Image preprocessing in spatial domain

convolution, convolution theorem, cross-correlation

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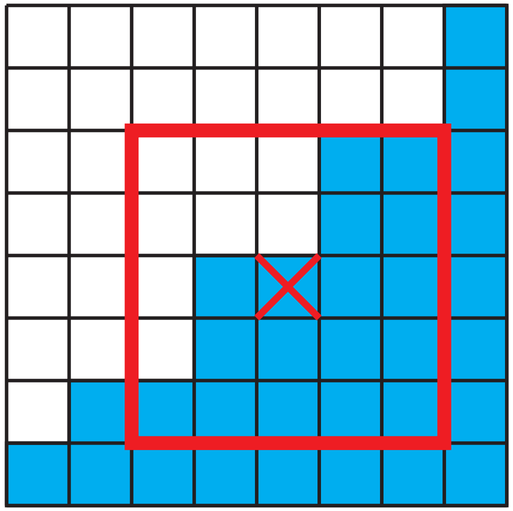
Spatial processing—idea

Replace a value of the image function (pixel) by a new one computed from the immediate neighbourhood.



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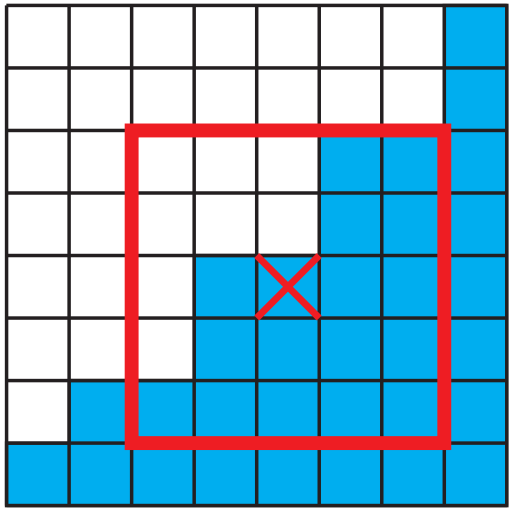

What is it good for?

- spatial relationships are important in images
- may be faster than a frequency filter
- more natural formulation in some problems
- robust statistics may be applied

Noise in images

- deterioration of analog signal
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- typically, the smaller active surface, the more noise

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How to suppress noise?

- digital only, ie. no A/D and D/A conversion. $\longrightarrow$ OK
- larger chips $\longrightarrow$ EXPENSIVE, EXPENSIVE LENSES
- cooled cameras (astronomy) $\longrightarrow$ SLOW, EXPENSIVE
- (local) image preprocessing

Example scene

Sample video[1] from a static camera

[1] http://cmp.felk.cvut.cz/cmp/courses/EZS/Demos/noise_in_camera.avi

Statistical point of view

Suppose we can acquire N images of the same scene. For each pixels we obtain N results $x_i, i = 1 \ldots N$. Assume:

- observations independent
- each x_i has $\mathrm{E}\{x_i\} = \mu$ and $\mathrm{var}\{x_i\} = \sigma^2$

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- Expectation: $\mathsf{E}\{s_N\} = \frac{1}{N}\sum_1^N \mathsf{E}\{x_i\} = \mu$
- Variance: We know that $\mathrm{var}\{x_i/N\} = \mathrm{var}\{x_i\}/N^2$, thus

$$\mathrm{var}\{s_N\} = \frac{\mathrm{var}\{x_1\}}{N^2} + \frac{\mathrm{var}\{x_2\}}{N^2} + \ldots + \frac{\mathrm{var}\{x_N\}}{N^2} = \frac{\sigma^2}{N} .$$

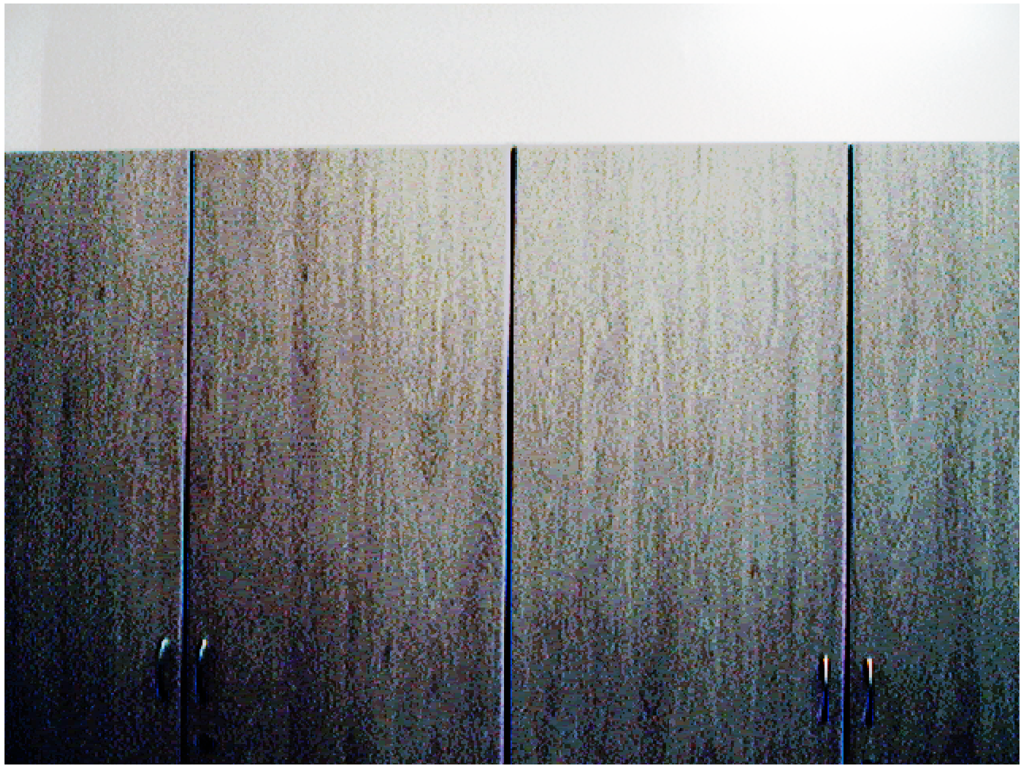
which means that standard deviation of s_N decreases as $\frac{1}{\sqrt{N}}$.

Example

a noisy image

average from $\approx$ 60 observations.

Example — equalized



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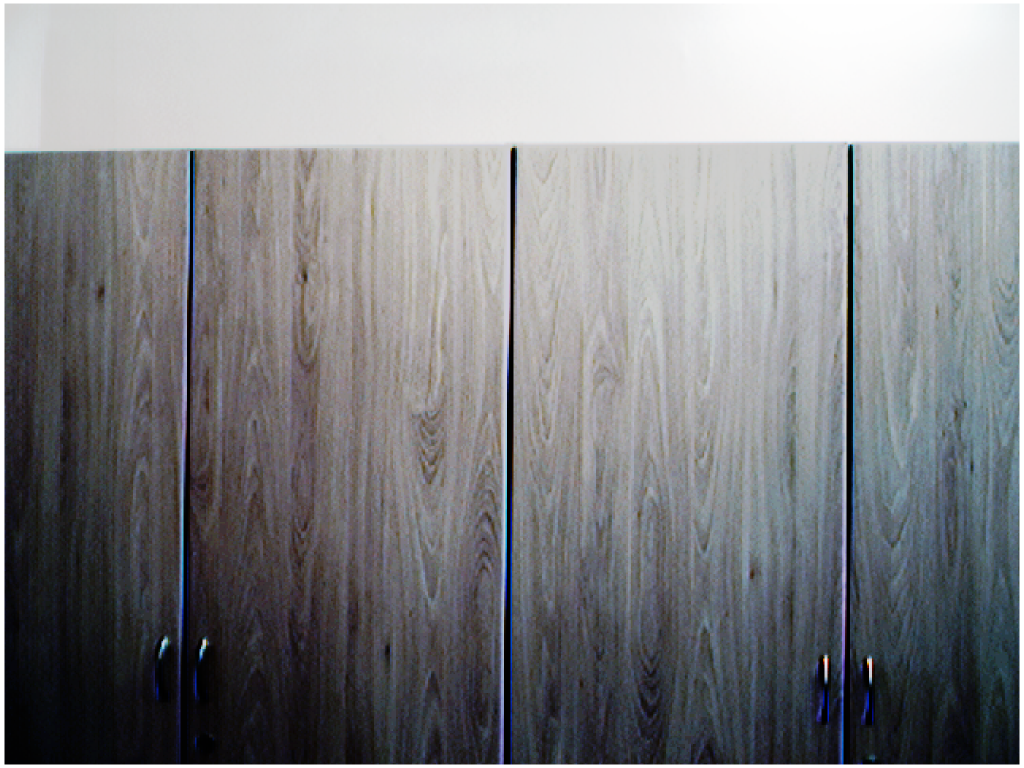

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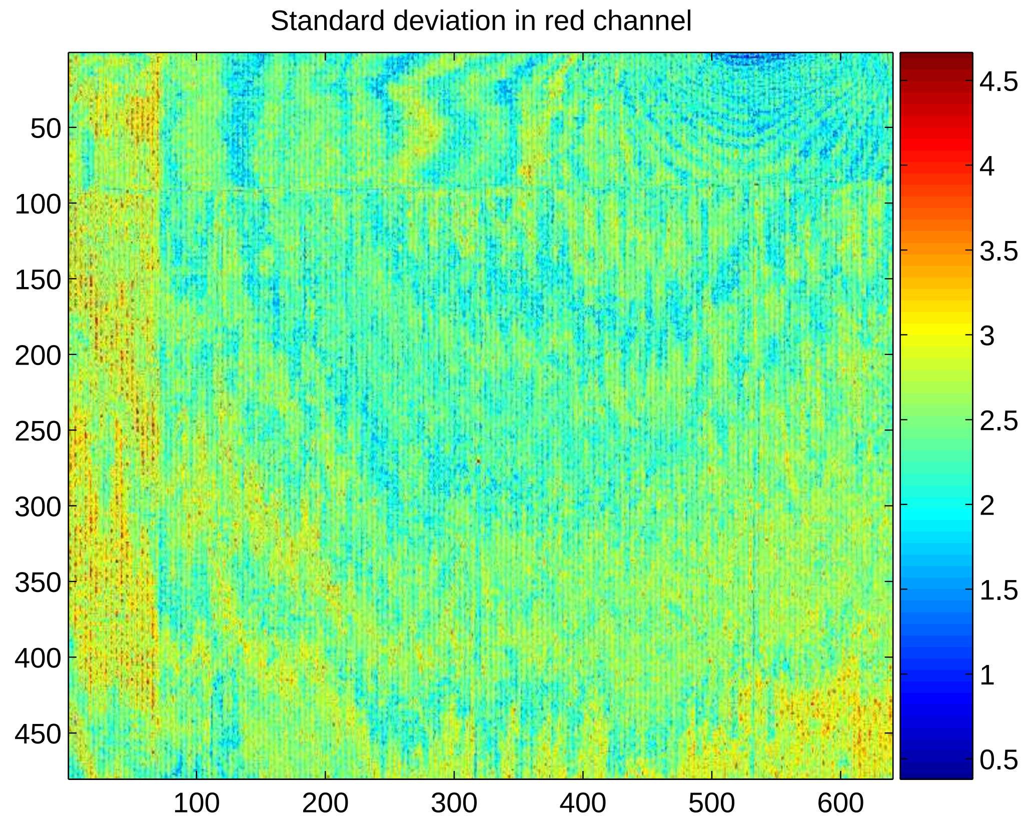
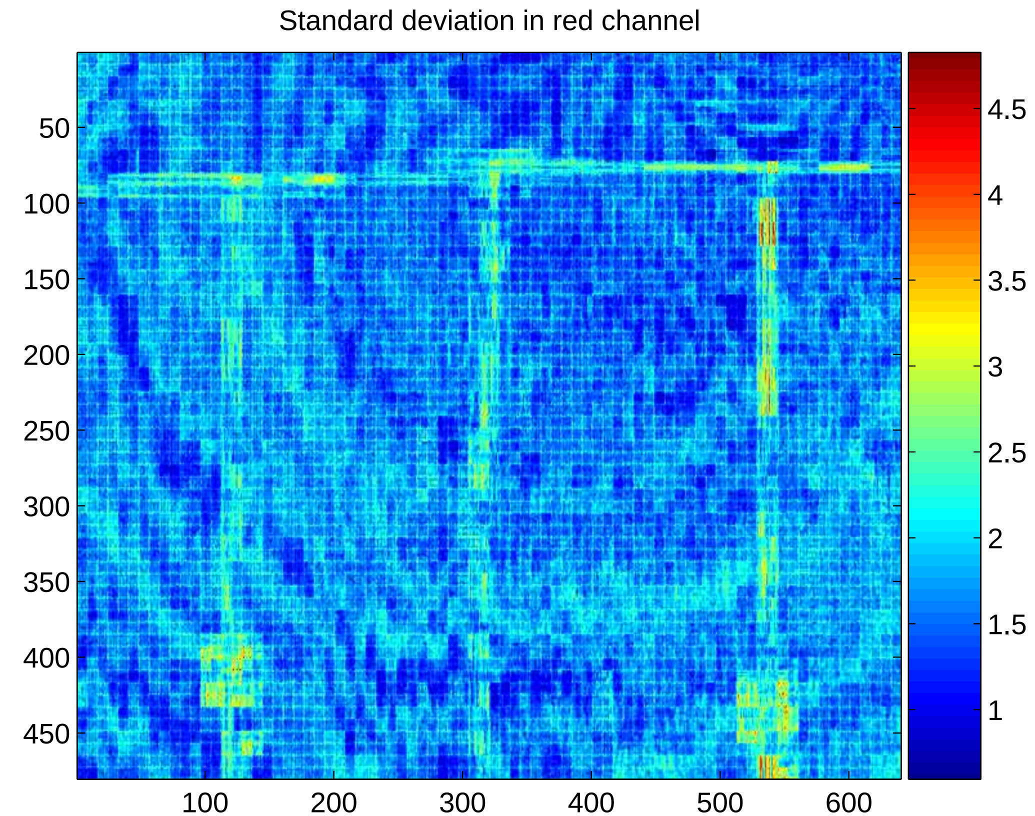
Standard deviations in pixels

for images:



without compression



lossy compressed (jpg)

Lossy compression is generally not a good choice for machine vision!

Problem: noise suppression from just one image

- redundancy in images
- neighbouring pixels have mostly the same or similar value
- correction of the pixel value based on an analysis of its neighbourhood
- leads to image blurring

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spatial filtering

Spatial filtering — informally

Idea: Output is a function of a pixel value and those of its neighbours.

Example for $8-$connected region.

$$g(x,y) = \mathrm{Op}\begin{bmatrix} f(x-1,y-1) & f(x,y-1) & f(x+1,y-1) \\ f(x-1,y) & f(x,y) & f(x+1,y) \\ f(x-1,y+1) & f(x,y+1) & f(x+1,y+1) \end{bmatrix}$$

Possible operations: sum, average, weighted sum, min, max, median . . .

Spatial filtering by masks

- Very common neighbour operation is per-element multiplication with a set of weights and sum together.
- Set of weights is often called mask or kernel.

Local neighbourhood

f(x-1,y-1)	f(x,y-1)	f(x+1,y-1)
f(x-1,y)	f(x,y)	f(x+1,y)
f(x-1,y+1)	f(x,y+1)	f(x+1,y+1)

mask

w(-1,-1)	w(0,-1)	w(+1,-1)
w(-1,0)	w(0,0)	w(+1,0)
w(-1,+1)	w(0,+1)	w(+1,+1)

$$g(x,y) = \sum_{k=-1}^{1} \sum_{l=-1}^{1} w(k,l) f(x+k, y+l)$$

2D convolution

- Spatial filtering is often referred to as convolution.
- We say, we convolve the image by a kernel or mask.
- Though, it is not the same. Convolution uses a flipped kernel.

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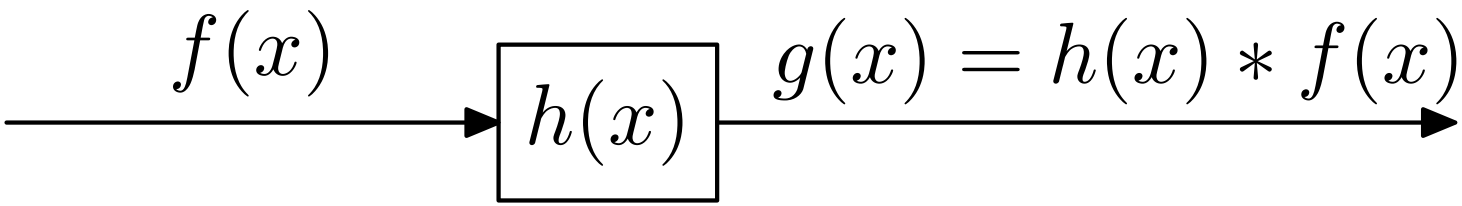
2D Convolution — Why is it important?

- Input and output signals need not to be related through convolution, but if they are (and only if) the system is linear and time invariant.



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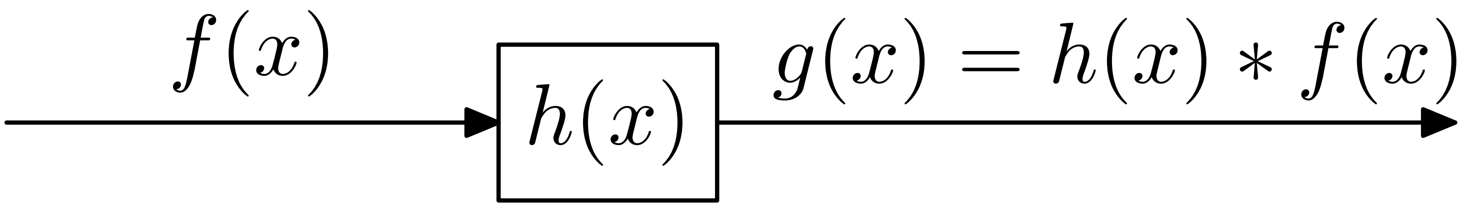

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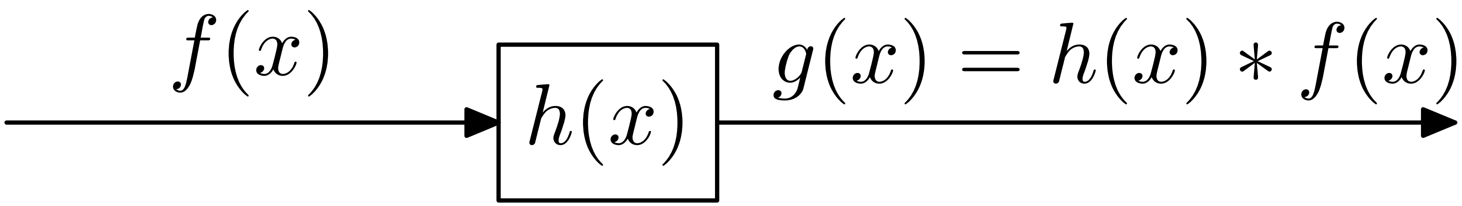

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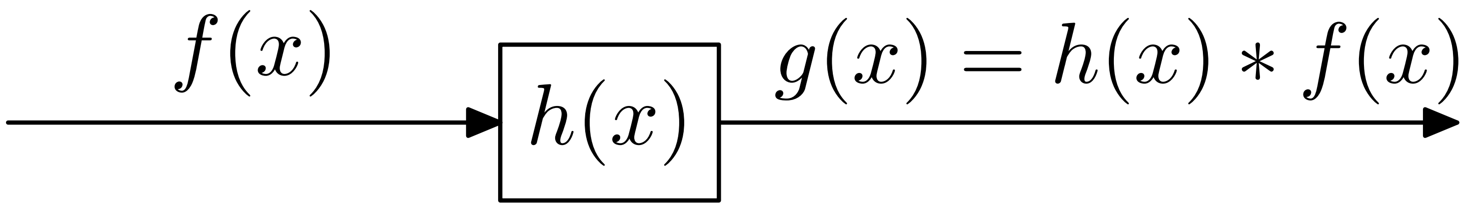

- 2D convolution describes well the formation of images.
- Many image distortions made by imperfect acquisition may be modelled by 2D convolution, too.
- It is a powerful thinking tool.

2D convolution — definition

Convolution integral

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x-k, y-l)h(k,l)dkdl$$

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Symbolic abbreviation

$$g(x,y) = f(x,y) * h(x,y)$$

Discrete 2D convolution

$$g(x,y) = f(x,y) * h(x,y) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f(x-k, y-l) h(k,l)$$

What with missing values $f(x-k, y-l)$?

Zero-padding: add zeros where needed.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} =$$

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The result is zero elsewhere. The concept is somehow contra-intuitive, practice with a pencil and paper.

Thinking about convolution

$$g(x) = f(x) * h(x) = \sum_k f(k)h(x-k)$$

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- send each one individually through h to produce blurred points

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Shifting h:

- shift a copy of h to each position k
- multiply by the value at that position $f(k)$
- add shifted, multiplied copies for all k

Thinking about convolution II

$$g(x) = f(x) * h(x) = \sum_k f(x-k)h(k)$$

Mask filtering:

- flip the function h around zero

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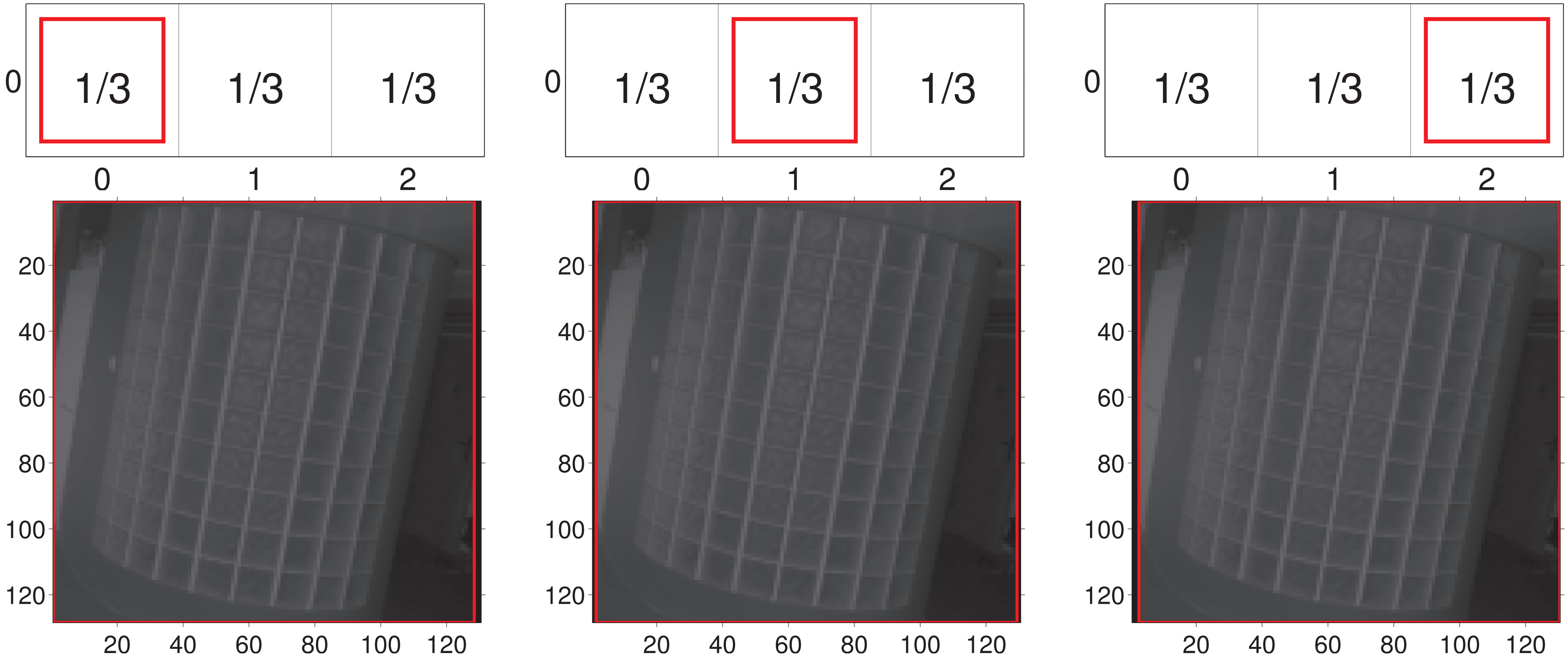
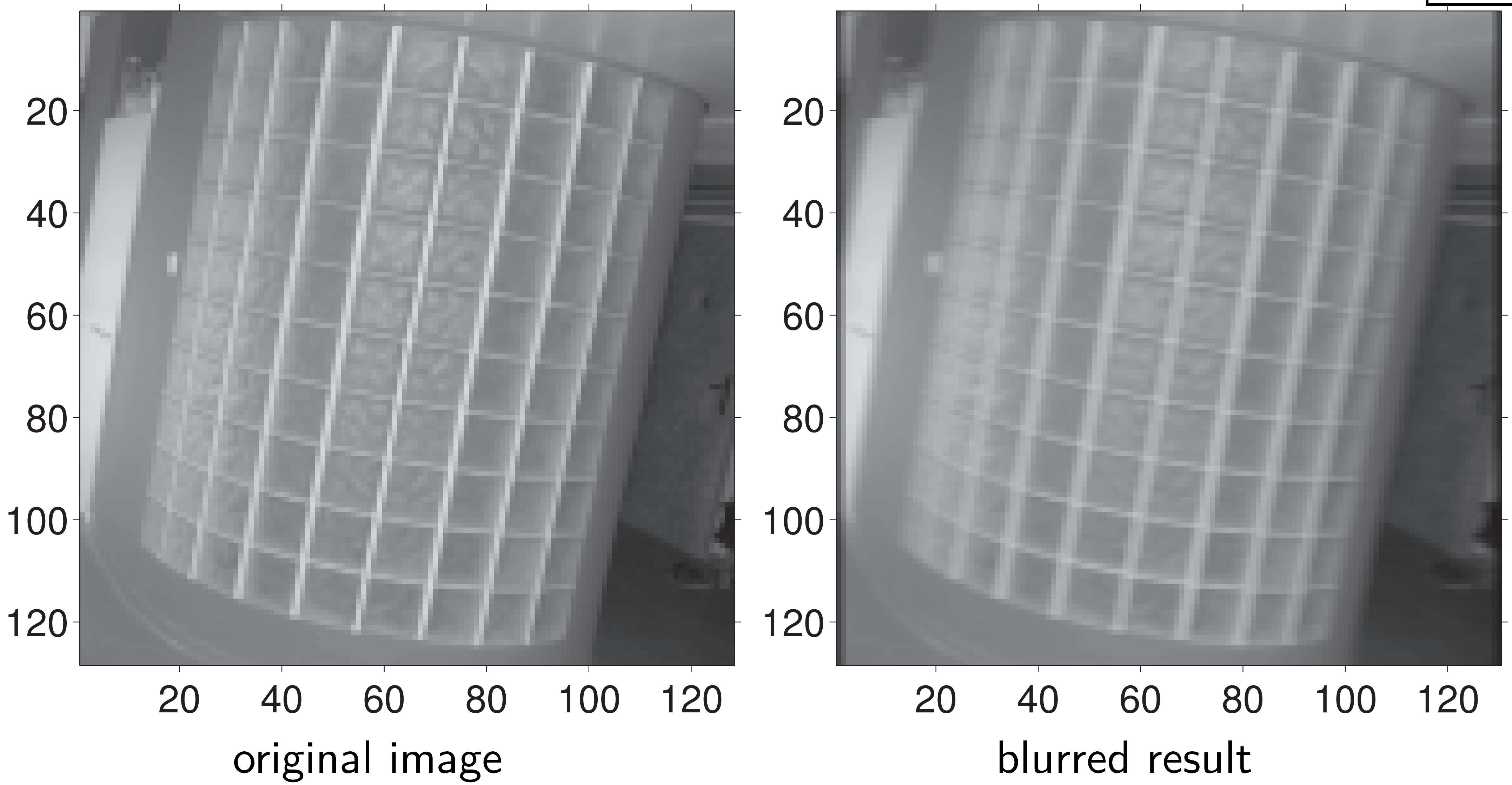
Mask filtering:

- flip the function h around zero
- shift to output position x
- point-wise multiply for each position k value $f(x-k)$ and the shifted flipped copy of h.
- sum for all k and write that value at position x

Camera moves 3 pixels during acquisition . . .



. . . produces blurred result



Motion blur modelled by convolution II

Camera moves along x axis during acquisition.

$$g(x) = \sum_k f(x-k)h(k)$$

- $g(x)$ is the image we get
- $f(x)$ say to be the (true) 2D function
- g does not depend only on $f(x)$ but also on all k previous values of f
- $\#k$ measures the amount of the motion
- if the motion is steady then $h(k) = 1/(\#k)$

h is impulse response of the system (camera), we will come to that later

Spatial filtering vs. convolution — Flipping kernel

Why not $g(x) = \sum_k f(x+k)h(k)$ as in spatial filtering but
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Solution: $h(-k)$

Convolution theorem

The Fourier transform of a convolution is the product of the Fourier transforms.

$$\mathcal{F}\{f(x,y) * h(x,y)\} = F(u,v)H(u,v)$$

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The Fourier transform of a product is the convolution of the Fourier transforms.

$$\mathcal{F}\{f(x,y)h(x,y)\} = F(u,v) * H(u,v)$$

Convolution theorem — proof

$$\mathcal{F}\{f(x,y) * h(x,y)\} = F(u,v)H(u,v)$$

$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)\exp\left(-i2\pi ux/M\right)$ and $g(x) = \sum_{k=0}^{M-1} f(k)h(x-k)$

$\mathcal{F}\{g(x)\} = \ldots$

- $\frac{1}{M}\sum_{x=0}^{M-1}\sum_{k=0}^{M-1} f(k)h(x-k)e^{(-i2\pi ux/M)}$
- introduce new (dummy) variable $w = x - k$
- $\frac{1}{M}\sum_{k=0}^{M-1} f(k)\sum_{w=-k}^{(M-1)-k} h(w)e^{(-i2\pi u(w+k)/M)}$
- remember that all functions g, h, f are assumed to be periodic with period M
- $\frac{1}{M}\sum_{k=0}^{M-1} f(k)e^{(-i2\pi uk/M)}\sum_{w=0}^{M-1} h(w)e^{(-i2\pi uw/M)}$
- which is indeed $F(u)H(u)$

Convolution theorem — what is it good for?

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Enough theory for now. Go for examples . . .

Spatial filtering

What is it good for?

- smoothing
- sharpening
- noise removal
- edge detection
- pattern matching
- ...

Smoothing

Output value is computed as an average of the input value and its neighbourhood.

