

Frequency analysis in images

Fourier pair of a rectangle function

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Intro

- ◆ auxiliary introductory text
- ◆ excerpt from a longer [lecture](#)¹

¹<http://cmp.felk.cvut.cz/cmp/courses/EZS/Lectures/lininteg.pdf>

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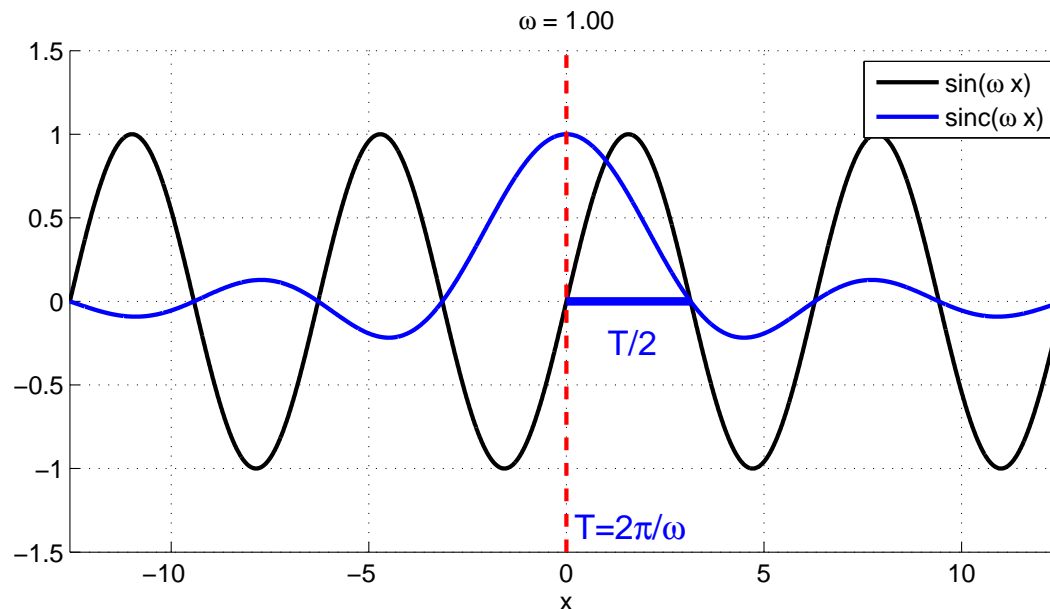
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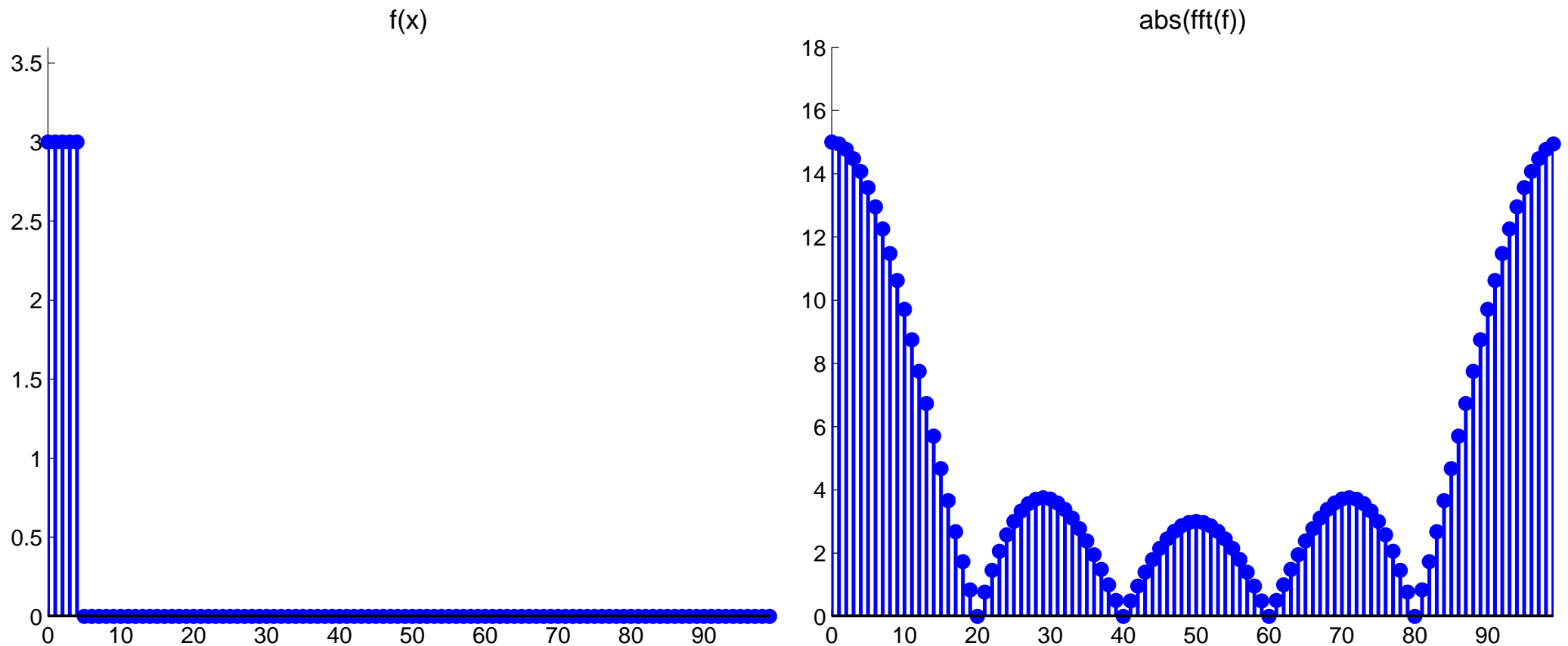
Fourier pair of rectangle function

$$\begin{aligned}
 F(u) &= \int_{-\infty}^{\infty} f(x) e^{-i2\pi ux} dx &&= \frac{A}{\pi u} e^{-i\pi Xu} \sin \pi Xu \\
 &= A \int_0^X e^{-i2\pi ux} dx &&= AX e^{-i\pi Xu} \frac{\sin \pi Xu}{\pi Xu} \\
 &= A \left[\frac{e^{-i2\pi ux}}{-i2\pi u} \right]_0^X &&= AX e^{-i\pi Xu} \operatorname{sinc}(\pi Xu) \\
 &= \frac{A}{-i2\pi u} (e^{-i2\pi Xu} - 1) && \\
 &= \frac{A}{-i2\pi u} (-e^{-i\pi Xu}) (e^{i\pi Xu} - e^{-i\pi Xu}) && \\
 &= \frac{A}{\pi u} e^{-i\pi Xu} \left(\frac{e^{i\pi Xu} - e^{-i\pi Xu}}{2i} \right) &&
 \end{aligned}$$

which implies

$$\|F(u)\| = AX \|\operatorname{sinc}(\pi Xu)\|$$

Fourier pair of discrete rectangle function



$A = 3, X = 5, M = 100$ makes $T/2$ of sinc equal to 20.

$$\mathcal{F}\{f_A(x) = A \text{ for } 0 \leq x \leq X\} = AX e^{-i\pi Xu} \text{sinc}\left(\frac{\pi X}{M}u\right)$$

