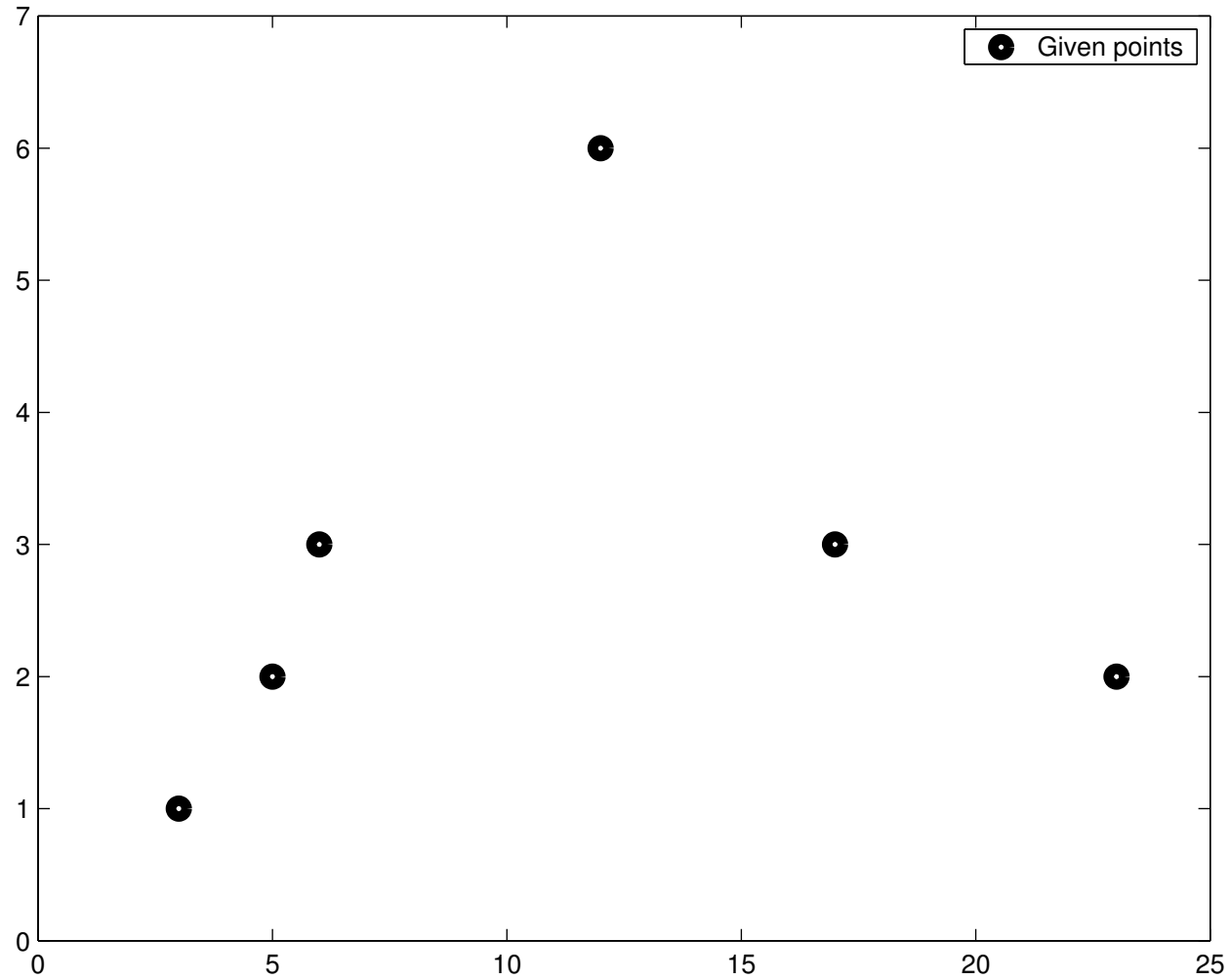
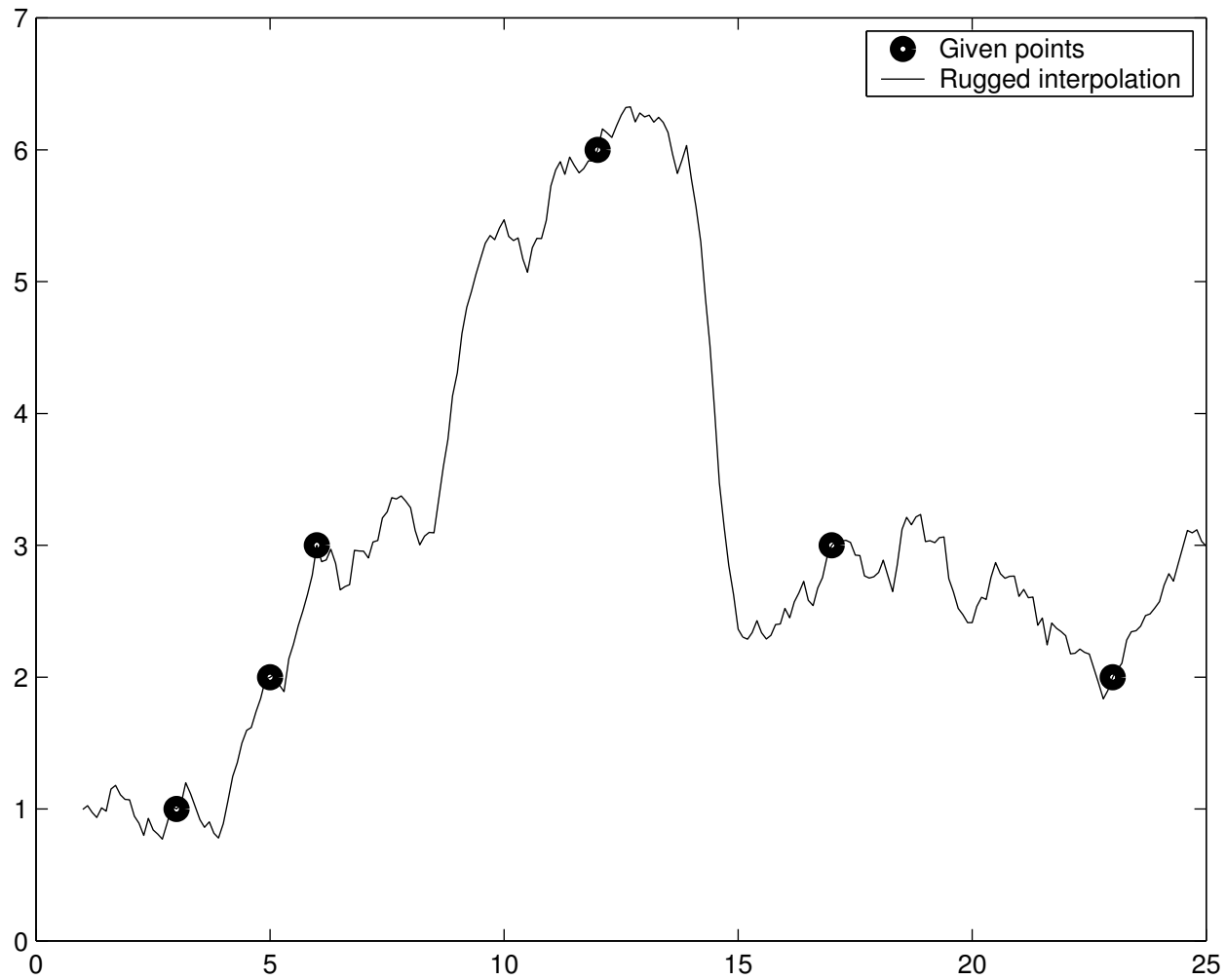


