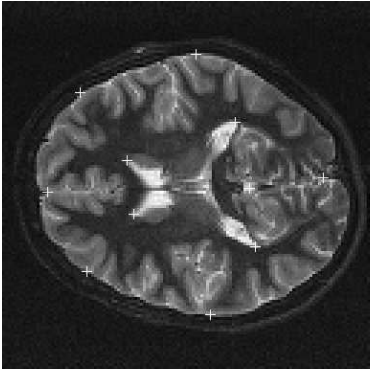
James Bond

Lupe

Registration example 4

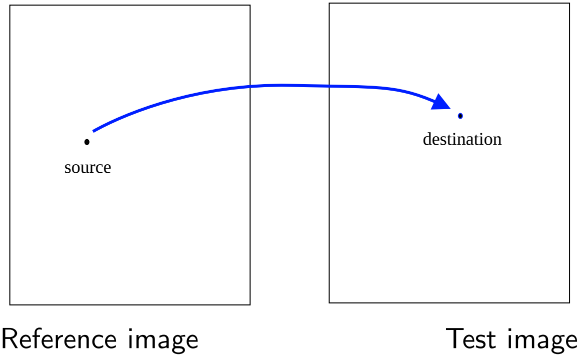


EPI MRI

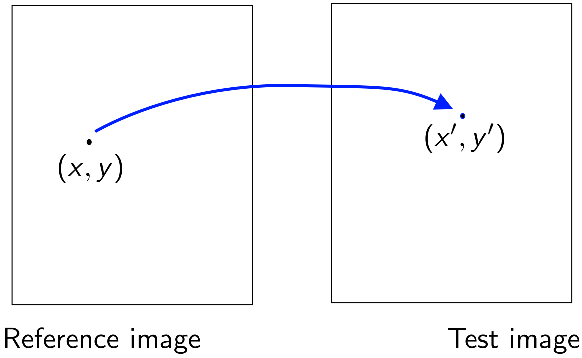


anatomical MRI

Correspondence function

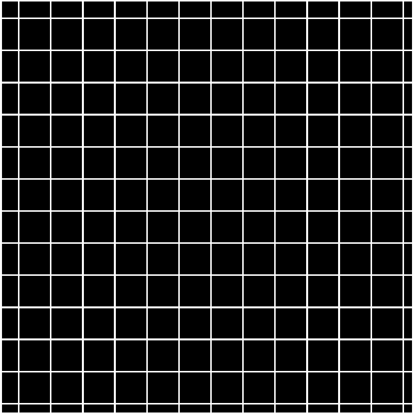


Correspondence function



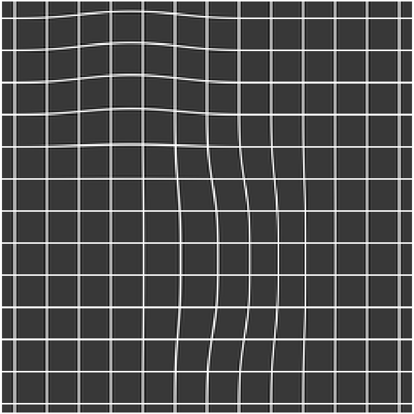
$$\mathbf{g}\left([x\ y]^T\right) = [x'\ y']^T$$

