Variational interpolation
Find a function
Find a function
Find a function

Variational interpolation, B-splines – p. 2/1
Rank functions

ugliest 

\[ Rm(R) + 0 \]

\[ Jf0 \]

\[ \ldots \]

prettiest
Rank functions

ugliest \ldots prettiest

Variational criterion

\[ J : \left( \mathbb{R}^m \to \mathbb{R}^n \right) \to \mathbb{R}_0^+ \]

\[ J(f) \geq 0 \]
Variational reconstruction

Find the best function satisfying the constraints.
Tunable 1D interpolation
Tunable 1D interpolation

\[ J(f) = \left\| \frac{\partial^M f}{\partial x^M} \right\|^2 \]
Tunable 2D interpolation
Tunable 2D interpolation

Test

Test

test
Tunable 2D interpolation

\[ \int \left\| \nabla^{0.5} g(x) \right\|^2 dx \]
Tunable 2D interpolation

\[ \int \| \nabla^{0.9} g(x) \|^{2} dx \]
Tunable 2D interpolation

\[ \int \| \nabla^{1.3} g(x) \|^2 dx \]
Tunable 2D interpolation

\[ \int \| \nabla^{2.5} g(x) \|^2 dx \]

Variational interpolation, B-splines – p. 6/1
Splines, B-splines
The splines

What are splines, anyway?
The splines

What are splines, anyway?

The best functions in the world!
(Uniform) splines
(Uniform) splines

- Piecewise polynomial of degree $n$
- Continuous $(n - 1)^{th}$ derivative
- (Uniform) knots
Uniform B-splines

Haar $\beta_0$
linear $\beta_1$
quadratic $\beta_2$
cubic $\beta_3$
Uniform B-splines

Haar $\beta_0$
linear $\beta_1$
quadaratic $\beta_2$
cubic $\beta_3$

Generation: $\beta_{n+1} = \beta_n \ast \beta_0$

Basis for splines: $s(x) = \sum_i c_i \beta(x - i)$
Practical B-splines

- Separability → speed
- B-spline transform (finding coefficients) fast through IIR filtering
- Interpolation fast (small support)