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Averaging:

$$g(x,y) = \frac{\sum_k \sum_l w(k,l) f(x+k, y+l)}{\sum_k \sum_l w(k,l)}$$

Smoothing kernels

Can be of any size, any shape

$$h = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad h = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix},$$

$$h = \frac{1}{25}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

Averaging ones($n \times n$) — increasing mask size

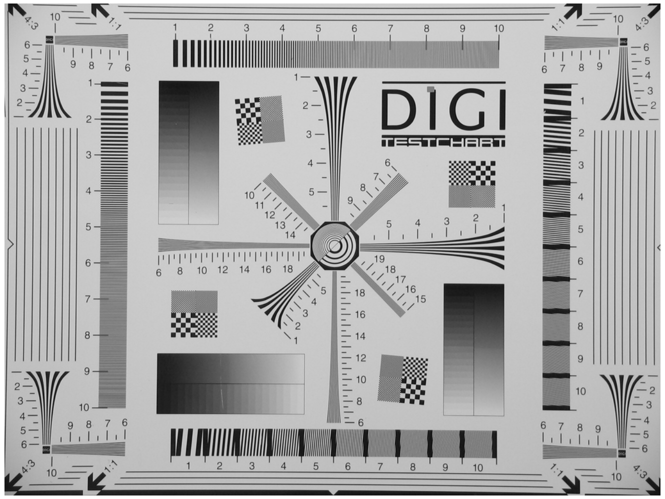
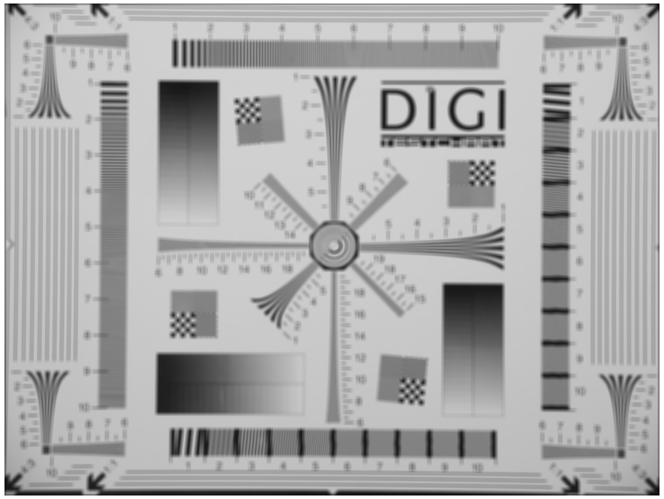
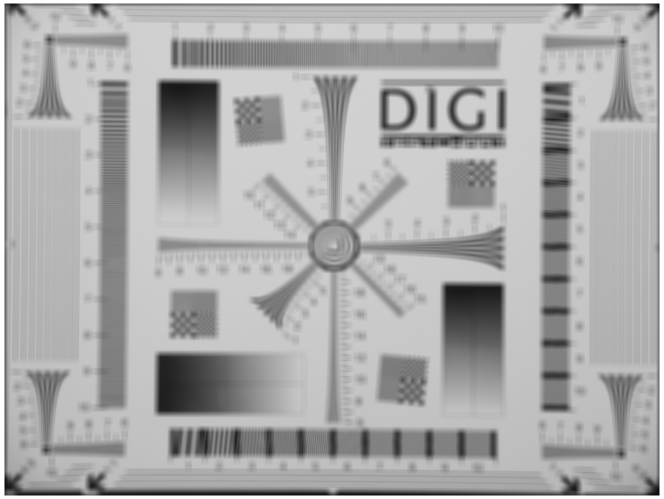


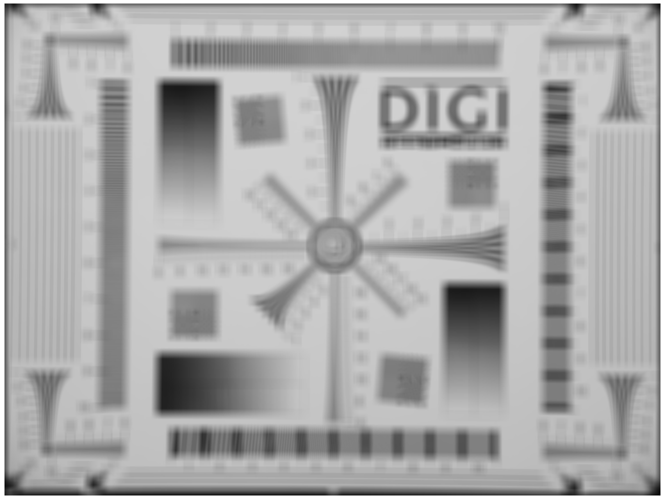
image 1024×768



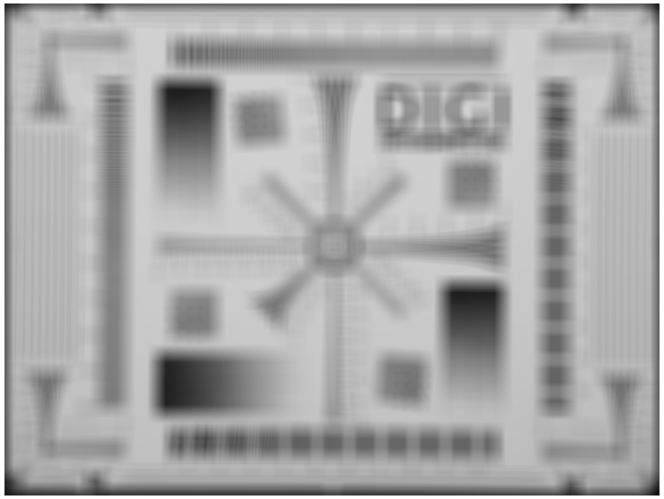
7×7



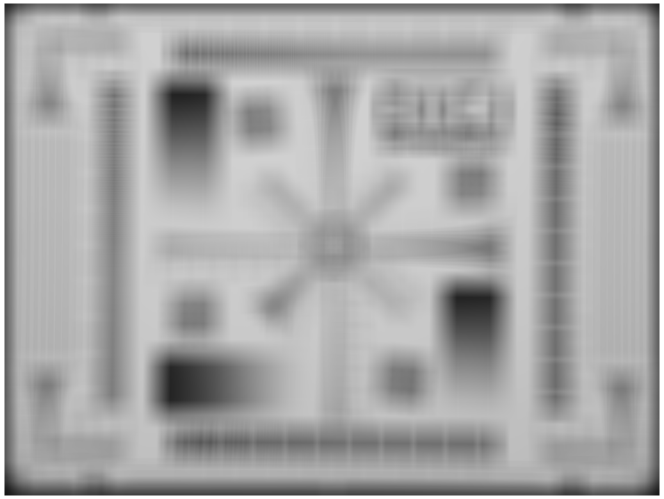
11×11



15×15



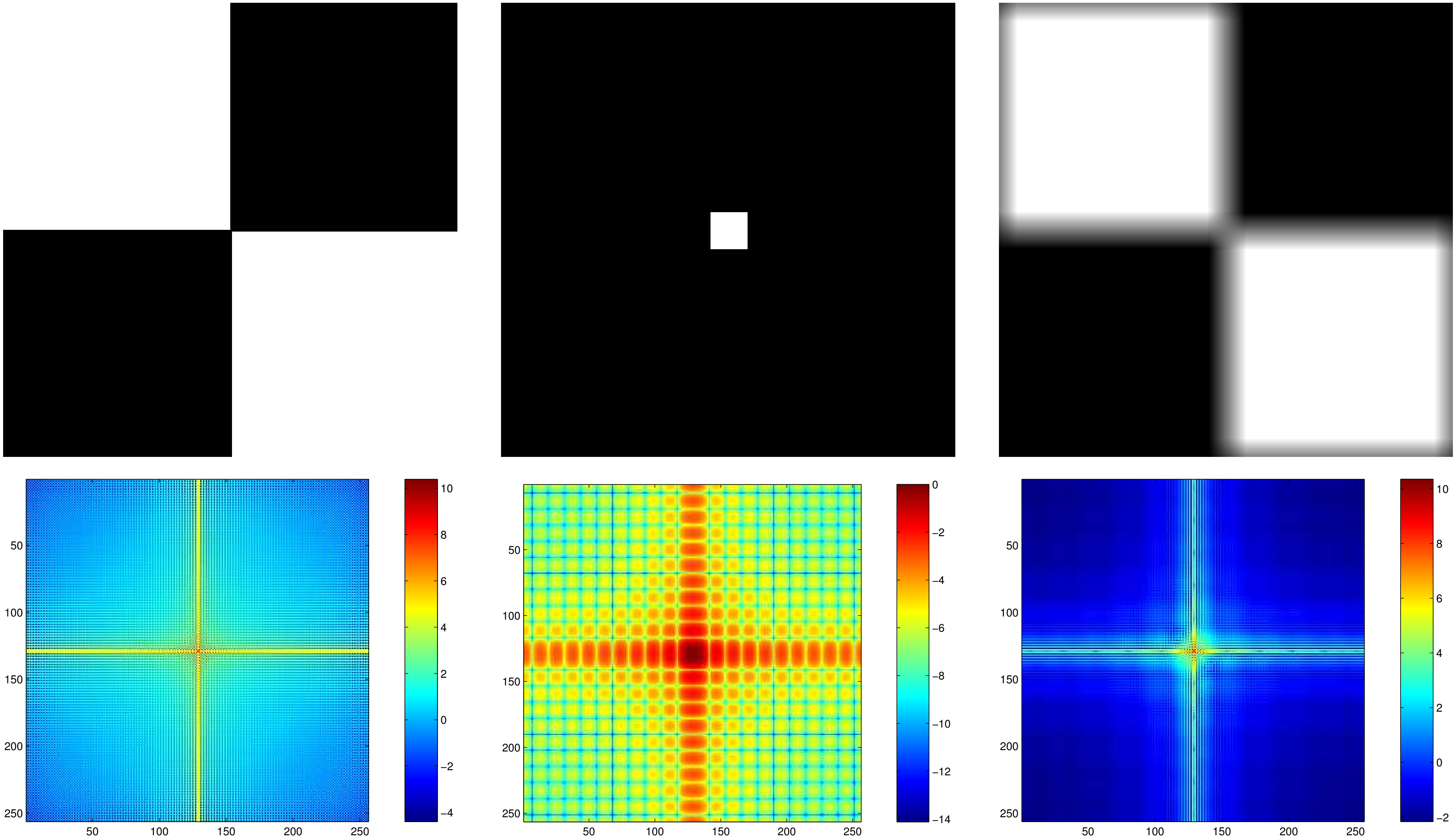
29×29



43×43

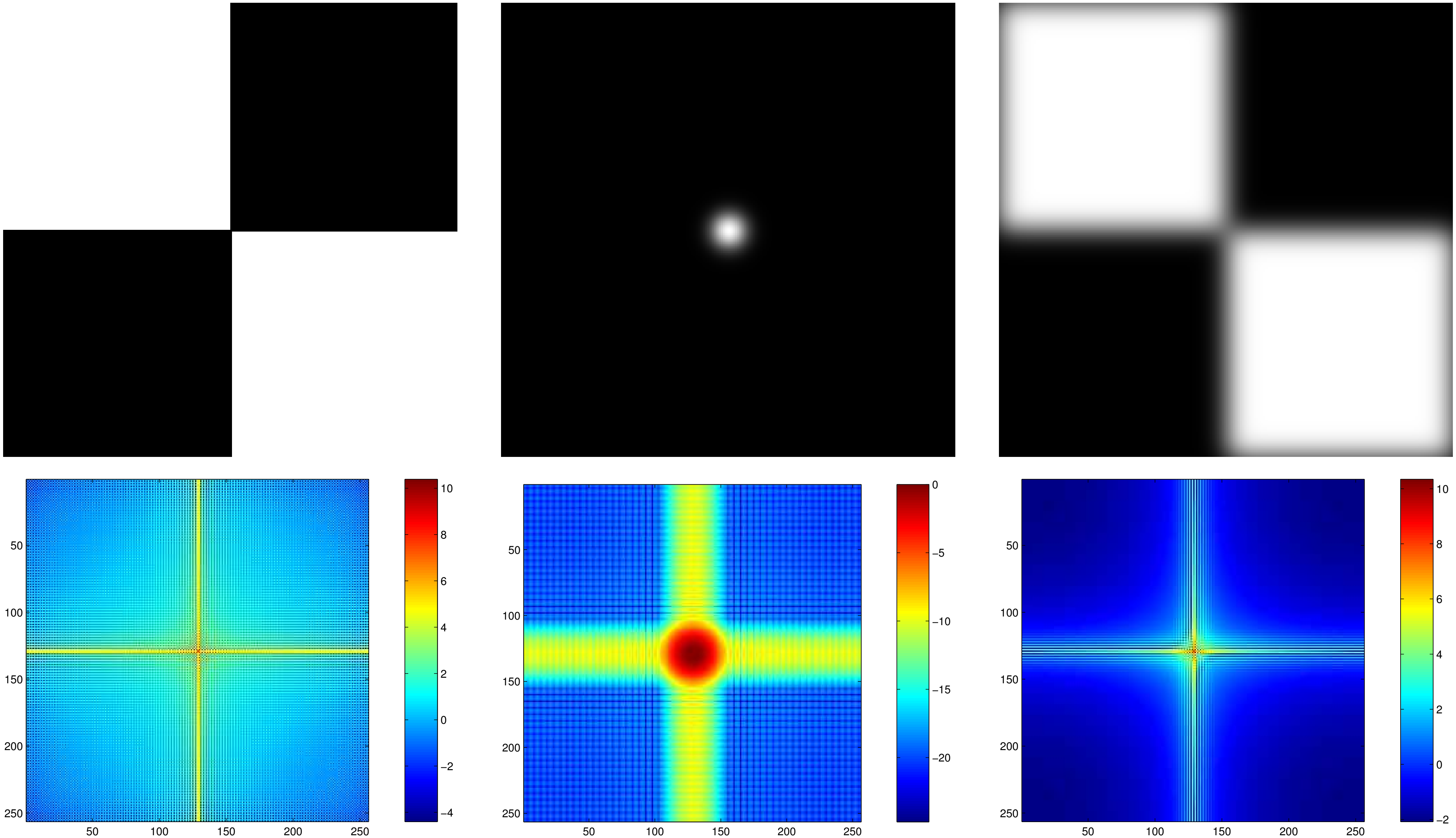
Frequency analysis of the spatial convolution — Simple averaging

Original image $\qquad$ 21×21 const. mask $\qquad$ filtered image

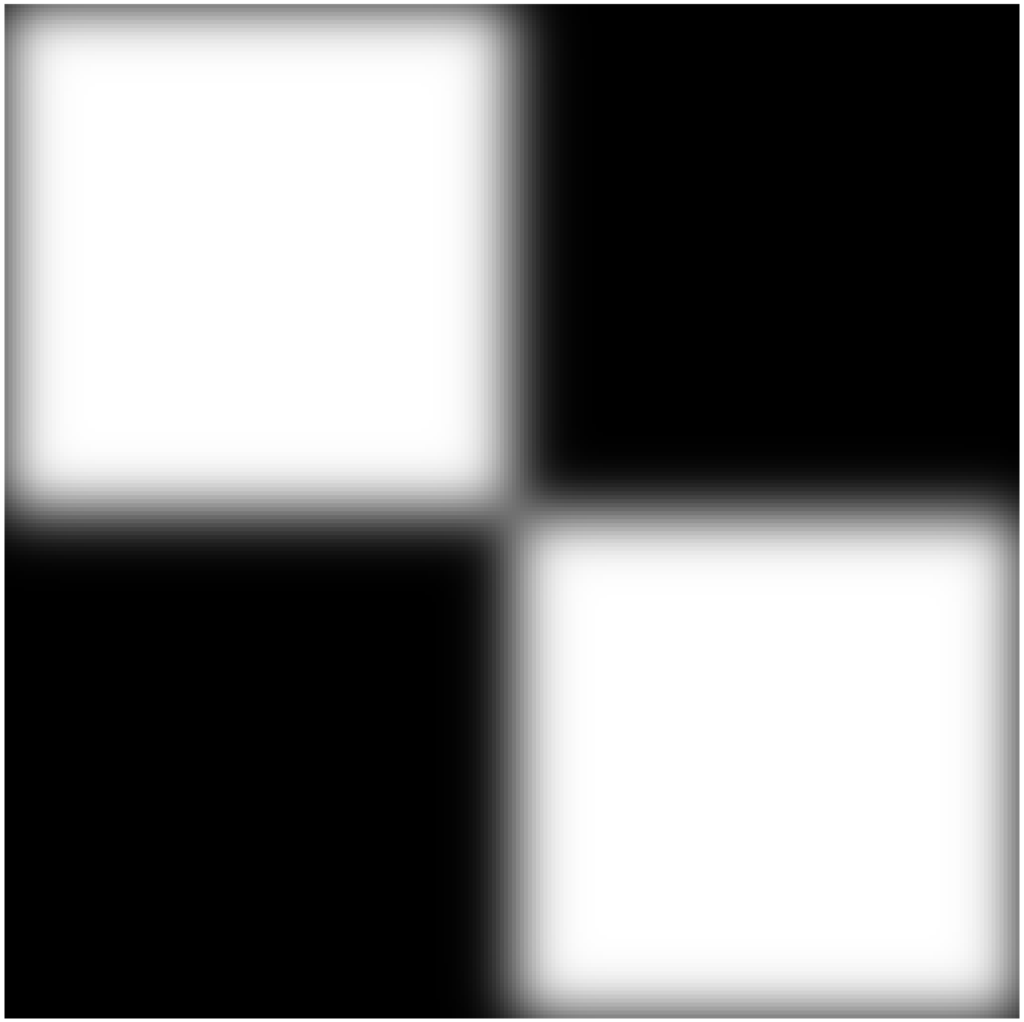


Frequency analysis of the spatial convolution — Gaussian smoothing

Original image | 21×21 Gauss. mask | filtered image



Simple averaging vs. Gaussian smoothing



Both images blurred but filtering by a constant mask still shows up some high frequencies!

Frequency analysis of the spatial convolution — Simple averaging

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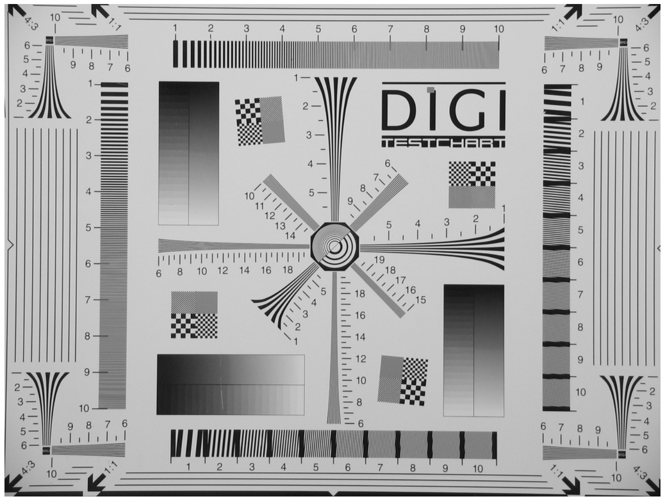
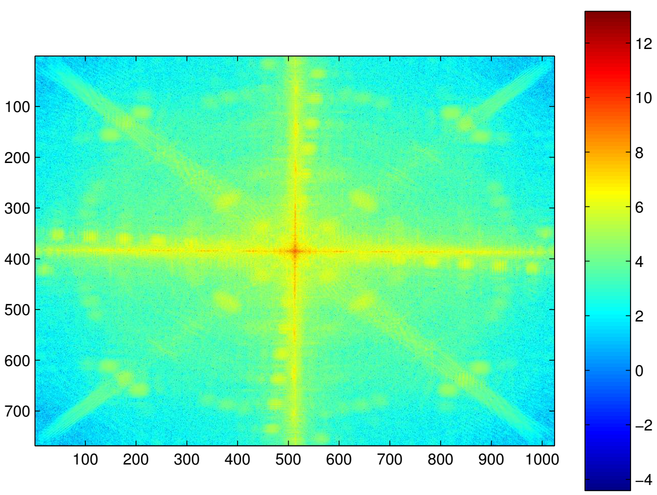
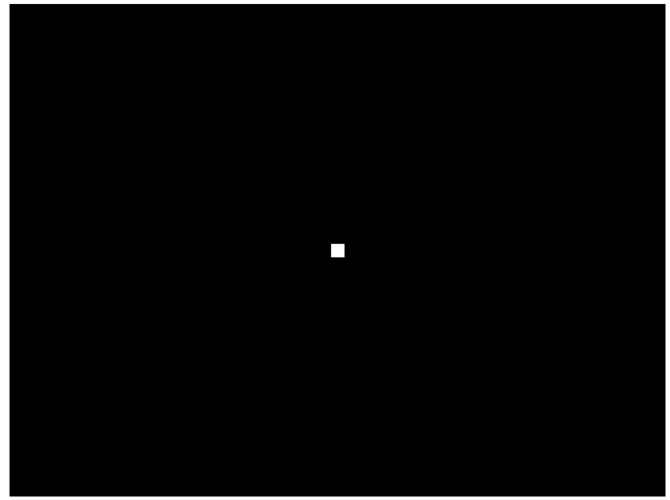
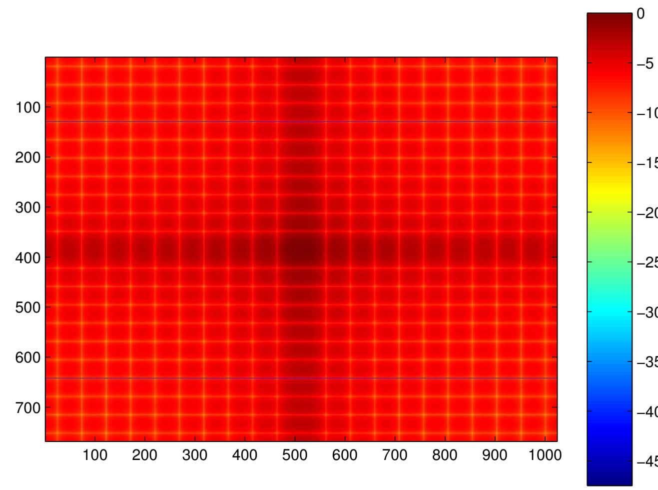


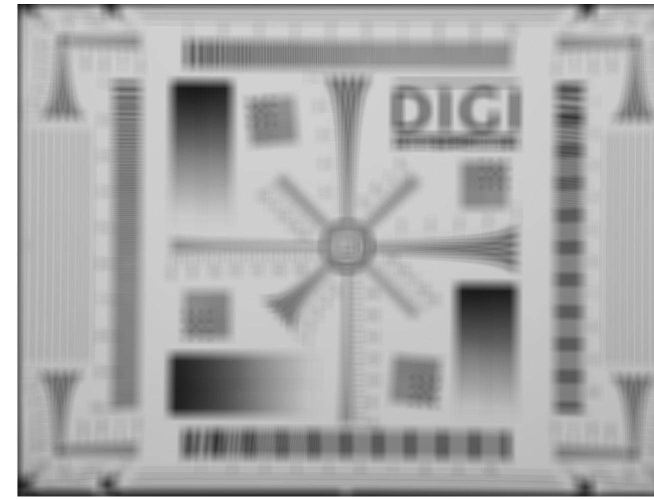


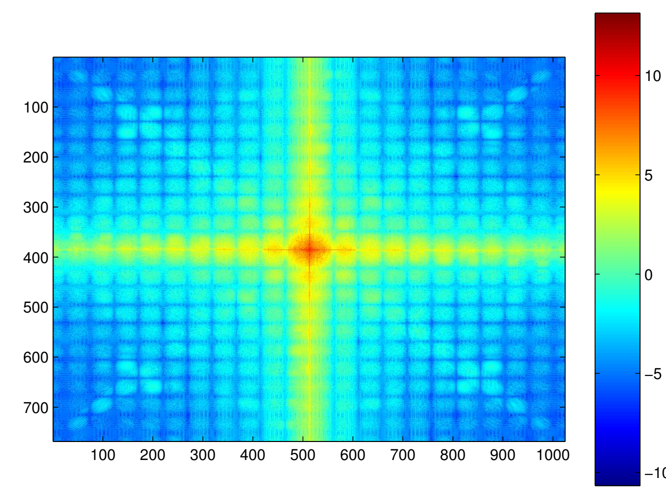
21×21 const. mask



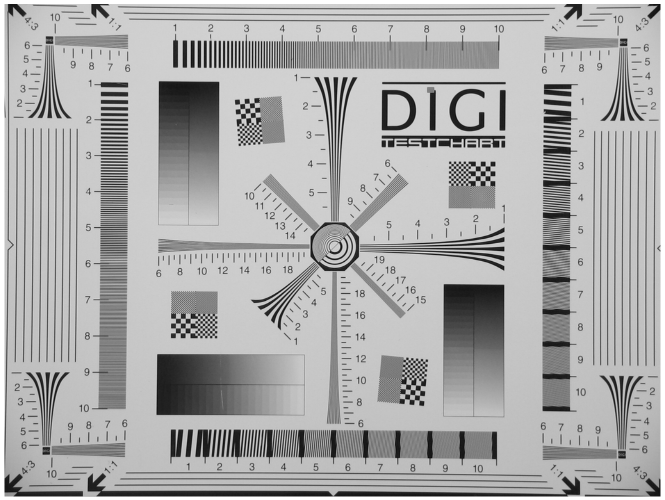
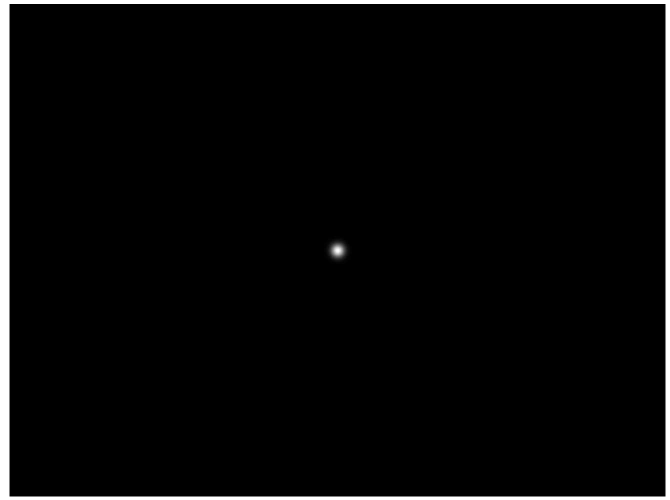
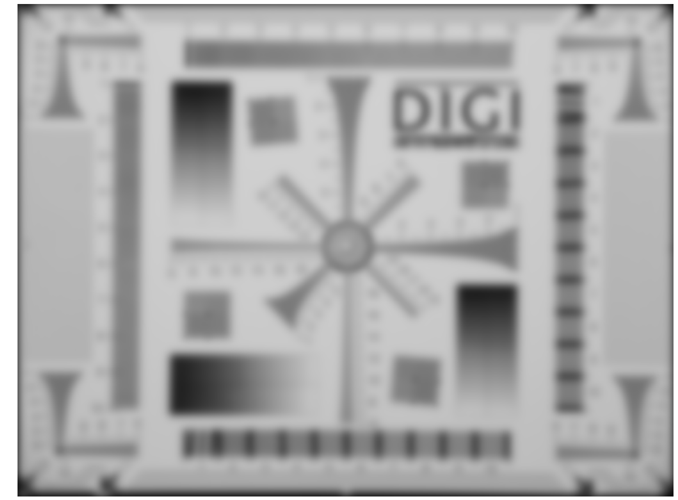
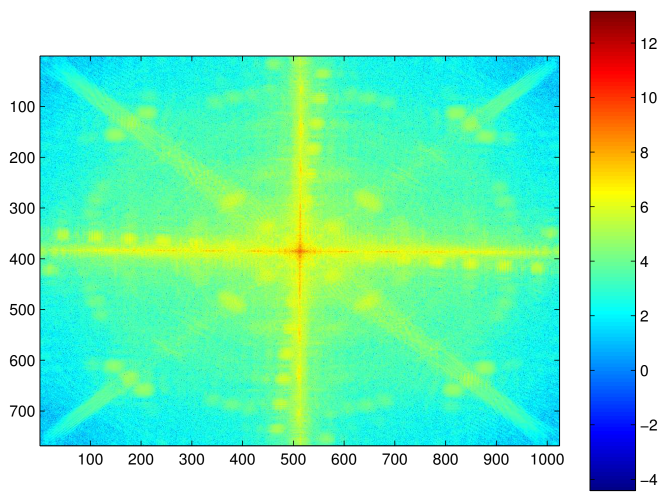
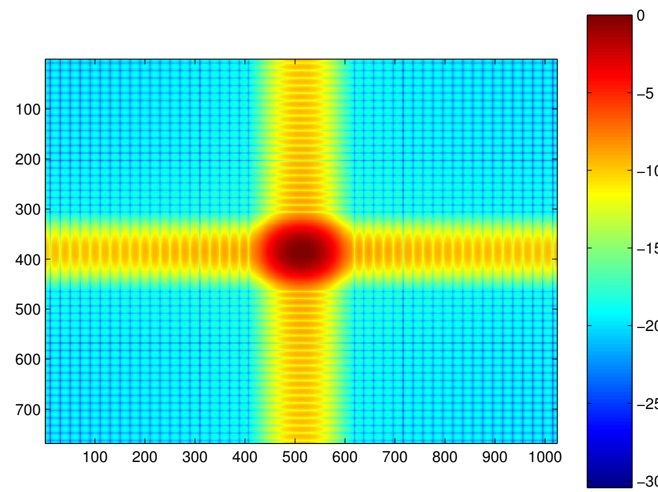
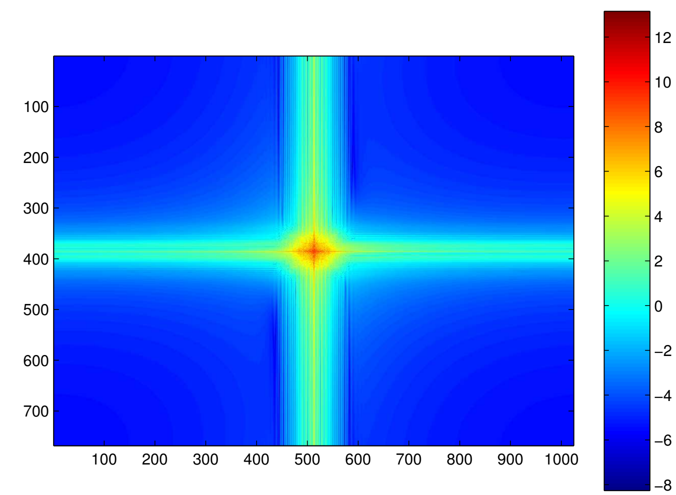


filtered image



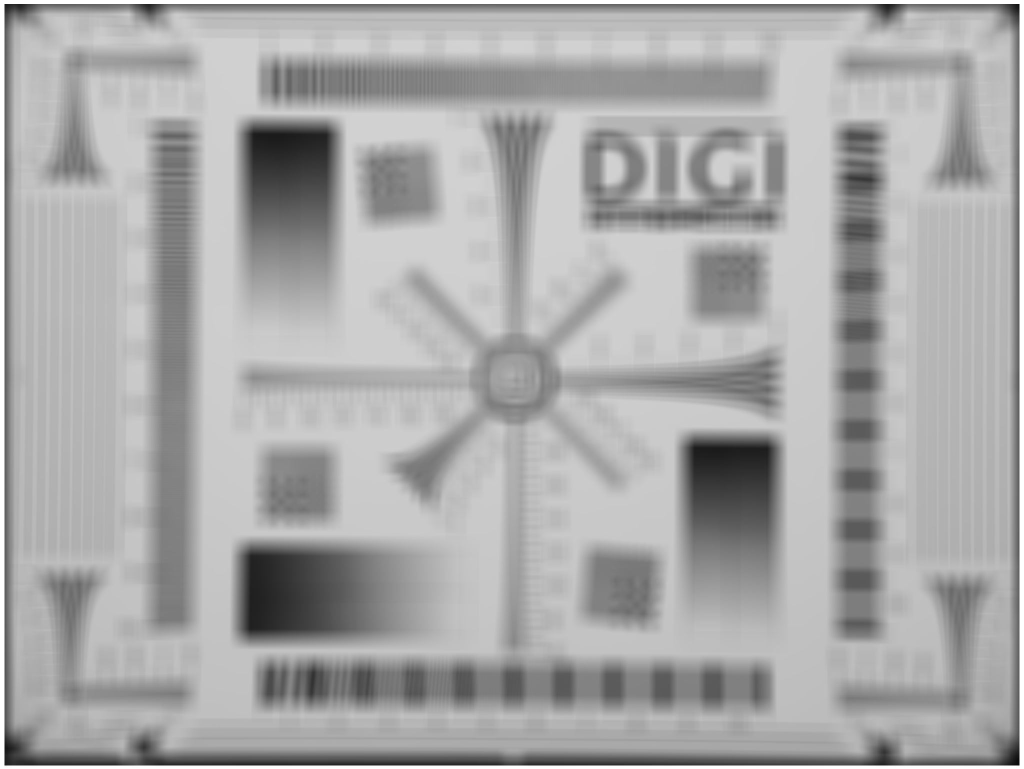


Frequency analysis of the spatial convolution — Gaussian smoothing

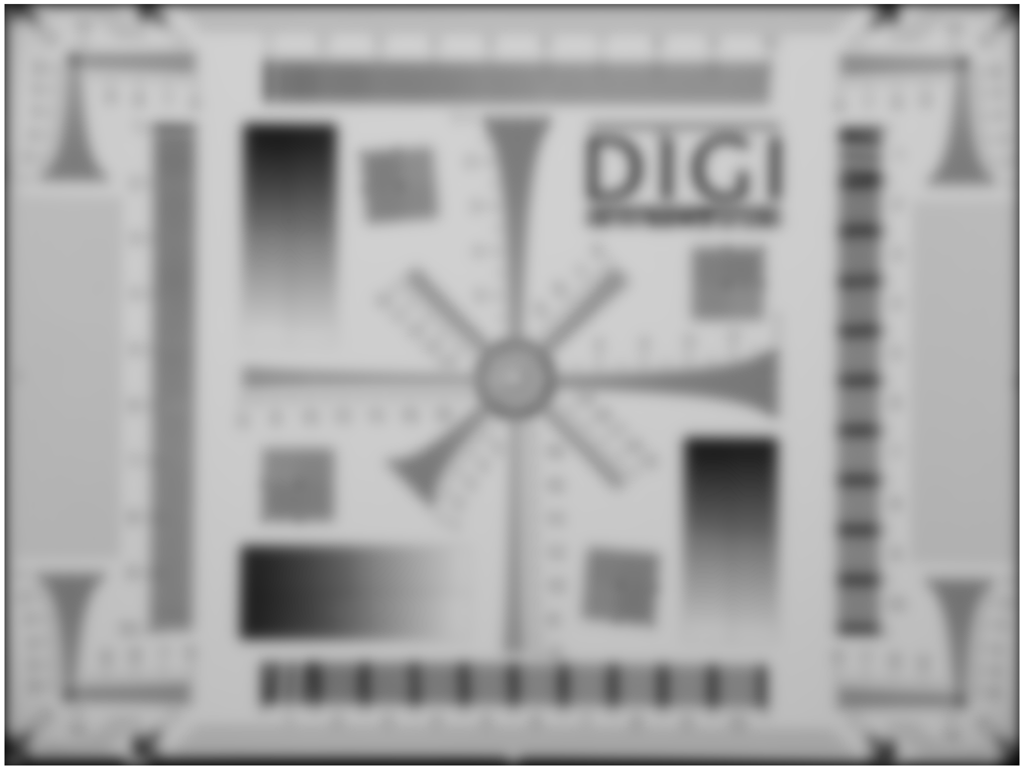
Original image	21×21 Gauss. mask	filtered image
		
		

Simple averaging vs. Gaussian smoothing

simple averaging



Gaussian smoothing



Both images blurred but filtering by a constant mask still shows up some high frequencies!

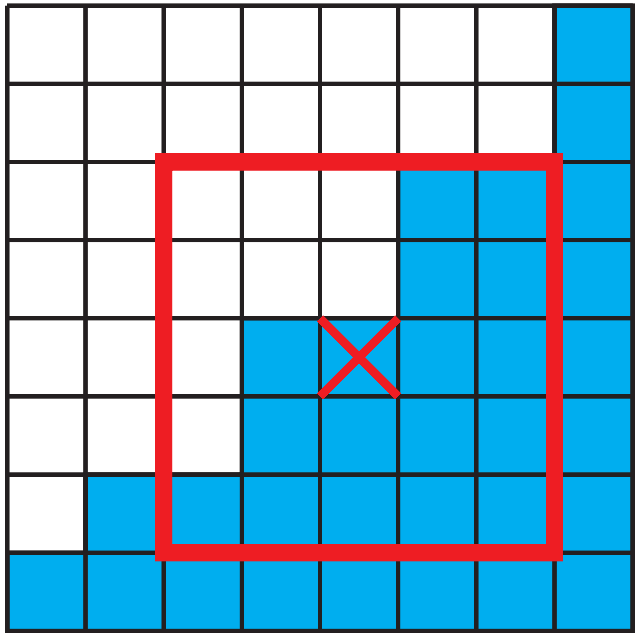
Non-linear smoothing

Goal: reduce blurring of image edges during smoothing

Non-linear smoothing

Goal: reduce blurring of image edges during smoothing

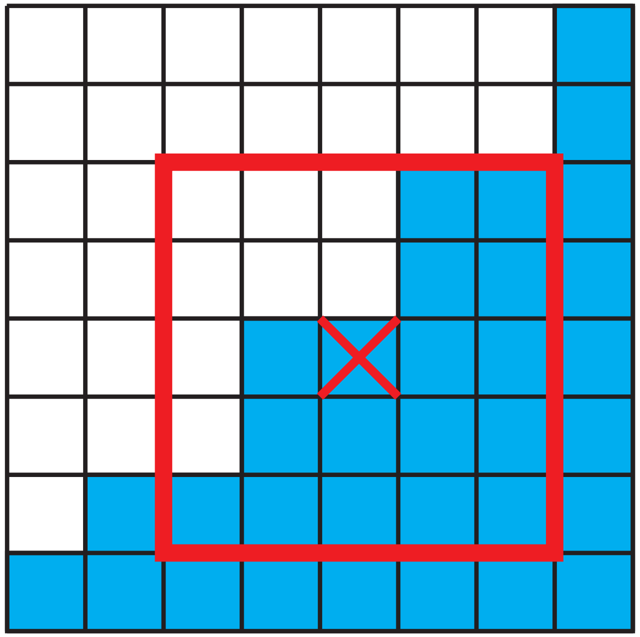
Homogeneous neighbourhood: find a proper neighbourhood where the values have minimal variance.



Non-linear smoothing

Goal: reduce blurring of image edges during smoothing

Homogeneous neighbourhood: find a proper neighbourhood where the values have minimal variance.

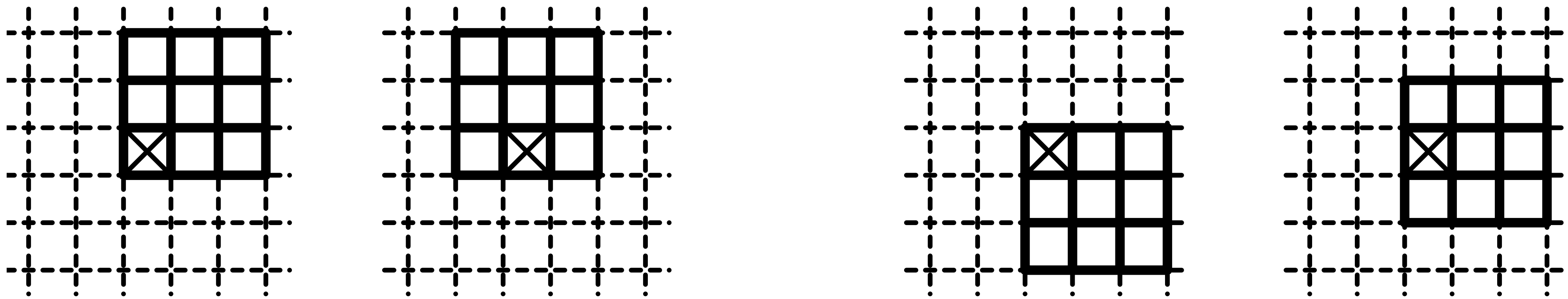


Robust statistics: something better than the mean.

Rotation mask

Rotation mask 3×3 seeks a homogeneous part at 5×5 neighbourhood.

Together 9 positions, 1 in the middle + 8 on the image



1 2 ... 7 8

The mask with the lowest variance is selected as the proper neighbourhood.