Deformation field



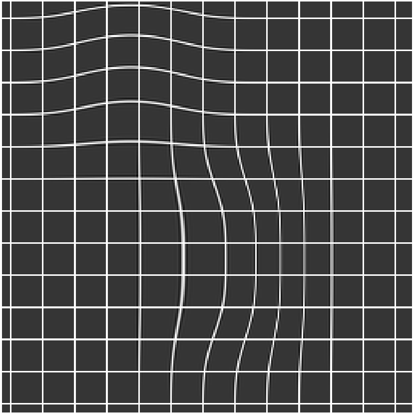
0 % deformation

Deformation field



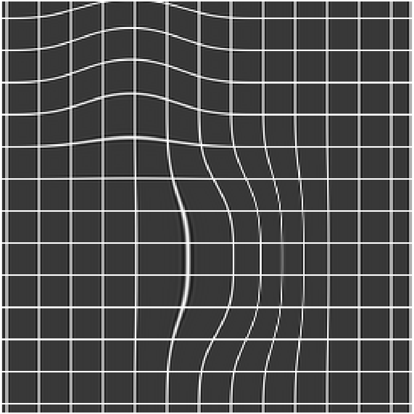
25 % deformation

Deformation field



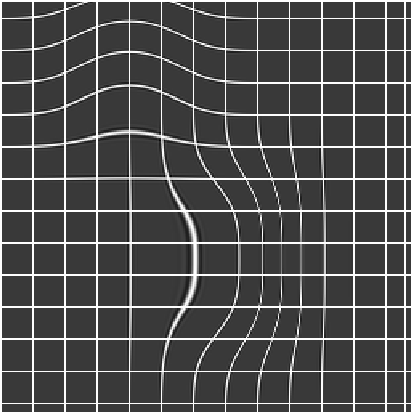
50 % deformation

Deformation field



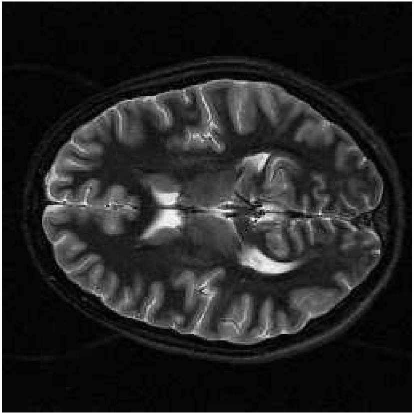
75 % deformation

Deformation field



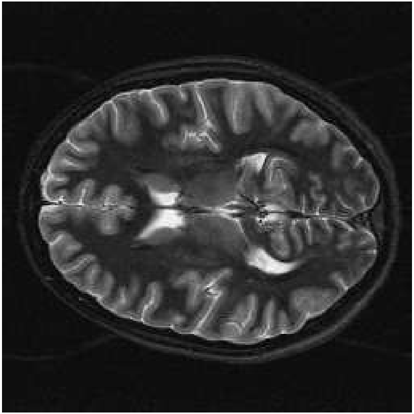
100 % deformation

Image warping



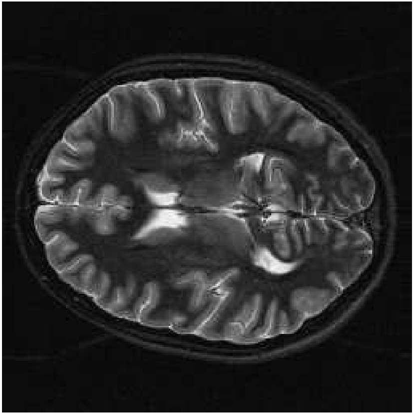
0 % deformation

Image warping



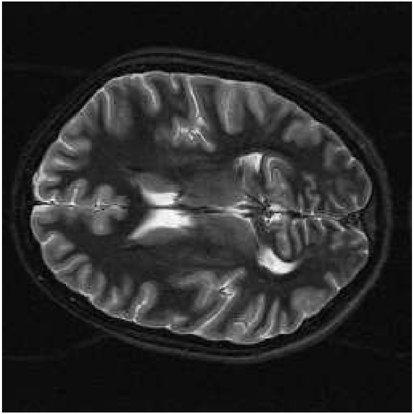
25 % deformation

Image warping



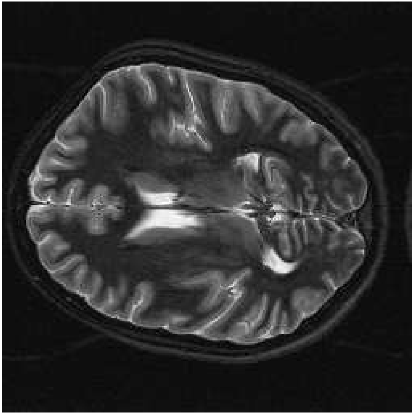
50 % deformation

Image warping



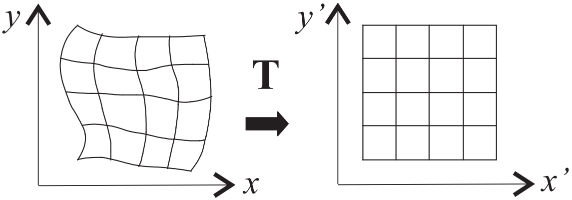
75 % deformation

Image warping



100 % deformation

Image warping = geometric transformation



Rozepíšeme vektorovou transformaci T do dvou složek

$$\mathbf{x}' = T(\mathbf{x}) \quad \rightarrow \quad x' = T_x(x,y)\,, \qquad y' = T_y(x,y)\,.$$

Transformace souřadnic + interpolace

- *Transformace souřadnic bodů* počítá se nová poloha každého bodu ve spojitých souřadnicích (reálná čísla). $\mathbf{x}' = T(\mathbf{x})$
- *Aproximace jasové funkce* z neceločíselných pozic x', y' hledá hodnotu jasu v celočíselných pozicích

Problémy:

- Někdy $\mathbf{x}'$ mimo obraz.
- Interpolace z neuspořádaných (scattered) dat je těžká úloha.

Duální transformace

Souřadnice bodů (x, y) ve vstupním obraze lze vypočítat invertováním vztahu $(x', y') = \mathbf{T}(x, y)$, tj.

$$(\mathbf{x}, \mathbf{y}) = \mathbf{T^{-1}}(\mathbf{x}', \mathbf{y}') .$$

Aproximuje se jas ve vstupním obraze, který odpovídá jasu hledaného bodu x', y' ve výstupní mřížce.
Interpolace z uspořádaných dat (pravoúhlá síť) je 'lehká' úloha.

Transformace souřadnic bodů

Obvykle se $x' = T_x(x, y)$, $y' = T_y(x, y)$ aproximuje polynomem m-tého stupně.

$$x' = \sum_{r=0}^{m} \sum_{k=0}^{m-r} a_{rk}\, x^r\, y^k \,, \qquad y' = \sum_{r=0}^{m} \sum_{k=0}^{m-r} b_{rk}\, x^r\, y^k \,.$$

Vztah je lineární vzhledem ke koeficientům a_{rk}, b_{rk} $\Rightarrow$ odhad metodou nejmenších čtverců.
Potřebné dvojice sobě odpovídajících (vlícovacích/klíčových) bodů x, y a x', y'.

Bilineární, afinní transformace souřadnic

Když se geometrická transformace v závislosti na pozici v obraze příliš náhle nemění, postačují aproximační polynomy nízkého stupně $m = 2$ nebo $m = 3$.

Bilineární transformace

$$
\begin{aligned}
x' &= a_0 + a_1 x + a_2 y + a_3 xy \, , \\
y' &= b_0 + b_1 x + b_2 y + b_3 xy \, .
\end{aligned}
$$

Ještě speciálnější je **afinní transformace** která zahrnuje v praxi potřebnou rotaci, translaci a zkosení.

$$
\begin{aligned}
x' &= a_0 + a_1 x + a_2 y \, , \\
y' &= b_0 + b_1 x + b_2 y \, .
\end{aligned}
$$

Homogenní souřadnice

- Homogenní souřadnice jsou obvyklé v teoretické mechanice, projektivní geometrii, počítačové grafice a robotice.
- Základní myšlenkou je reprezentovat bod ve vektorovém prostoru o jednu dimenzi větším.
- Bod $[x, y]^T$ se v homogenních souřadnicích vyjádří ve 3D vektorovém prostoru jako $[\lambda x, \lambda y, \lambda]^T$, kde $\lambda \neq 0$.
- Pro jednoduchost se obvykle používá jedno z nekonečně mnoha vyjádření $[x, y, 1]^T$.

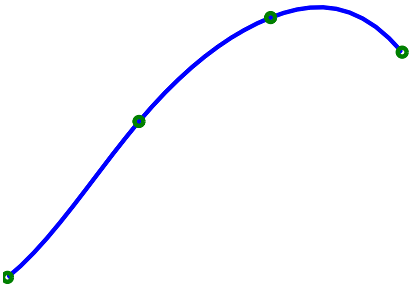
Afinní transformace

Affinní zobrazení se po zavedení homogenních souřadnic vyjádří

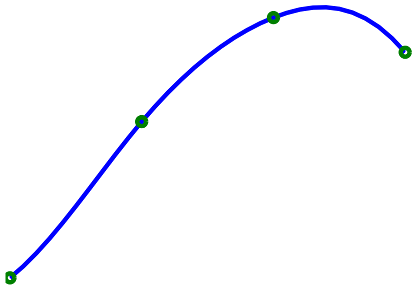
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_0 \\ b_1 & b_2 & b_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} .$$

- Po částech afinní transformace.