Variational interpolation

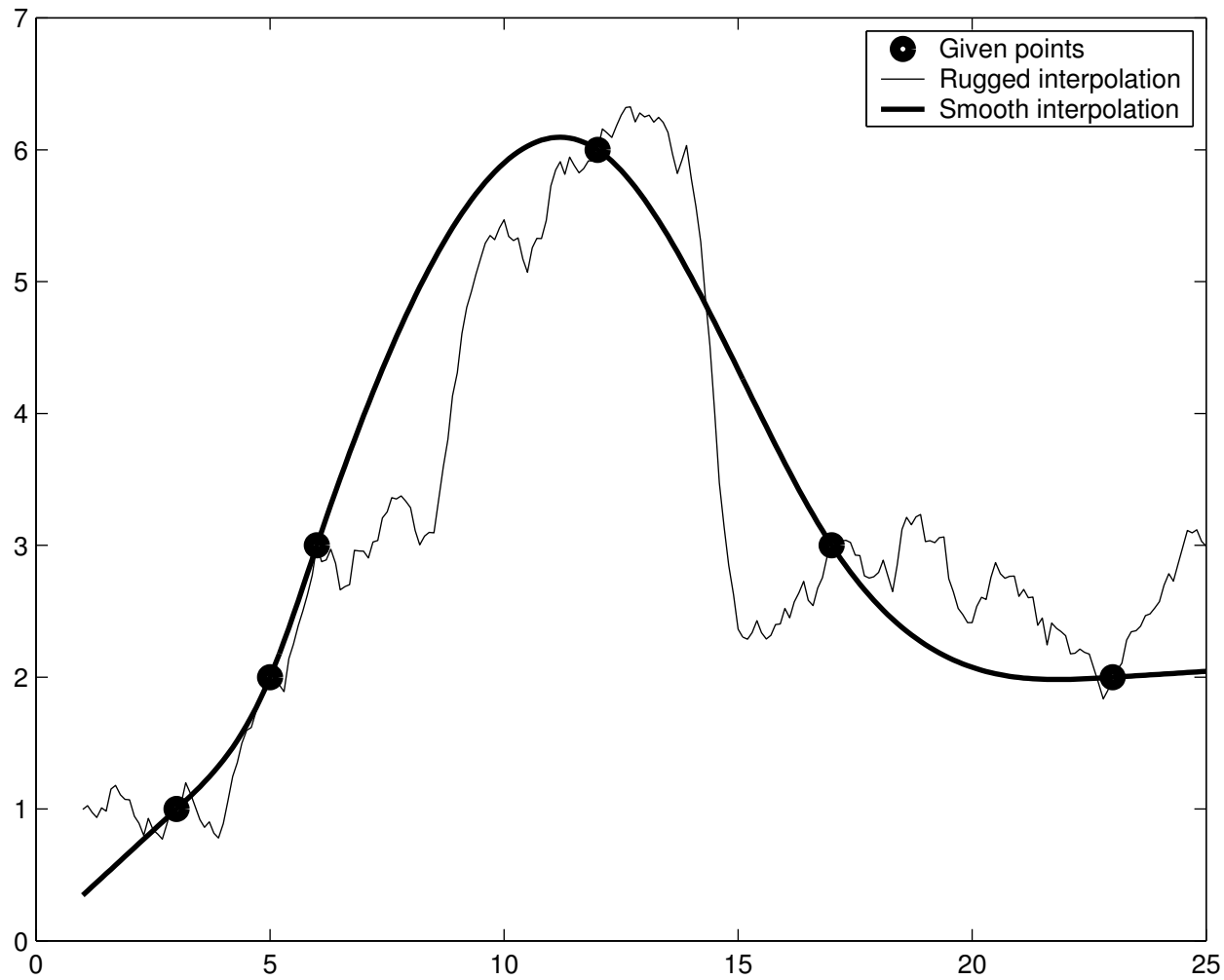
Find a function



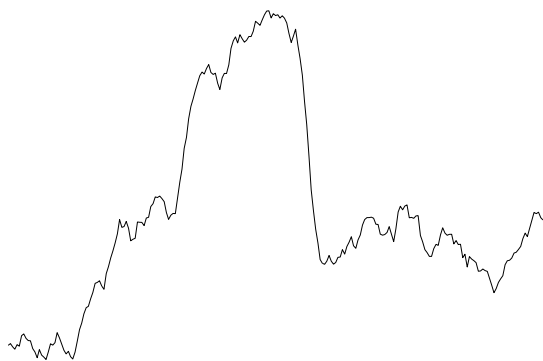
Find a function



Find a function

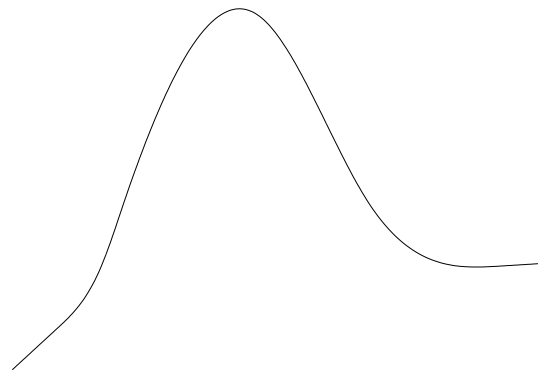


Rank functions



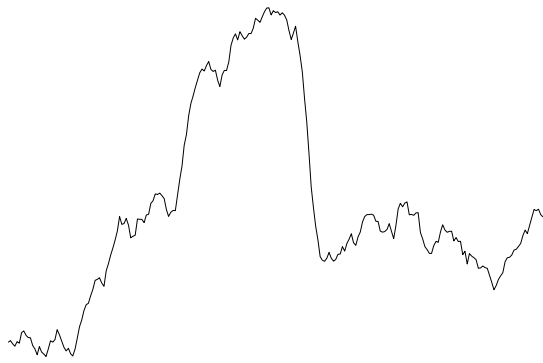