Rotation mask—original image

Rotation mask—first filtration

Rotation mask—second filtration

Rotation mask—third filtration

Rotation mask—fourth filtration

Rotation mask—fifth filtration

Nonlinear smoothing — Robust statistics

Order-statistic filters

Nonlinear smoothing — Robust statistics

Order-statistic filters

- median

Nonlinear smoothing — Robust statistics

Order-statistic filters

- median
  - Sort values and select the middle one.

Nonlinear smoothing — Robust statistics

Order-statistic filters

- median
  - Sort values and select the middle one.
  - A method of edge-preserving smoothing.
  - Particularly useful for removing salt-and-pepper, or impulse noise.

Nonlinear smoothing — Robust statistics

Order-statistic filters

- median
  - Sort values and select the middle one.
  - A method of edge-preserving smoothing.
  - Particularly useful for removing salt-and-pepper, or impulse noise.
- trimmed mean
  - Throw away outliers and average the rest.
  - More robust to a non-Gaussian noise than a standard averaging.

Median filtering

100	98	102
99	105	101
95	100	255

Median filtering

100	98	102
99	105	101
95	100	255

Mean $= 117.2$

Median filtering

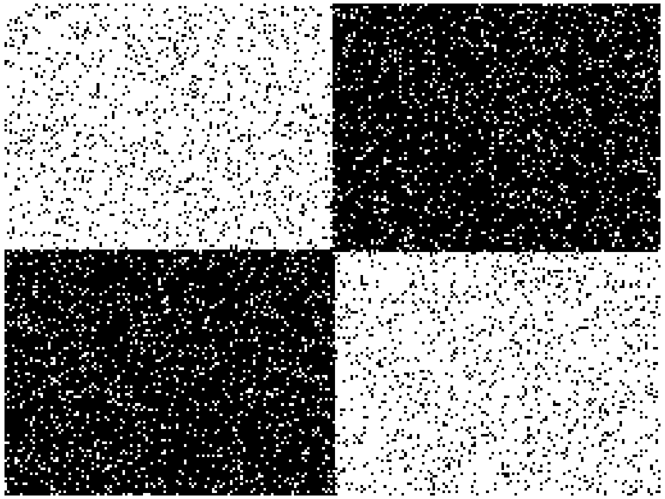
100	98	102
99	105	101
95	100	255

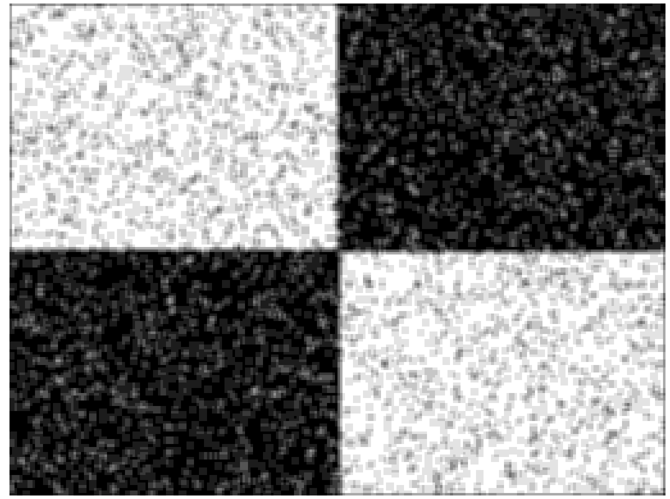
Mean = 117.2

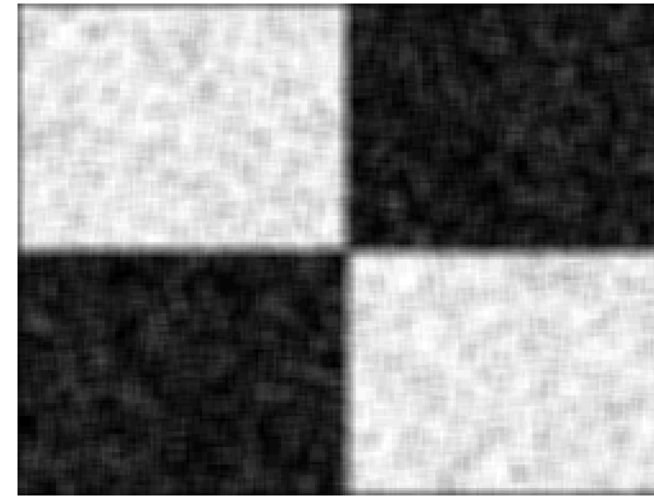
median: 95 98 99 100 100 101 102 105 255

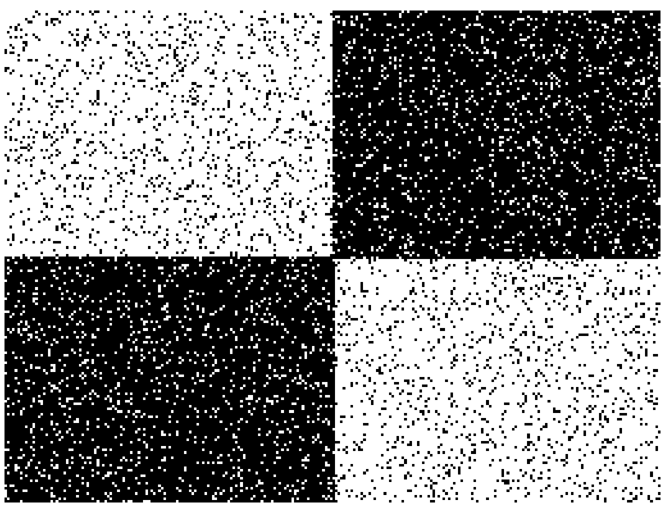
Very robust, up to 50% of values may be outliers.

Nonlinear smoothing examples


noisy image


averaging 3×3


averaging 7×7


noisy image


median 3×3


median 7×7

The median filtering damage corners and thin edges.

Cross-correlation

$$g(x,y) = \sum_k \sum_l h(k,l) f(x+k, y+l) = h(x,y) \star f(x,y)$$

Cross-correlation is not, unlike convolution, commutative

$$h(x,y) \star f(x,y) \neq f(x,y) \star h(x,y)$$

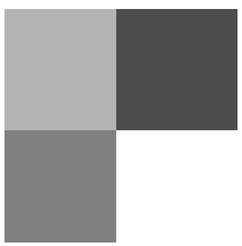
When $h(x,y) \star f(x,y)$ we often say that h scans f.

Cross-correlation is related to convolution through

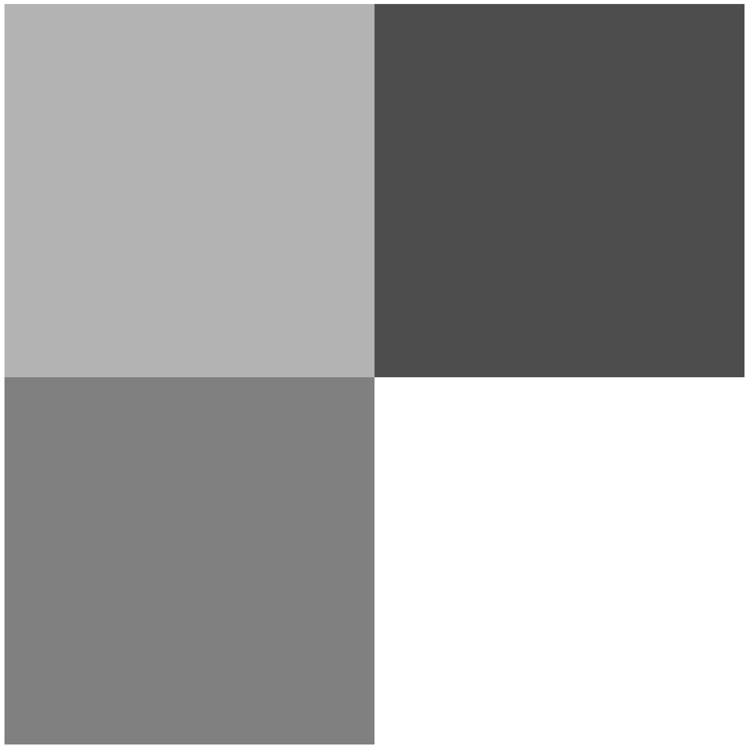
$$h(x,y) \star f(x,y) = h(x,y) * f(-x,-y)$$

Cross-correlation is useful for pattern matching

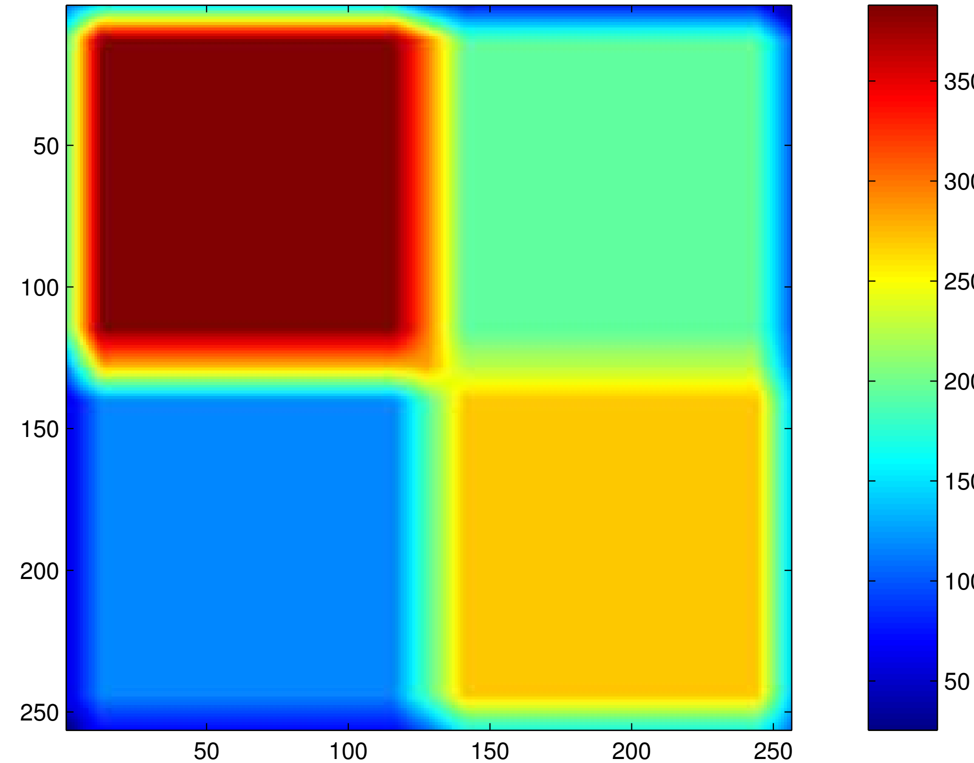
Cross-correlation



$h(x, y)$



scans $f(x, y)$



$g(x, y)$

This is perhaps not exactly what we expected and what we want. The result depend on the amplitudes. Do we have some normalisation?

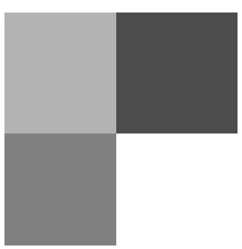
Normalised cross-correlation

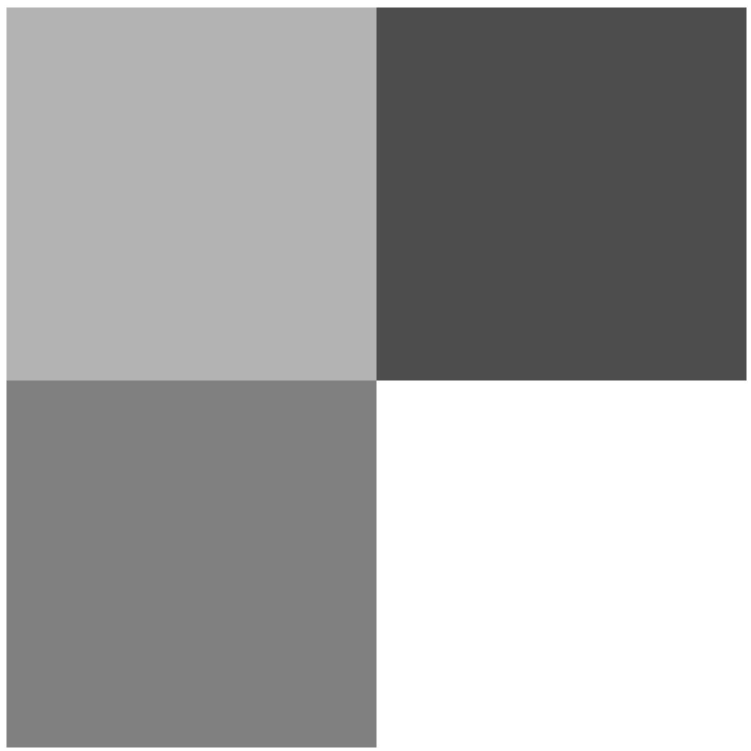
Sometimes called correlation coefficient

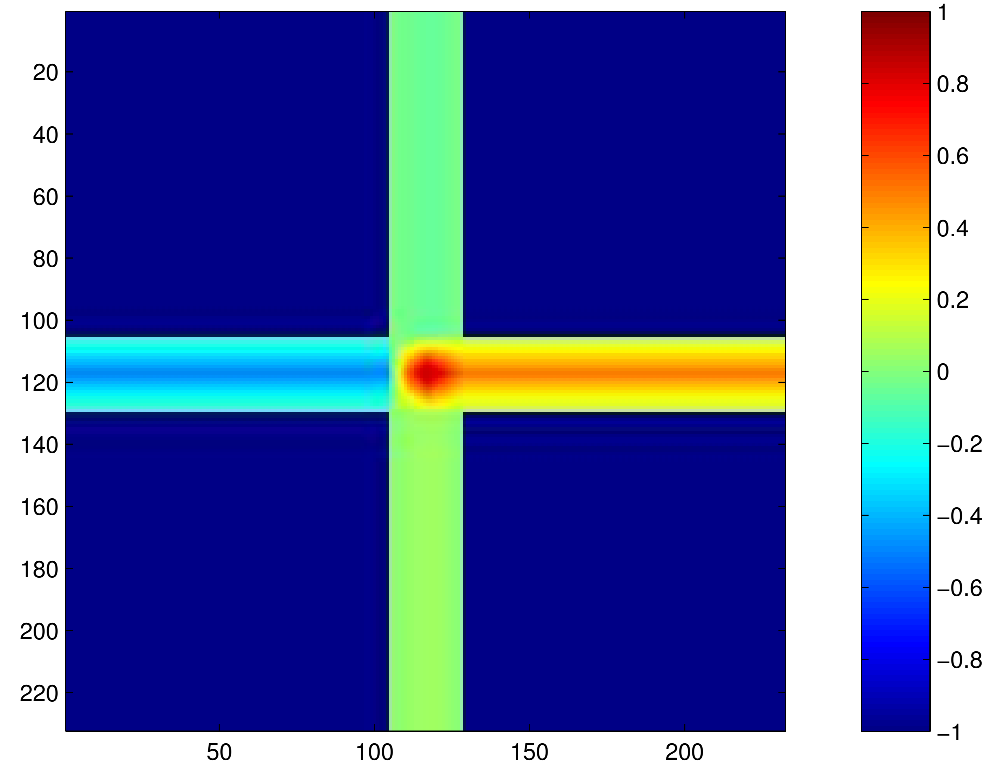
$$c(x,y) = \frac{\sum_k \sum_l \left(h(k,l) - \overline{h}\right)\left(f(x+k,y+l) - \overline{f(x,y)}\right)}{\sqrt{\sum_k \sum_l \left(h(k,l) - \overline{h}\right)^2 \sum_k \sum_l \left(f(x+k,y+l) - \overline{f(x,y)}\right)^2}}$$

- $\overline{h}$ is the mean of h
- $\overline{f(x,y)}$ is the mean of the k, l neighbourhood around (x,y)
- $\sum_k \sum_l \left(h(k,l) - \overline{h}\right)^2$ and $\sum_k \sum_l \left(f(x+k,y+l) - \overline{f(x,y)}\right)^2$ are indeed the variances.
- $-1 \le c(x,y) \le 1$

Normalised cross-correlation



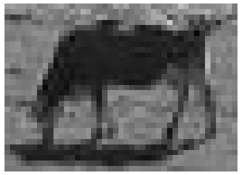




$h(x, y)$ $f(x, y)$ $g(x, y)$

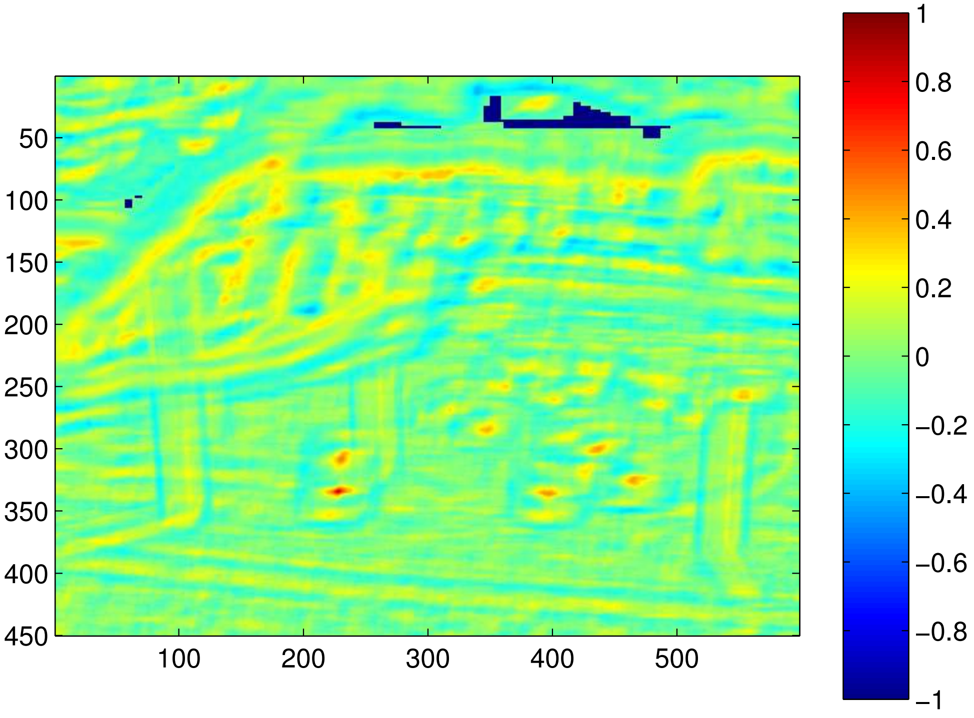
The -1s are in fact undefined, NaN. The maximum response is indeed where we expected.

Normalised cross-correlation — real images



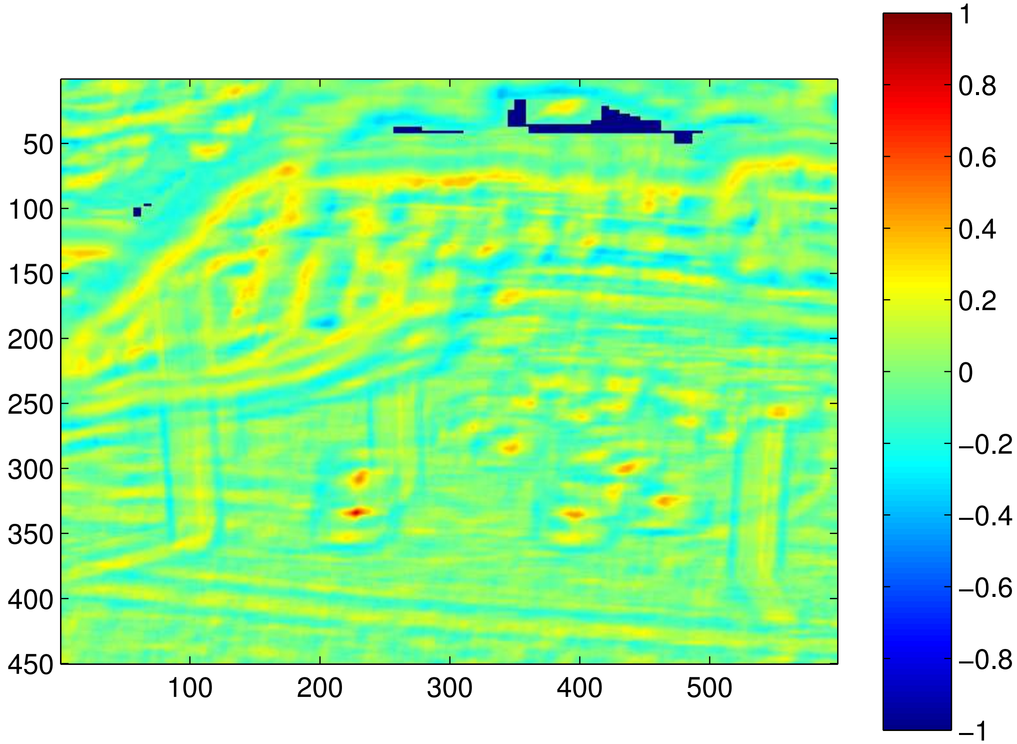
$h(x, y)$

$f(x, y)$



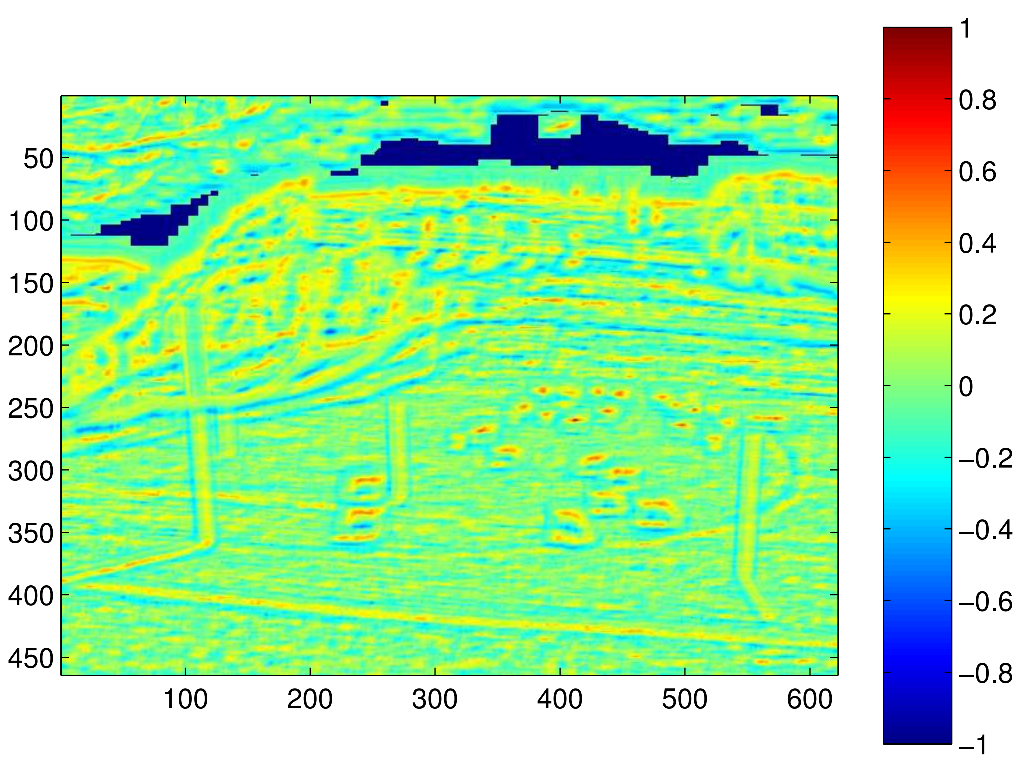
$g(x, y)$

Normalised cross-correlation — non-maxima suppression



Red rectangle denotes the pattern. The crosses are the 5 highest values of ncc after non-maxima suppression.

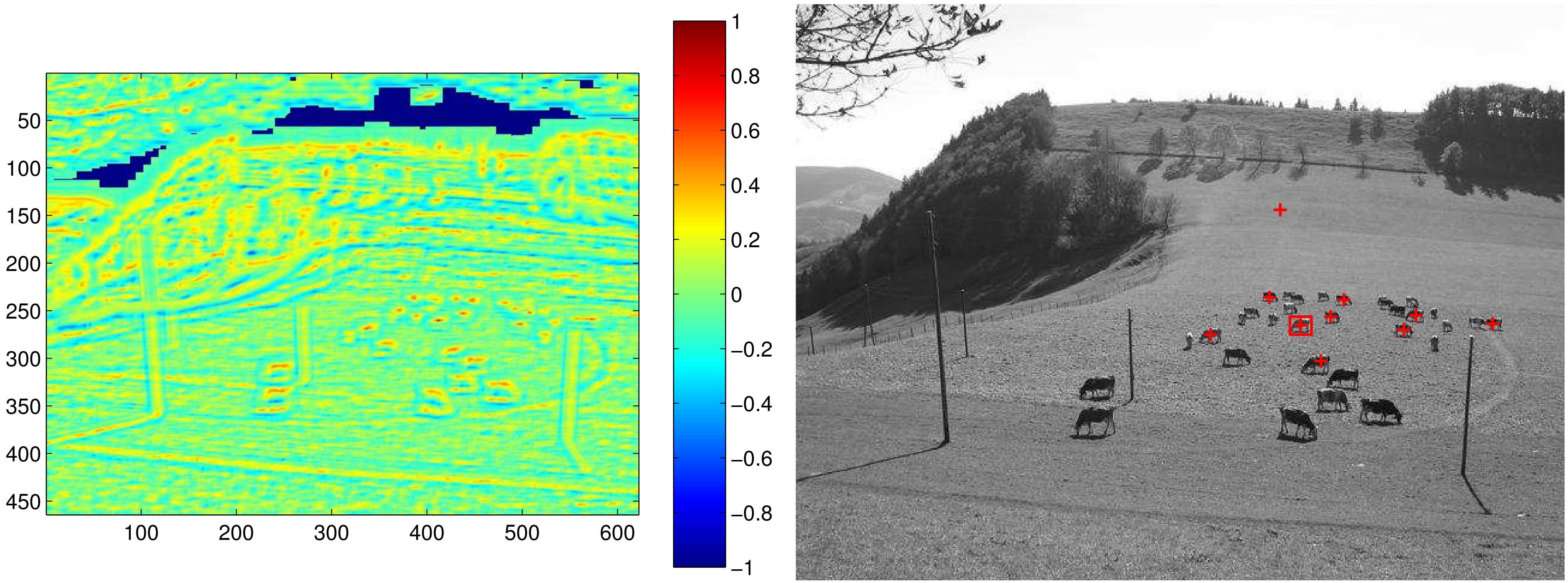
Normalised cross-correlation — non-maxima suppression



Red rectangle denotes the pattern. The crosses are the 10 highest values of ncc after non-maxima suppression.

We see the problem. The algorithm finds the cow in any position in the image. However, it does not scale.

Normalised cross-correlation — non-maxima suppression



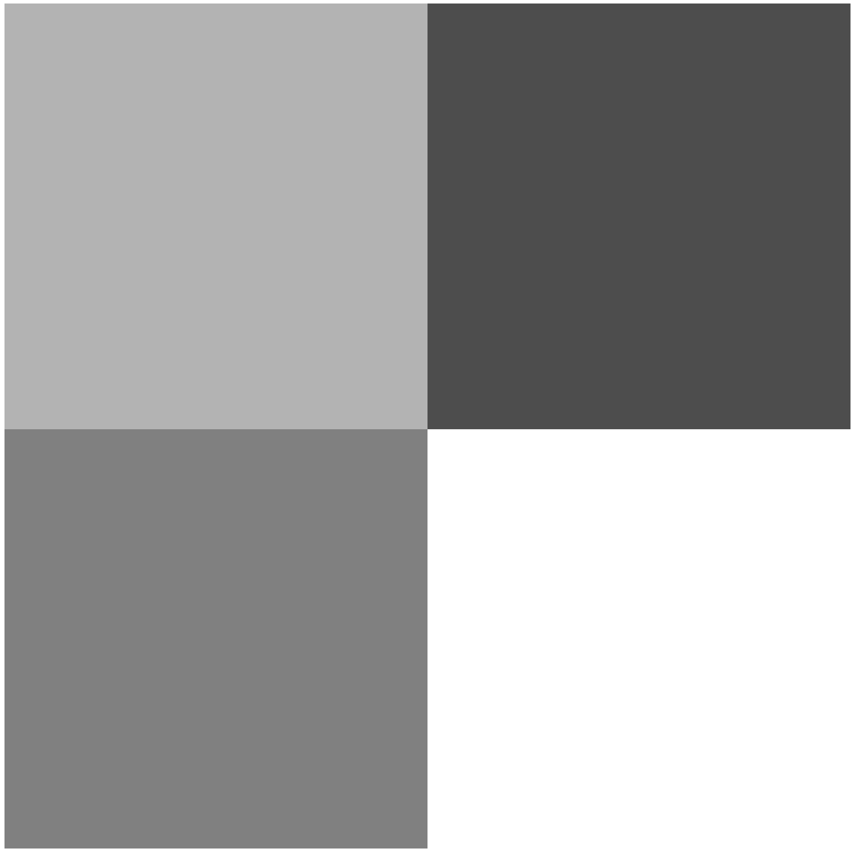
Red rectangle denotes the pattern. The crosses are the 10 highest values of ncc after non-maxima suppression.

We see the problem. The algorithm finds the cow in any position in the image. However, it does not scale.

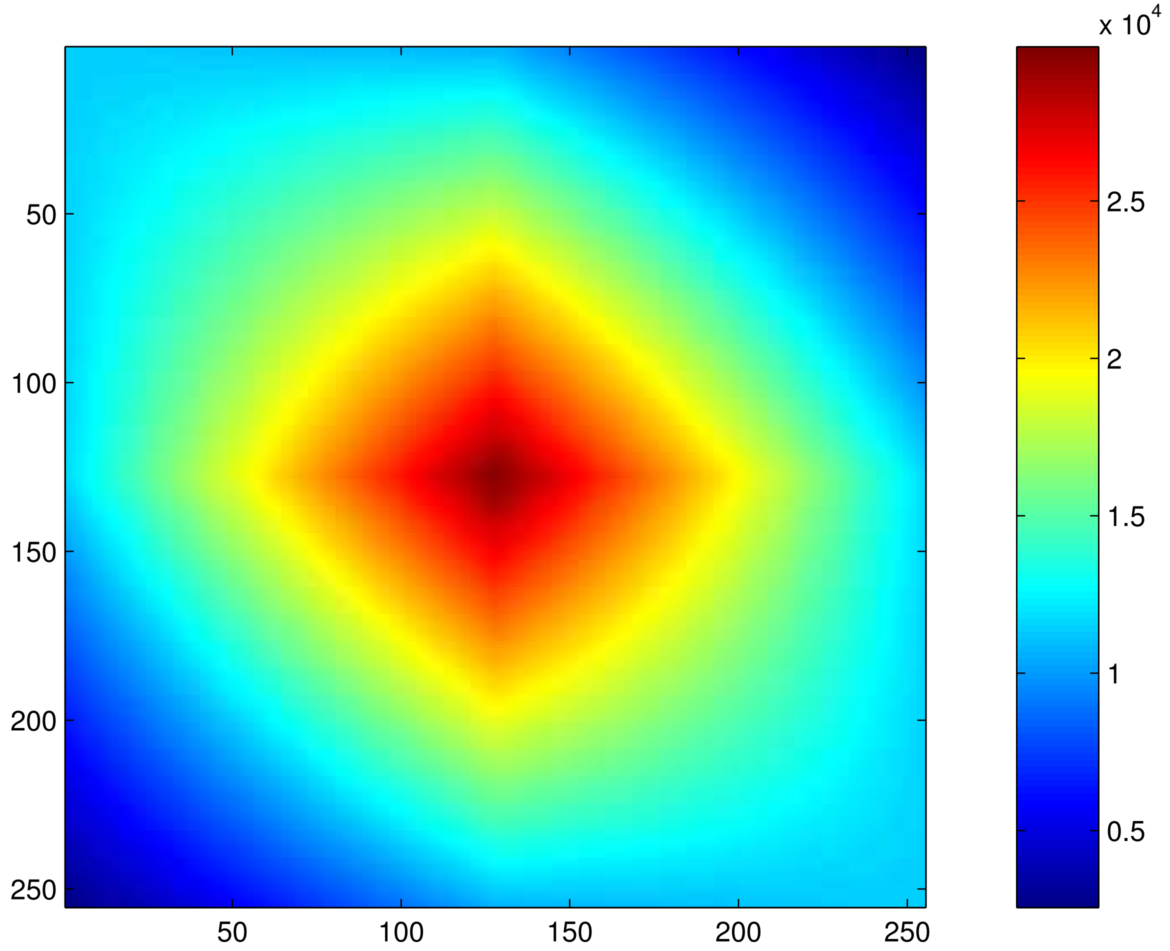
But we leave the problem for some advanced computer vision course.