Remember that $T = 2\pi/\omega$, hence

$$X = \frac{2M}{T}$$

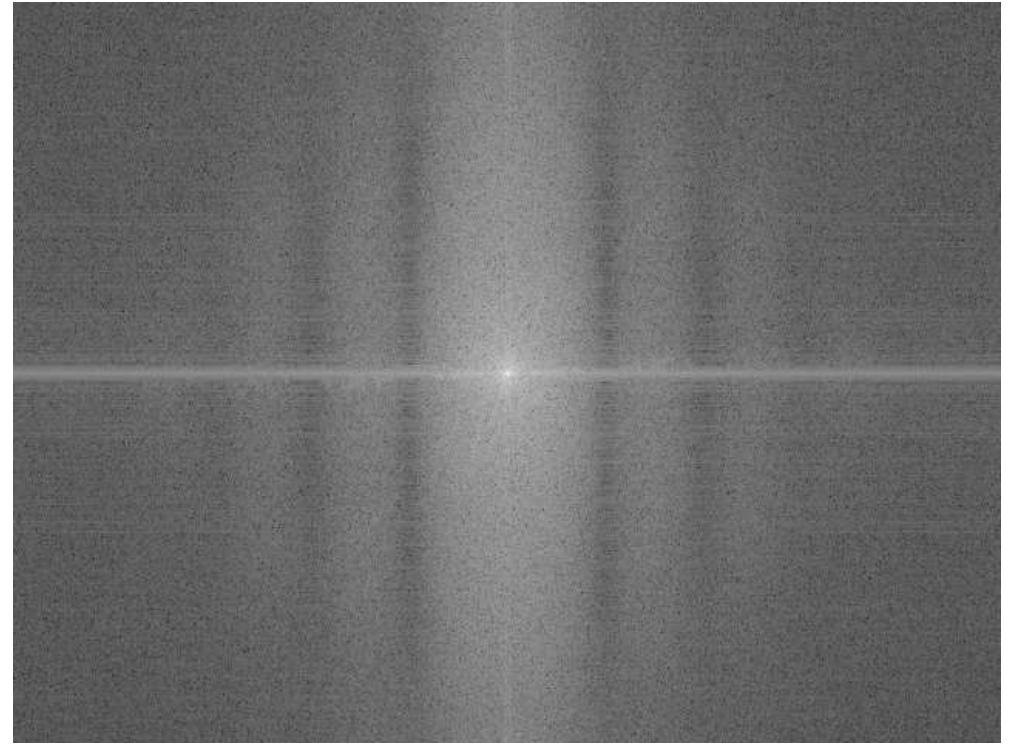
Use of Rectangle function — Image Restoration



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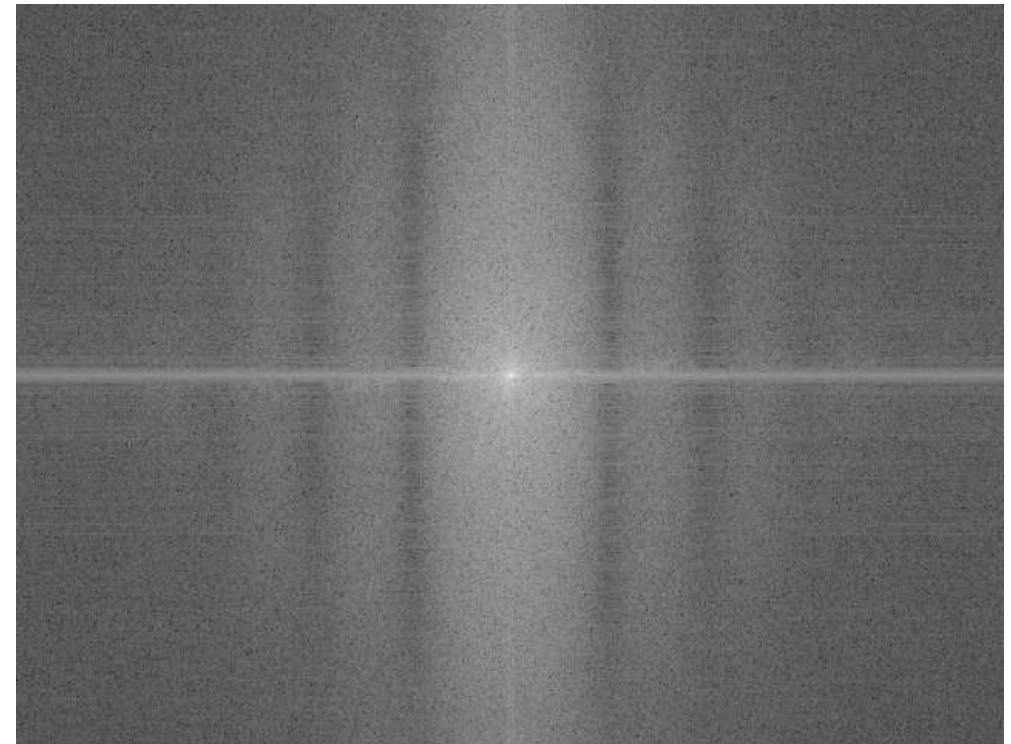
spectrum of blurred image



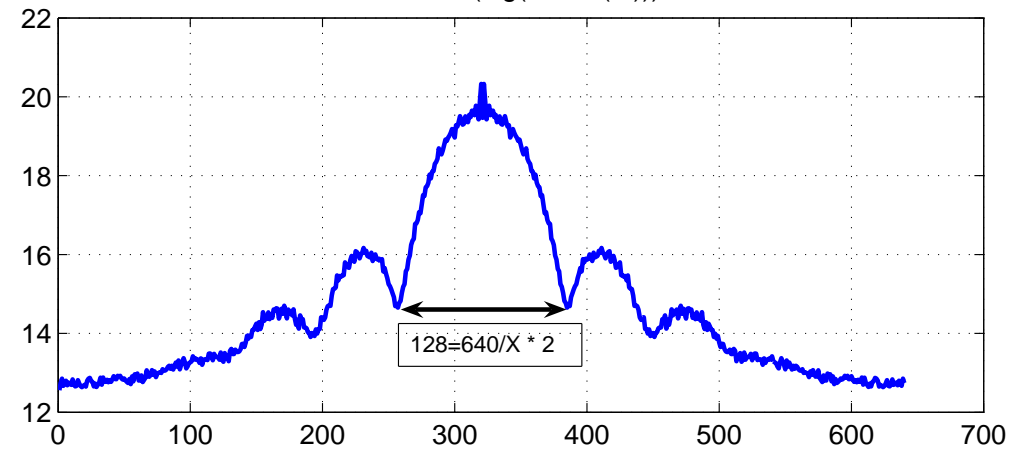
Use of Rectangle function — Image Restoration



spectrum of blurred image



median(log(1+abs(fft)))



Use of Rectangle function — Image Restoration



Blurred image.

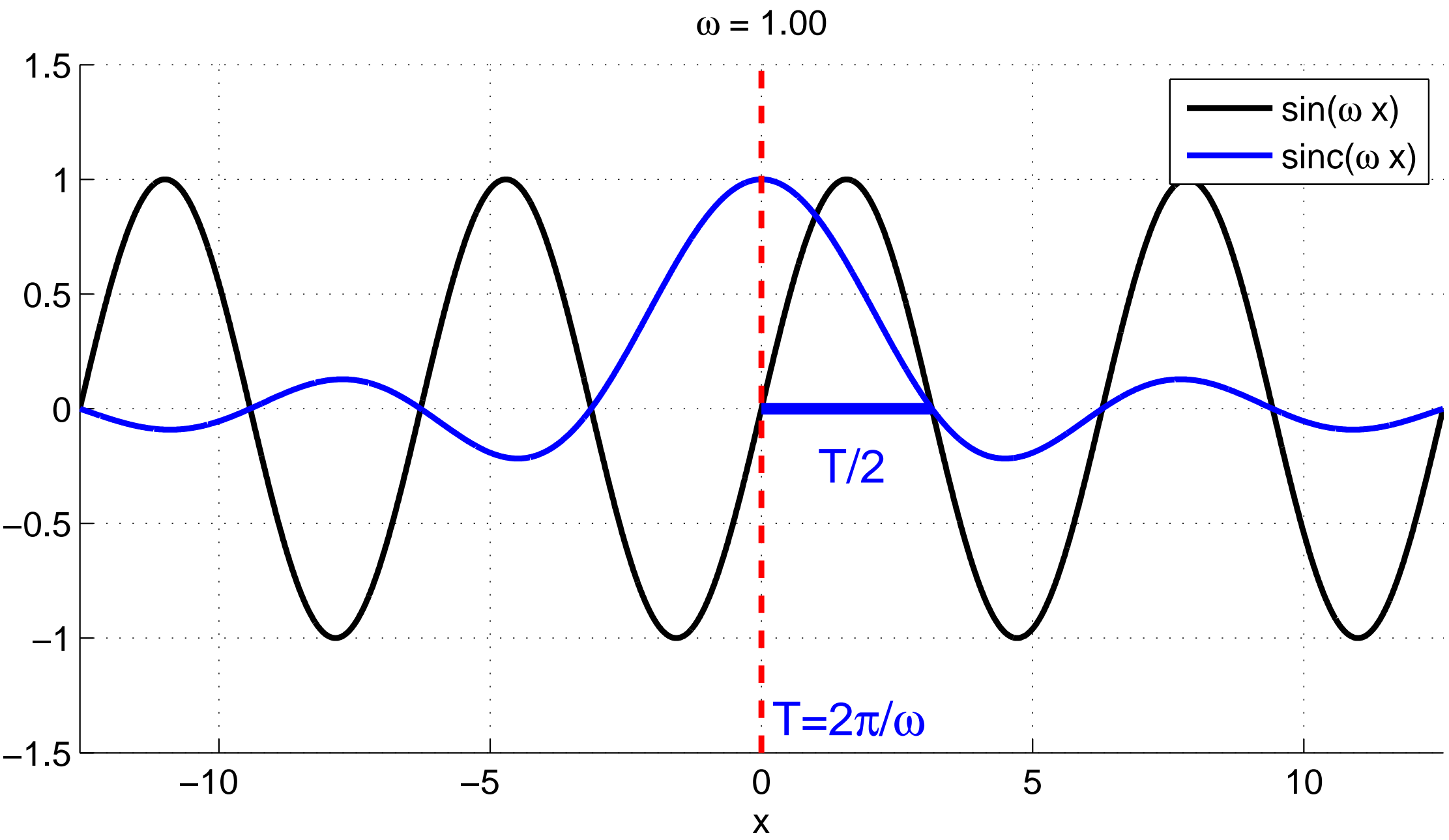
Use of Rectangle function — Image Restoration