(Uniform) splines



(Uniform) splines



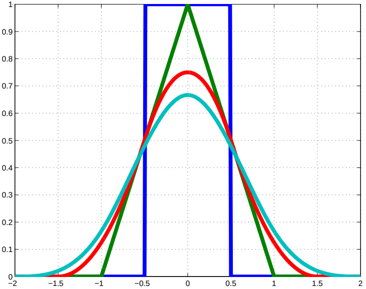
- Piecewise polynomial of degree n
- Continuous $(n-1)^{\text{th}}$ derivative
- (Uniform) knots

Non-uniform splines (1D)

- Polynomial in each interval
- Continuous derivatives
- Boundary conditions (natural)
- $\longrightarrow$ band system of linear equations
- Example: cubic splines

Uniform B-splines

Haar	β_0
linear	β_1
quadratic	β_2
cubic	β_3



Uniform B-splines

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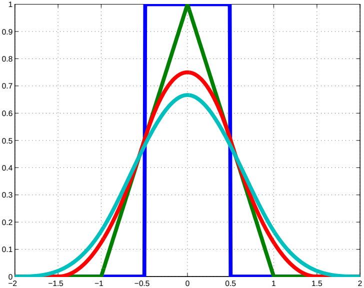


- Generation: $\beta_{n+1} = \beta_n * \beta_0$
- Basis for splines: $s(x) = \sum_i c_i \, \beta(x - i)$

Practical B-splines

- Separability ➢ speed
- B-spline transform (finding coefficients) fast through IIR filtering
- Interpolation fast (small support)
- Extension to n-D by Cartesian product. Separability.

Software

Spliny Matlab, Numerical Recipes, ...

B-spliny Unser, Thevenaz, `bigwww.epfl.ch`

Interpolace = Aproximace jasové funkce

- Transformované souřadnice x', y' leží mimo rastr. Máme jen informaci o vstupním obraze $f(x, y)$ v celočíselných vzorcích.
- Také geometricky transformovaný obraz se má reprezentovat maticí. Proto i zde máme předepsanou pravoúhlou vzorkovací mřížku.
- Příklad: topografická představa.
- Principiálně správnou odpověď poskytuje **teorie aproximace**. Ze vzorků odhadneme spojitou funkci.
- Obvykle se aproximuje polynomem.

Interpolace jako 2D konvoluce

Vzorkovaná verze $g_s(l\,\Delta\,x, k\,\Delta\,y)$ spojité funkce $f(x, y)$.
Interpolace $\longrightarrow$ spojitá $f_n(x, y)$ aproximující f.
Linearita + nezávislost na posunutí (LTI) $\longrightarrow$ konvoluce.

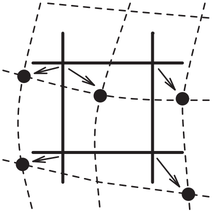
$$f_n(x, y) = \sum_{l=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} g_s(l\,\Delta\,x,\ k\ \ \Delta\,y)\, h_n(x - l\,\Delta\,x,\ y - k\,\Delta\,y)\,.$$

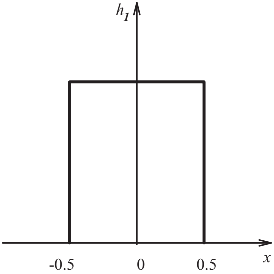
h_n je **interpolační jádro**, obvykle malé pokrytí (support).

Nejbližší soused

Přiřadí bodu (x, y) hodnotu jasu nejbližšího bodu g_s v diskrétní mřížce.

$$f_1(x, y) = g_s(round\,(x),\; round\,(y))\,.$$



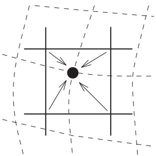


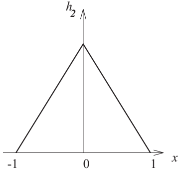
P0 - po částech konstantní.

Lineární interpolace

4 body v okolí (x, y), lineární kombinace. Vliv každého ze čtyř bodů klesá se vzdáleností.

$$
\begin{aligned}
f_2(x,y) = \quad & (1-a)(1-b)\, g_s(l,k) + a(1-b)\, g_s(l+1,k) \\
& + b(1-a)\, g_s(l,k+1) + ab\, g_s(l+1,k+1)\,, \\
\text{kde} \quad & l = round\,(x)\,, \quad a = x - l\,, \\
& k = round\,(y)\,, \quad b = y - k\,.
\end{aligned}
$$



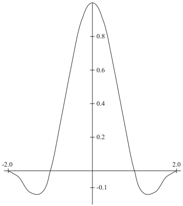


P1 - po částech lineární.

Bikubická interpolace

Lokálně interpolován obrazovou funkci bikubickým polynomem z 16 bodů v okolí. 1D interpolační jádro (pro přehledné zobrazení)

$$h_3 = \begin{cases} 1 - 2|x|^2 + |x|^3 & \text{pro } 0 \leq |x| < 1\,, \\ 4 - 8|x| + 5|x|^2 - |x|^3 & \text{pro } 1 \leq |x| < 2\,, \\ 0 & \text{jinde.} \end{cases}$$



Interpolace P3.

Interpolace obrazu B-spliny

$$f(x,y) = \sum_{i,j} c_{ij}\beta_n(x-i)\beta_n(x-j)$$

- Interpolace spliny je globální, zatímco P0,P1,P2,P3. . . jsou lokální v obou osách
- Pixely na pravidelné mřížce ➢ lze předpočítat B-spline coeficienty (FIR filtr)
- Vyhodnocení β_3 je stejně rychlé jako u P3, kvalita lepší.
- Okrajové podmínky (nulové, periodické, zrcadlo na bodu, mezi body)
- Vyšší řády vedou ke vzniku Gibbsových artefaktů na hranách (ringing).

Interpolace sinc

Bandlimited (frekvenčně omezený) signál f,
$\operatorname{supp} \hat{f} \subseteq [-F_s/2, F_s/2]$, $f \in L^2$
Jednoznačně určený svými vzorky $f(i/F_s)$:

$$f(x) = \sum_i f(i/F_s) \operatorname{sinc}(F_s x - n)$$
$$\operatorname{sinc}(\tau) = \frac{\sin(\pi\tau)}{\pi\tau}$$

Interpolační jádro sinc není kompaktní $\longrightarrow$ výpočetně náročné.
Někdy lze počítat pomocí FFT (zero-padding).