Autocorrelation

$$g(x,y) = f(x,y) \star f(x,y)$$

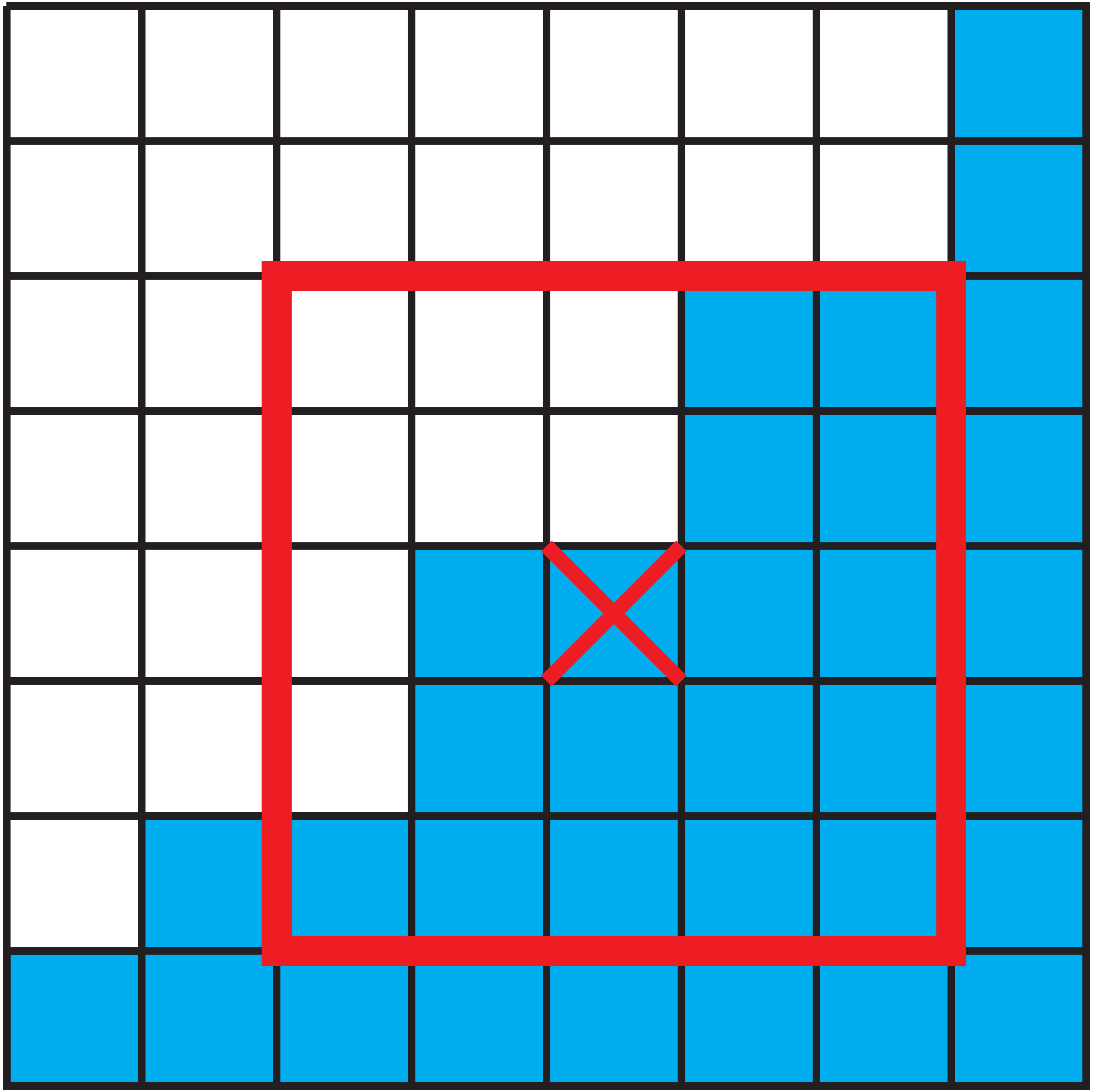


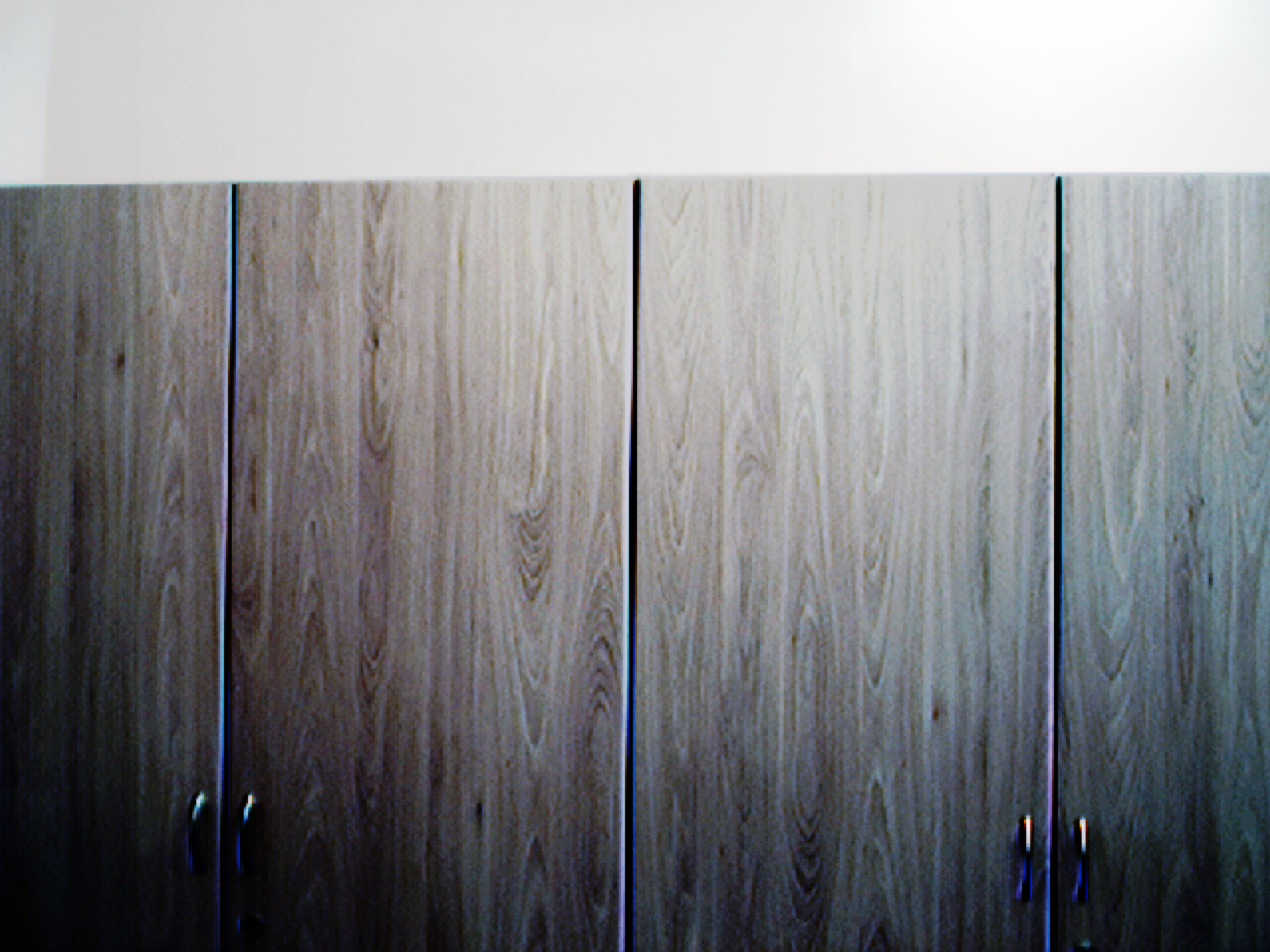
$f(x,y)$

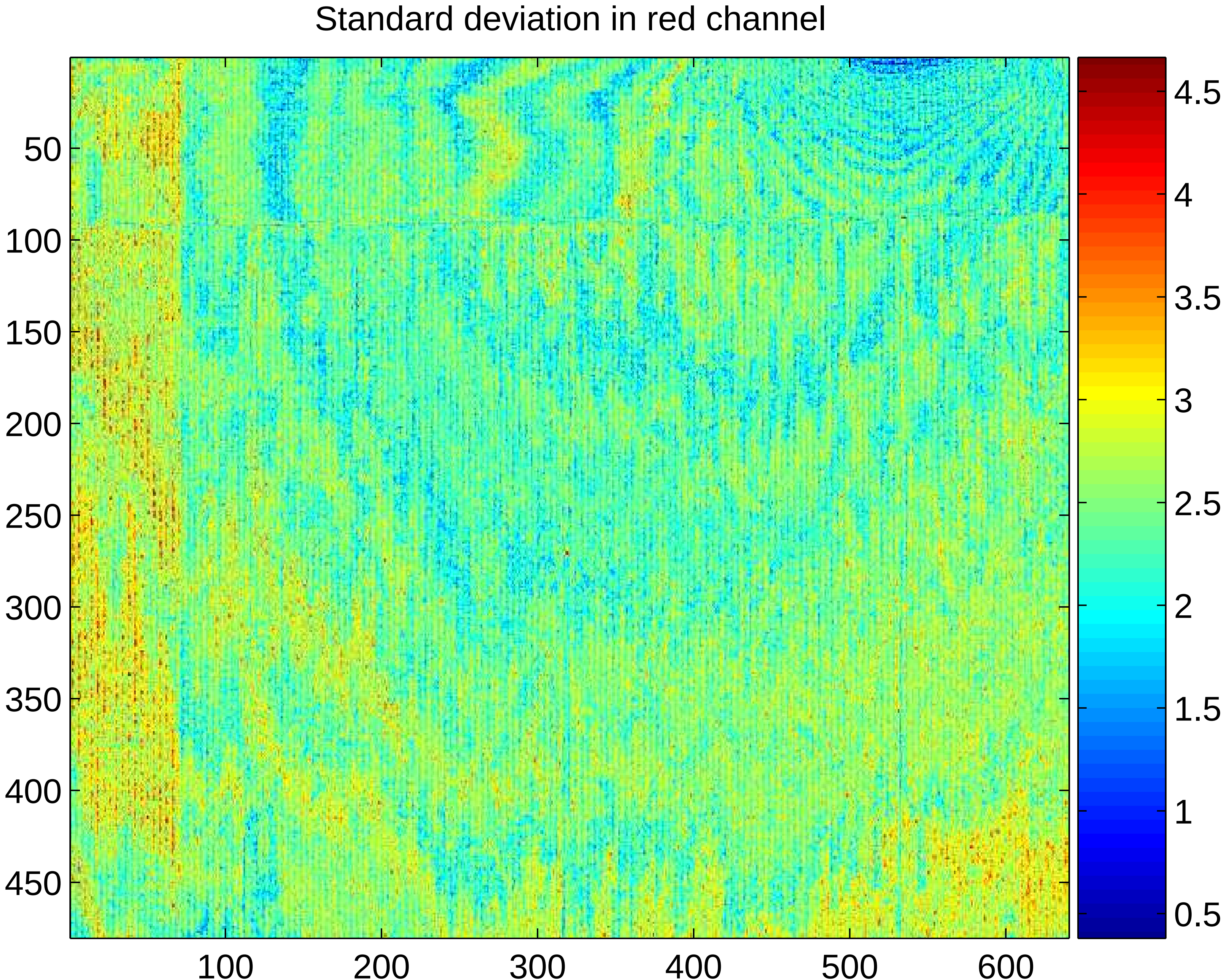


$f(x,y) \star f(x,y)$

References







Standard deviation in red channel
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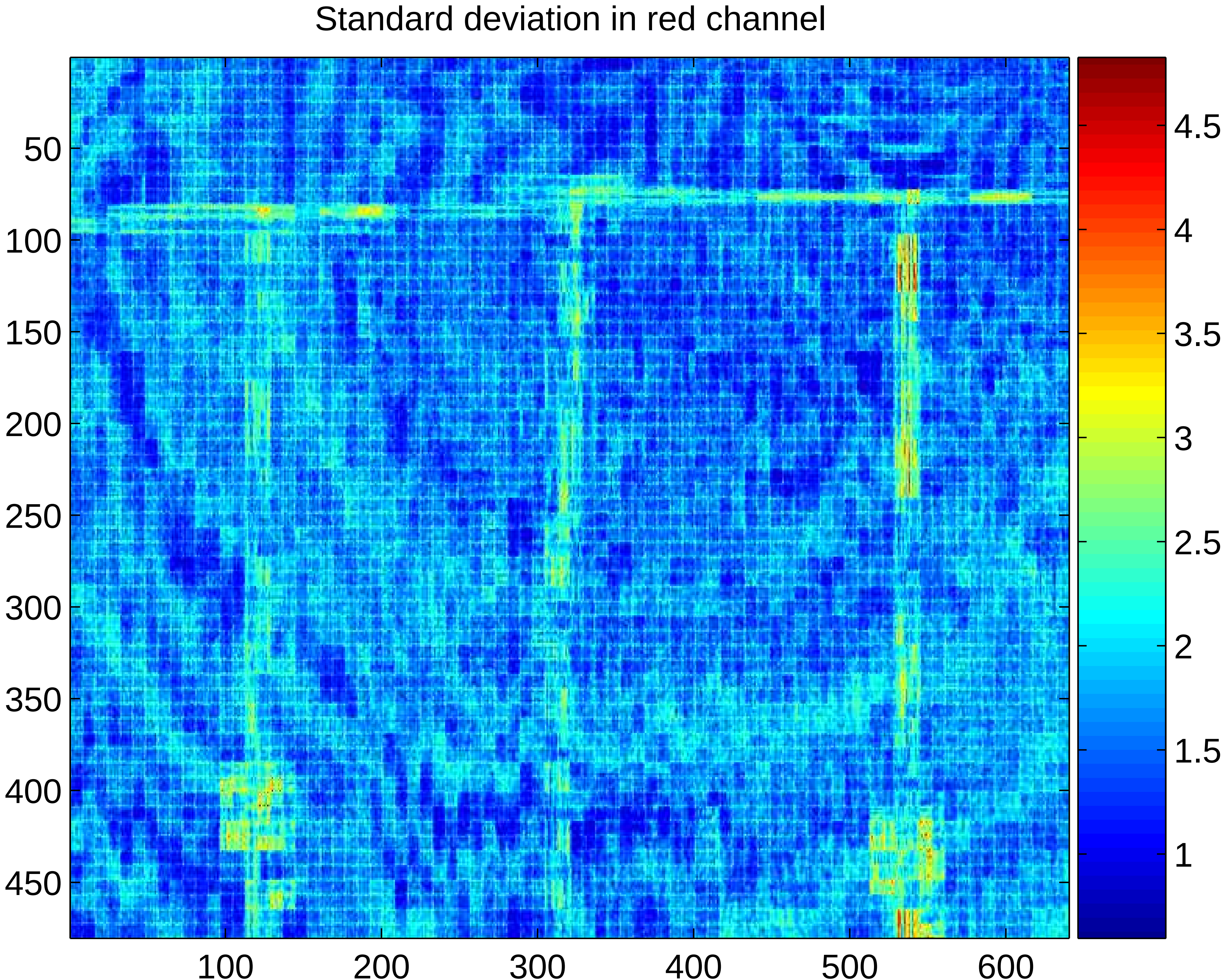

Standard deviation in red channel
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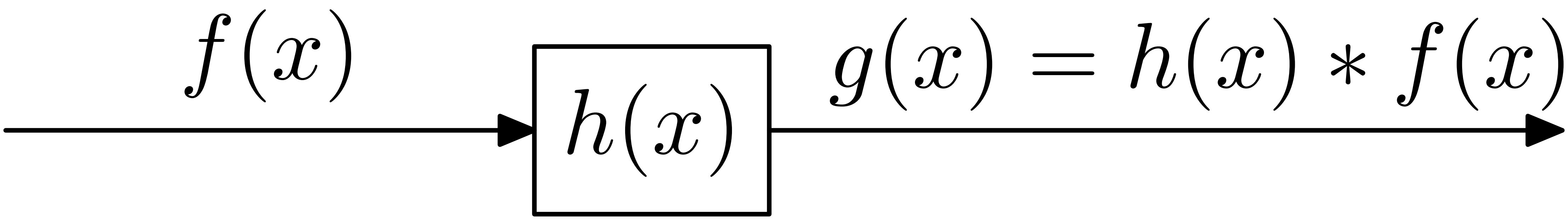
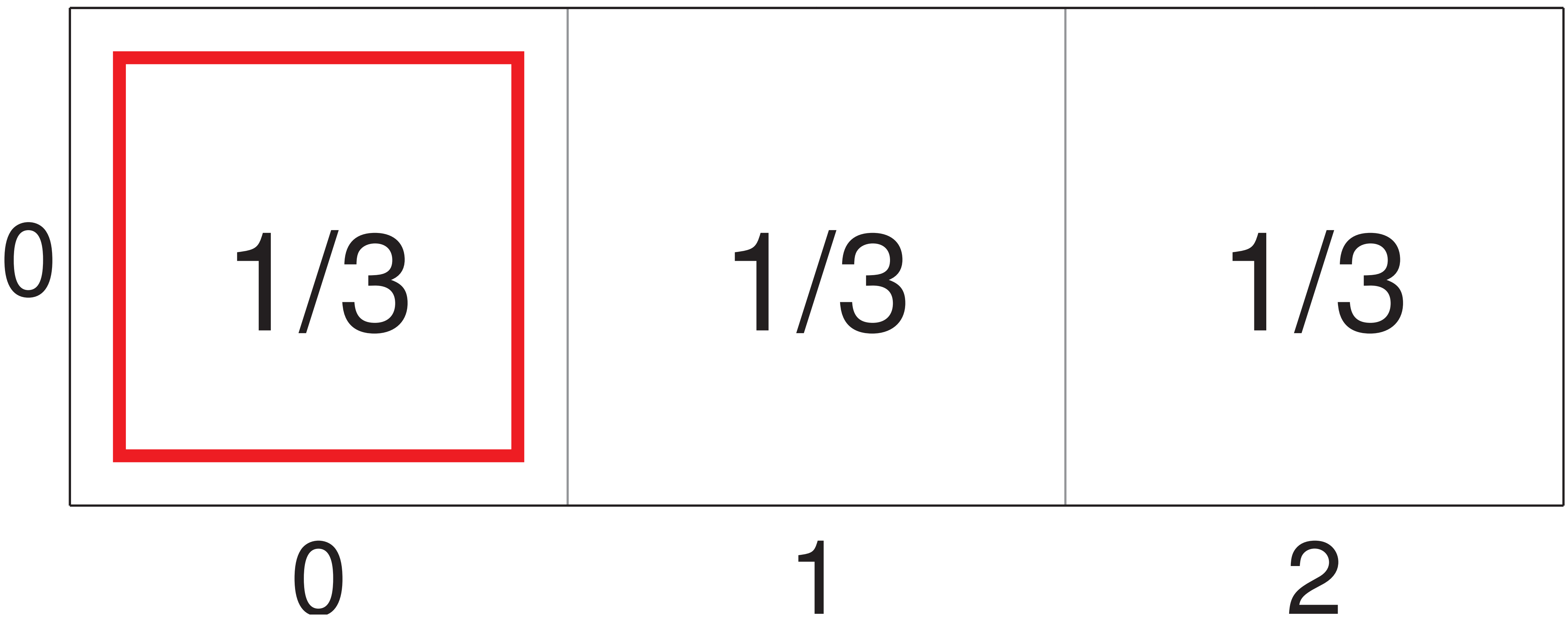

$f(x)$
$h(x)$
$g(x) = h(x) * f(x)$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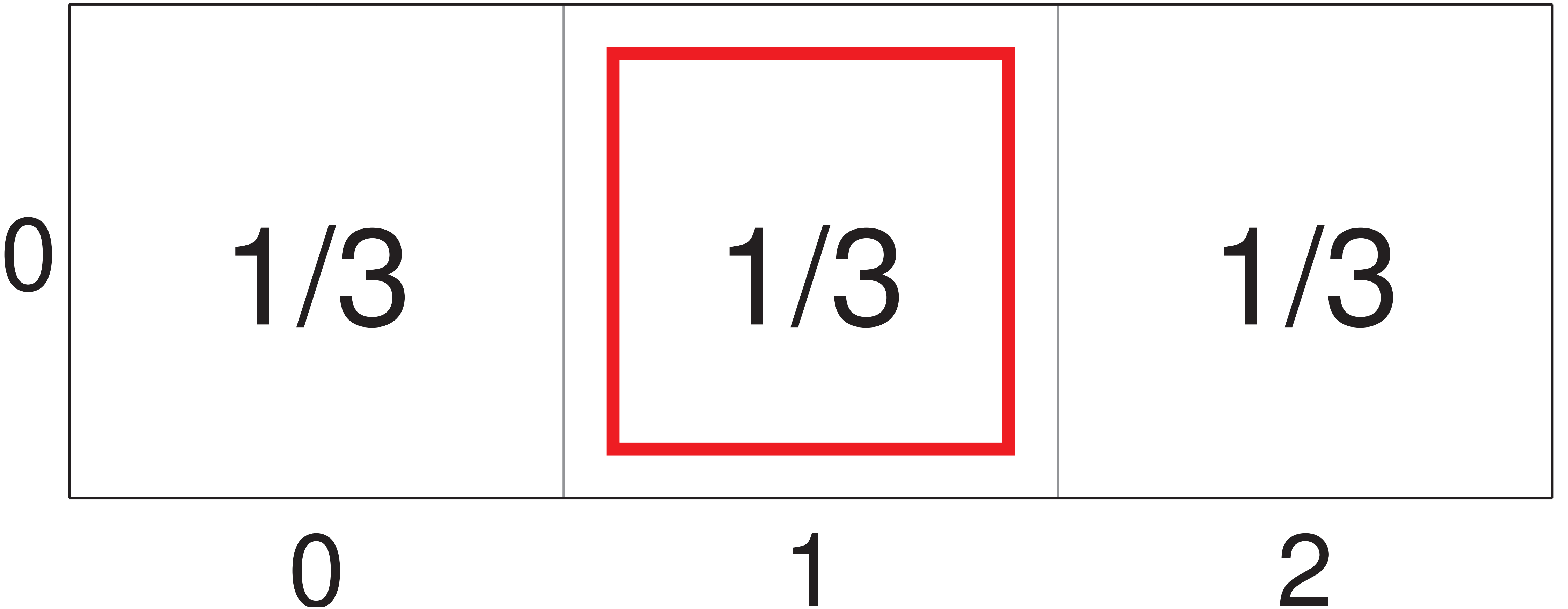
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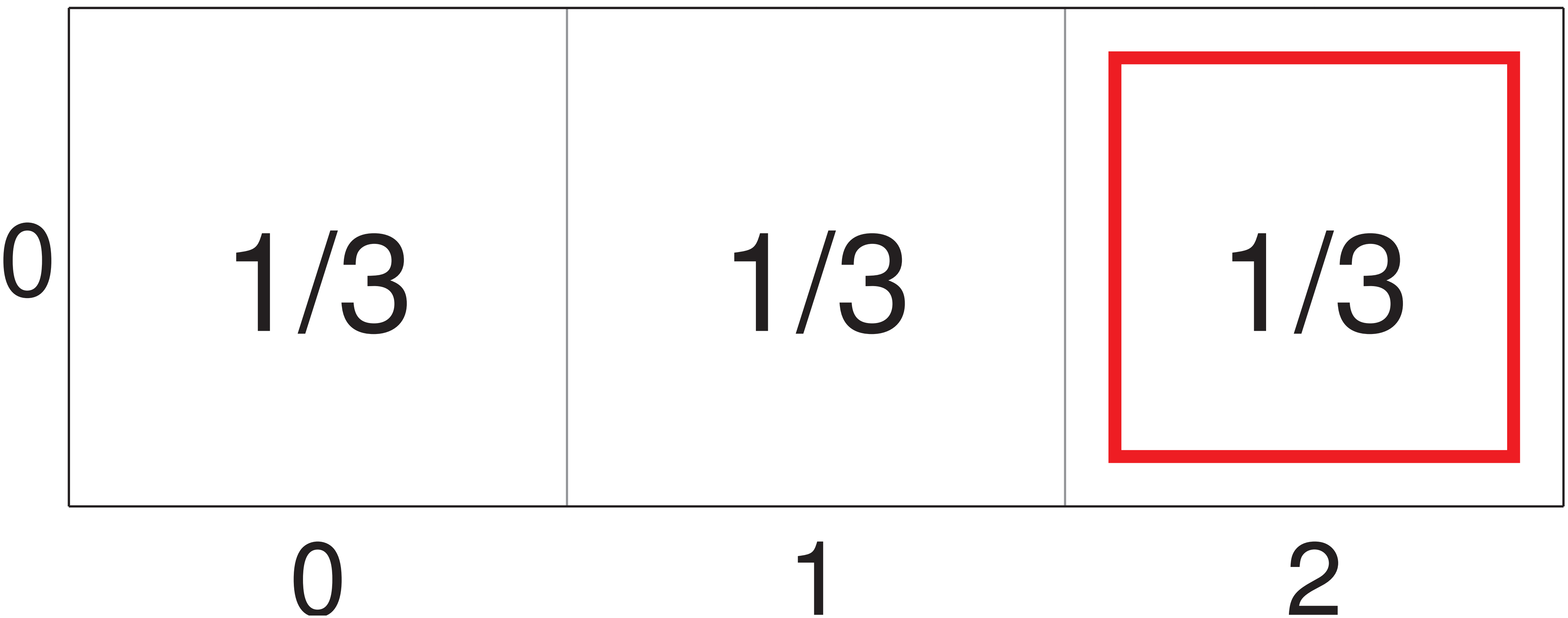
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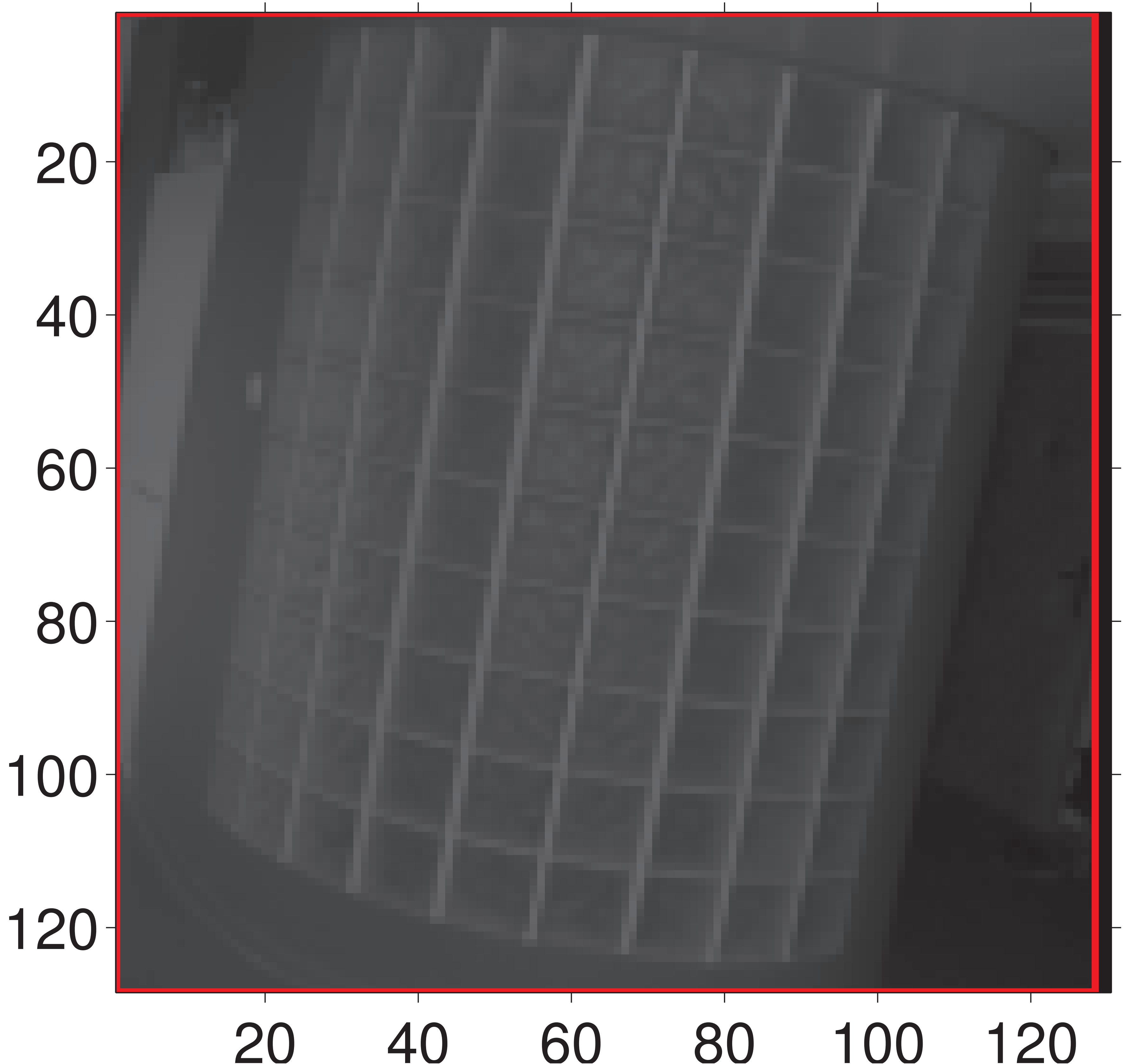
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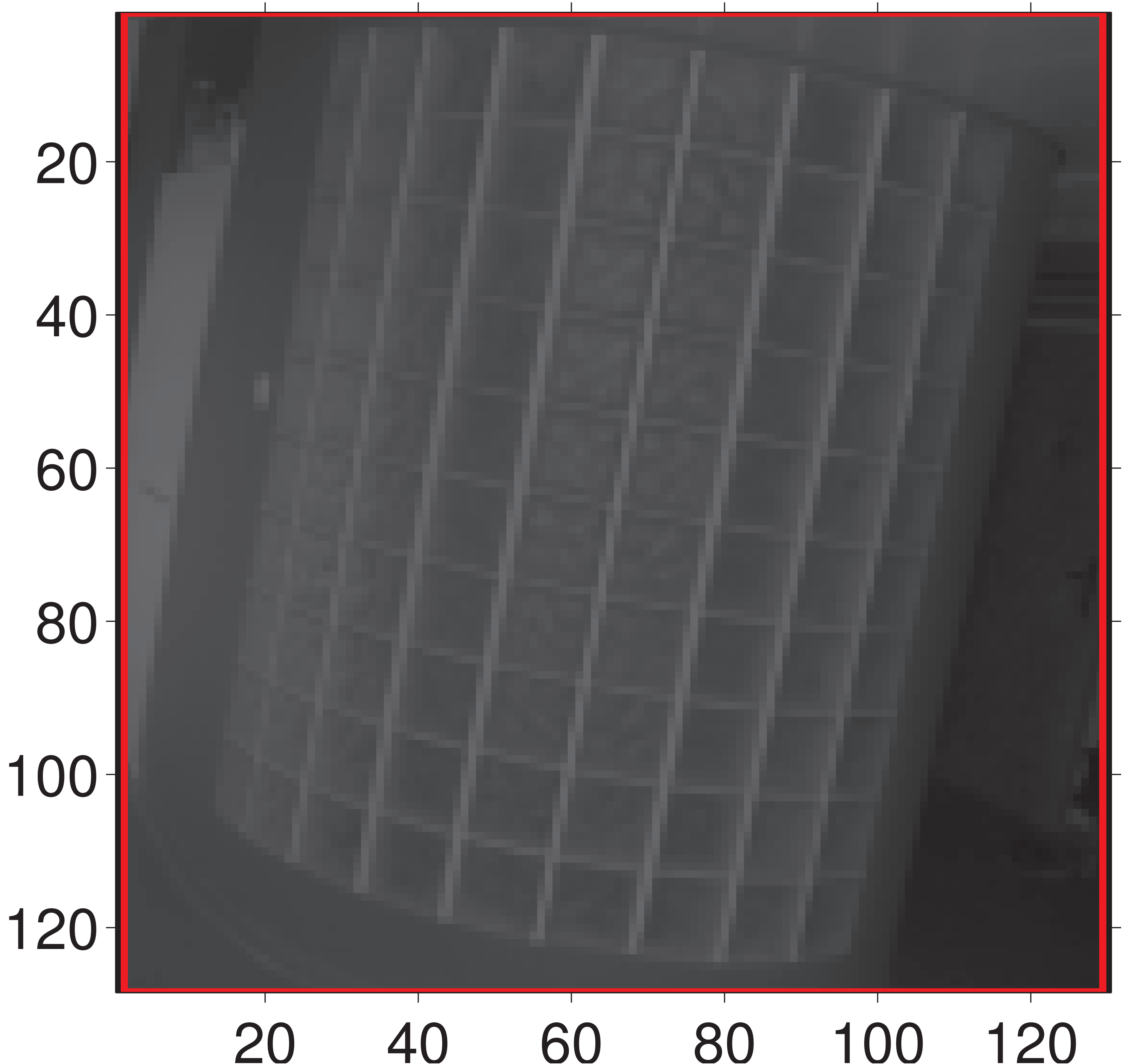
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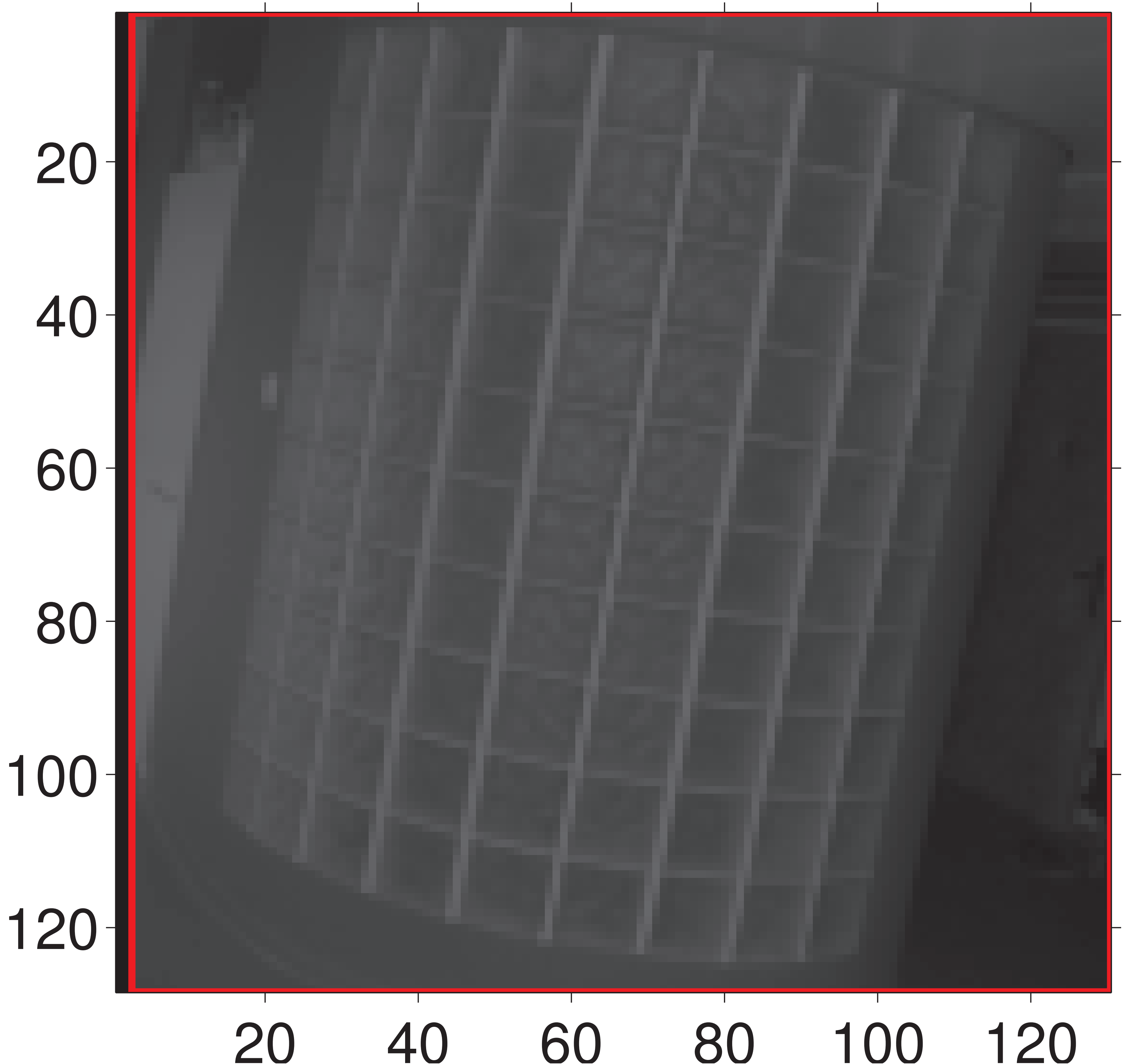