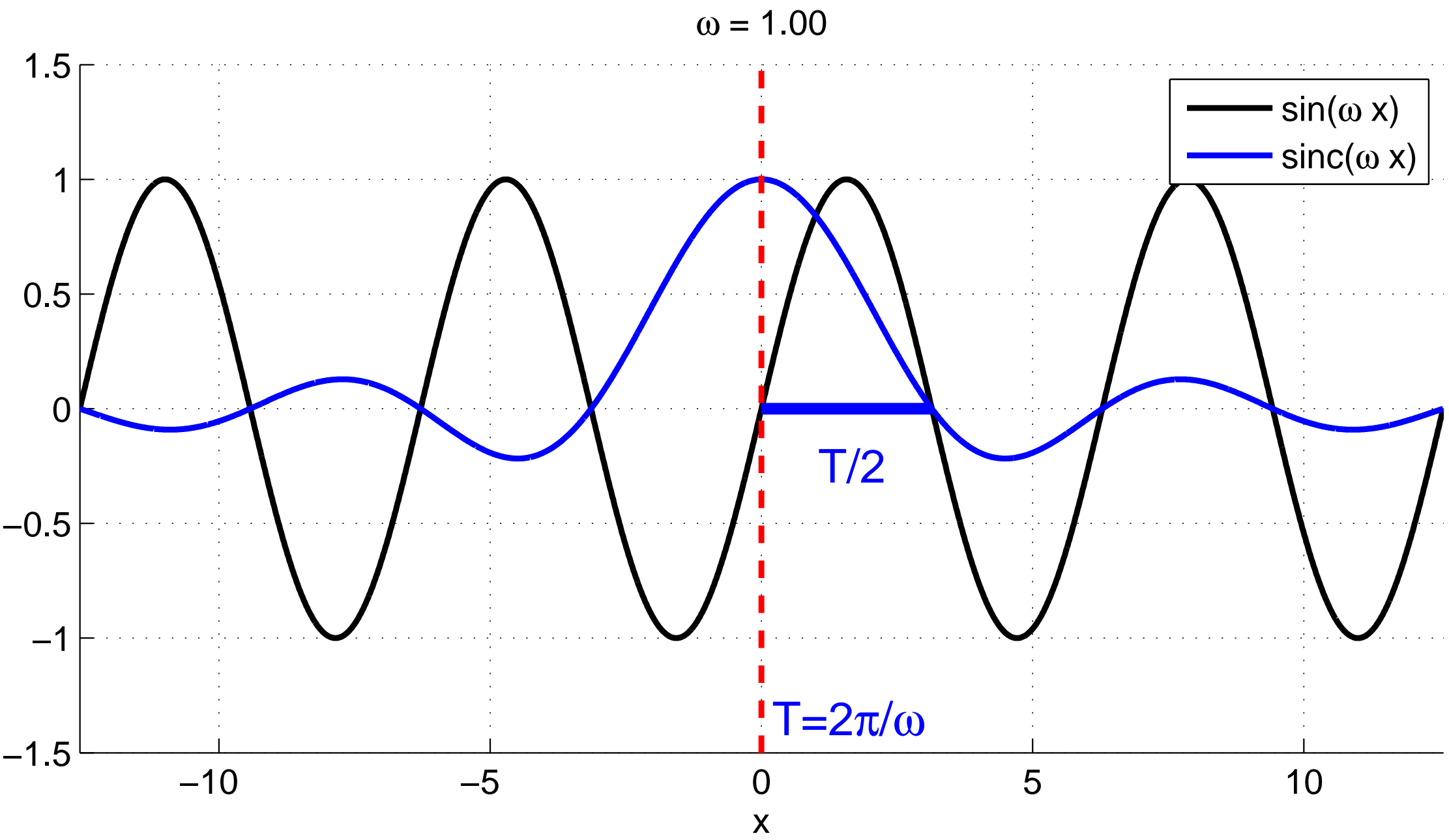


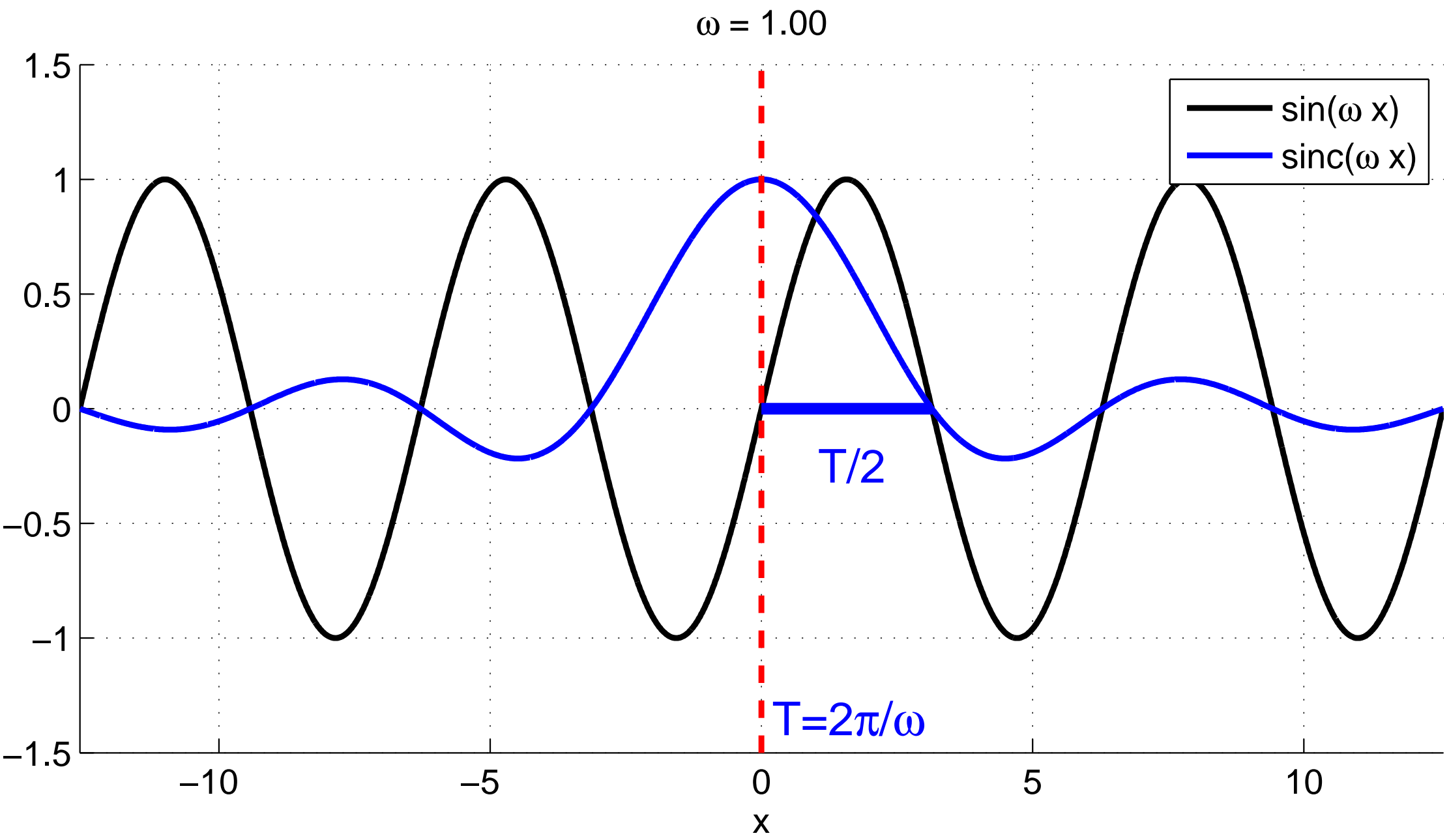
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