Image registration

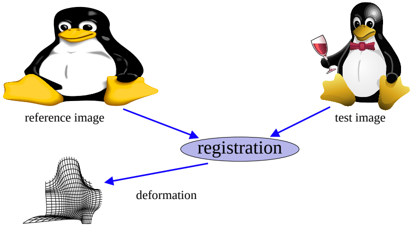
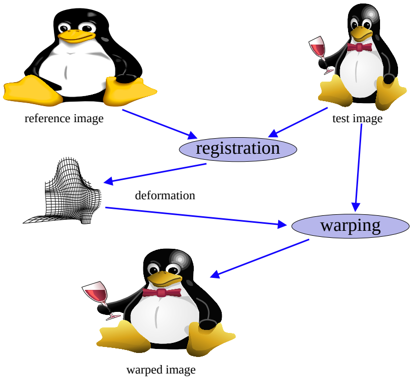


Image registration



(Biomedical) applications

...of image registration

- Comparing images
  - Different times
  - Different methods
  - Different subjects
- Analyzing sequences
  - Motion estimation
  - Segmentation

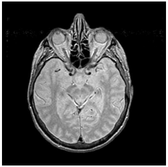
Qualitative and quantitative information.

Other applications of image registration

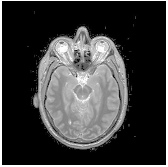
- video stabilization
- video compression
- image mosaicking
- stereo matching
- structure from motion

- video ultrazvuk srdce (2C,4C)

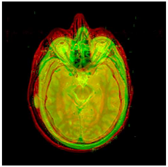
Image alignment



reference

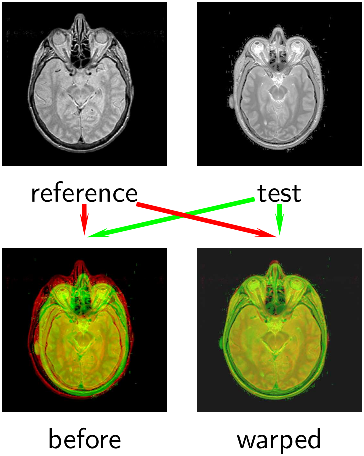


test



before

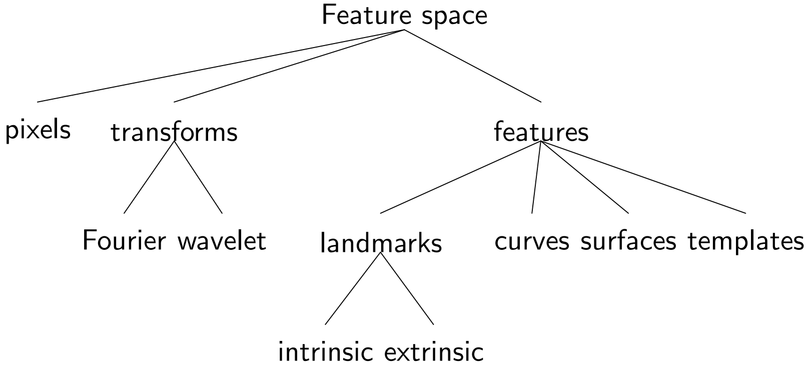
Image alignment



Classification of registration methods

- **Feature space** — intermediate data extracted from image
- **Search space** — representation of the deformation
- **Similarity metric** — measuring the dissimilarity
- **Search strategy** — how to find the minimum
- **User interaction level**

Registration methods – Feature space



Registration methods – Search space

- Local
  - Variational
  - PDE
- Semi-local
  - (Quad)tree
  - B-splines
  - Wavelets
- Global
  - linear
  - polynomial
  - harmonic
- RBF, krigging
- Image dependent models (e.g. adaptive quadtrees)

Similarity metrics

- **Data term for pixel based criteria**
  - l_2 norm (SSD)
  - l_1 norm
  - correlation, normalized correlation
  - mutual information, normalized mutual information
- **Other data terms**
  - image interpolation — important
  - feature-based methods — distance
  - template-based methods — windowed pixel-based criteria
  - transform-based methods — norm in the transform domain
  - *preprocessing* — filtering, histogram equalization,. . .
- **Regularization**
  - Norm (l_p) of the derivatives
  - Implicit regularization (constrained model)
  - Smoothing

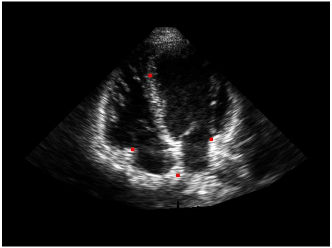
Search strategy

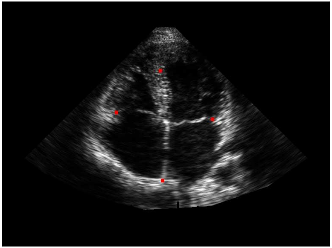
- Direct solution
- Exhaustive search
- Dynamic programming
- PDE evolution
- Multidimensional optimisation
  - gradient descent
  - Newton-like methods, exact/estimated Hessian, Marquardt-Levenberg, conjugated gradients, BFGS, ...
- Multiresolution

User interaction level

- Manual
- Automatic
- Semi-automatic

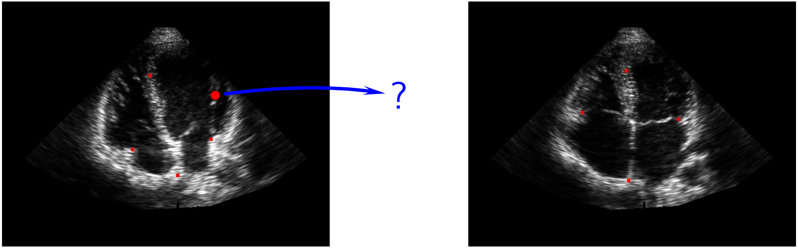
Manual registration





- Landmark identification

Manual registration



- Landmark identification
- Landmark interpolation

Landmark interpolation (2)