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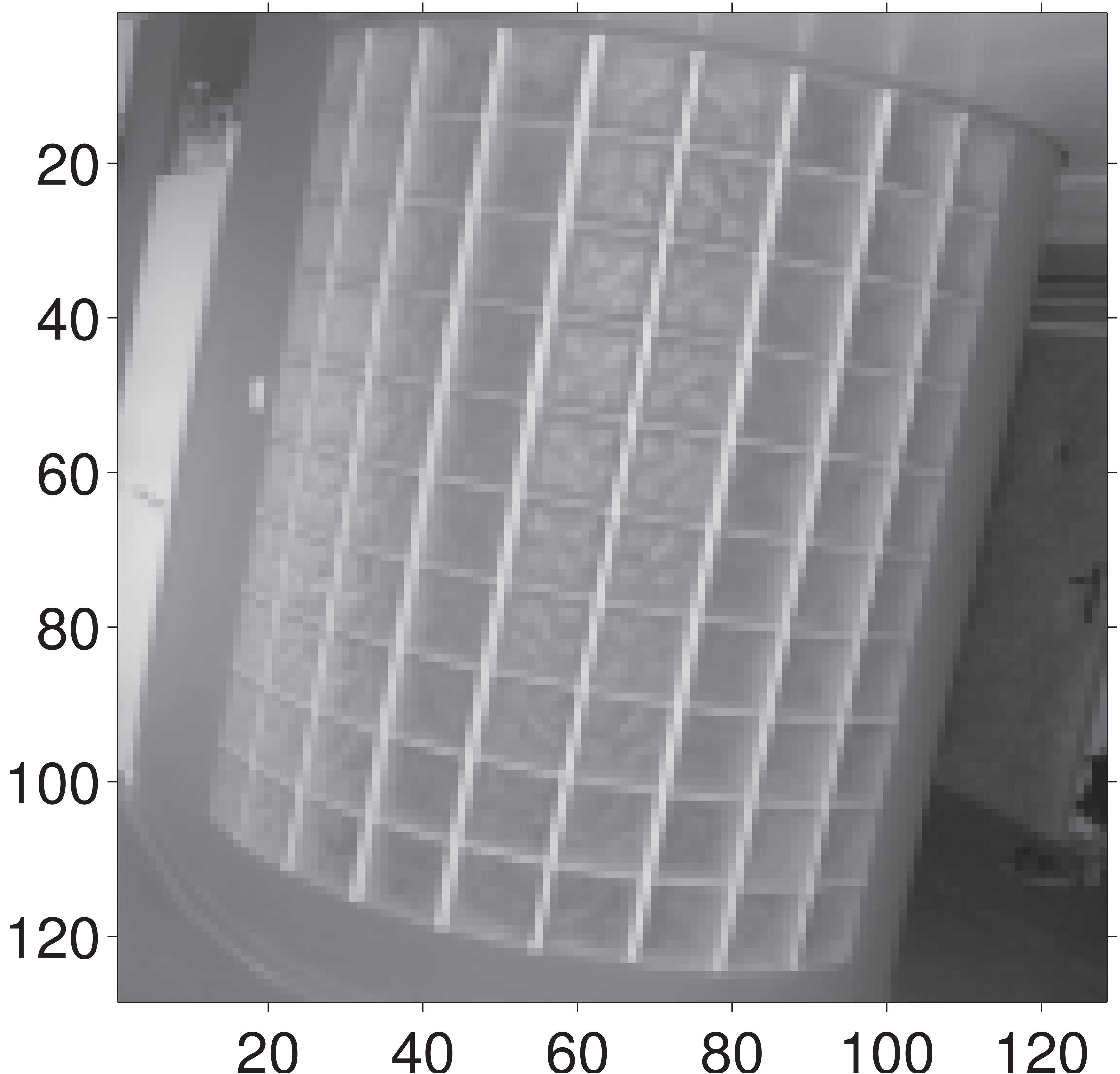

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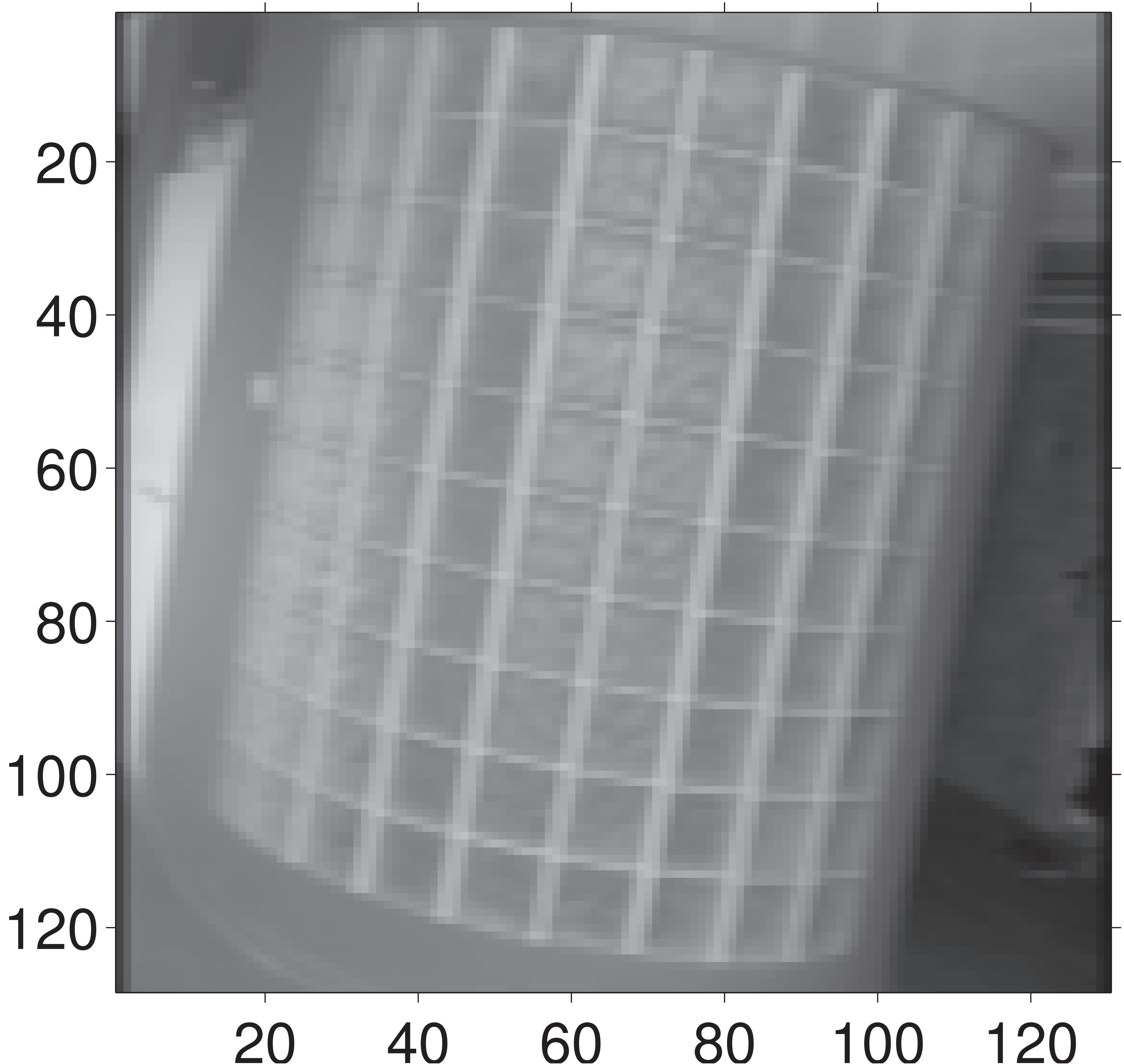
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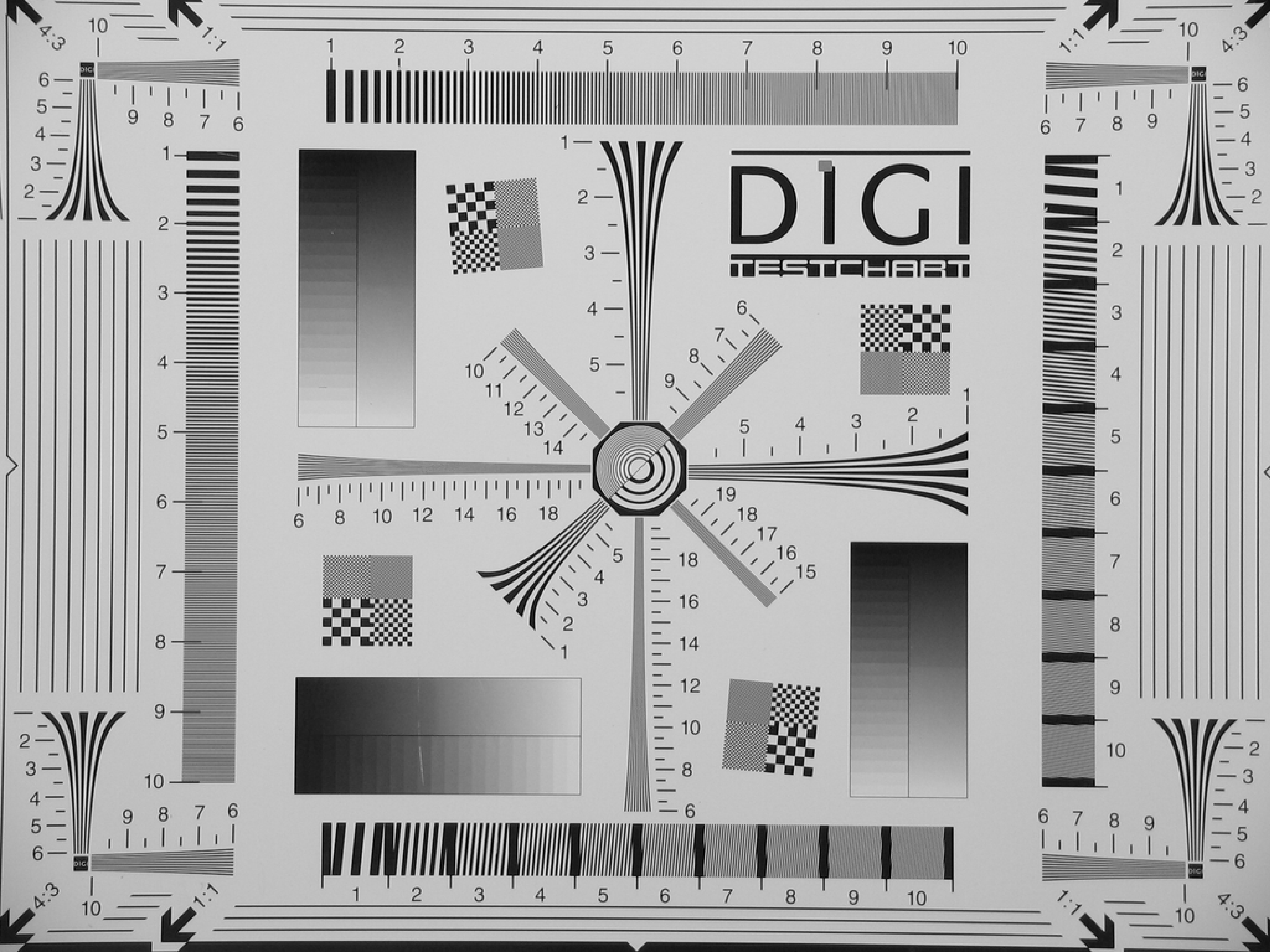
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TESTCHART
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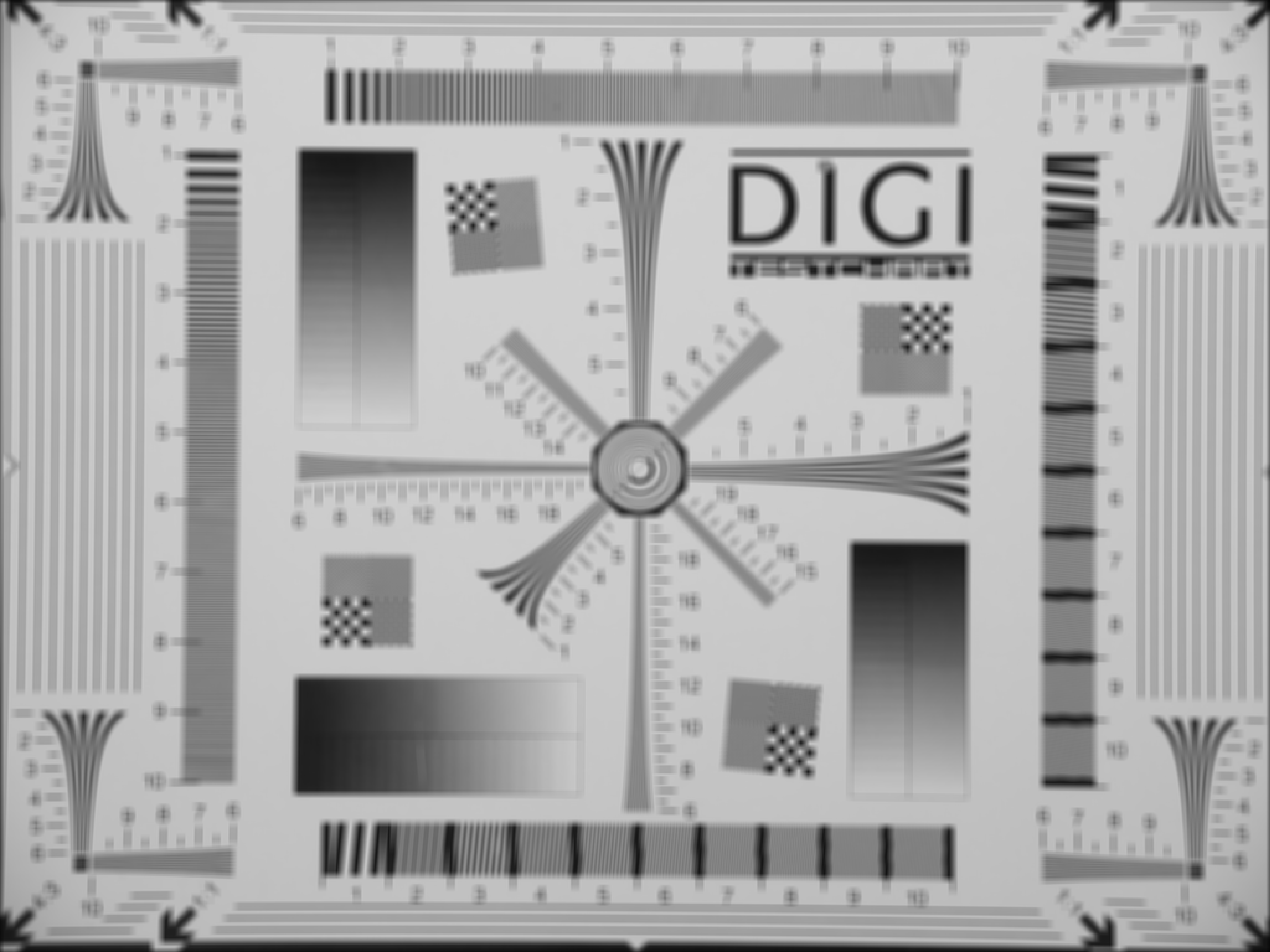
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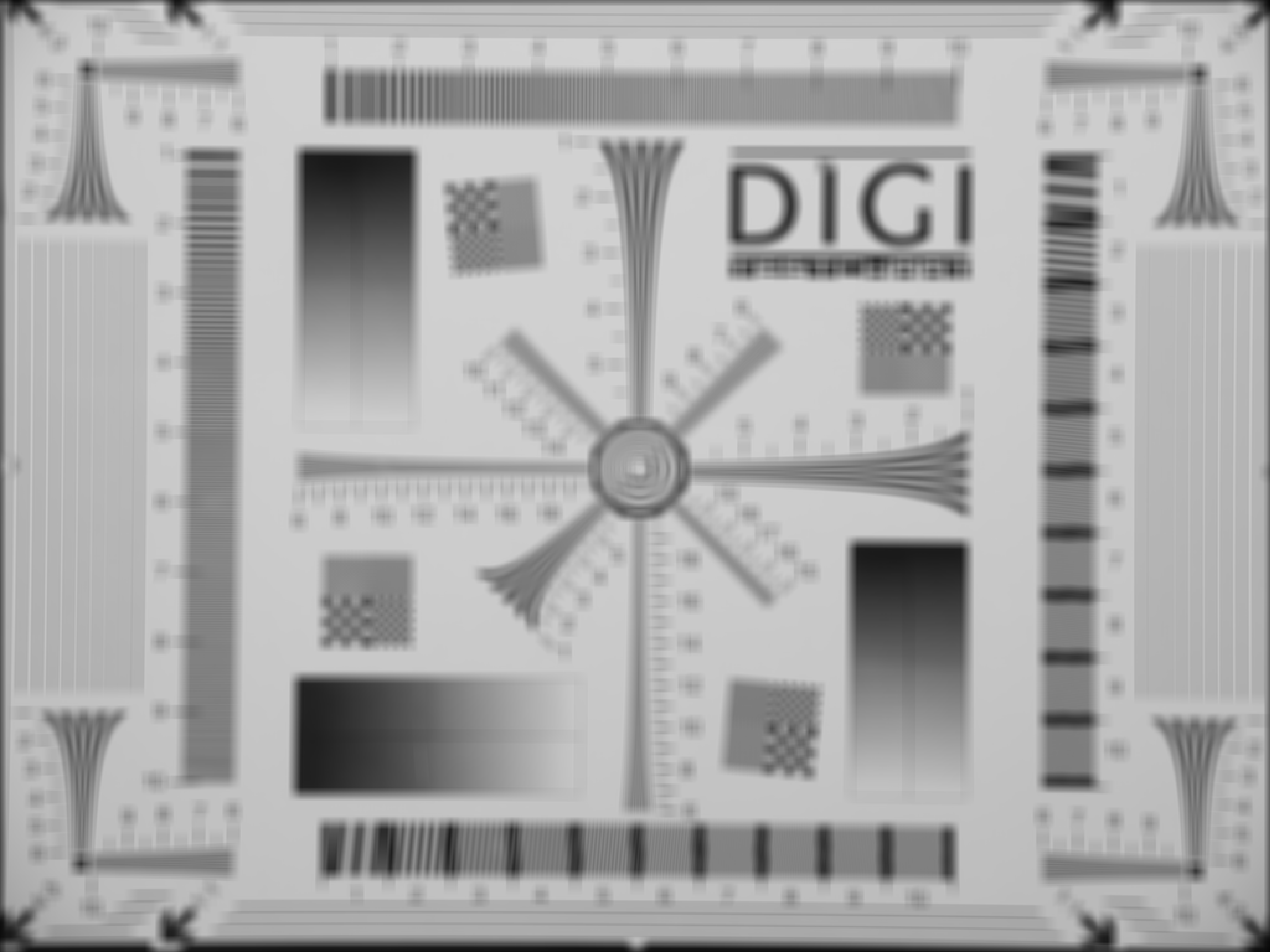
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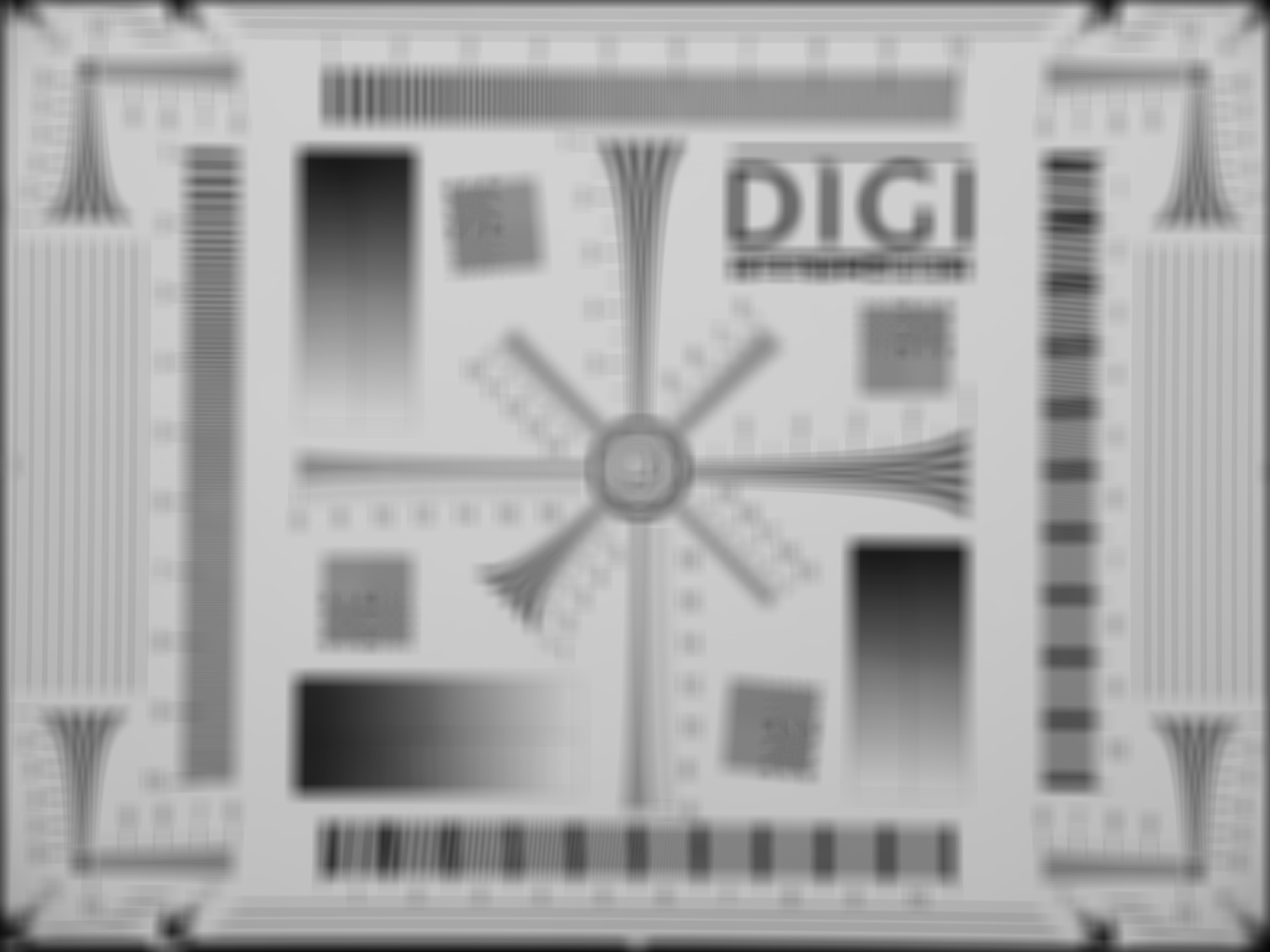
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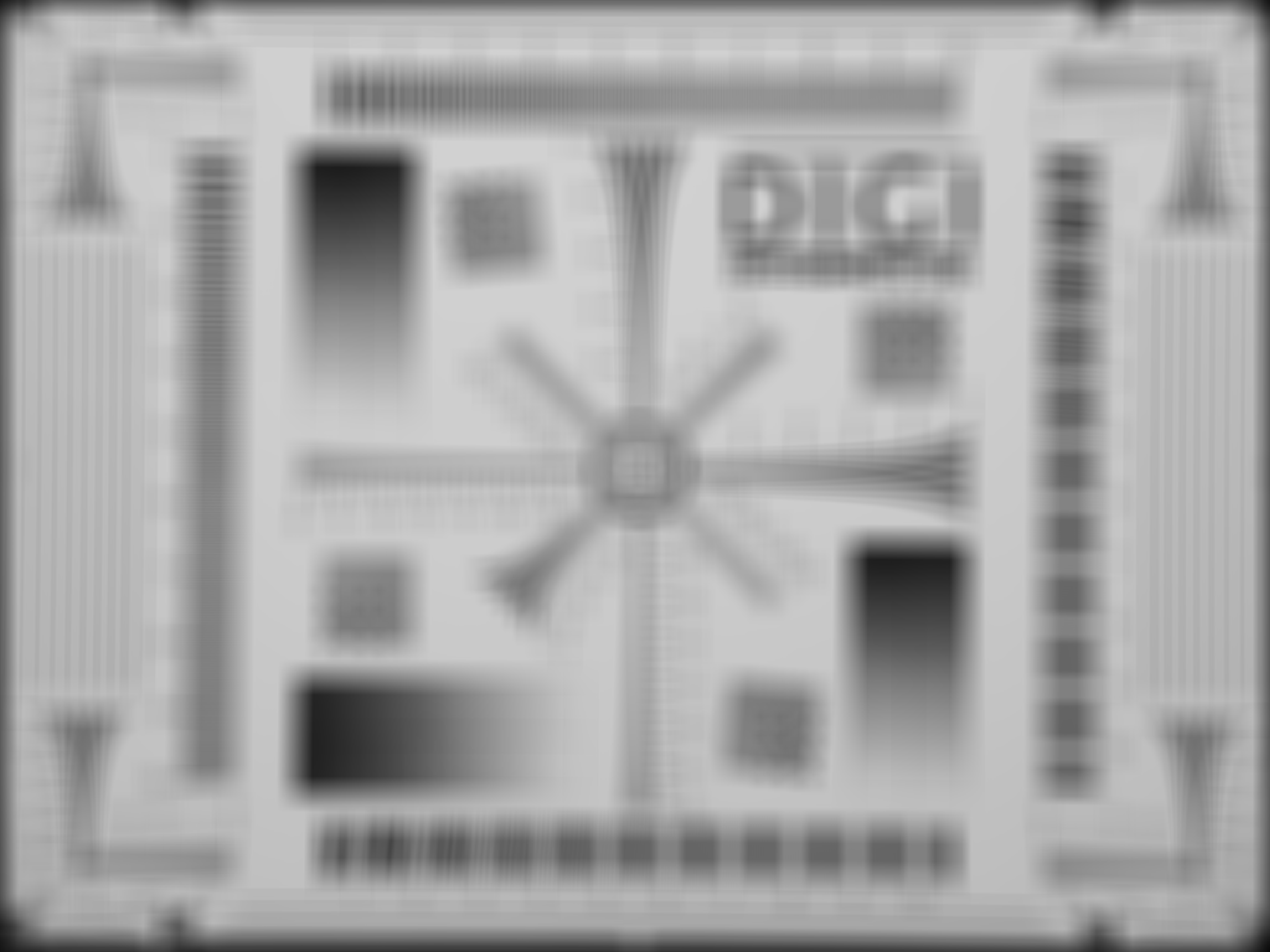
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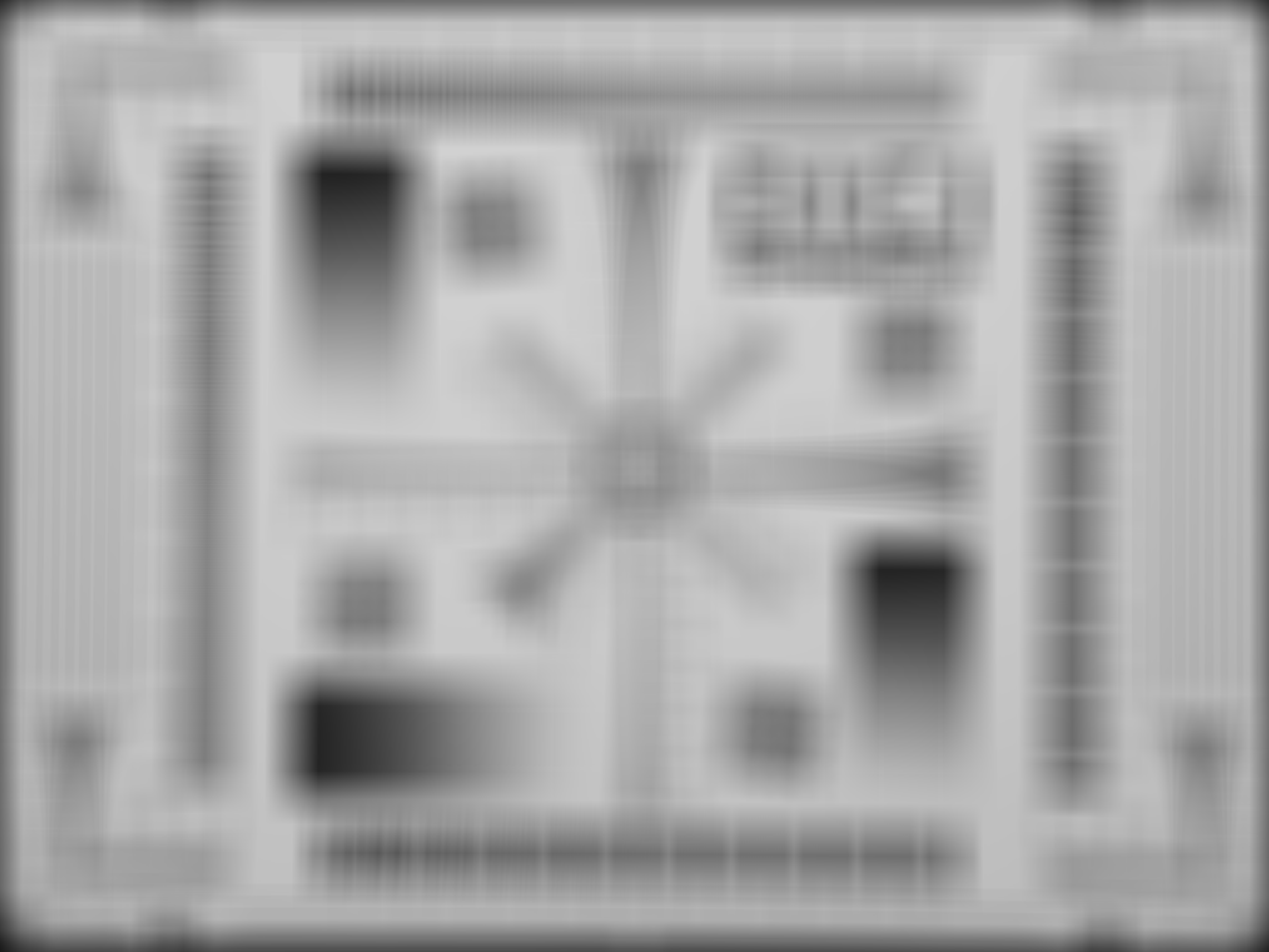

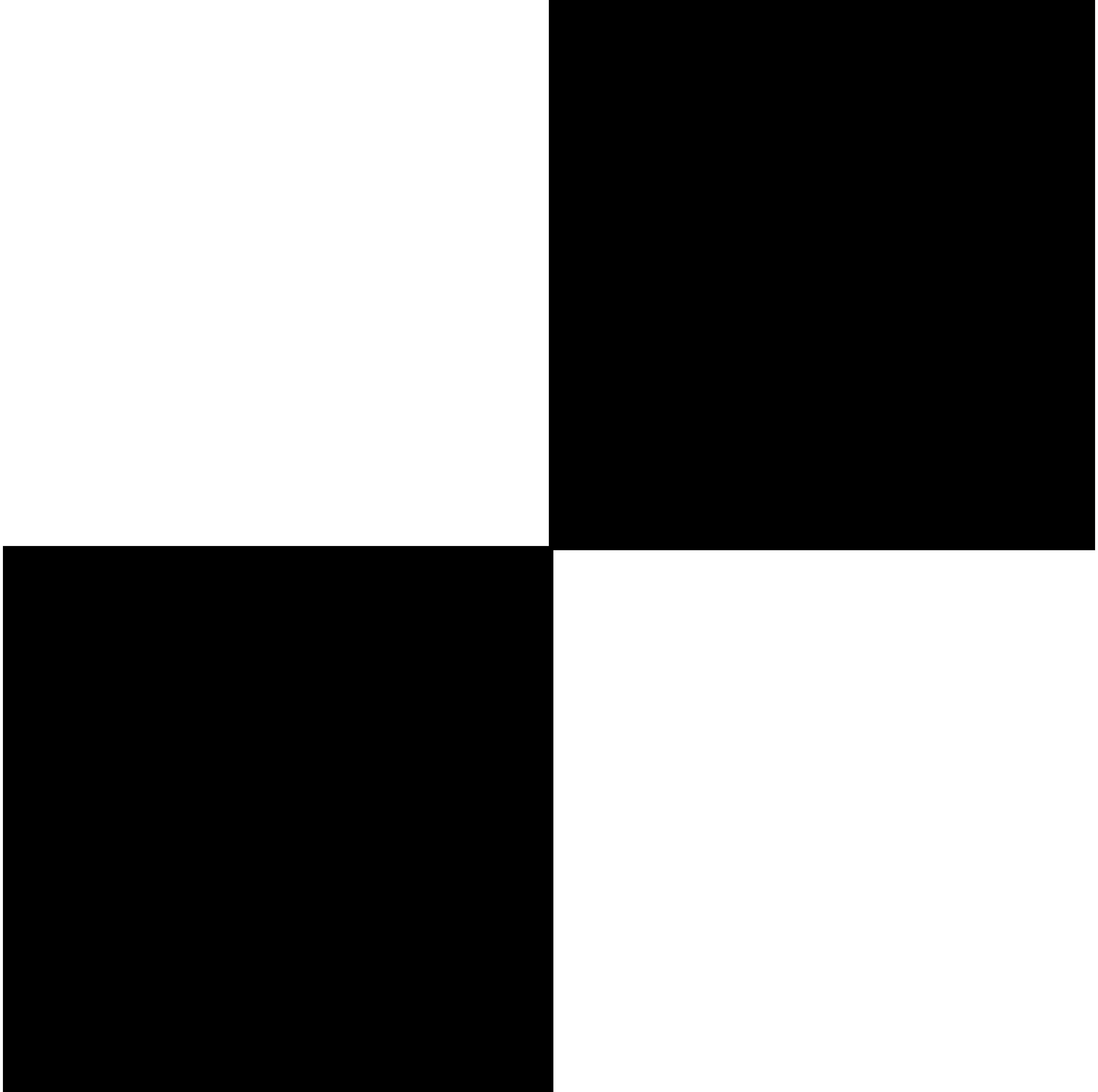

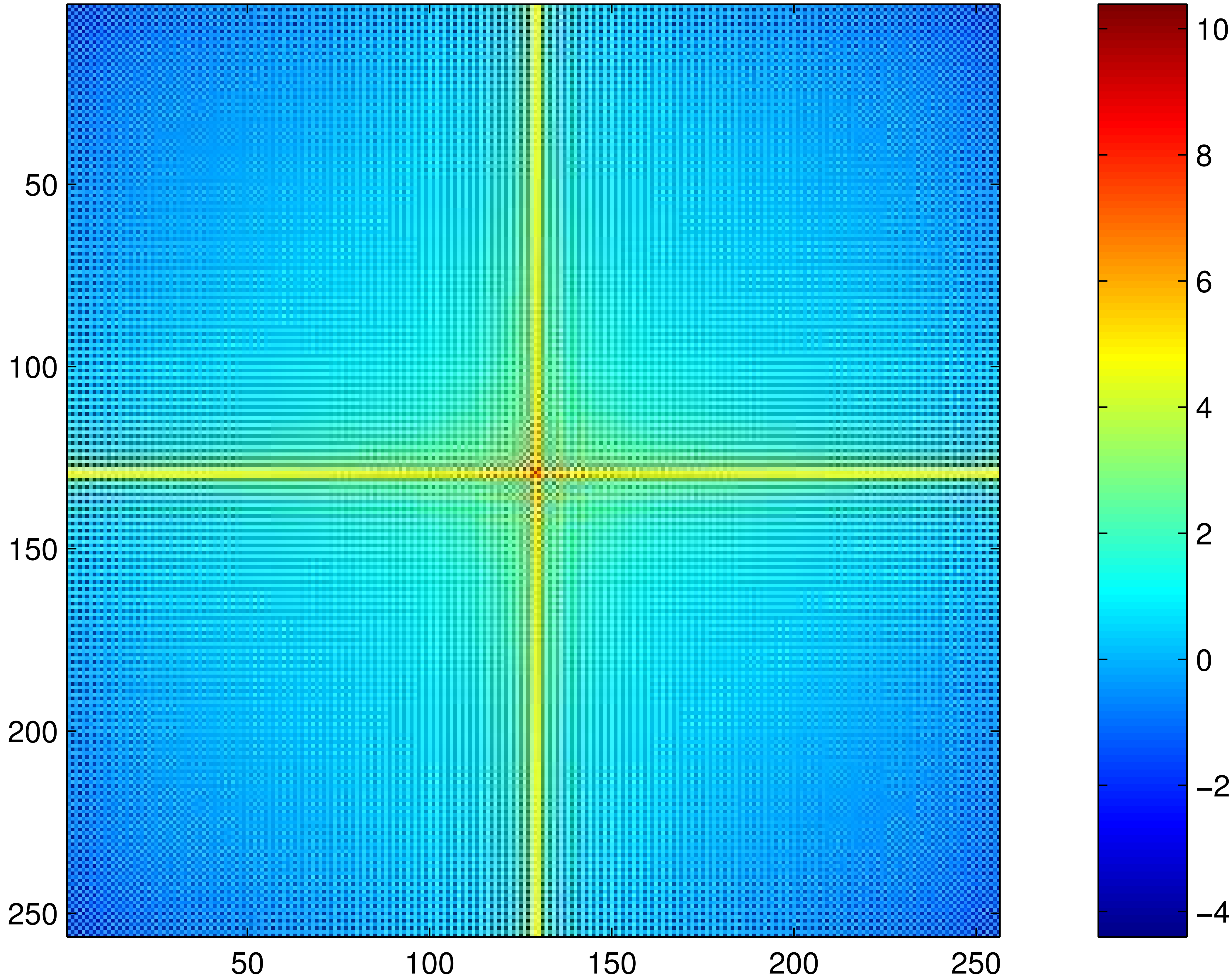
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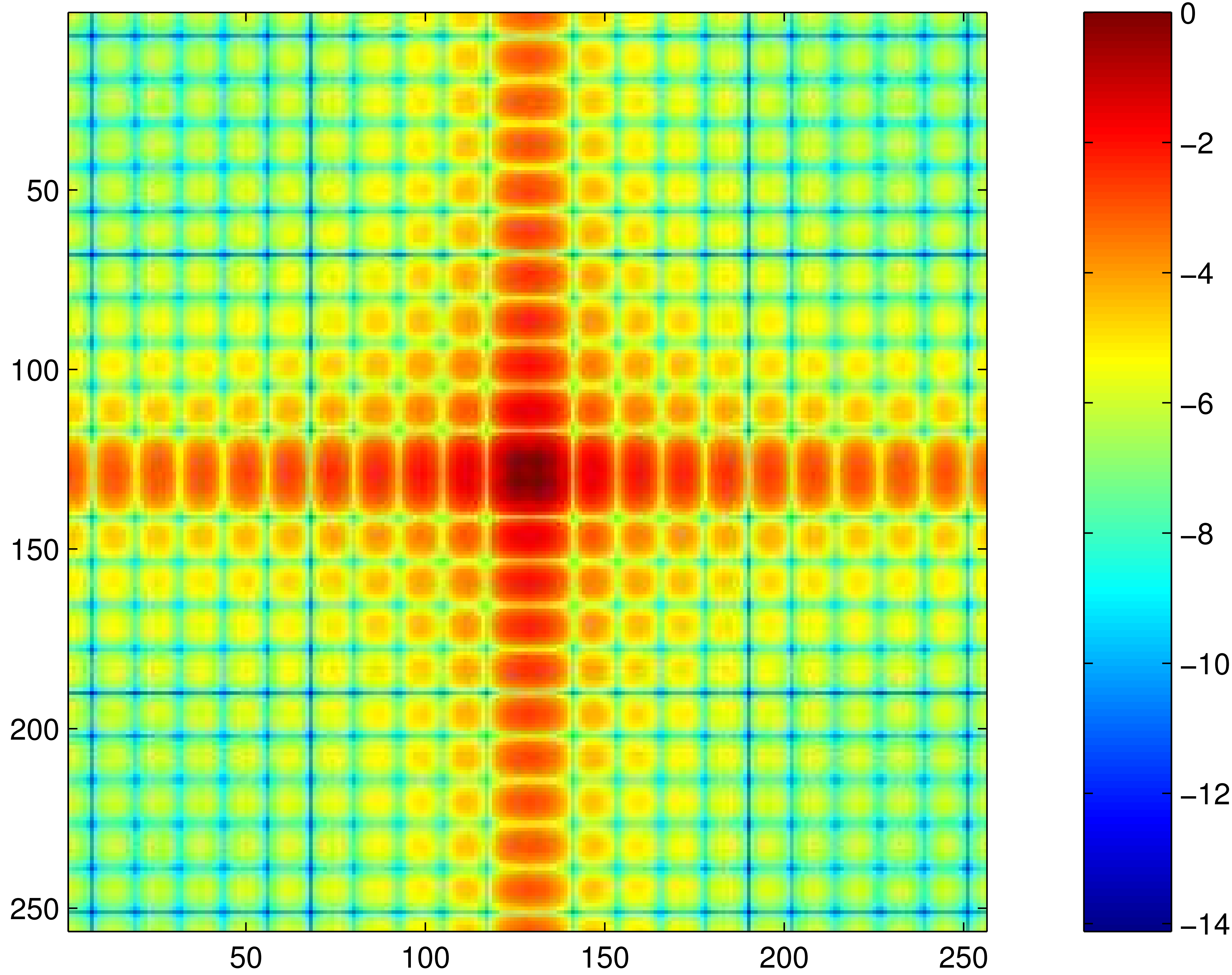