ugliest

...



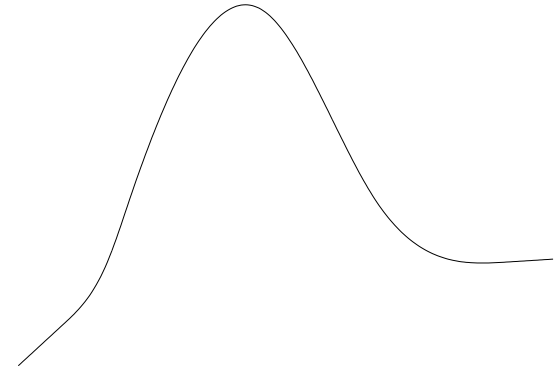
prettiest

Rank functions



ugliest

...



prettiest

Variational criterion



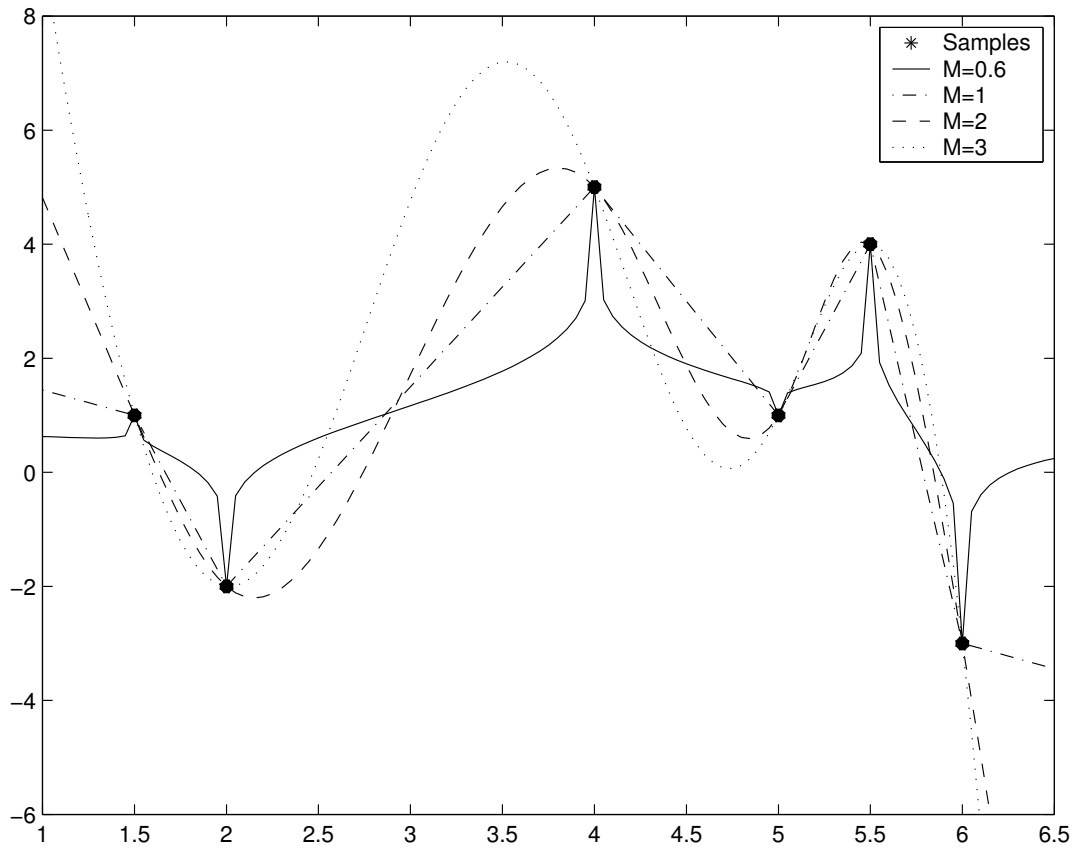
$$J : (\mathbb{R}^m \rightarrow \mathbb{R}^n) \rightarrow \mathbb{R}_0^+$$