- **Constraints**
  - Hard constraints

$$\mathbf{g}(\mathbf{x}_i) = \begin{bmatrix} g_x(\mathbf{x}_i) \\ g_y(\mathbf{x}_i) \end{bmatrix} = \mathbf{z}_i \quad \text{for all } i \in \{1, \ldots, N\}$$

  - Soft constraints

$$\sum_{i=1}^{N} \|\mathbf{g}(\mathbf{x}_i) - \mathbf{z}_i\|^2 \leq \varepsilon$$

- **Properties**
  - invariance to scale, shifts, rotations
  - representability of linear transforms

Thin-plate splines

- Minimize an energy

$$J(g) = \int \left(\frac{\partial^2 g}{\partial x^2}\right)^2 + 2\left(\frac{\partial^2 g}{\partial x \partial y}\right)^2 + \left(\frac{\partial^2 g}{\partial y^2}\right)^2 \mathrm{d}x\mathrm{d}y$$

$$J(\mathbf{g}) = J(g_x) + J(g_y)$$

under constraints

$$g(x_i, y_i) = z_i$$

Thin-plate splines

- Minimize an energy

$$J(g) = \int \left(\frac{\partial^2 g}{\partial x^2}\right)^2 + 2\left(\frac{\partial^2 g}{\partial x \partial y}\right)^2 + \left(\frac{\partial^2 g}{\partial y^2}\right)^2 \mathrm{d}x\mathrm{d}y$$
$$J(\mathbf{g}) = J(g_x) + J(g_y)$$

under constraints

$$g(x_i, y_i) = z_i$$

- Solution

$$g(x, y) = \sum_{i=1}^{N} \lambda_i \varrho(\|\mathbf{x} - \mathbf{x}_i\|) + a_0 x + a_1 y + a_2$$

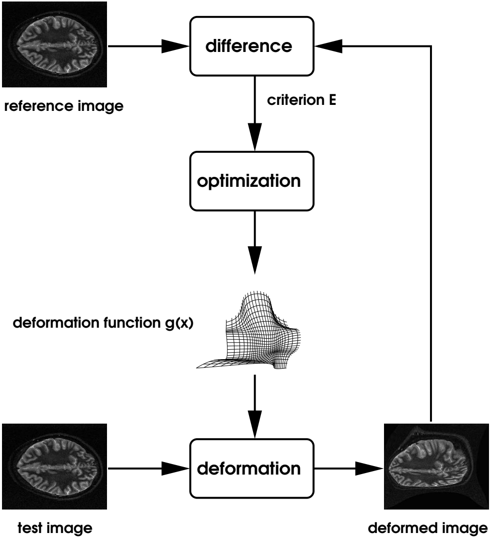
with $\|\mathbf{x} - \mathbf{x}_i\| = \sqrt{(x - x_i)^2 + (y - y_i)^2} = r$

where $\varrho(r)$ is a *radial basis function* and $\varrho(r) = r^2 \log r$

Automatic rigid registration

- Look for rigid (euclidean or affine) transformation
- To compensate different position, scale
- ... or to simplify a more complicated problem

Registration as minimization

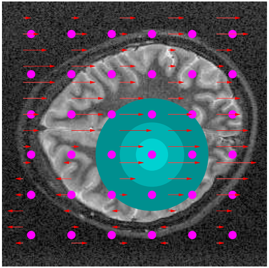


ITK (Insight Registration and segmentation toolbox)

Automatic elastic B-spline registration

- Look for elastic (non-linear) transformation
- Smooth deformation wanted
- Semi-local model with many parameters

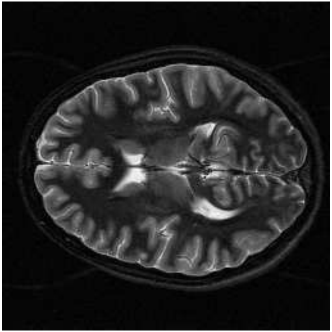
Spline based warping

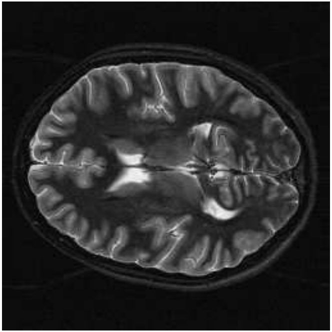


- Approximation properties ➢ precision
- Short support ➢ speed
- Scalability
- Representability of linear transforms

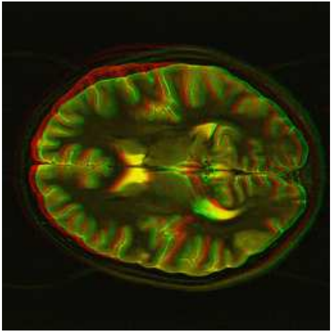
$$\mathbf{g}(\mathbf{x}) = \mathbf{x} + \sum_{\mathbf{i} \in \mathbb{Z}^2} \mathbf{c}(\mathbf{i})\, \beta(\mathbf{x}/\mathbf{h} + \mathbf{d} - \mathbf{i})$$

Evaluating the difference

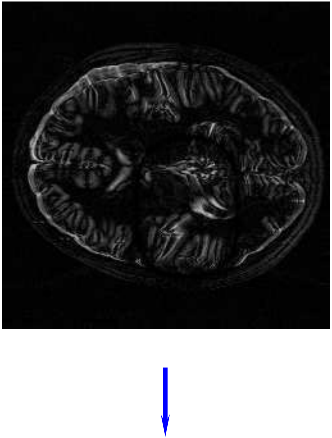




Evaluating the difference



Evaluating the difference



$$E = (1/N) \sum_{\mathbf{i}} \left(f_t^c(\mathbf{g}(\mathbf{i})) - f_r(\mathbf{i}) \right)^2$$

Multiresolution

32×32

Multiresolution

64×64

Multiresolution

128×128

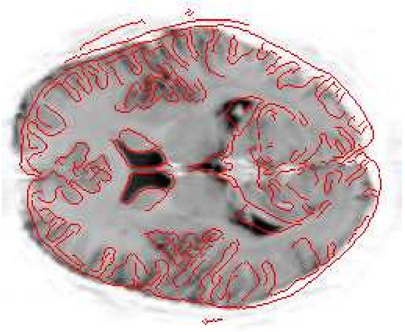
Multiresolution

256×256

video of the registration

Applications

- EPI distortion

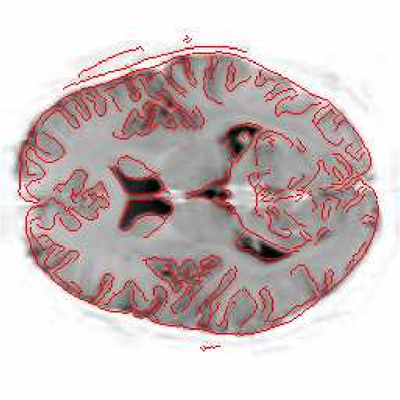


Before

(with Arto Nirkko)