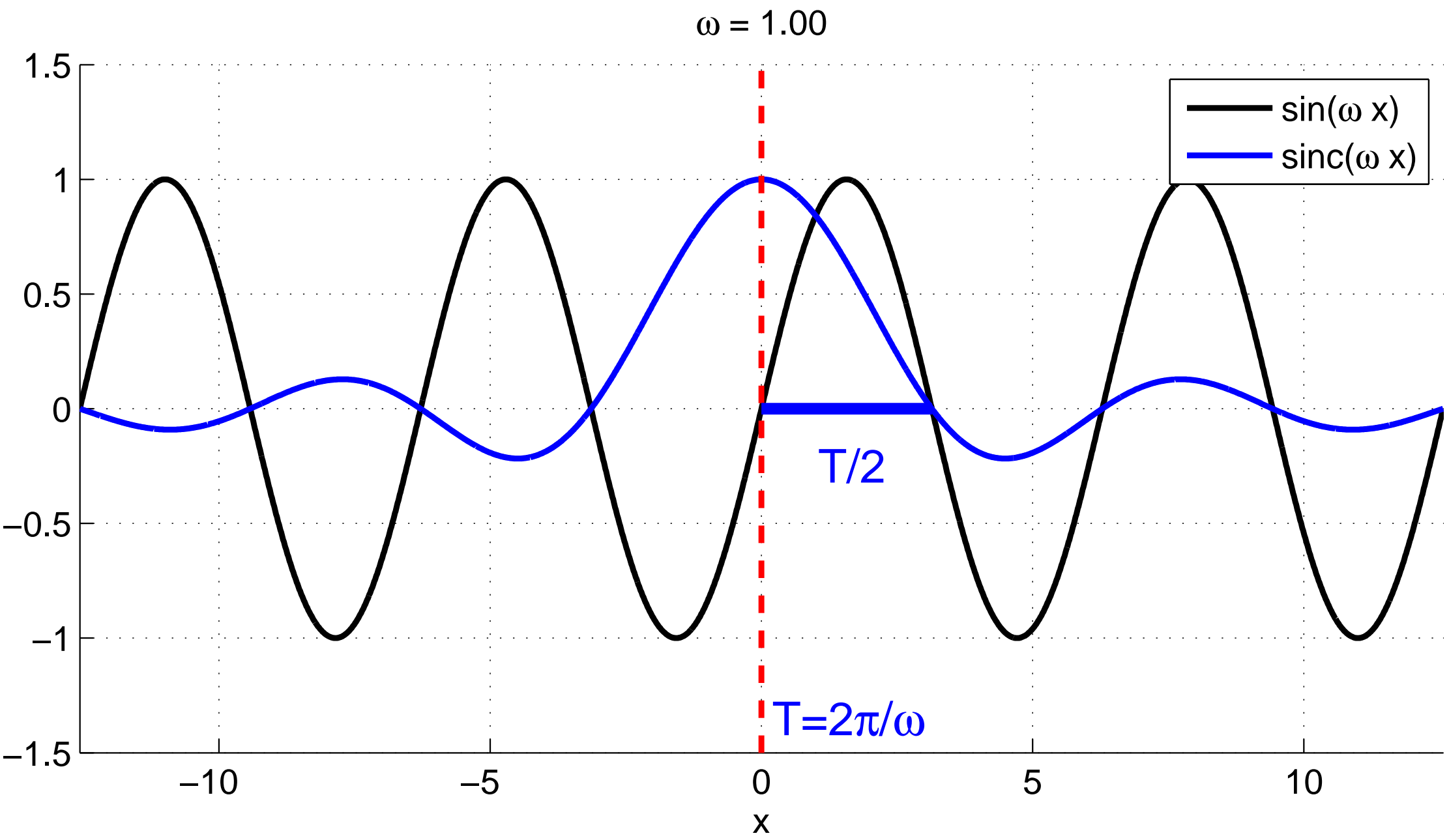


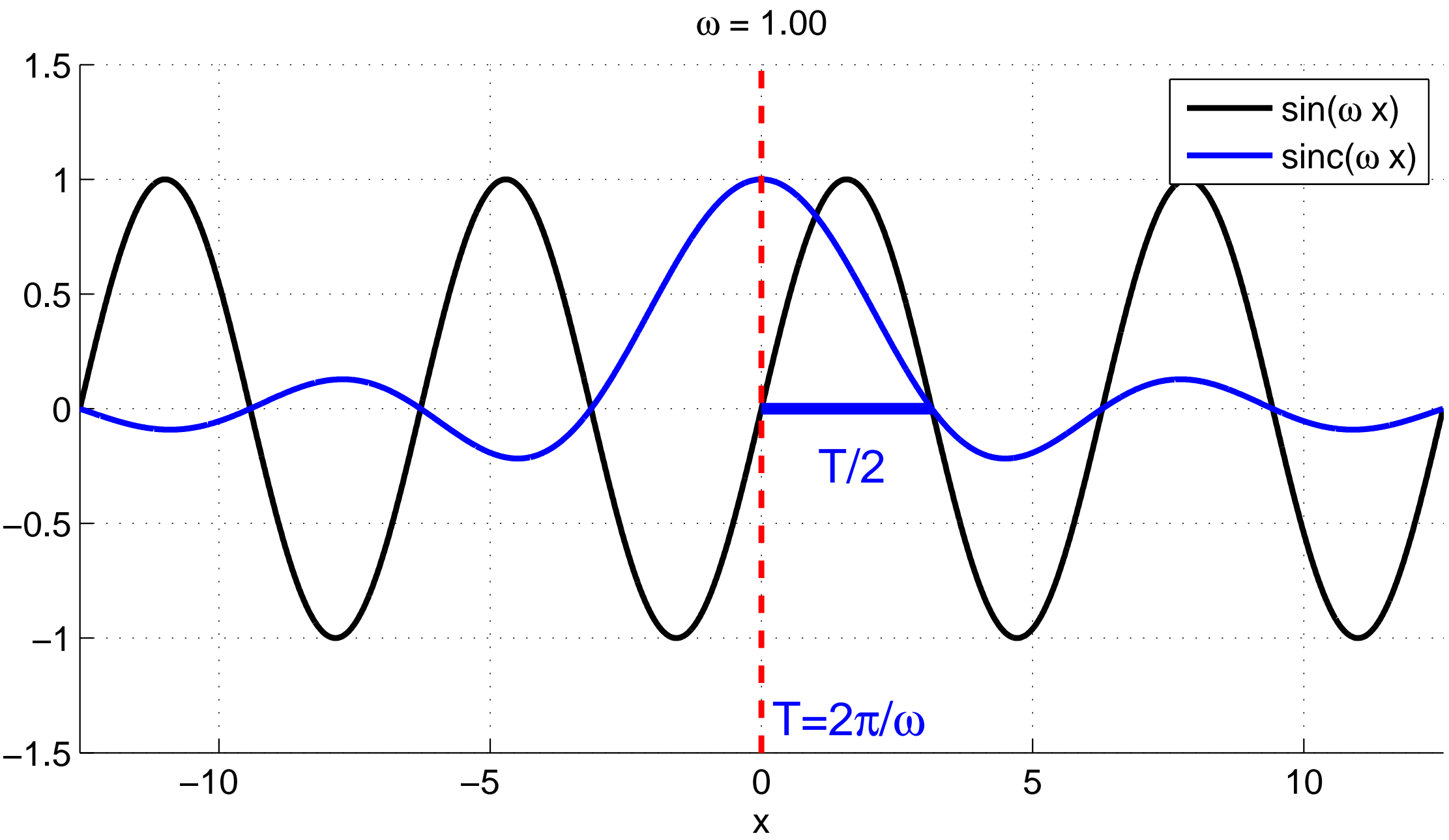
Restored image.