$$J(\mathbf{f}) \geq 0$$

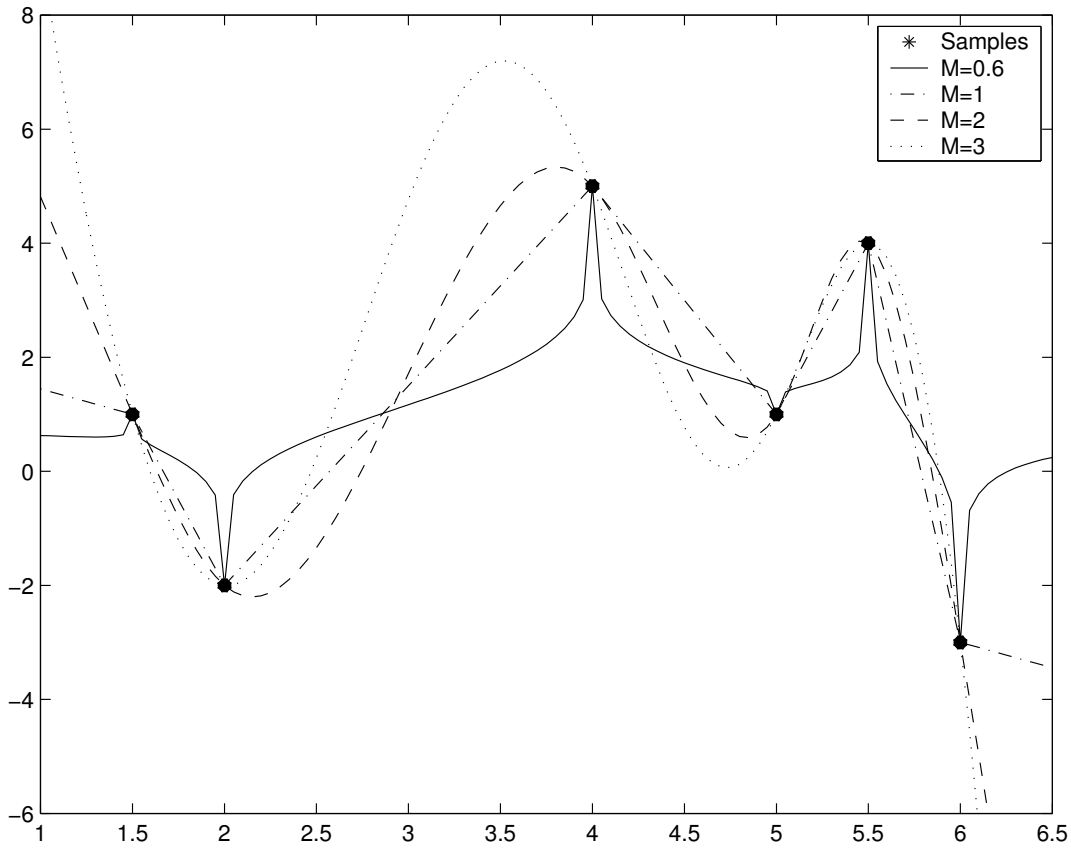
Variational reconstruction

Find the **best** function
satisfying the **constraints**.