Applications

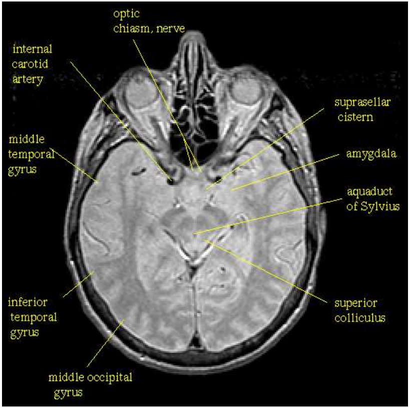
- EPI distortion



After

Applications

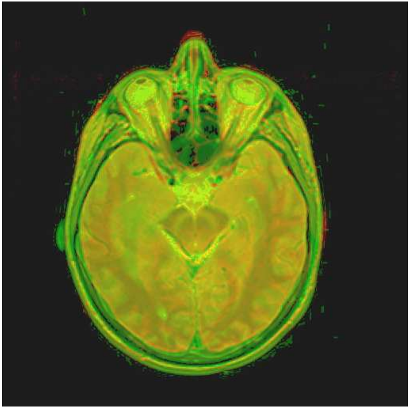
- EPI distortion
- MRI atlas



Atlas

Applications

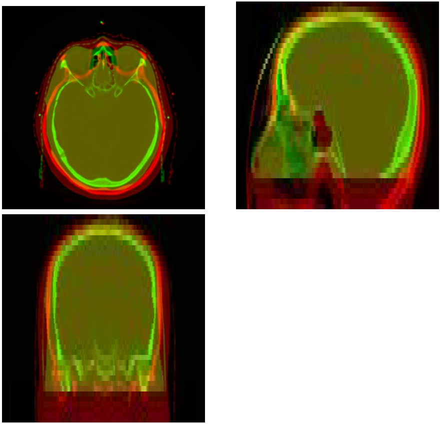
- EPI distortion
- MRI atlas



Aligned

Applications

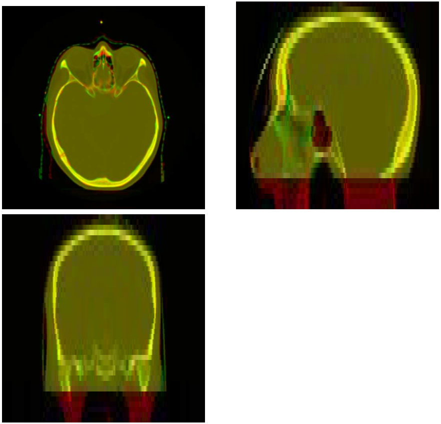
- EPI distortion
- MRI atlas
- CT alignment



Before

Applications

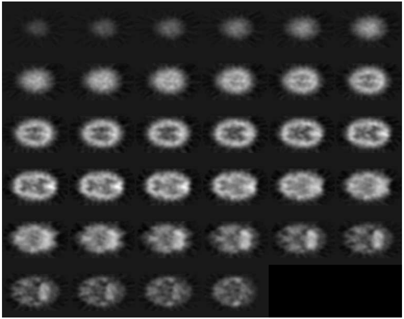
- EPI distortion
- MRI atlas
- CT alignment



After

Applications

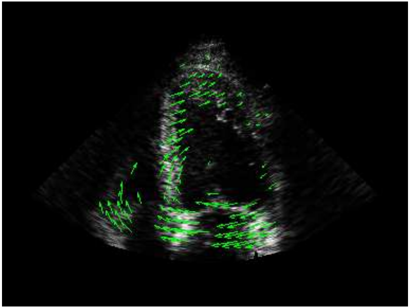
- EPI distortion
- MRI atlas
- CT alignment
- SPECT atlas



(with University Hospital in Geneva)

Applications

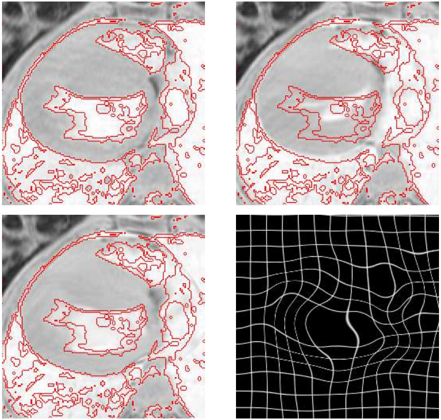
- EPI distortion
- MRI atlas
- CT alignment
- SPECT atlas
- Ultrasound



velocity (with María J. Ledesma-Carbayo)

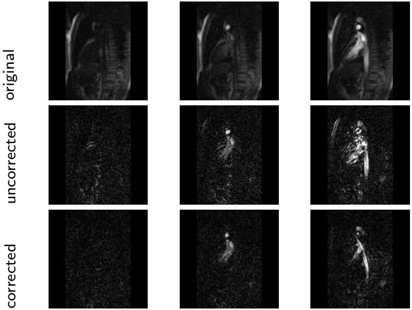
Applications

- EPI distortion
- MRI atlas
- CT alignment
- SPECT atlas
- Ultrasound
- MRI heart sequence



Applications

- EPI distortion
- MRI atlas
- CT alignment
- SPECT atlas
- Ultrasound
- MRI heart sequence
- MRI perfusion



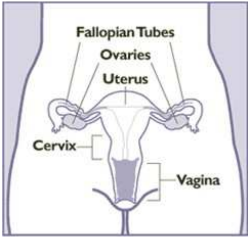
Automatic dense PDE-based registration

- Look for elastic (non-linear) transformation
- General motion (vector) field is sought
- Criteria formulated in the continuous domain
- Regularization to impose smoothness

Some facts about cervical cancer

- Cervical cancer is the second most common cancer among women worldwide
- Nearly 380,000 new cases are diagonosed yearly
- When detected early, cervical neoplasia is nearly 100% curable
- Papanicolau test (Pap Smear) and Colposcopy are the most widespread tests for cancer screening