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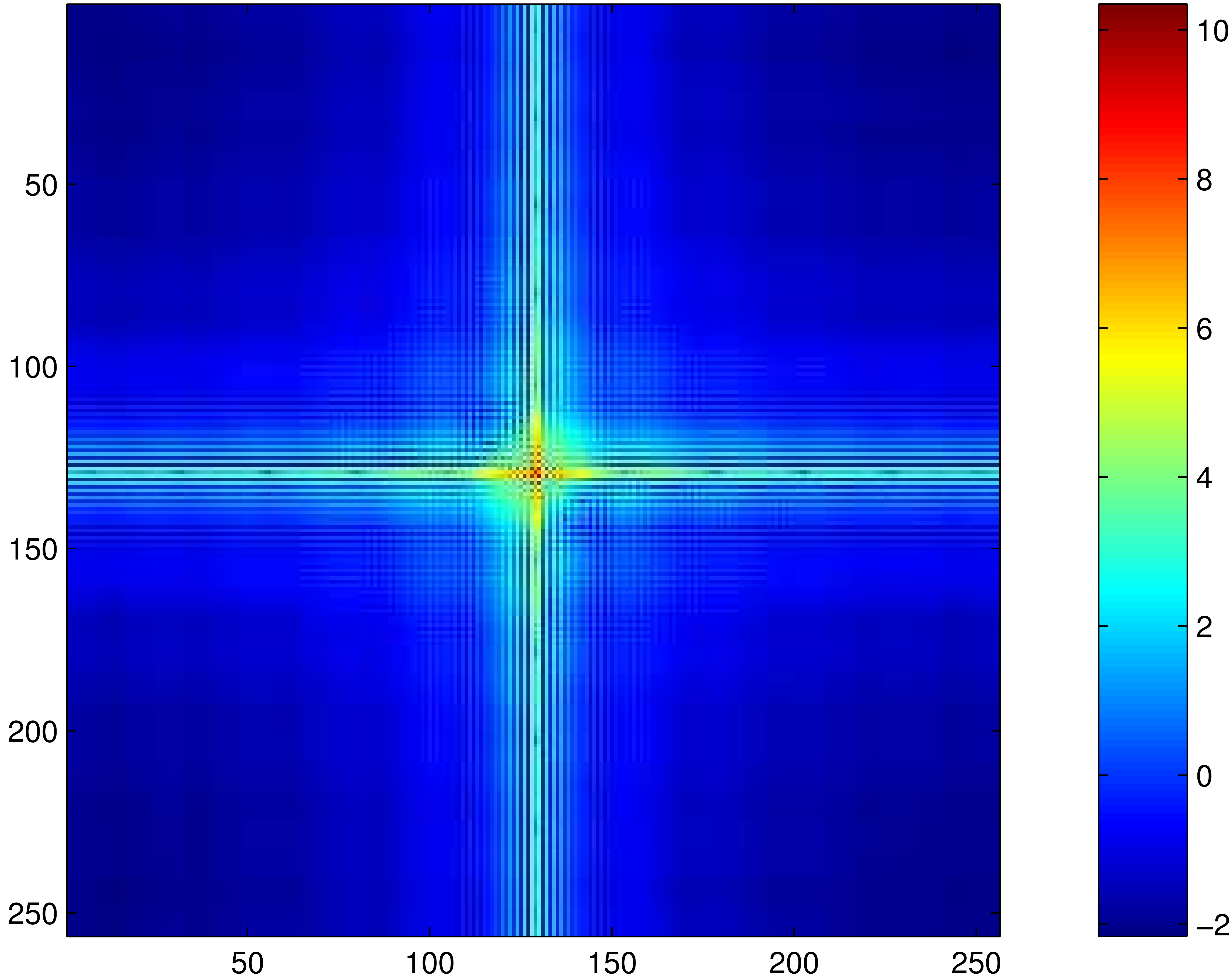

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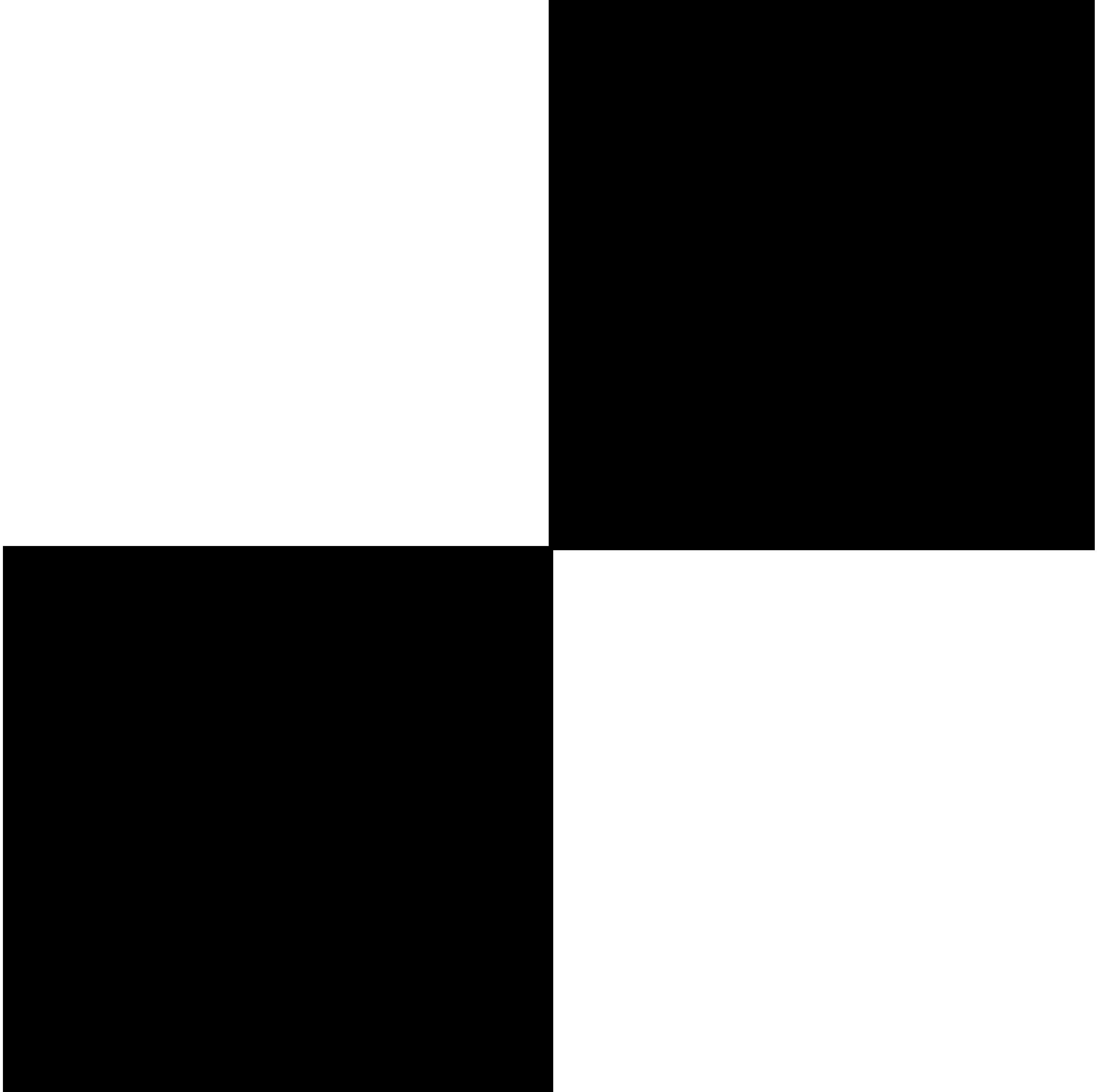

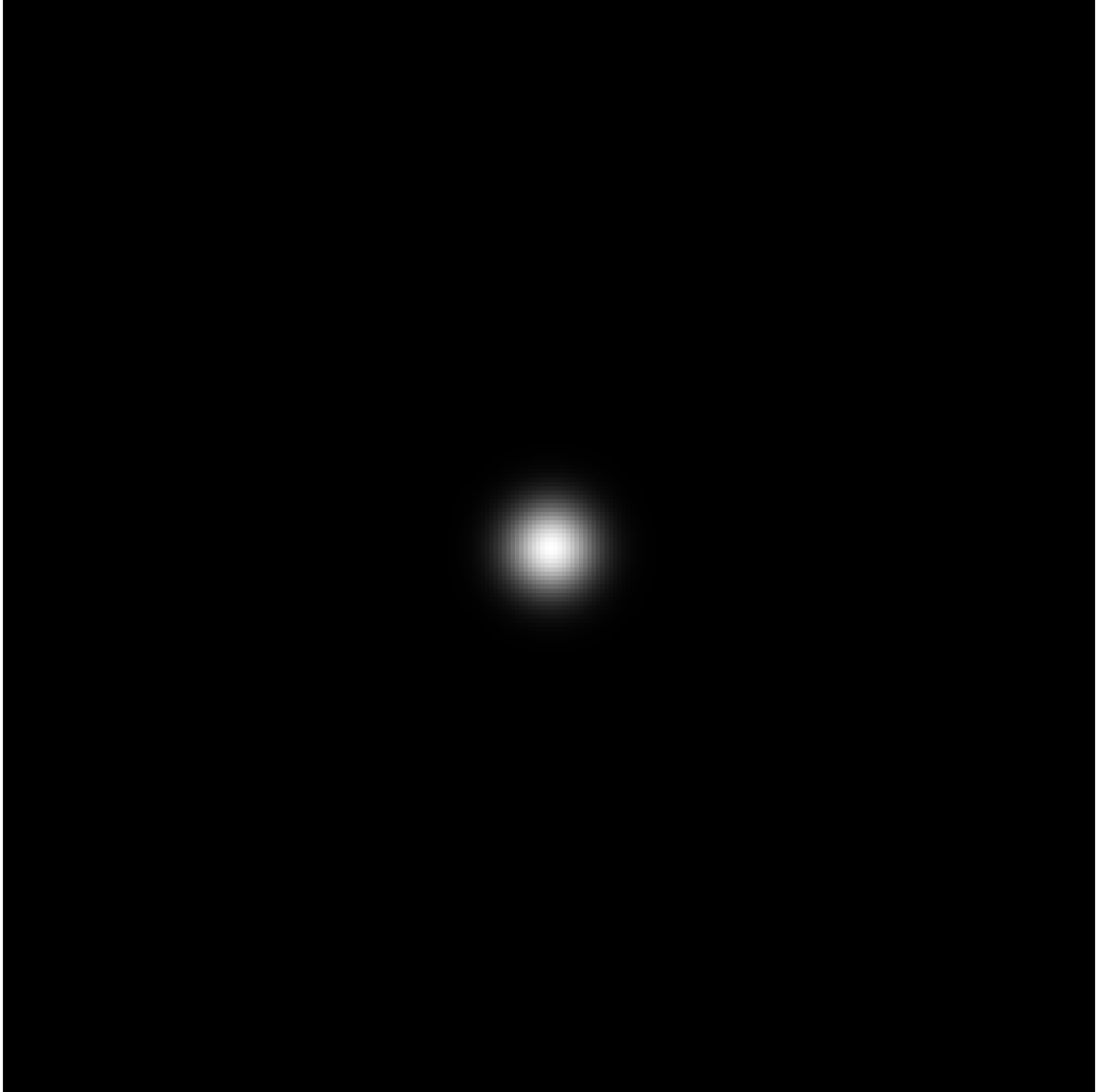

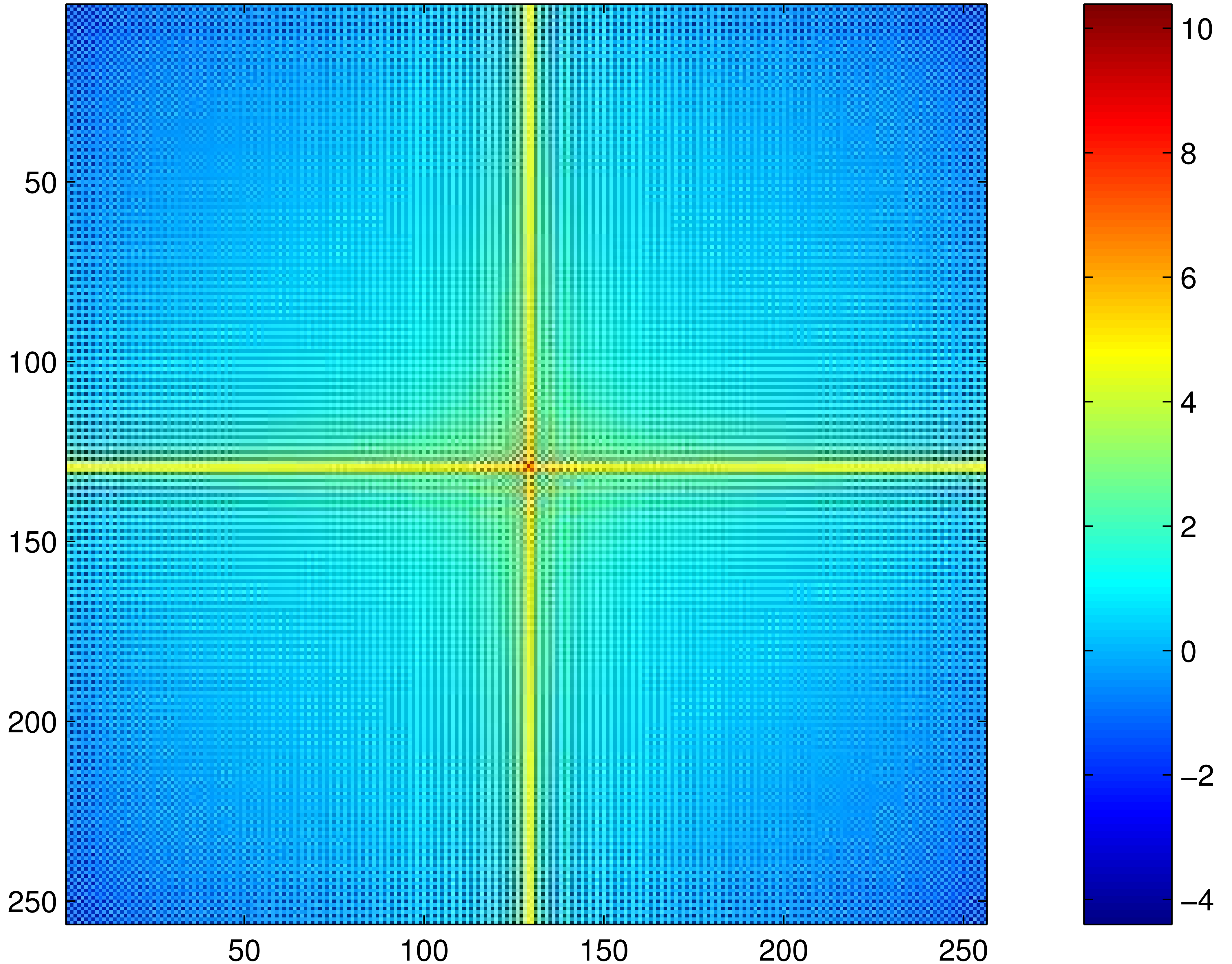

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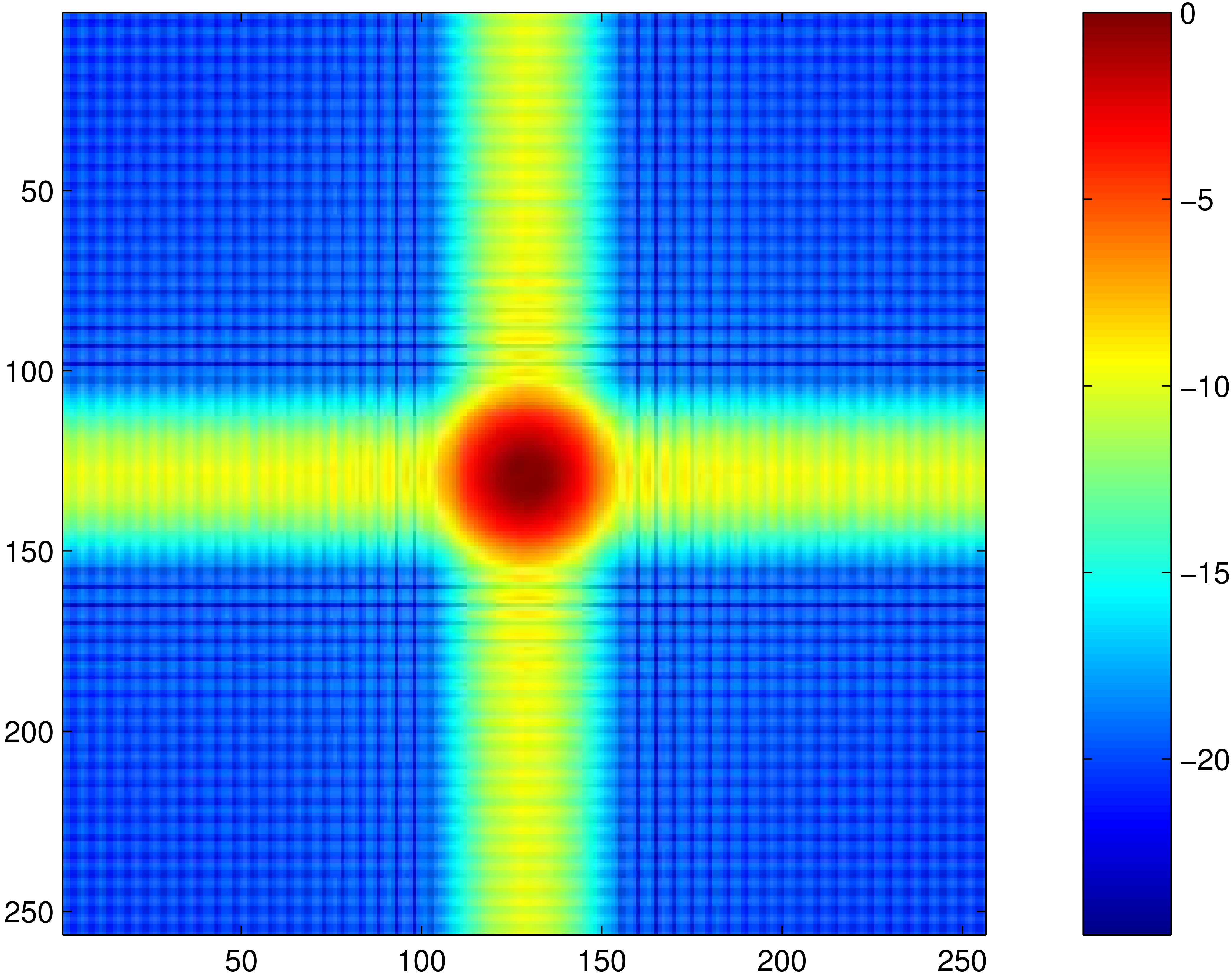
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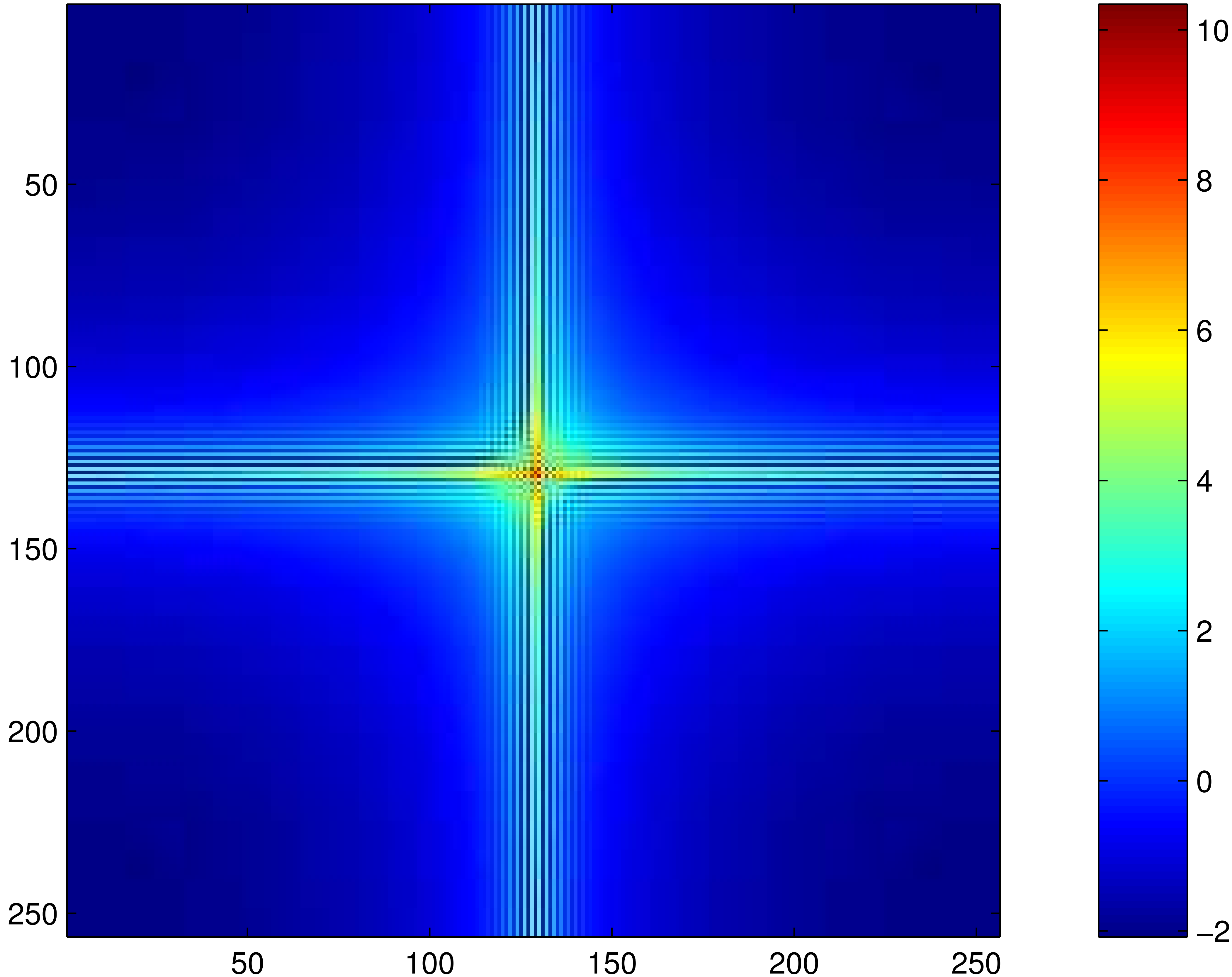

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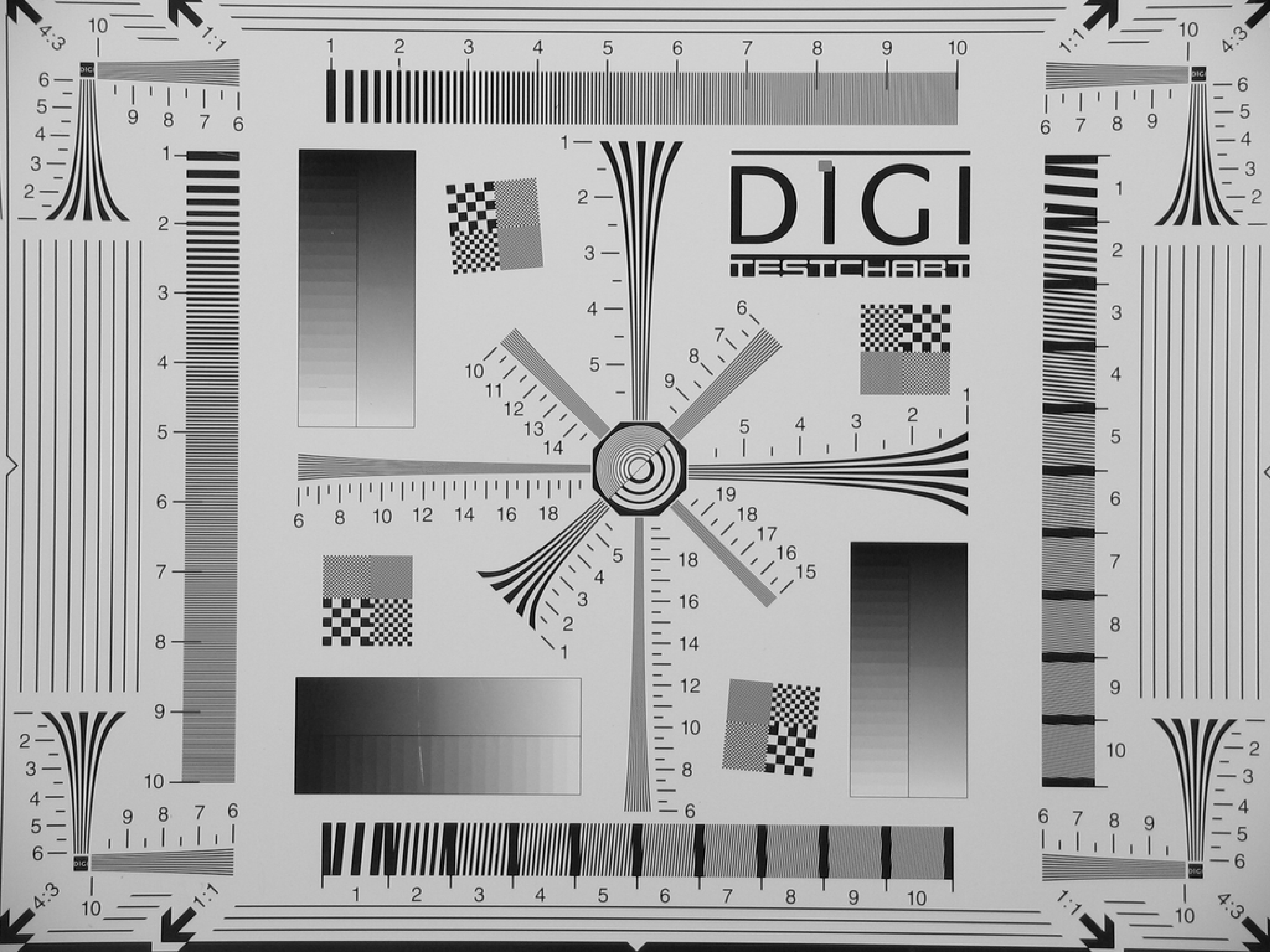
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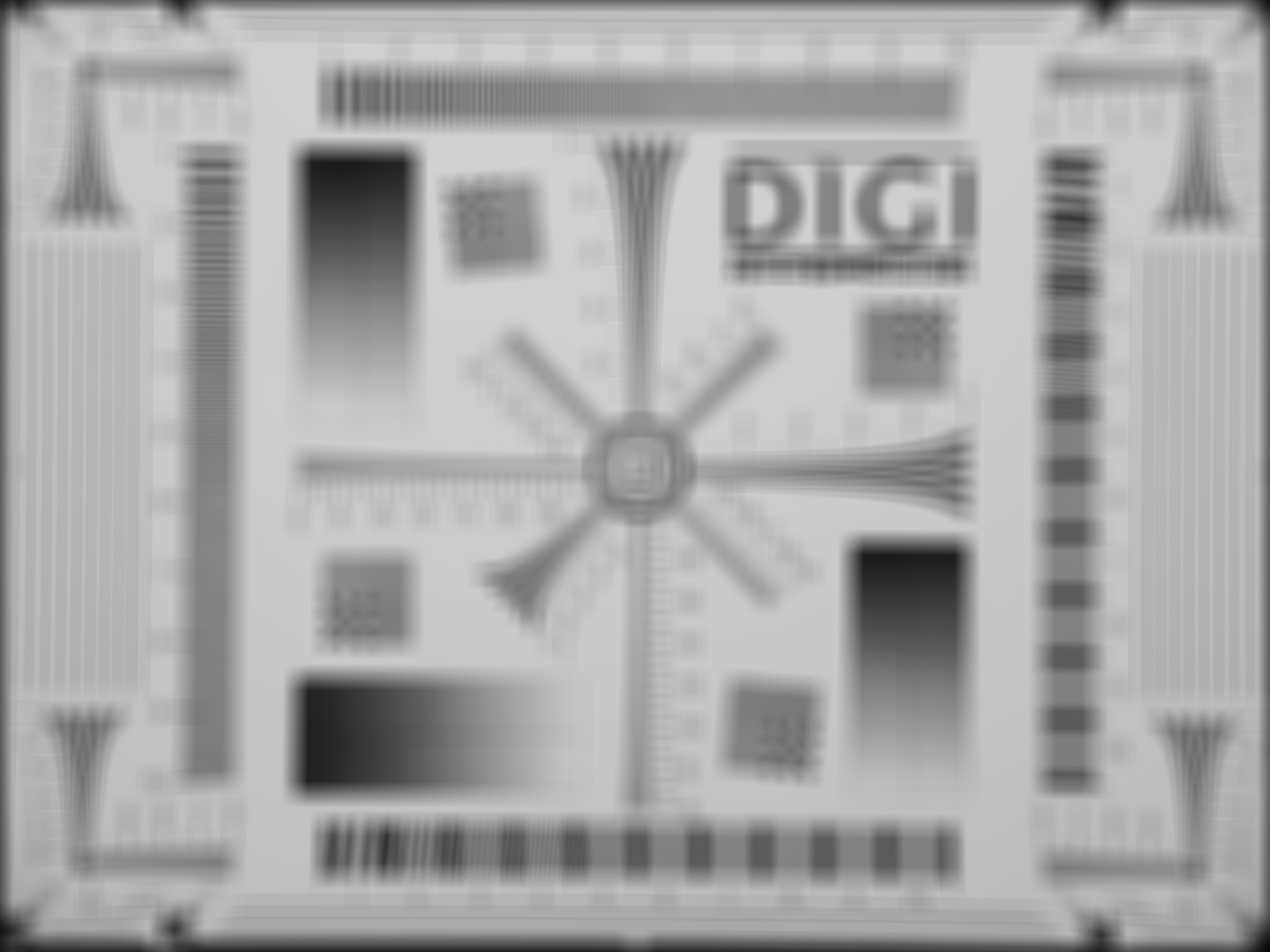
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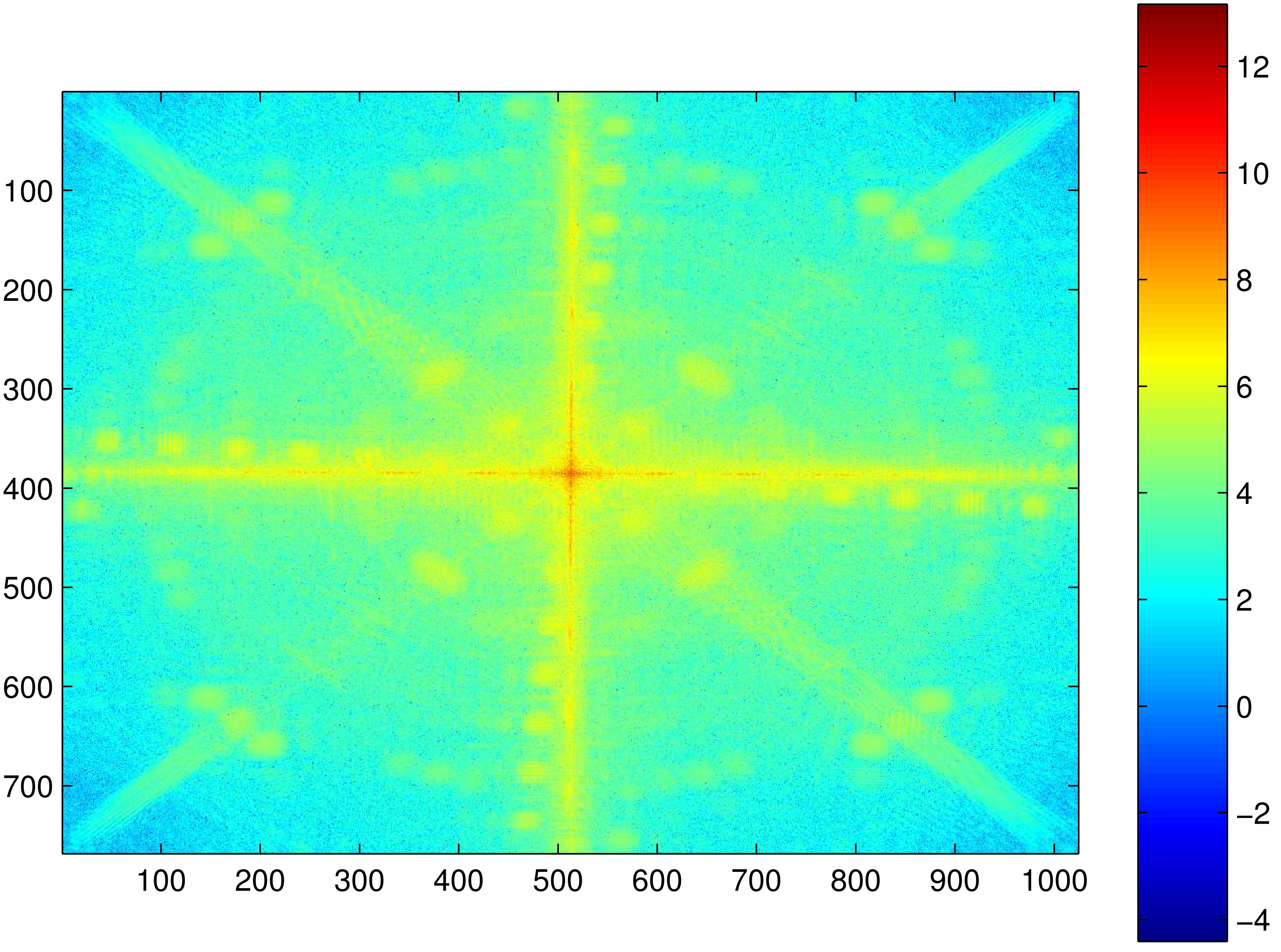
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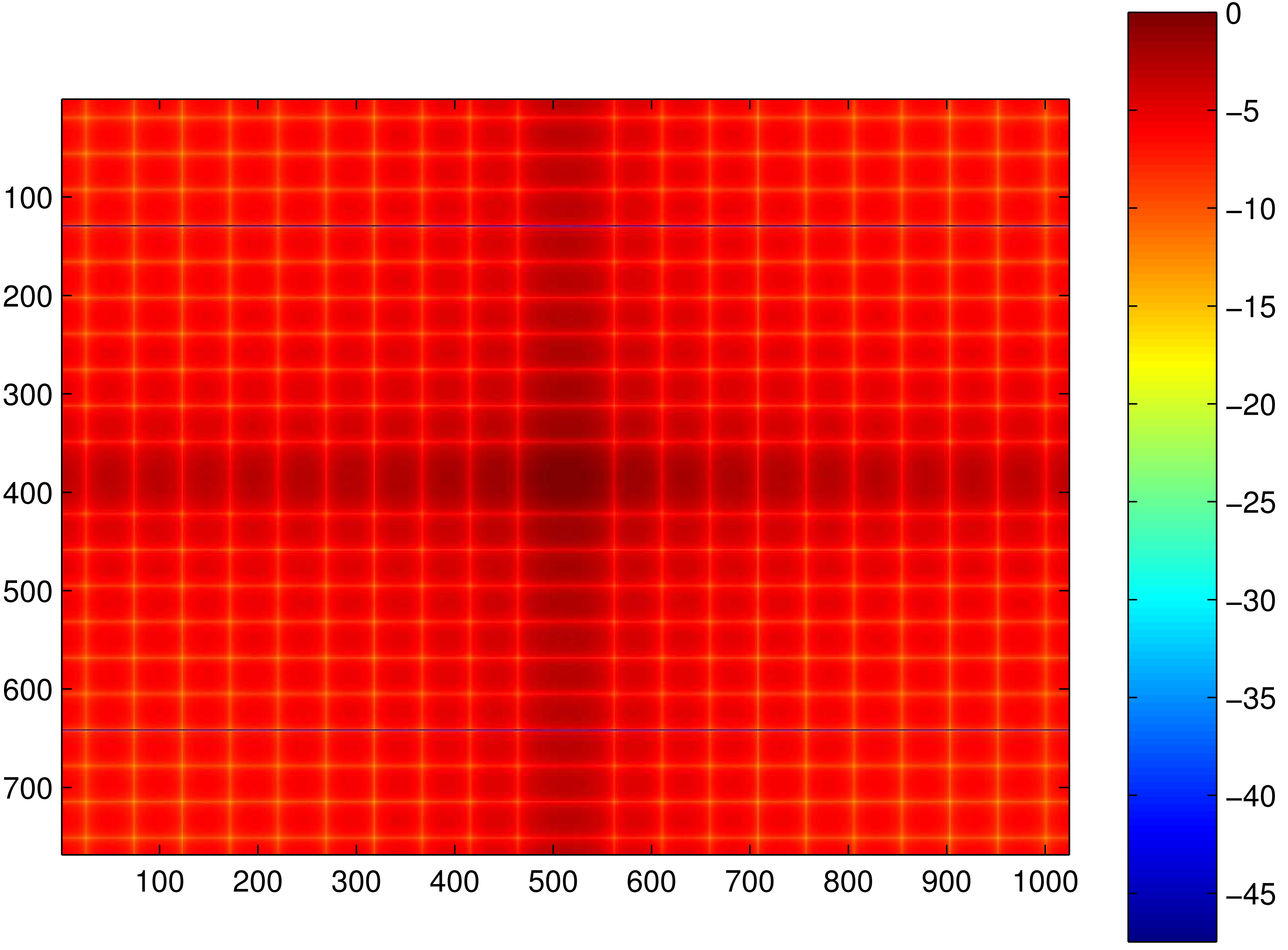