Tunable 1D interpolation



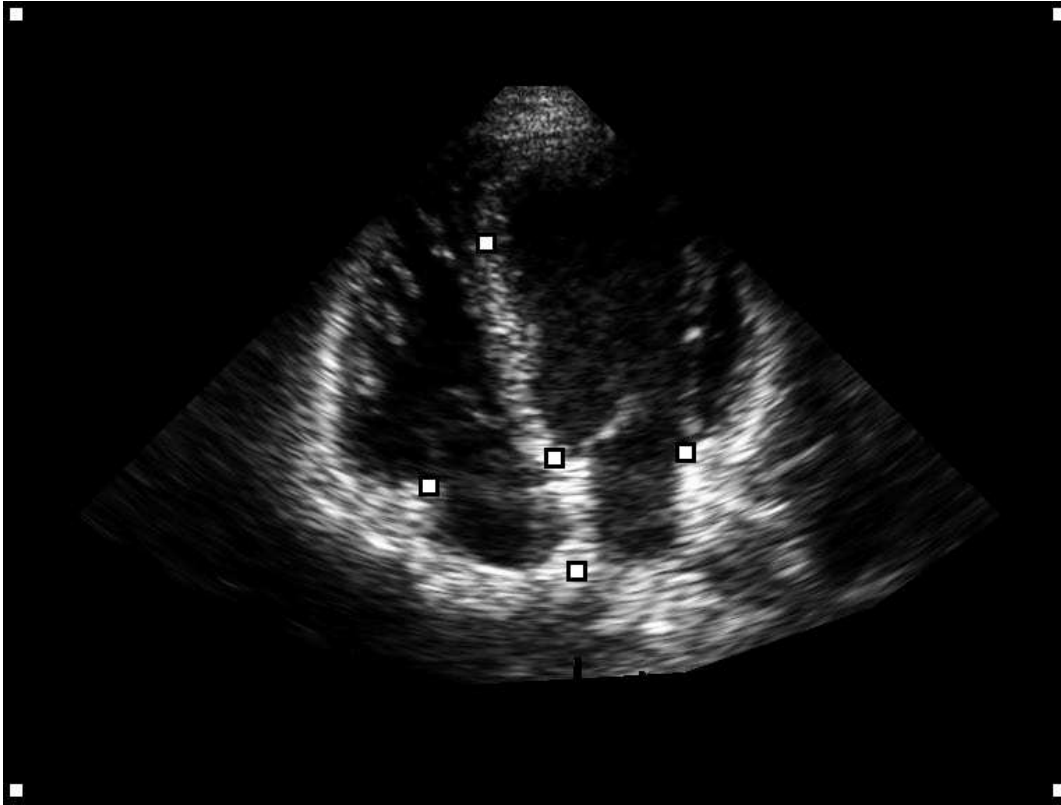
Tunable 1D interpolation



$$J(f) = \left\| \frac{\partial^M f}{\partial x^M} \right\|^2$$

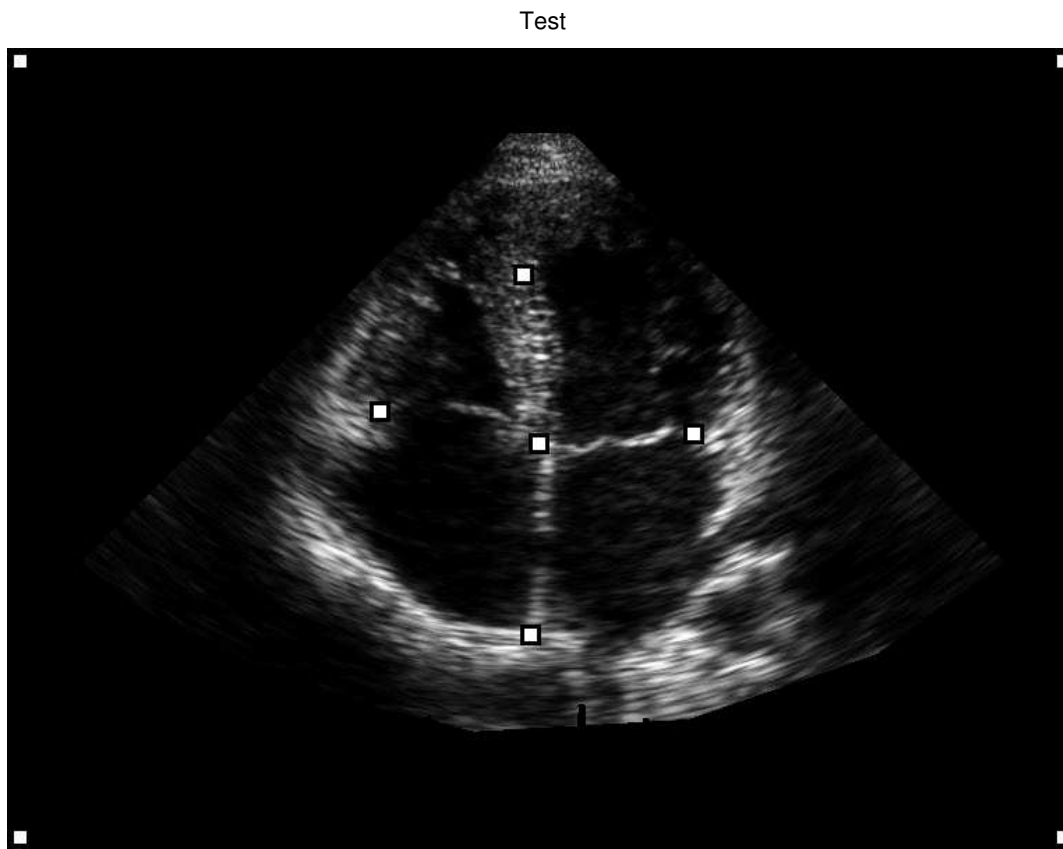
Tunable 2D interpolation

Reference



reference

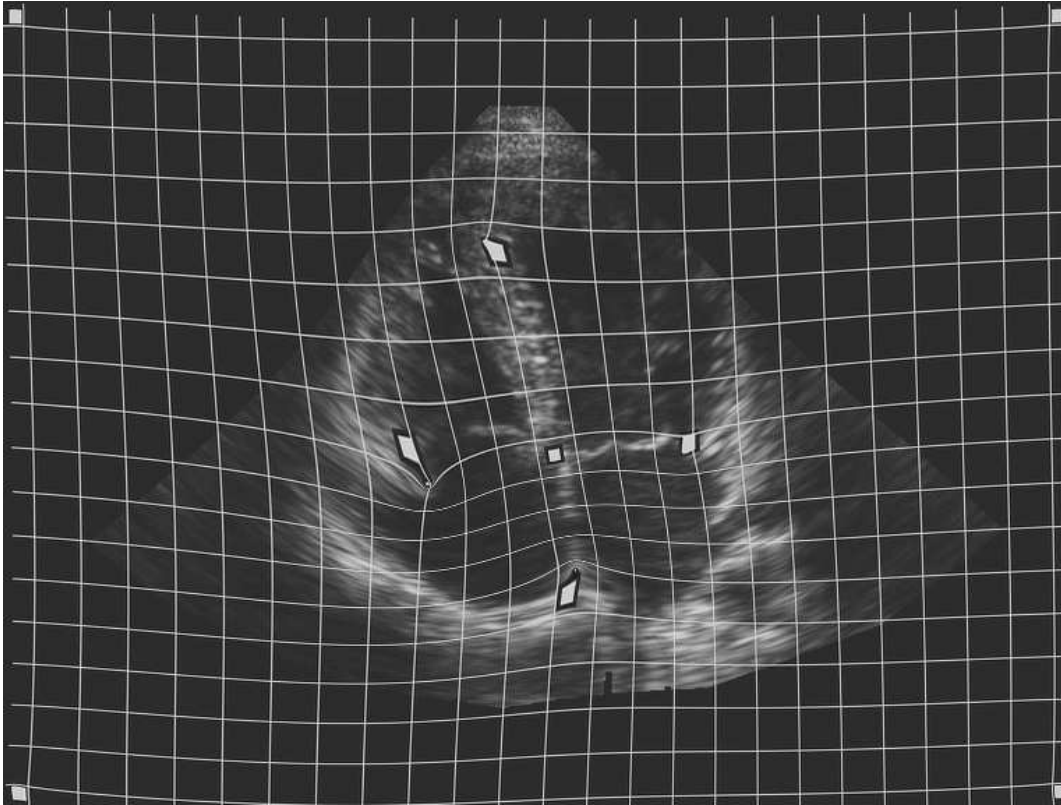
Tunable 2D interpolation