Female Anatomy



Diagnosis: Colposcopy

- Colposcopy visually inspects inspects the cervix area at low magnification
- The application of acetic-acid will temporally alter the appearence of cancerous tissue
- Colposcopists must subjectively asses appearence changes in small areas over prolonged periods of time

60 seconds

300 seconds

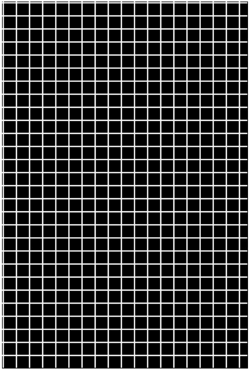
Deformation as a Vector Field

We represent correspondence function H as a dense vector field

$$H([x, y]) = [x', y']$$

Original Image

Deformation Field



Deformed Image

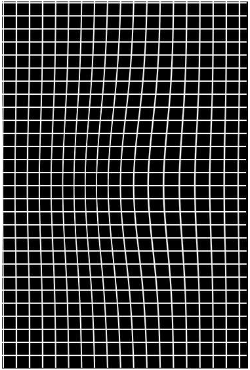
Deformation as a Vector Field

We represent correspondence function H as a dense vector field

$$H([x, y]) = [x', y']$$

Original Image

Deformation Field



Deformed Image

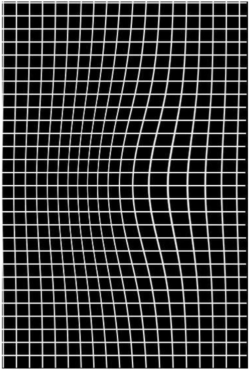
Deformation as a Vector Field

We represent correspondence function H as a dense vector field

$$H([x, y]) = [x', y']$$

Original Image1

Deformation Field



Deformed Image

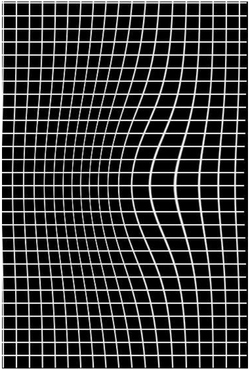
Deformation as a Vector Field

We represent correspondence function H as a dense vector field

$$H([x, y]) = [x', y']$$

Original Image1

Deformation Field



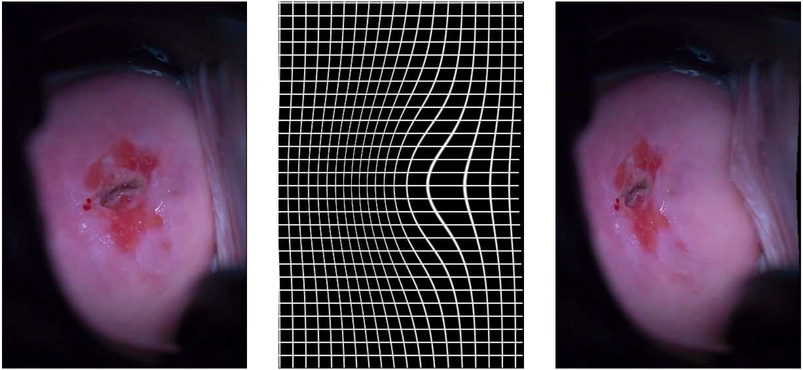
Deformed Image

Deformation as a Vector Field

We represent correspondence function H as a dense vector field

$$H([x, y]) = [x', y']$$

Original Image1 Deformation Field Deformed Image



Registration as optimization

- Correspondence function H and vector field $\mathbf{h}$ are related by:

$$H([i,j]) = [i,j] + \mathbf{h}(i,j) \tag{1}$$

- The problem is then formulated as the minimization of a criterion J with respect to vector field $\mathbf{h}$:

$$\mathbf{h}^* = \arg\min_{\mathbf{h}}(J(\mathbf{f} \circ \mathbf{h}, \mathbf{g}, \mathbf{h}\ )) \tag{2}$$

  where $\mathbf{h}^*$ is the optimal solution, $\mathbf{f}$ and $\mathbf{g}$ are the images to be registered and J is a cost function measuring the dissimilarity between the images and the likelyhood of the transformation.

- Cost function J is divided into a data and a regularization term multiplied by a proportionality constant:

$$J(\mathbf{f}, \mathbf{g}, \mathbf{h}) = J_D(\mathbf{f} \circ \mathbf{h}, \mathbf{g}) + \alpha J_R(\mathbf{h}) \tag{3}$$

Similarity criteria

- The data term J_D is the sum of squared differences (SSD) between the template image **g** and the moving image **f** deformed by **h**:

$$J_D(\mathbf{f}\circ\mathbf{h}, \mathbf{g}) = \int_{(x,y)\subset\Omega} (\mathbf{f}(\mathbf{h}(x,y)+[x,y]) - \mathbf{g}(x,y))^2 \,\mathrm{d}x\,\mathrm{d}y \quad (4)$$

Discretized version:

$$J_D(\mathbf{f}\circ\mathbf{h}, \mathbf{g}) = \sum_{(i,j)\subset\Omega} (\mathbf{f}(\mathbf{h}(i,j)+[i,j]) - \mathbf{g}(i,j))^2 \quad (5)$$

Regularization

- Regularization term penalizes un-smooth deformations and makes the optimization of J a well-posed problem
- Regularization criterion J_R is chosen so its gradient coincides with the linearized 2D elasticity operator describing equilibrium in an elastic material.

$$\nabla J_R(\mathbf{h}) = \xi \Delta \mathbf{h} + (1-\xi)\nabla(\nabla \cdot \mathbf{h}) \tag{6}$$

$$J_R(\mathbf{h}) = \frac{1}{2} \int_{(x,y)\subset\Omega} \left[\xi\left(\partial_x h_x\right)^2 + (1-\xi)\left(\left(\partial_x h_x\right)^2 + \partial_x h_x \cdot \partial_y h_y\right)\right] + \left[\xi\left(\partial_y h_y\right)^2 + (1-\xi)\left(\left(\partial_y h_y\right)^2 + \partial_x h_x \cdot \partial_y h_y\right)\right]$$

Gradient descent optimization

On every iteration:

- Calculate the new deformation field

$$\mathbf{h}' = \mathbf{h}_i - \lambda(\nabla J(\mathbf{f}, \mathbf{g}, \mathbf{h}_i)) \tag{8}$$

- If the step is succesful, then the step is accepted and the step size is increased

$$\lambda \leftarrow 2\lambda, \mathbf{h}_{i+1} \leftarrow \mathbf{h}', J_{i+1} \leftarrow J' \tag{9}$$

- Otherwise the step size is reduced

$$\lambda \leftarrow \lambda/10 \tag{10}$$

- We iterate until convergence (given by a suitable threshold).