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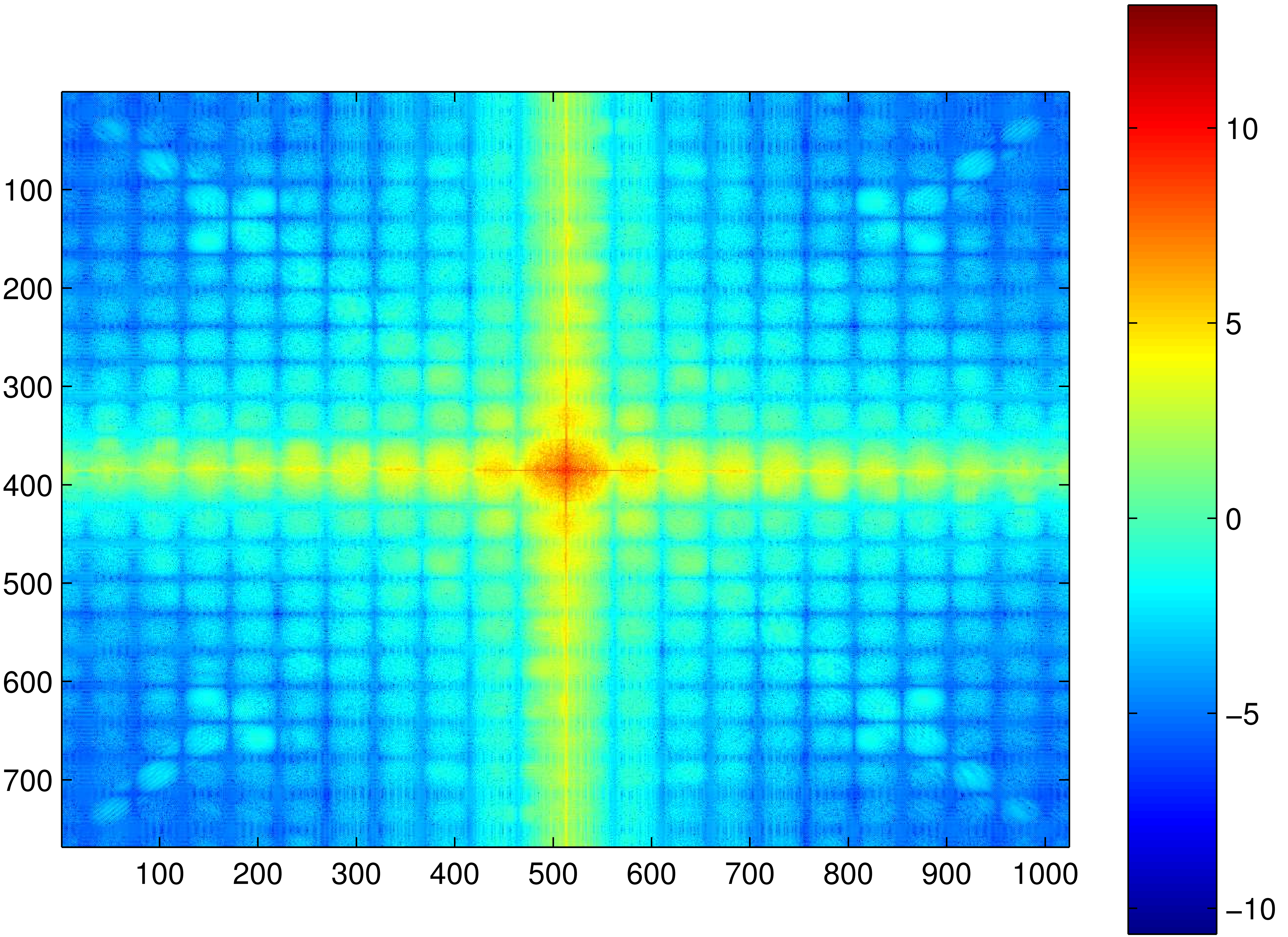
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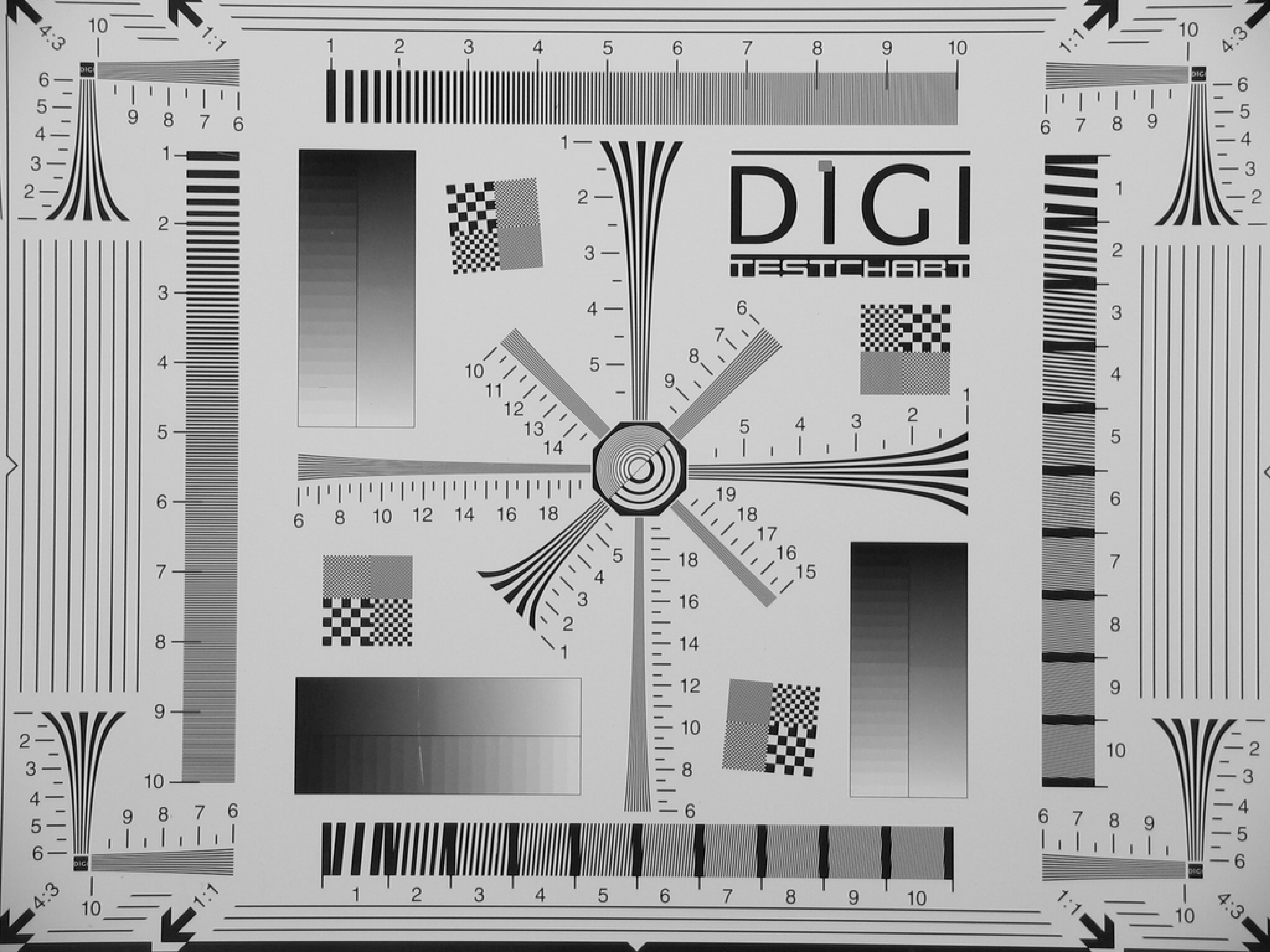
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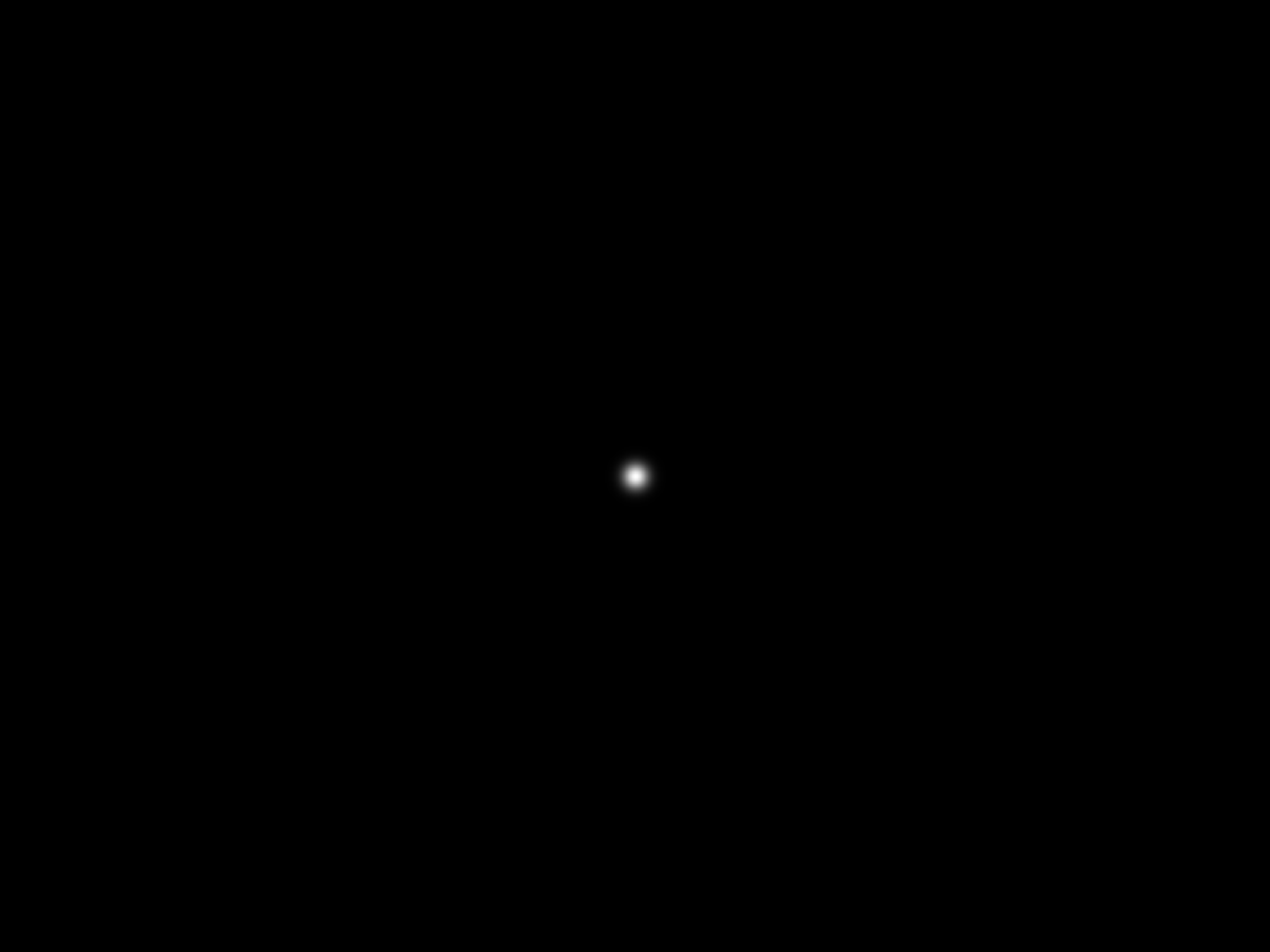


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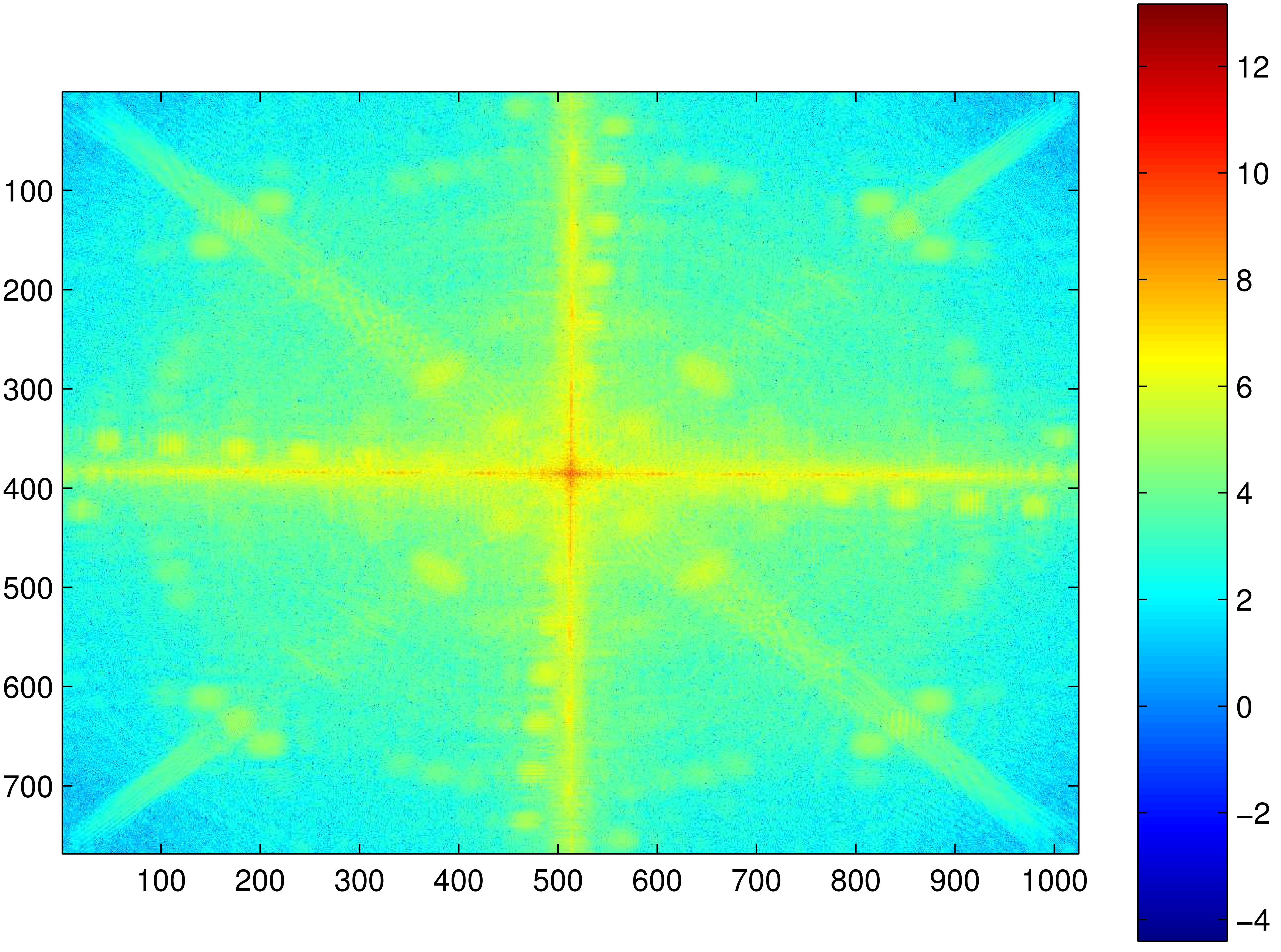
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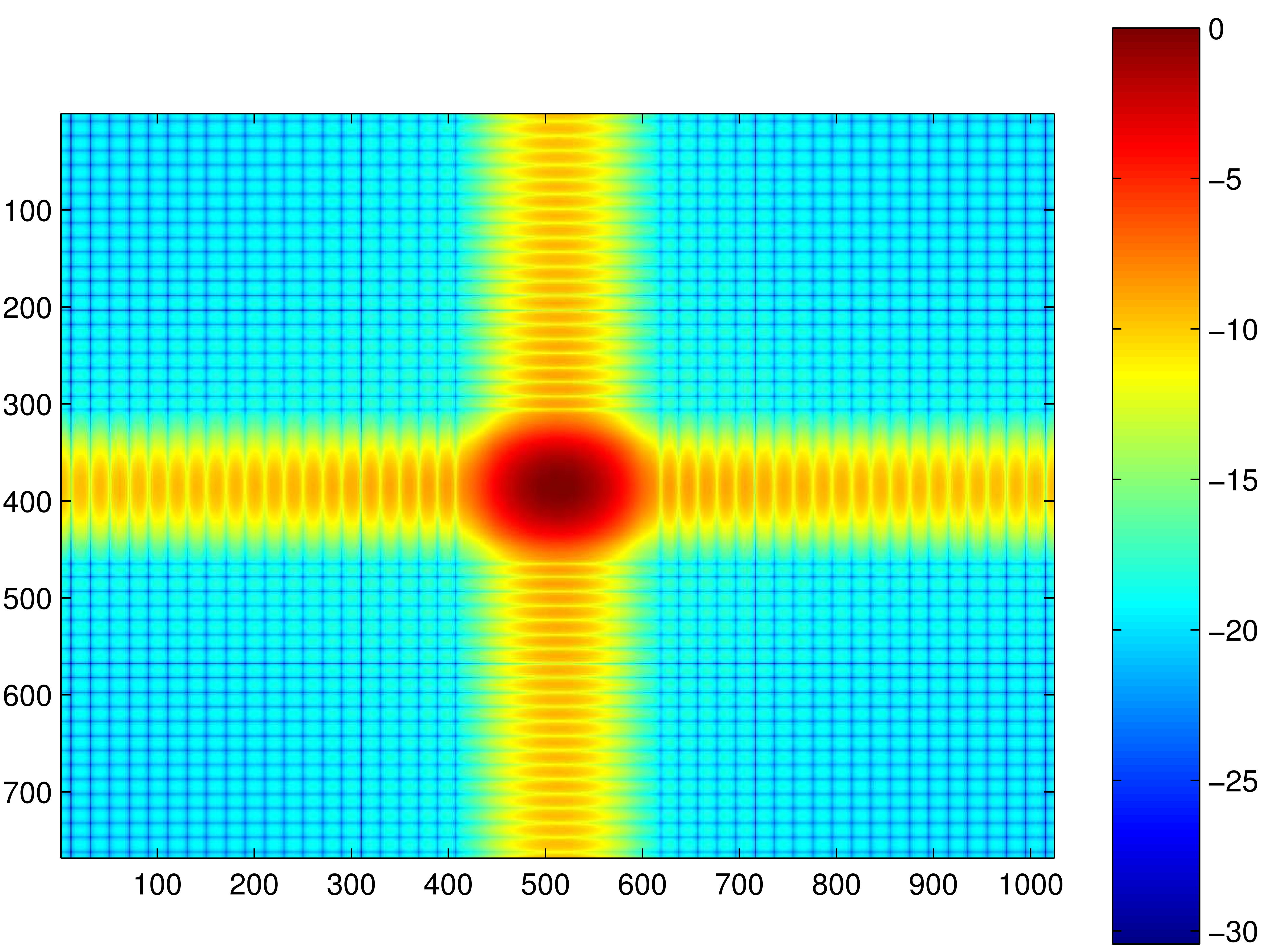

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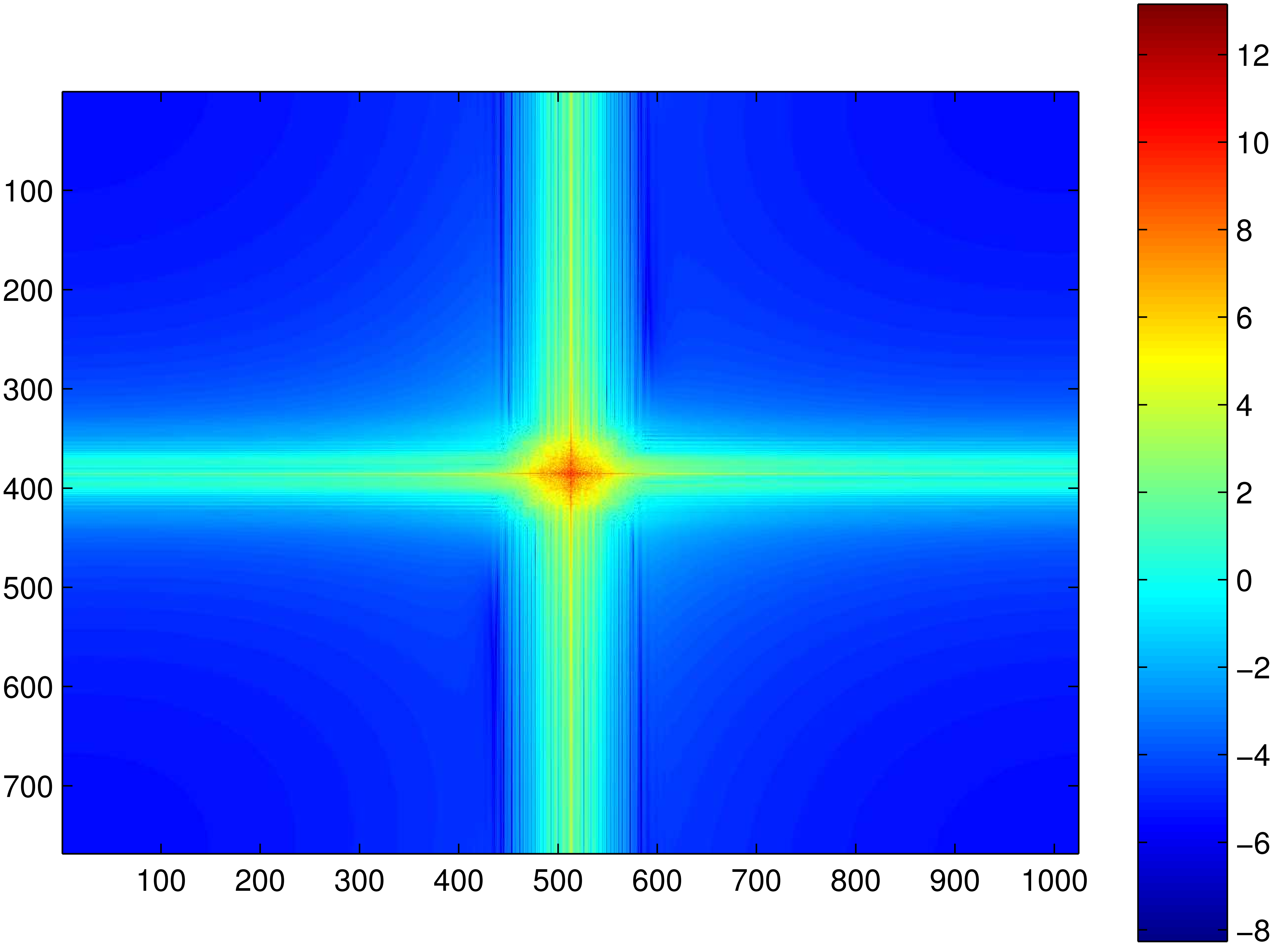

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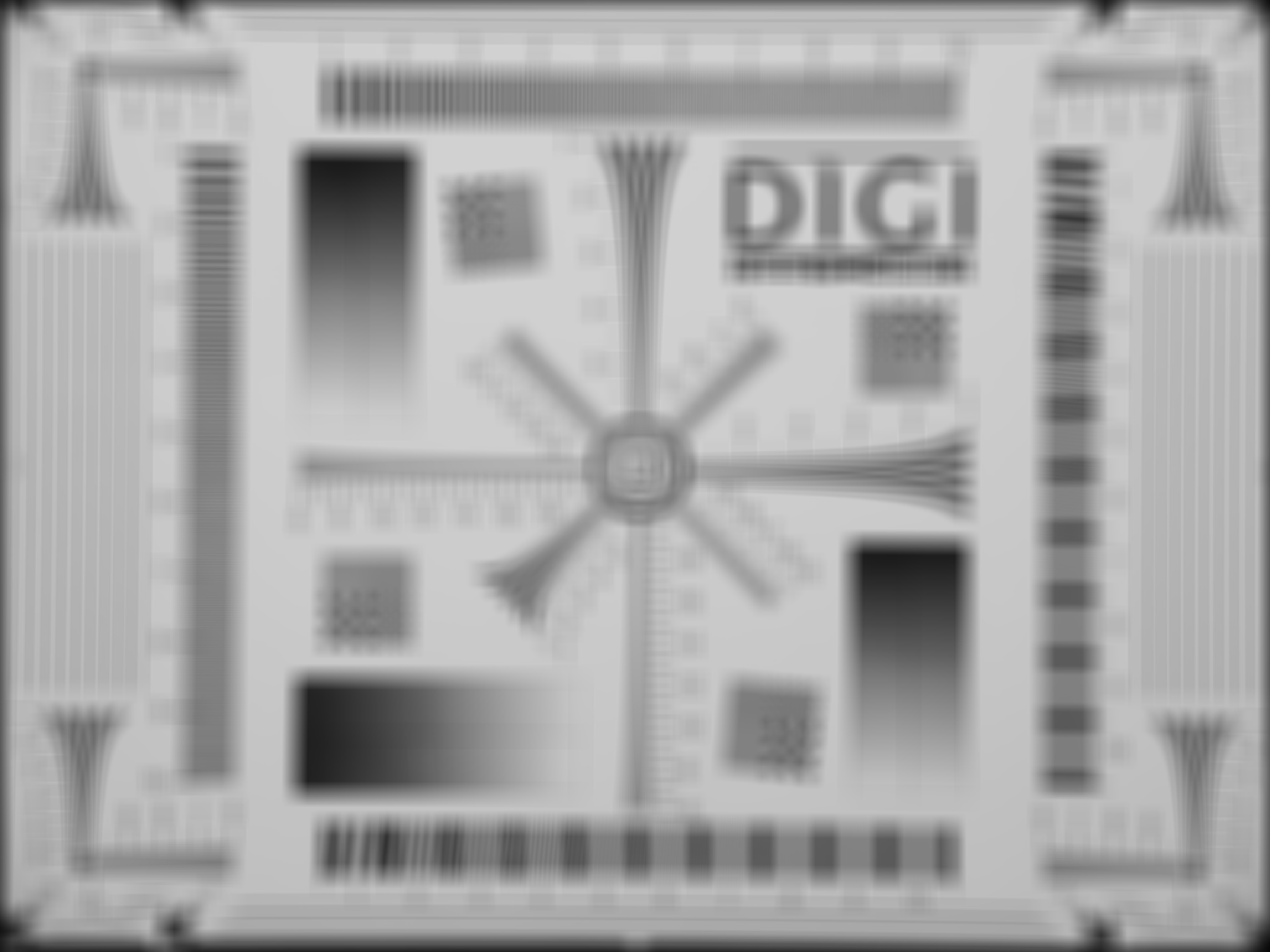
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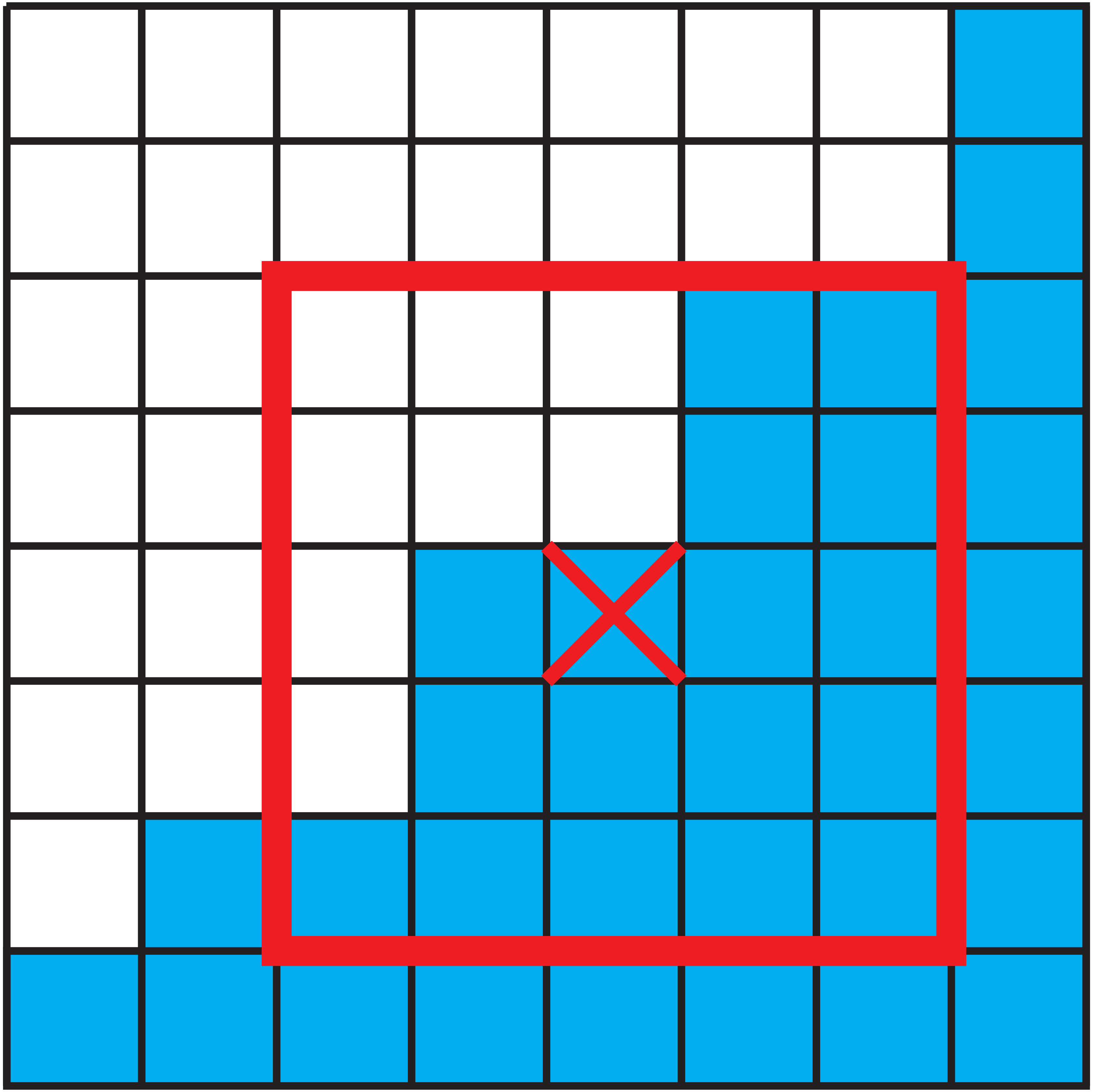