test

Tunable 2D interpolation

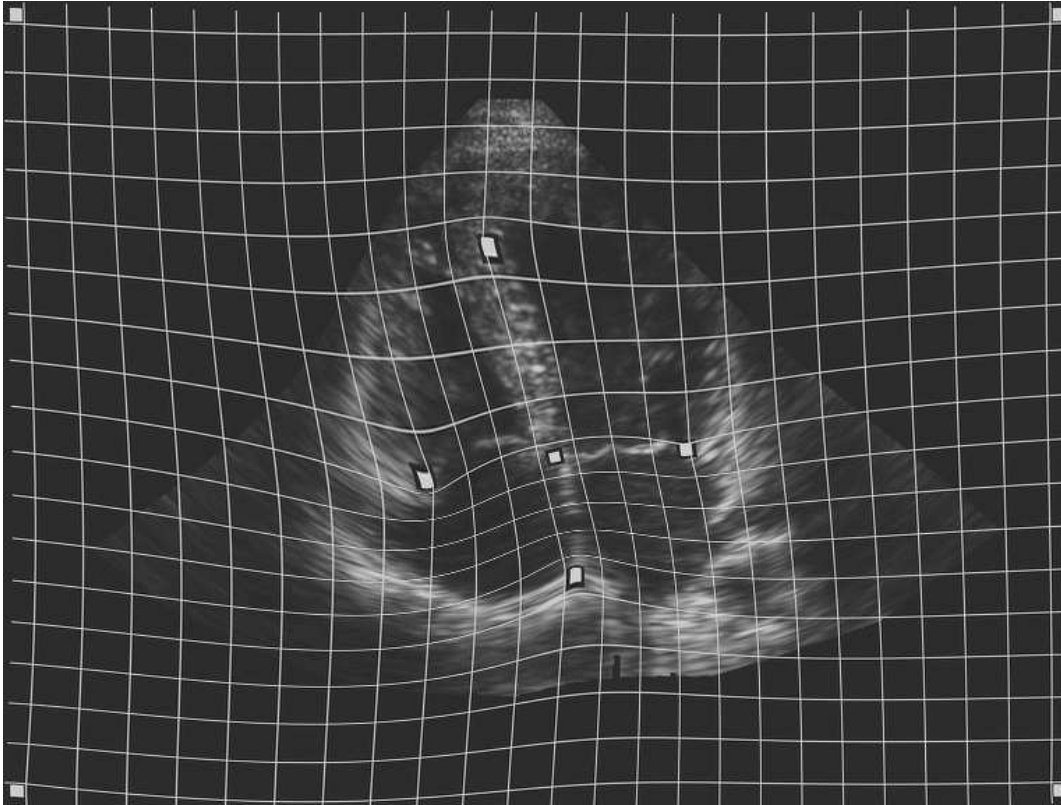
alpha=0.5



$$\int \|\nabla^{0.5} g(\mathbf{x})\|^2 d\mathbf{x}$$

Tunable 2D interpolation

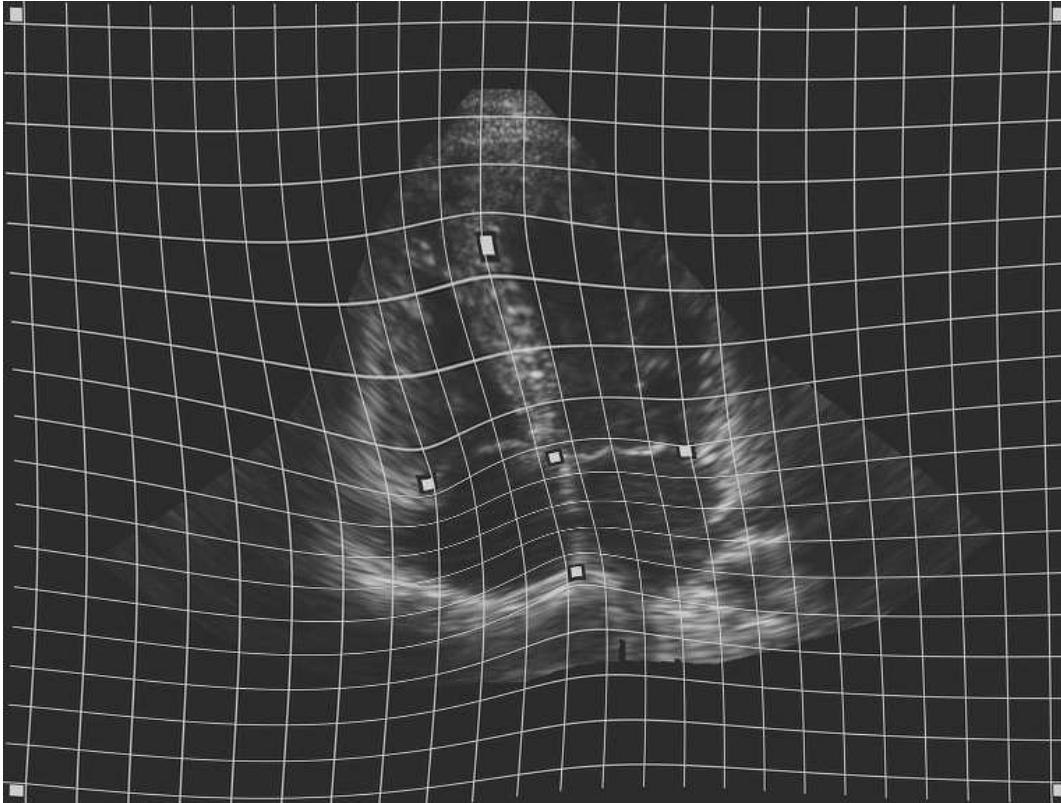
alpha=0.9



$$\int \|\nabla^{0.9} g(\mathbf{x})\|^2 d\mathbf{x}$$

Tunable 2D interpolation