Other implementation details

- Multi-resolution was used
- ROI masks were automatically generated
- Images were rigidly pre-registred
- Green color channel only

Experiments

- Algorithm tested with 45 image pairs
- Images taken before and 60 seconds after acetic-acid application
- Cross-polarization filters used to reduce the glint
- Uncompressed 1125x750 pixel 16-bit images were used

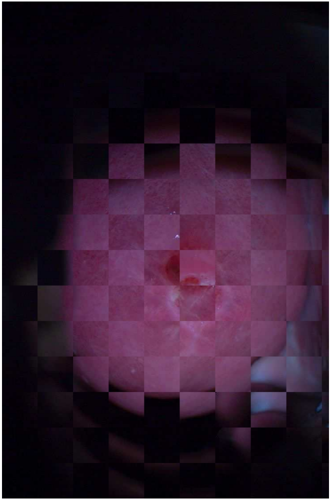
Results

Template

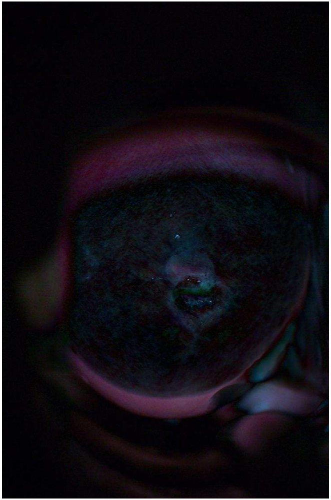
Moving

Results

Unregistered Checkerboard

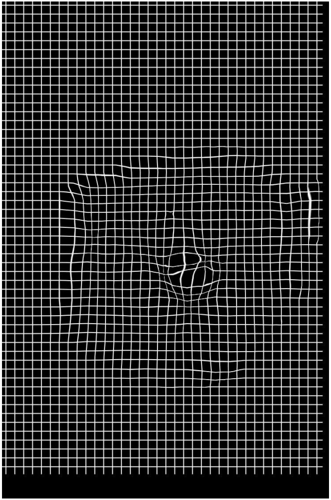


Unregistered Difference



Results

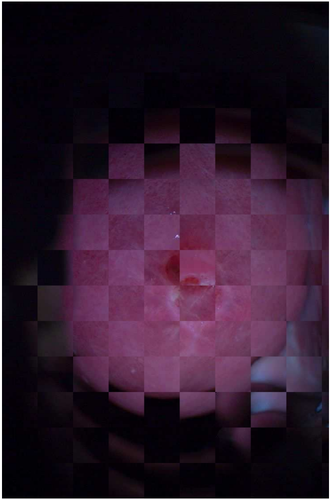
Deformation Field



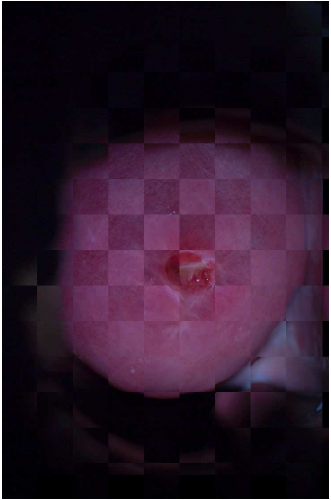
Registered

Results

Unregistered Checkerboard

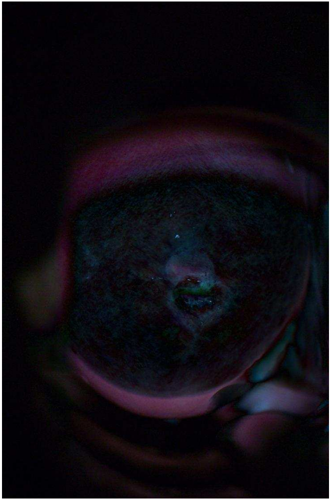


Registered Checkerboard

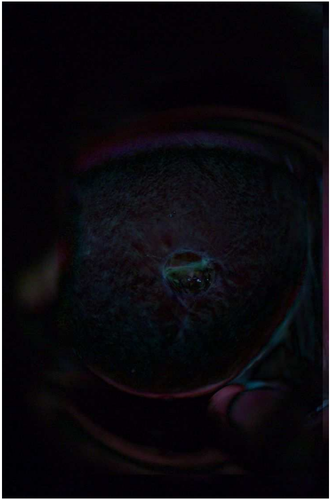


Results

Unregistered Difference

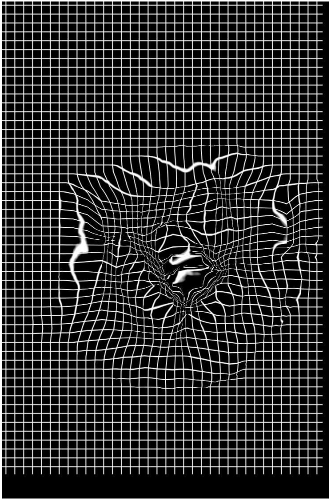


Registered Difference

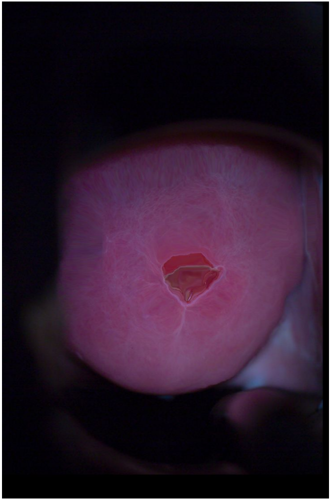


Insufficient Regularization

Deformation Field



Registered



Video cervix registration

Registration conclusions

- Many different approaches
- Many different applications
- Very frequent in medical imaging
- ... but also video processing, 3D reconstruction...
- Trade-off between robustness, speed and generality
- A priori knowledge always usefull, sometimes essential