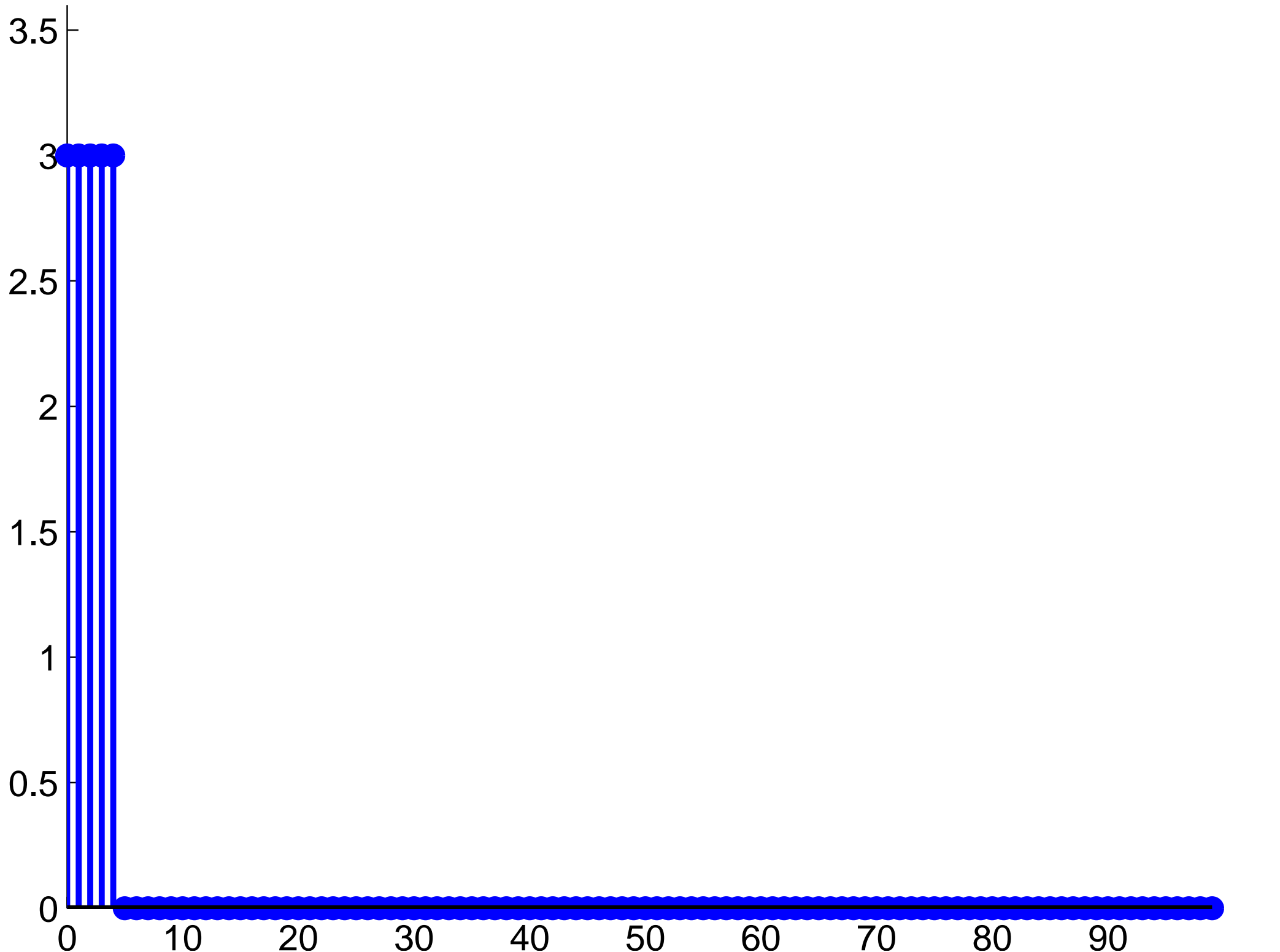




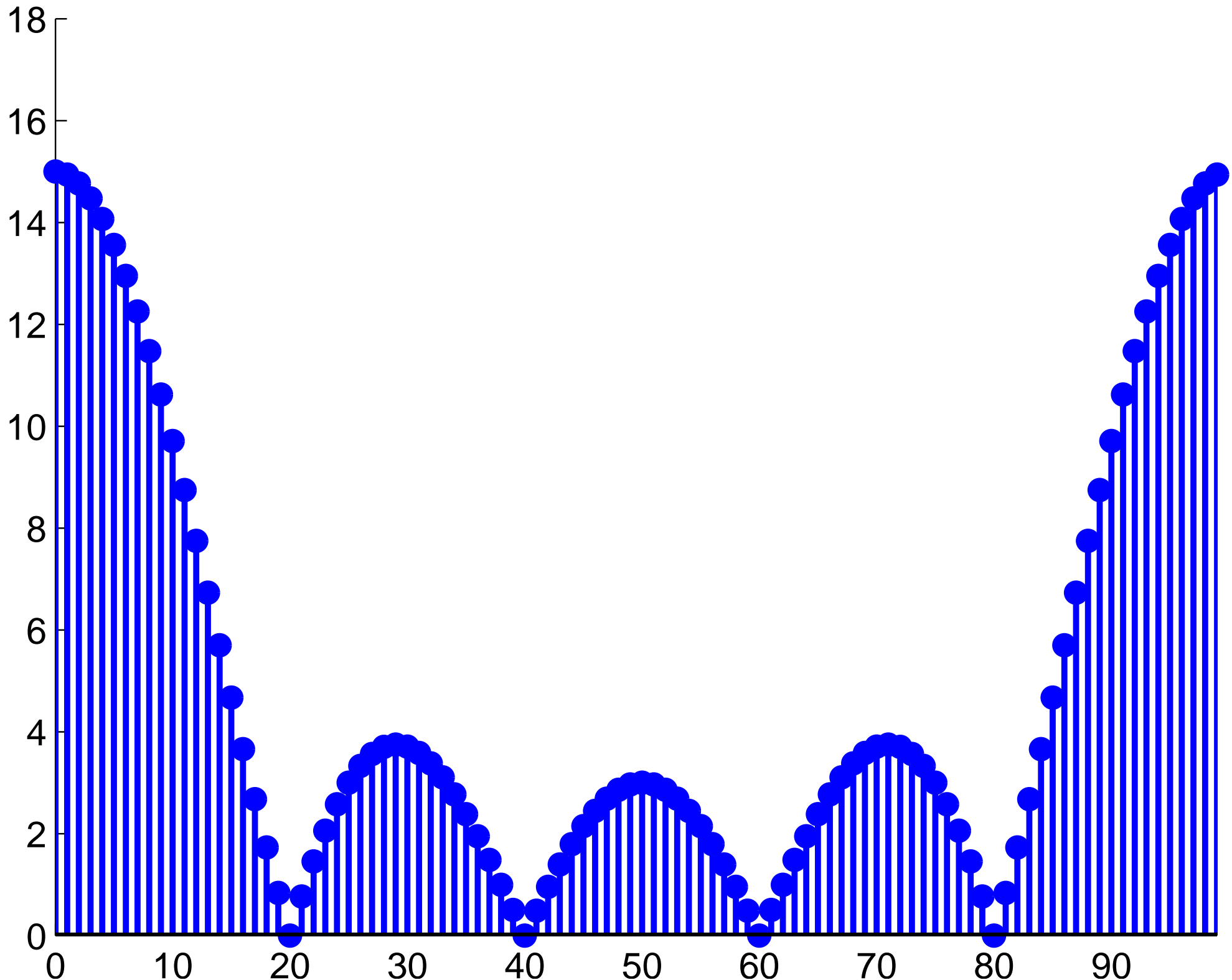




$f(x)$

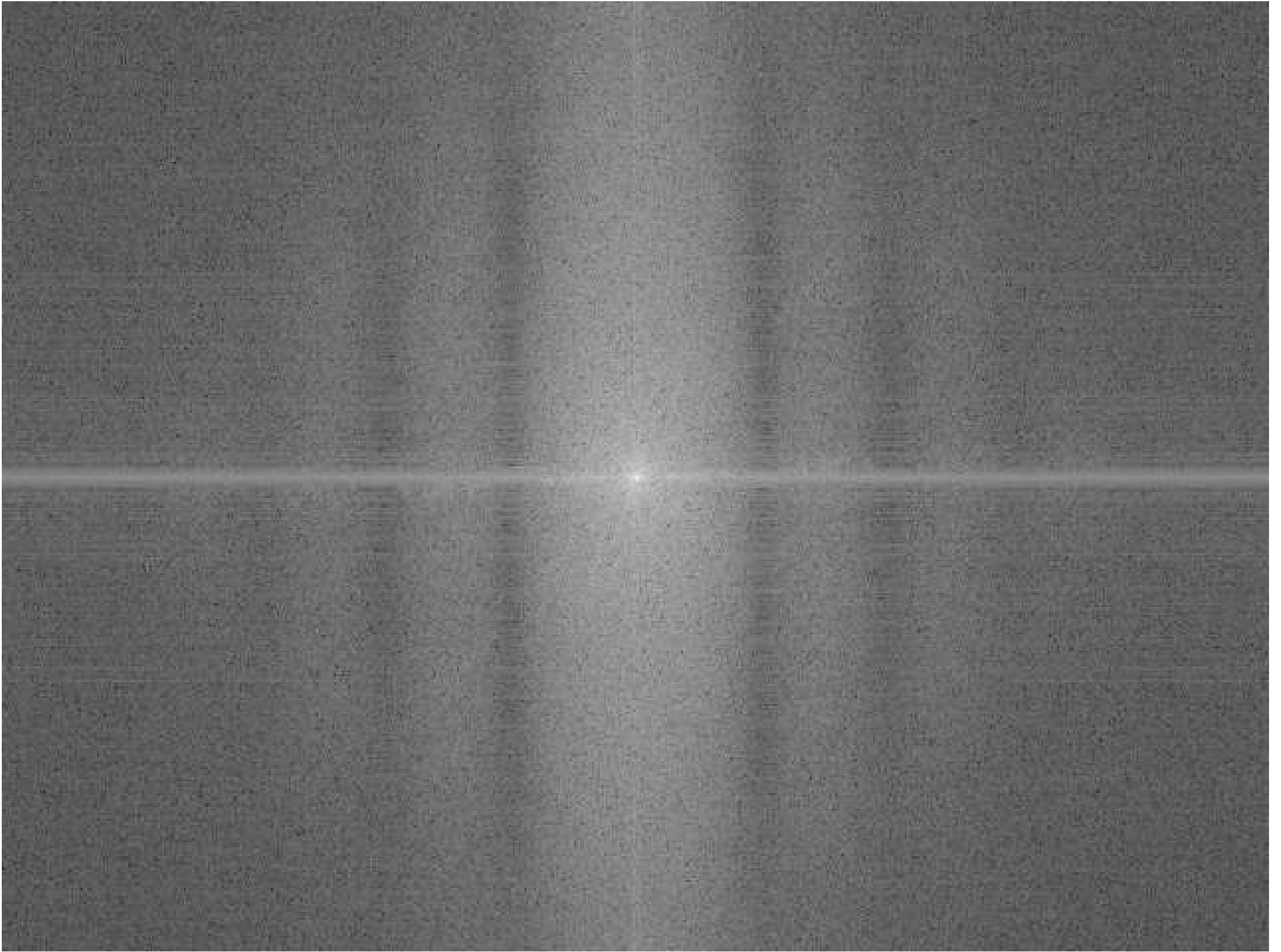