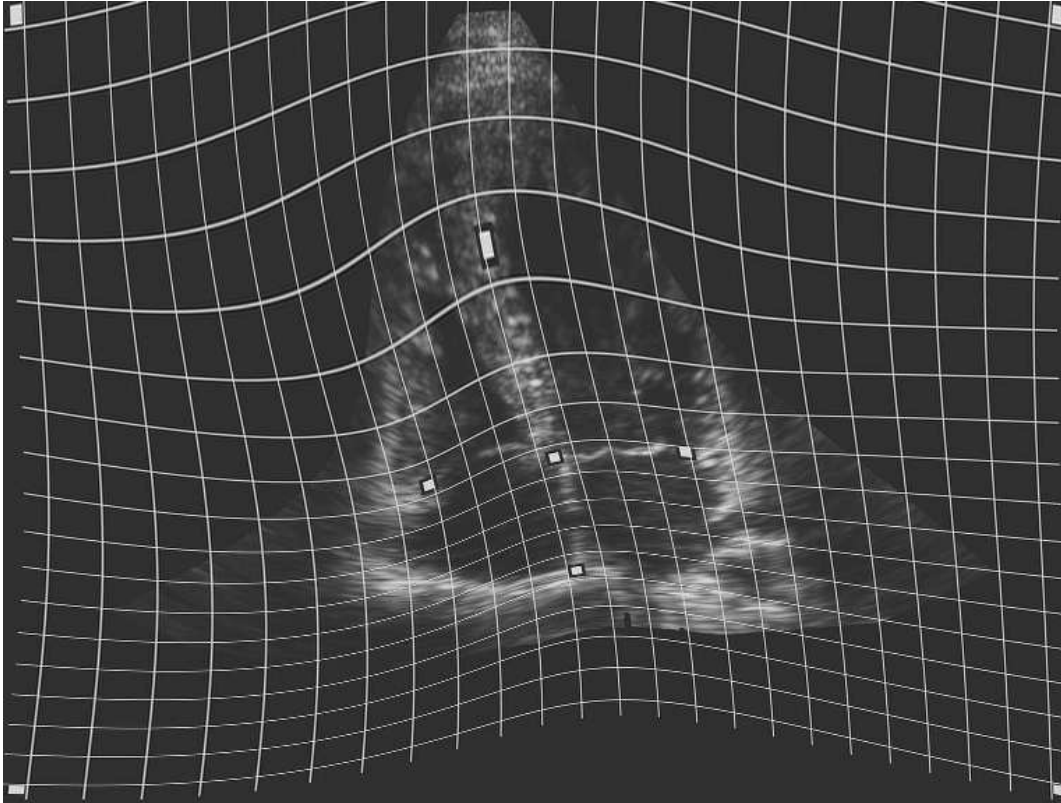
alpha=1.3



$$\int \|\nabla^{1.3} g(\mathbf{x})\|^2 d\mathbf{x}$$

Tunable 2D interpolation

alpha=2.5



$$\int \|\nabla^{2.5} g(\mathbf{x})\|^2 d\mathbf{x}$$

Splines, B-splines

The splines

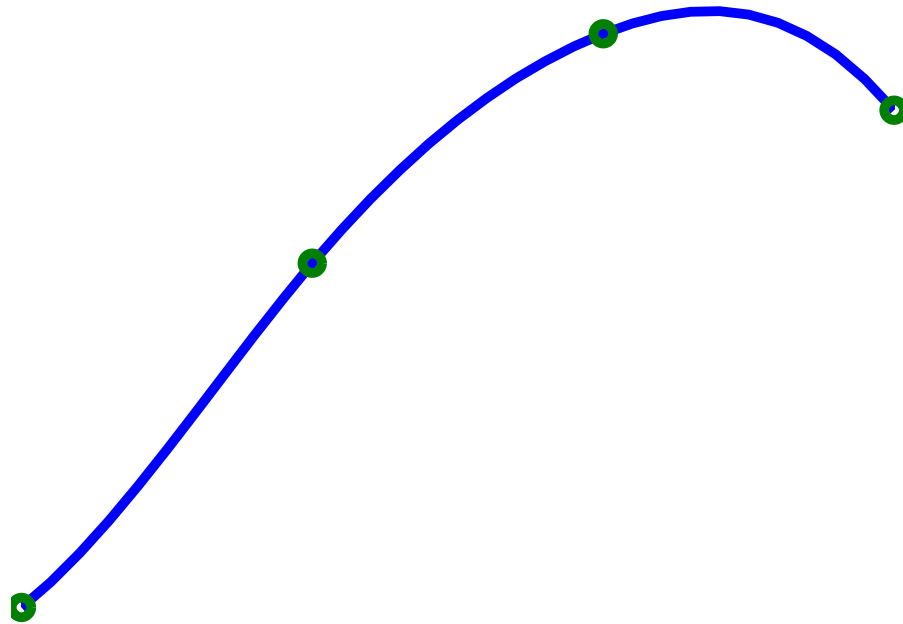
What are splines, anyway?