abs(fft(f))

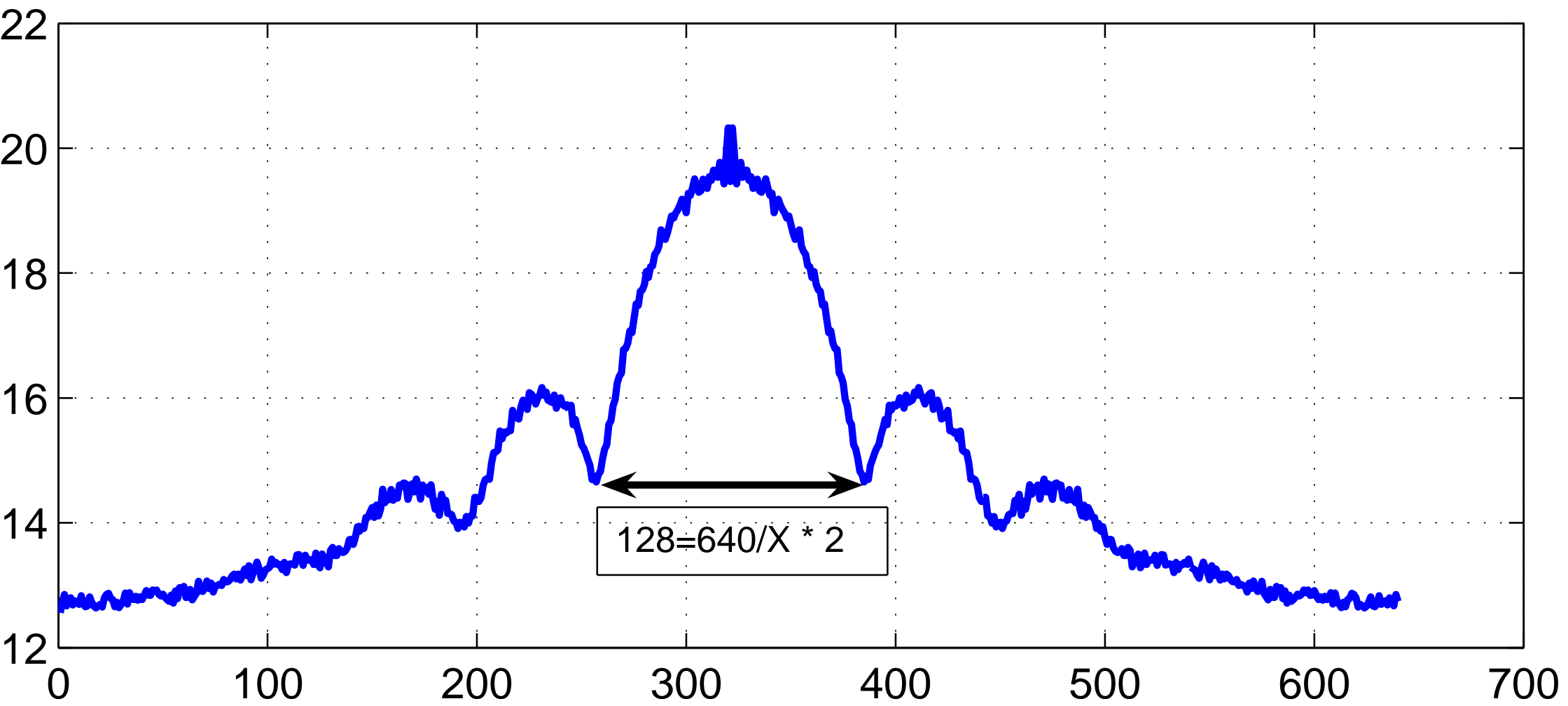




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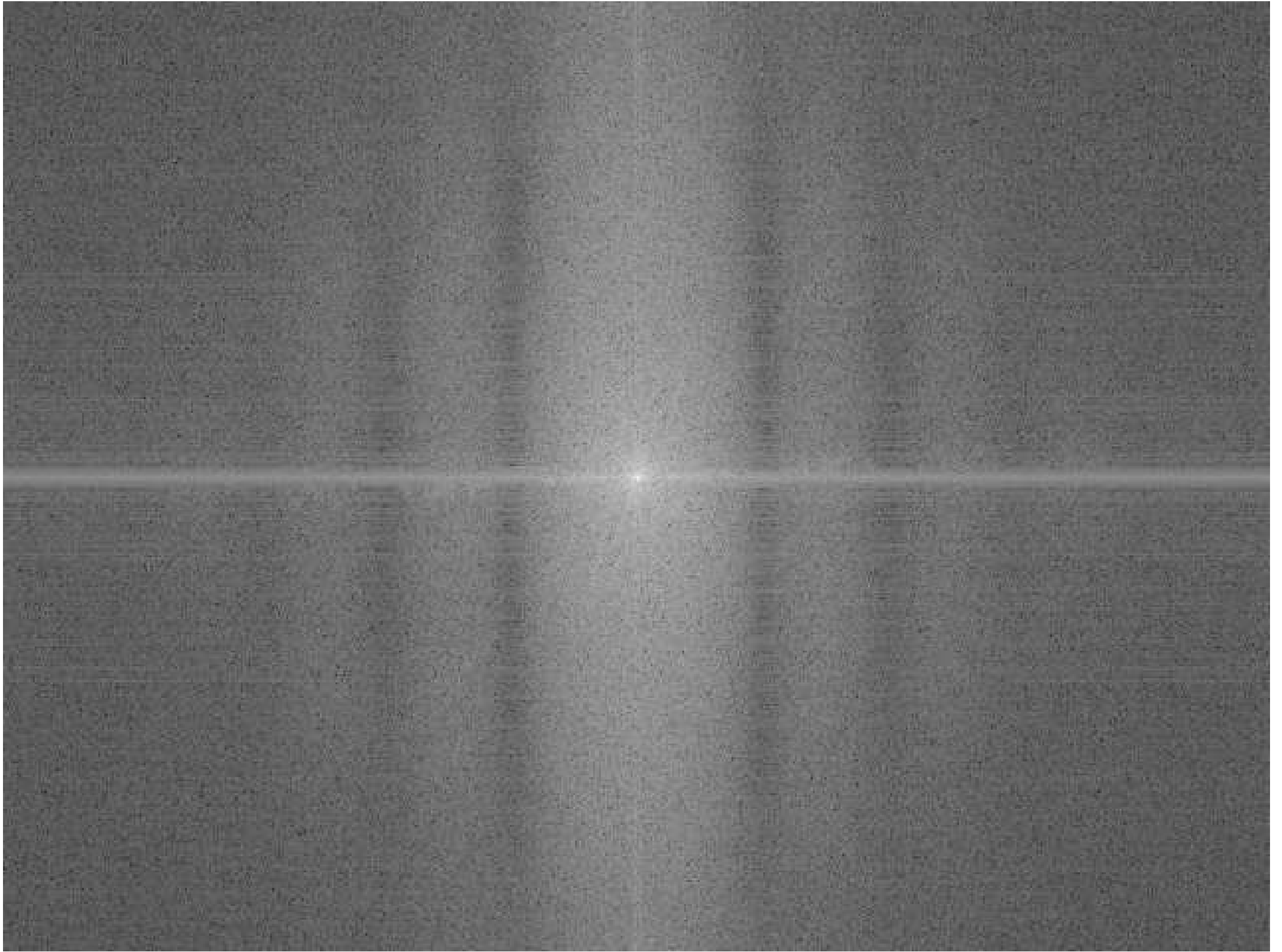