The splines

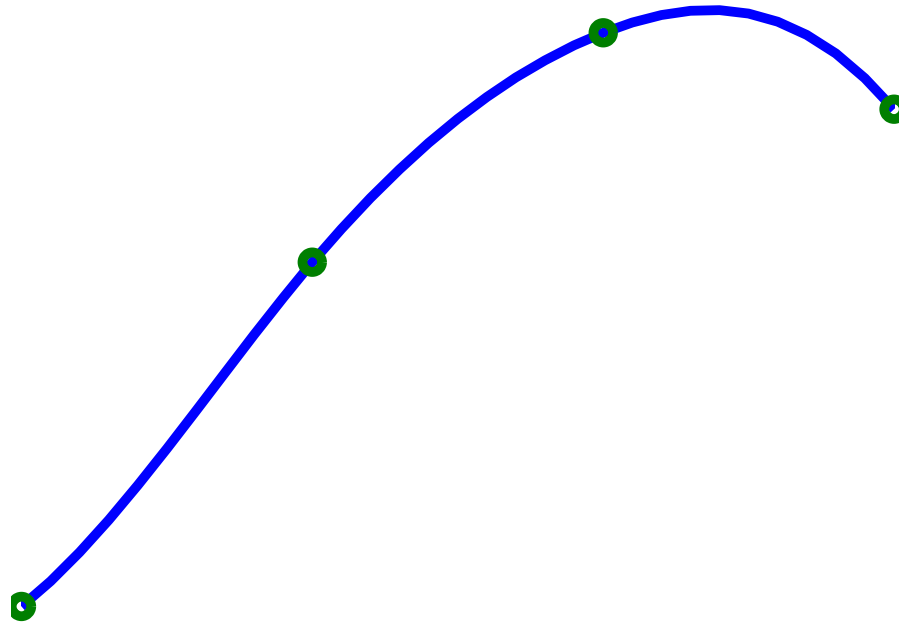
What are splines, anyway?

The best functions in the world!

(Uniform) splines



(Uniform) splines

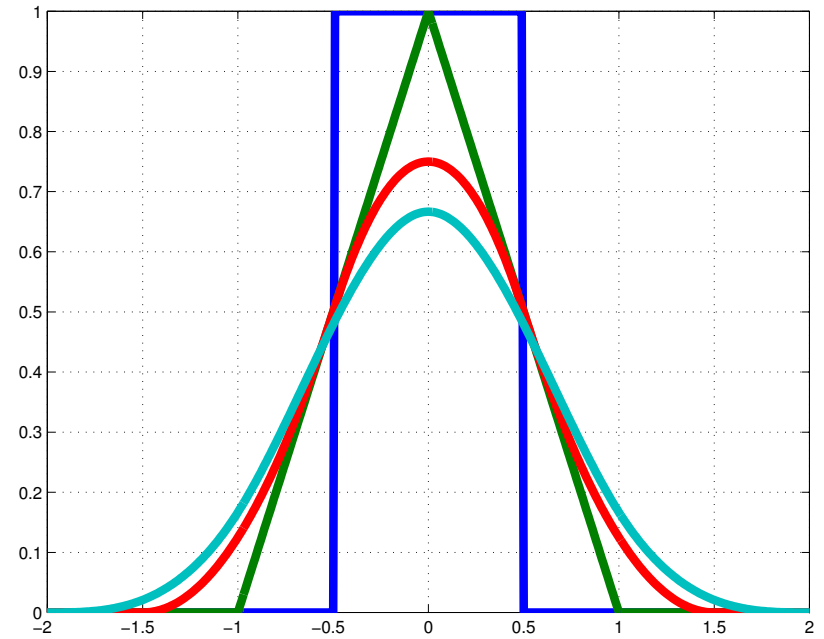


- Piecewise polynomial of degree n
- Continuous $(n - 1)^{\text{th}}$ derivative
- (Uniform) knots



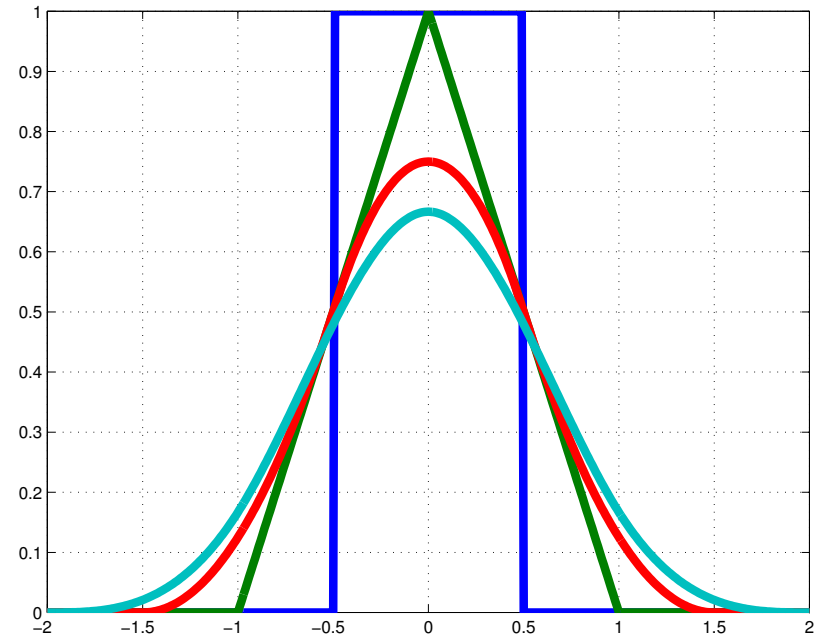
Uniform B-splines

Haar	β_0
linear	β_1
quadratic	β_2
cubic	β_3



Uniform B-splines

Haar β_0
linear β_1
quadratic β_2
cubic β_3



- Generation: $\beta_{n+1} = \beta_n * \beta_0$
- Basis for splines: $s(x) = \sum_i c_i \beta(x - i)$

Practical B-splines

- Separability → speed
- B-spline transform (finding coefficients) fast through IIR filtering
- Interpolation fast (small support)