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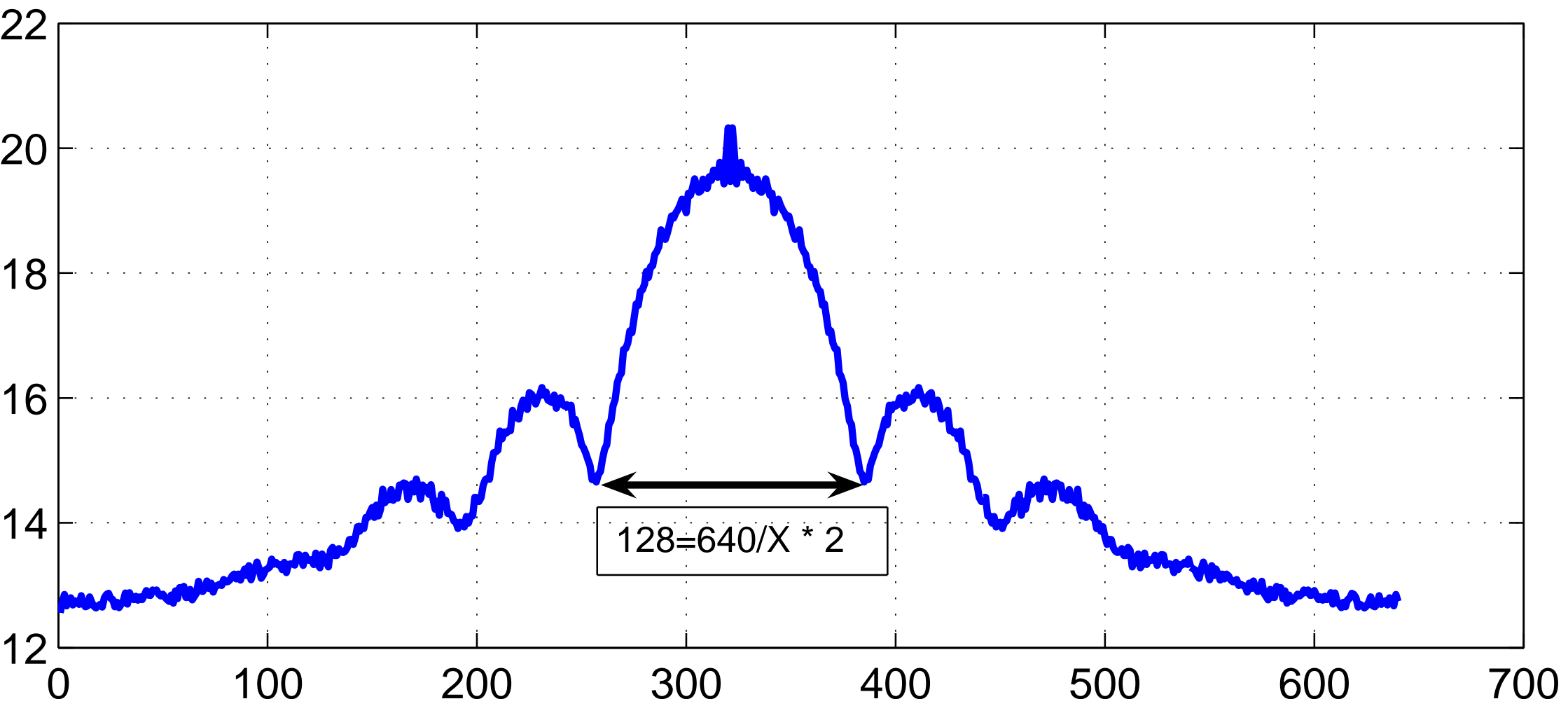




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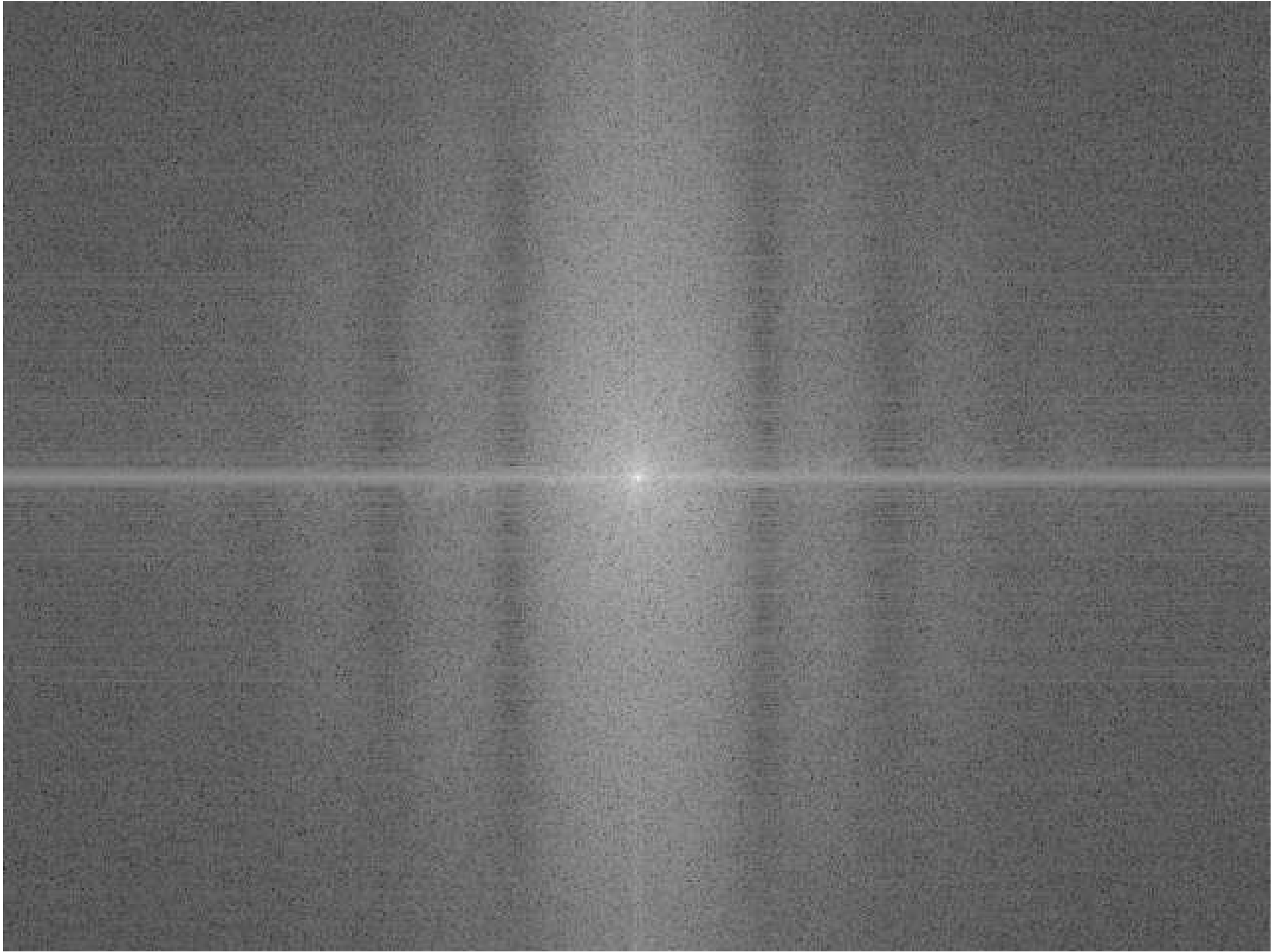


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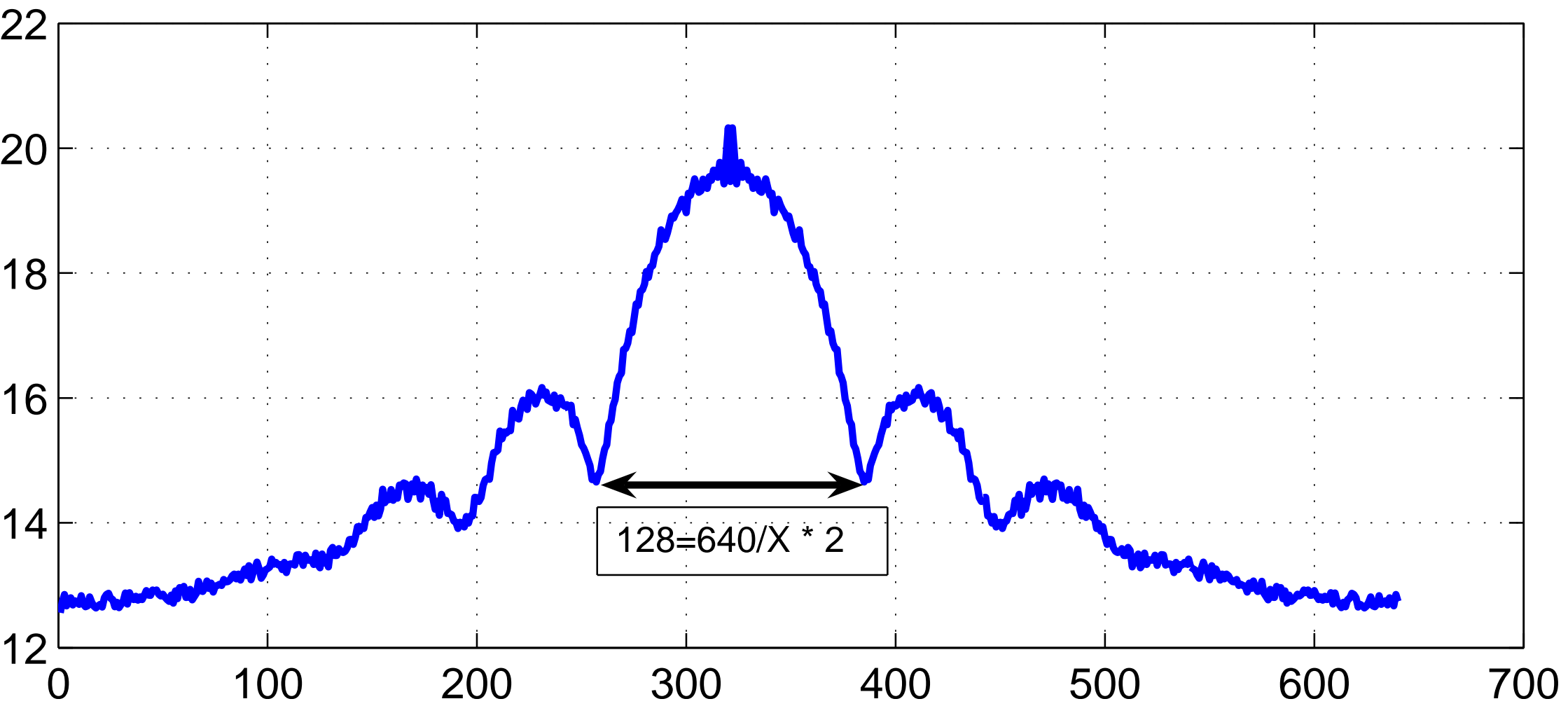




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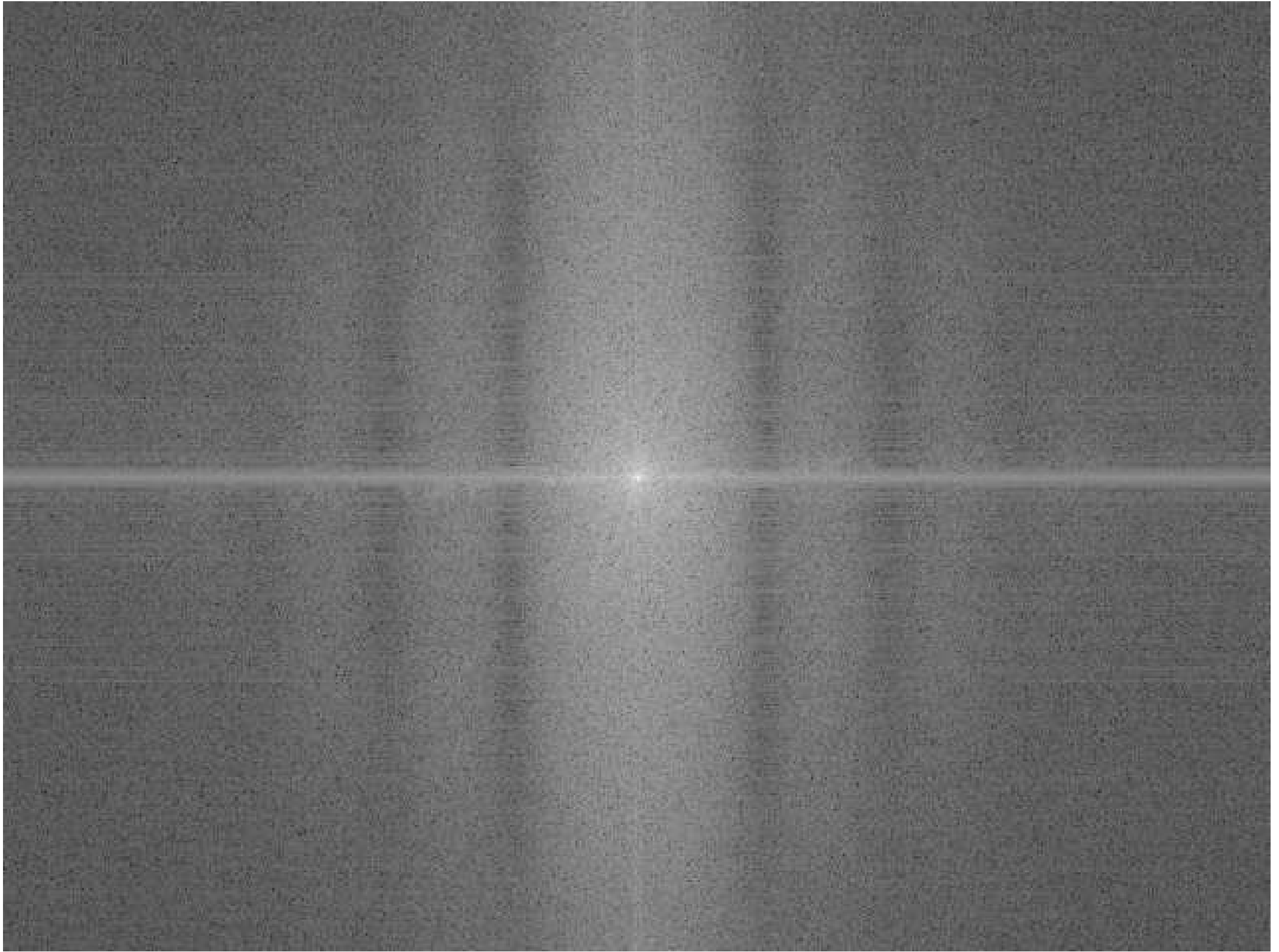


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