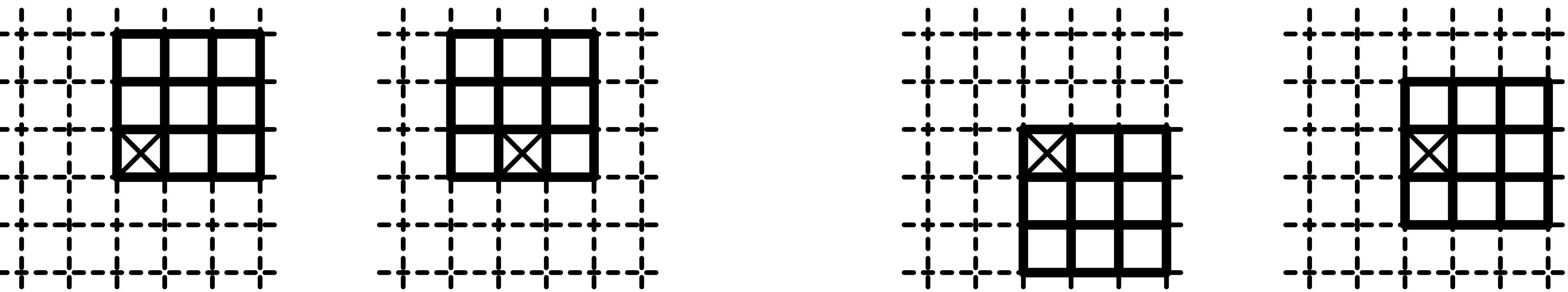

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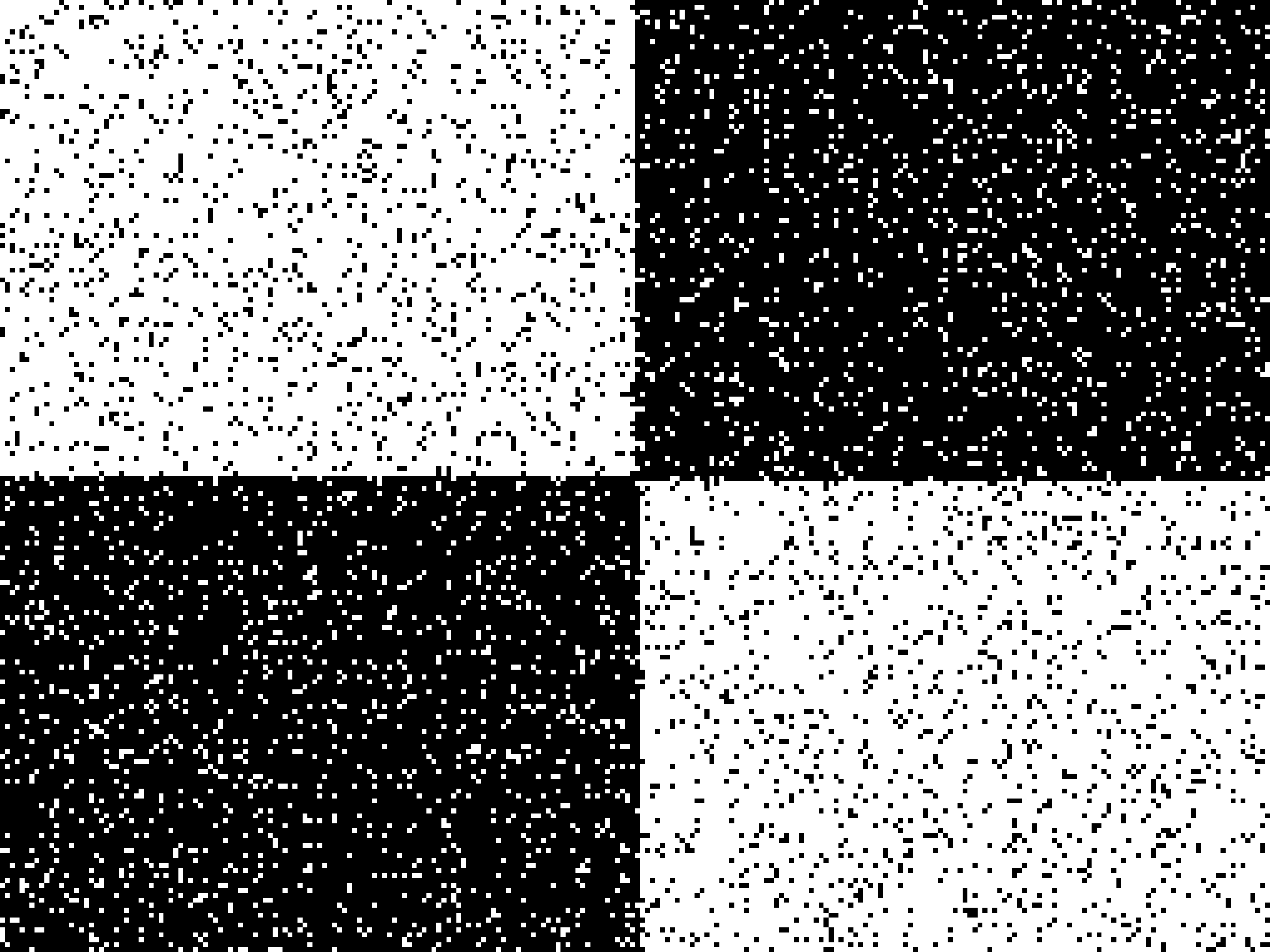

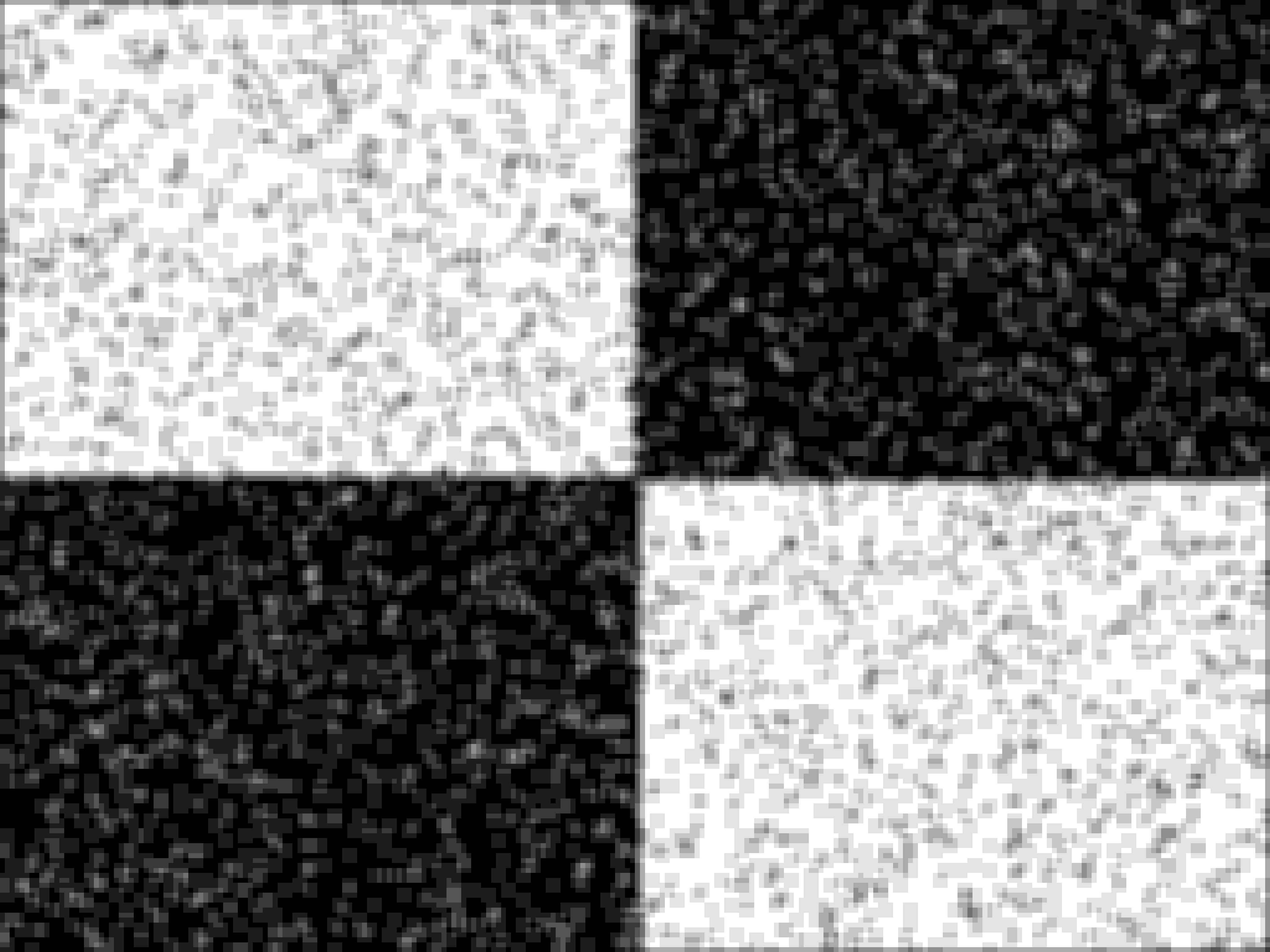

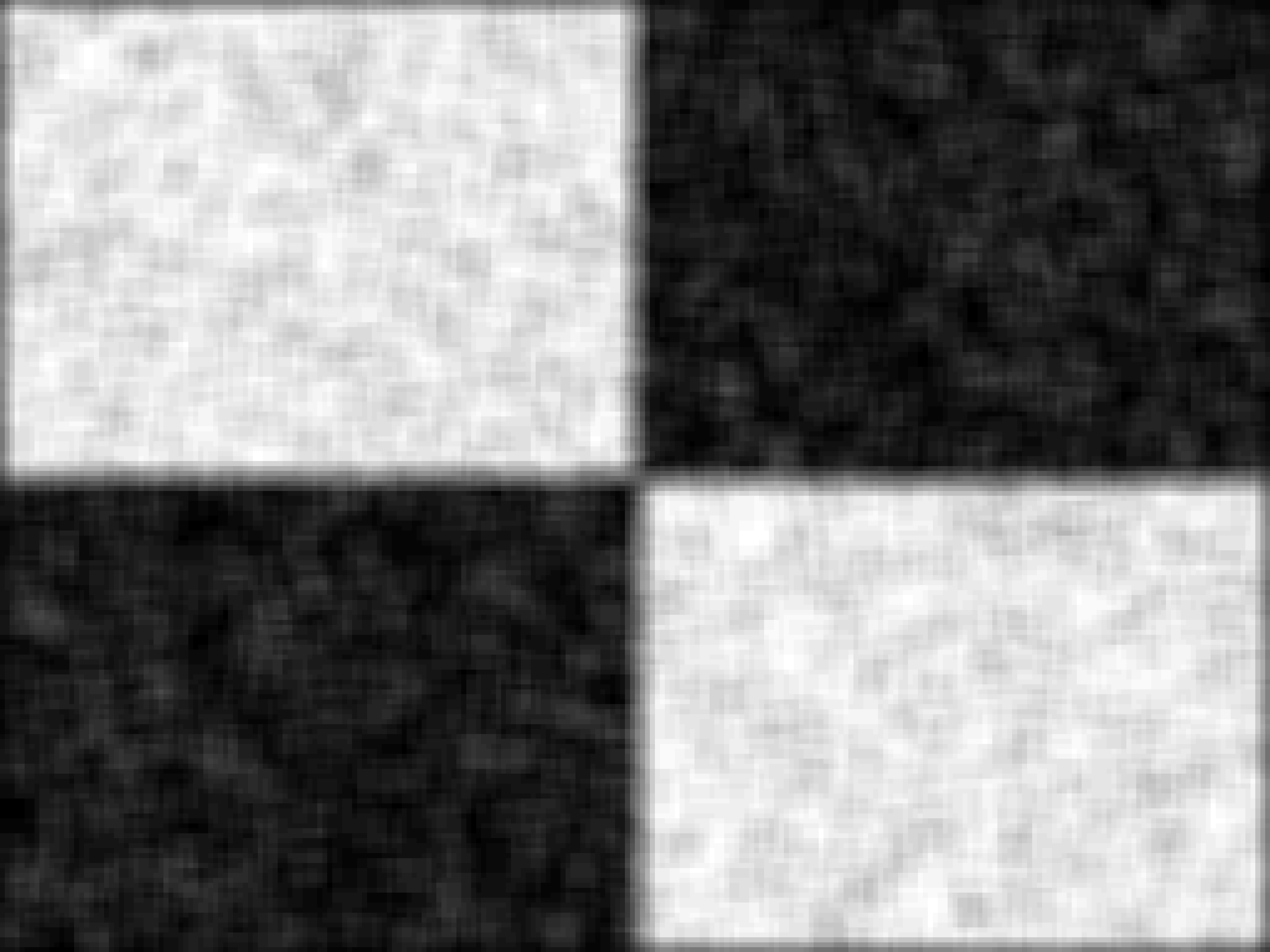

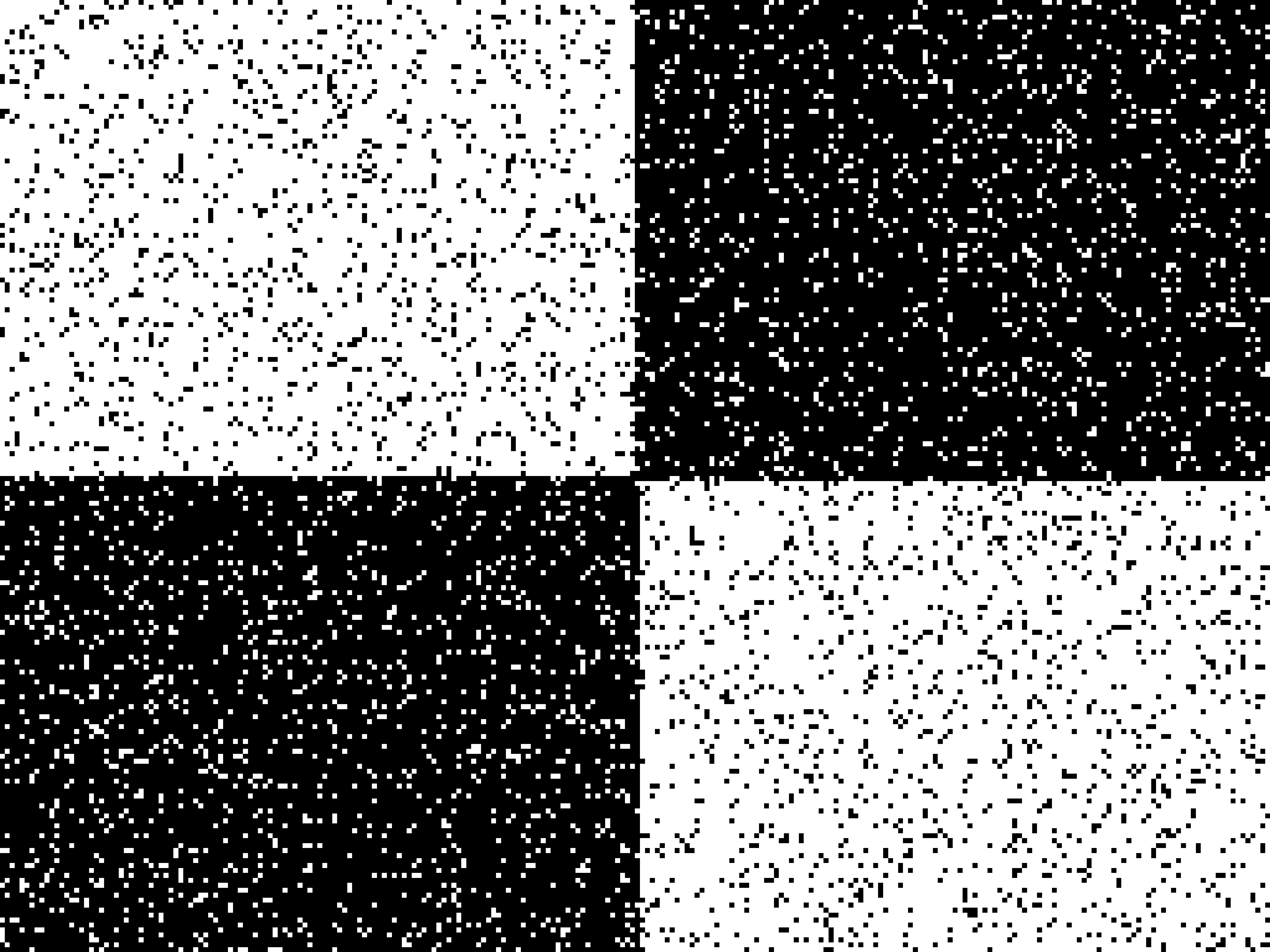

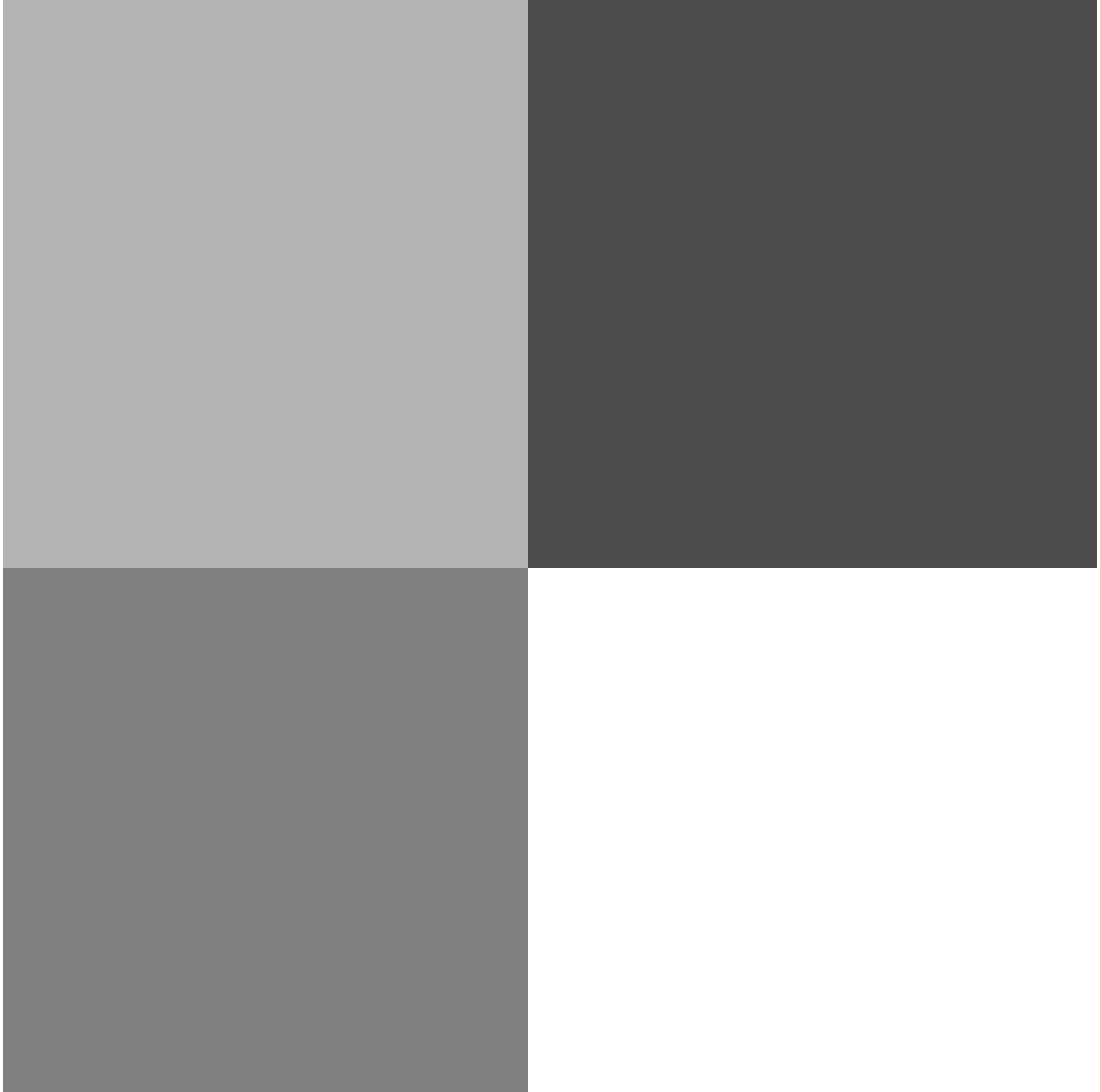

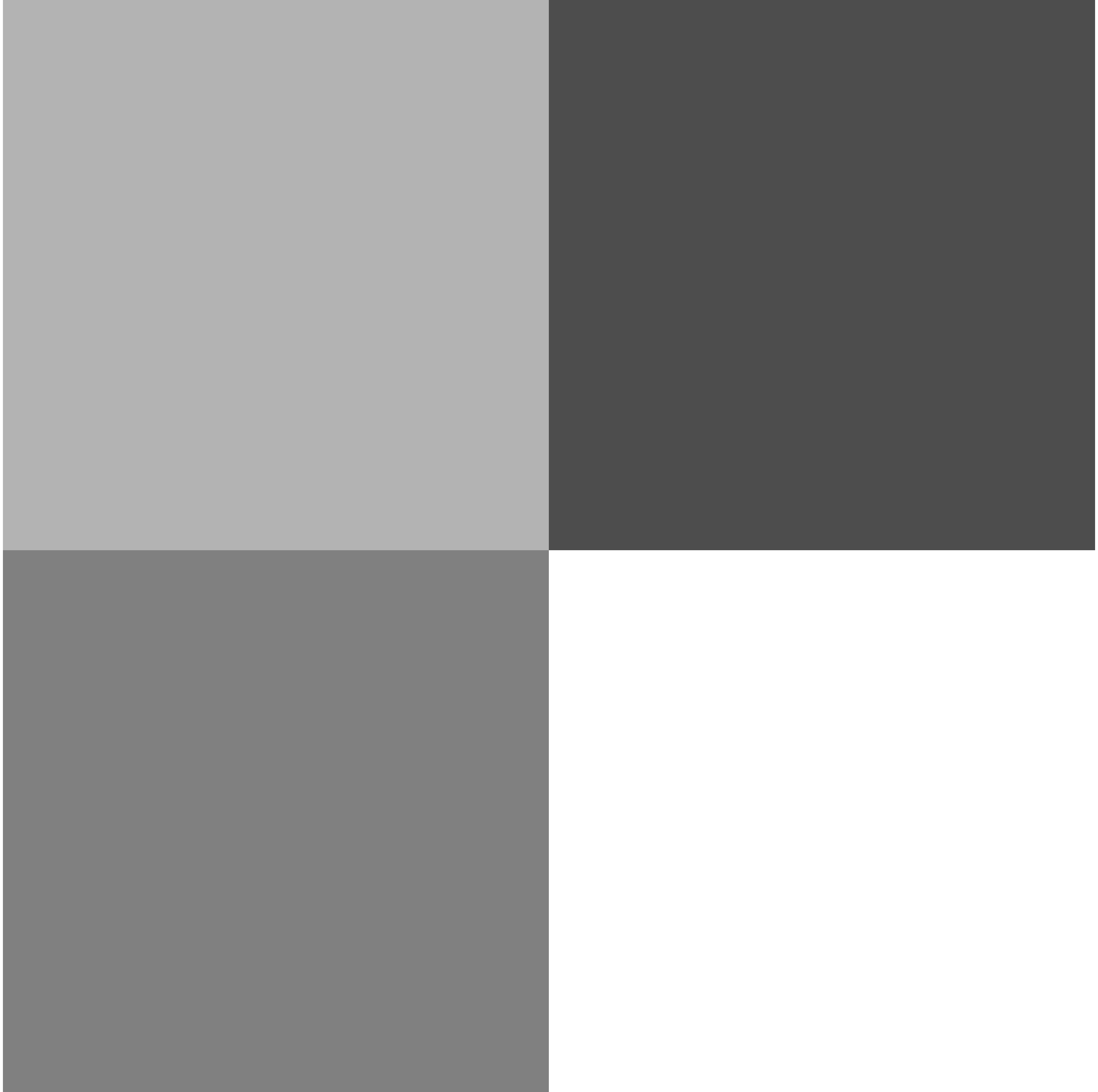

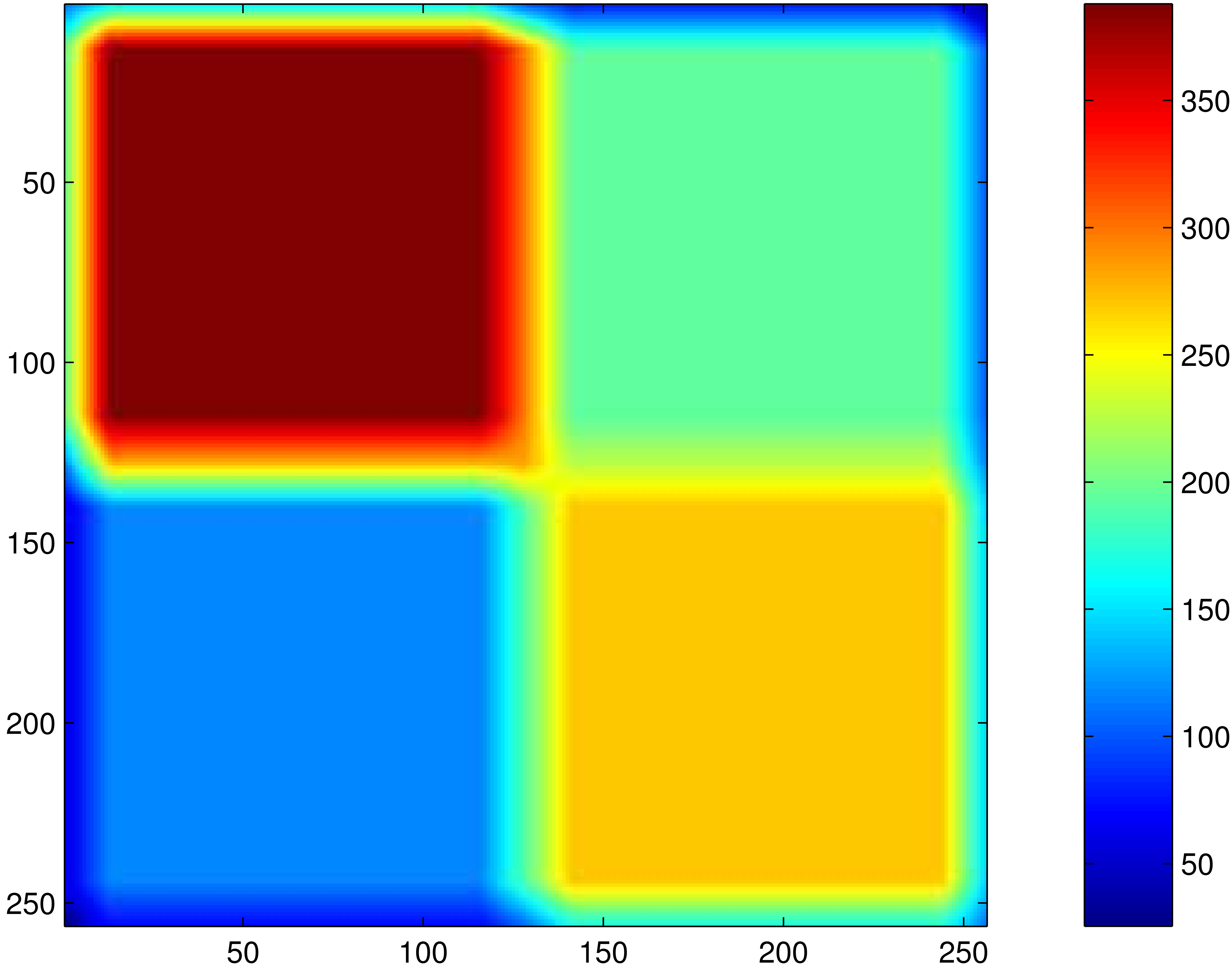

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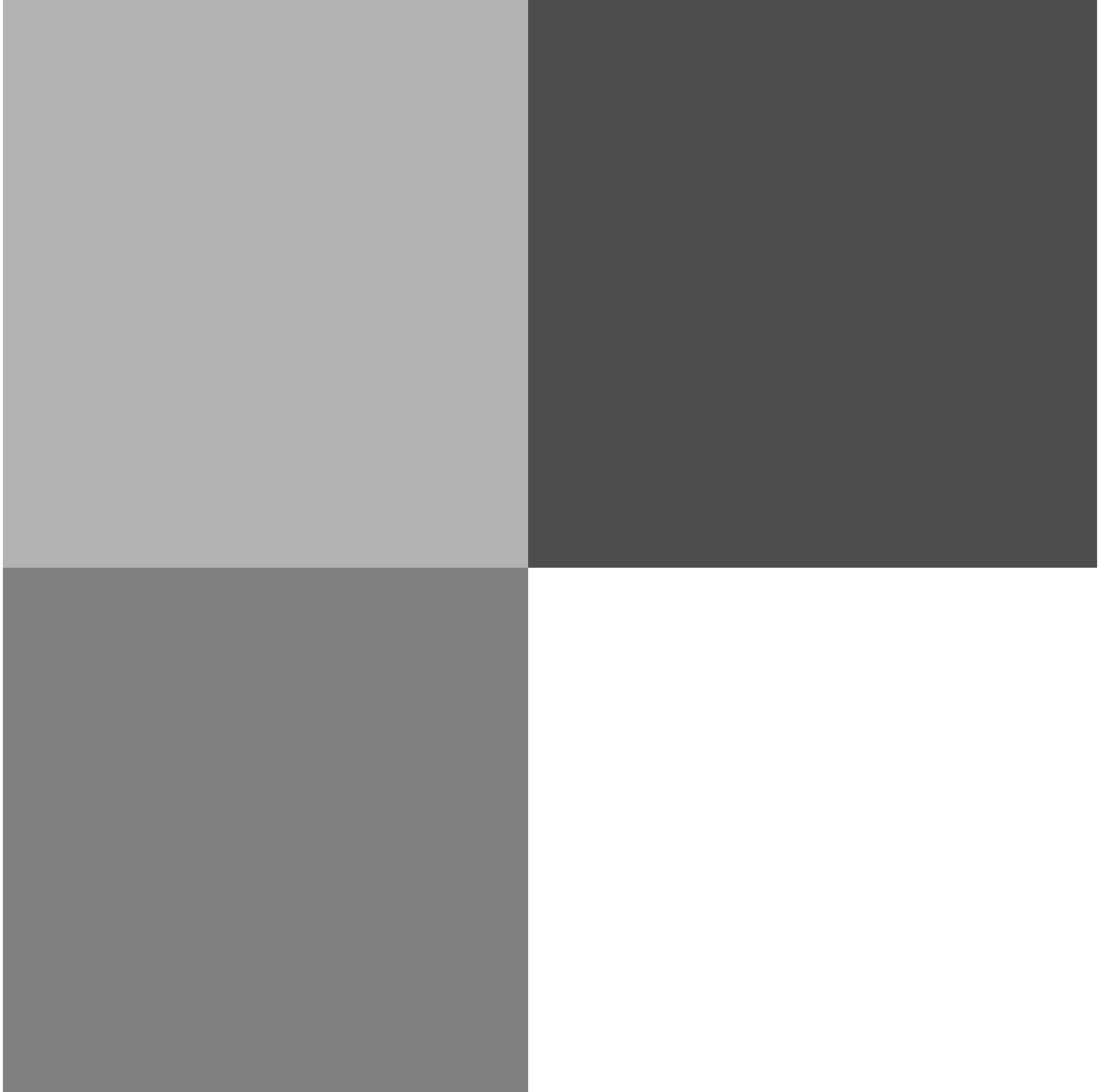

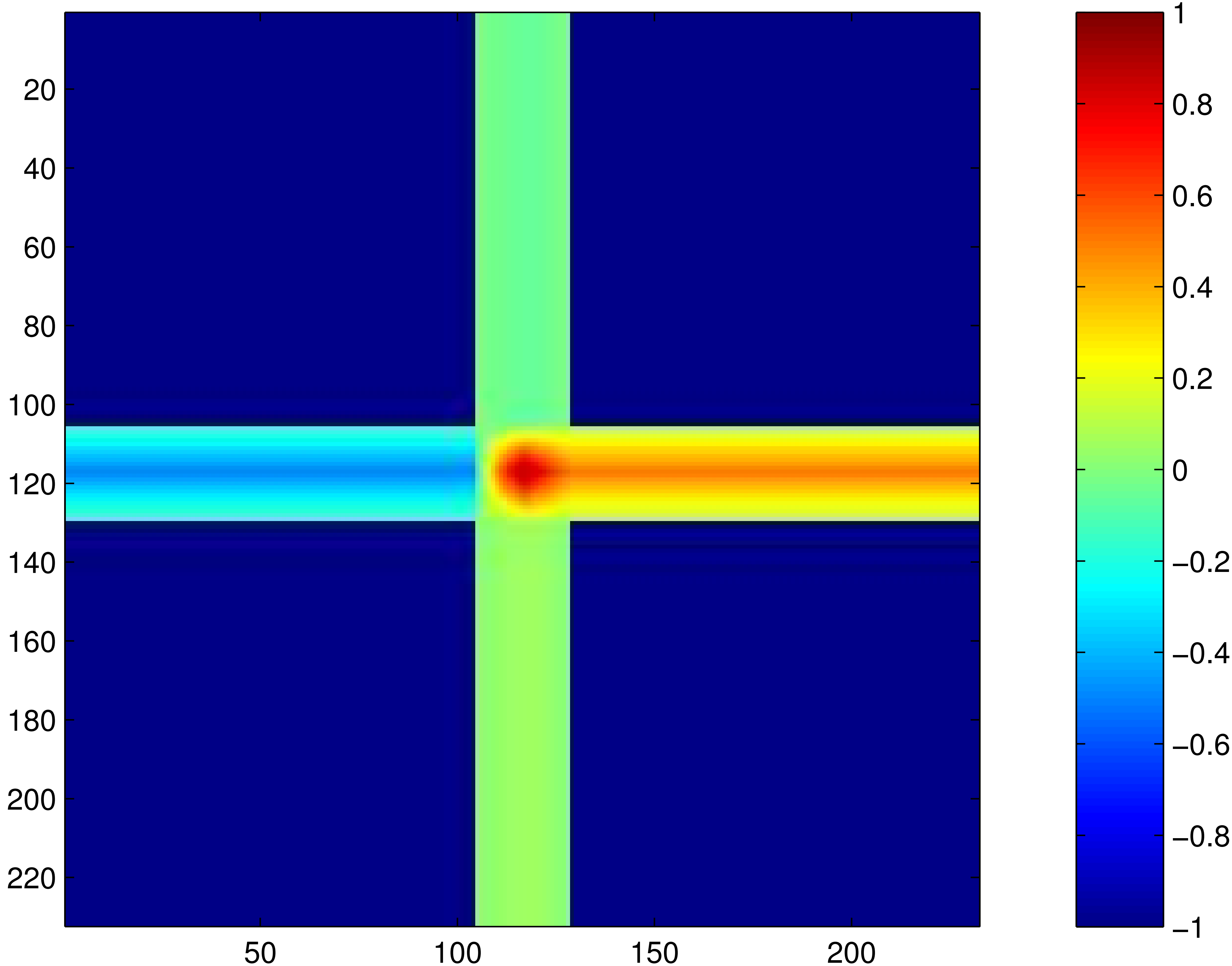

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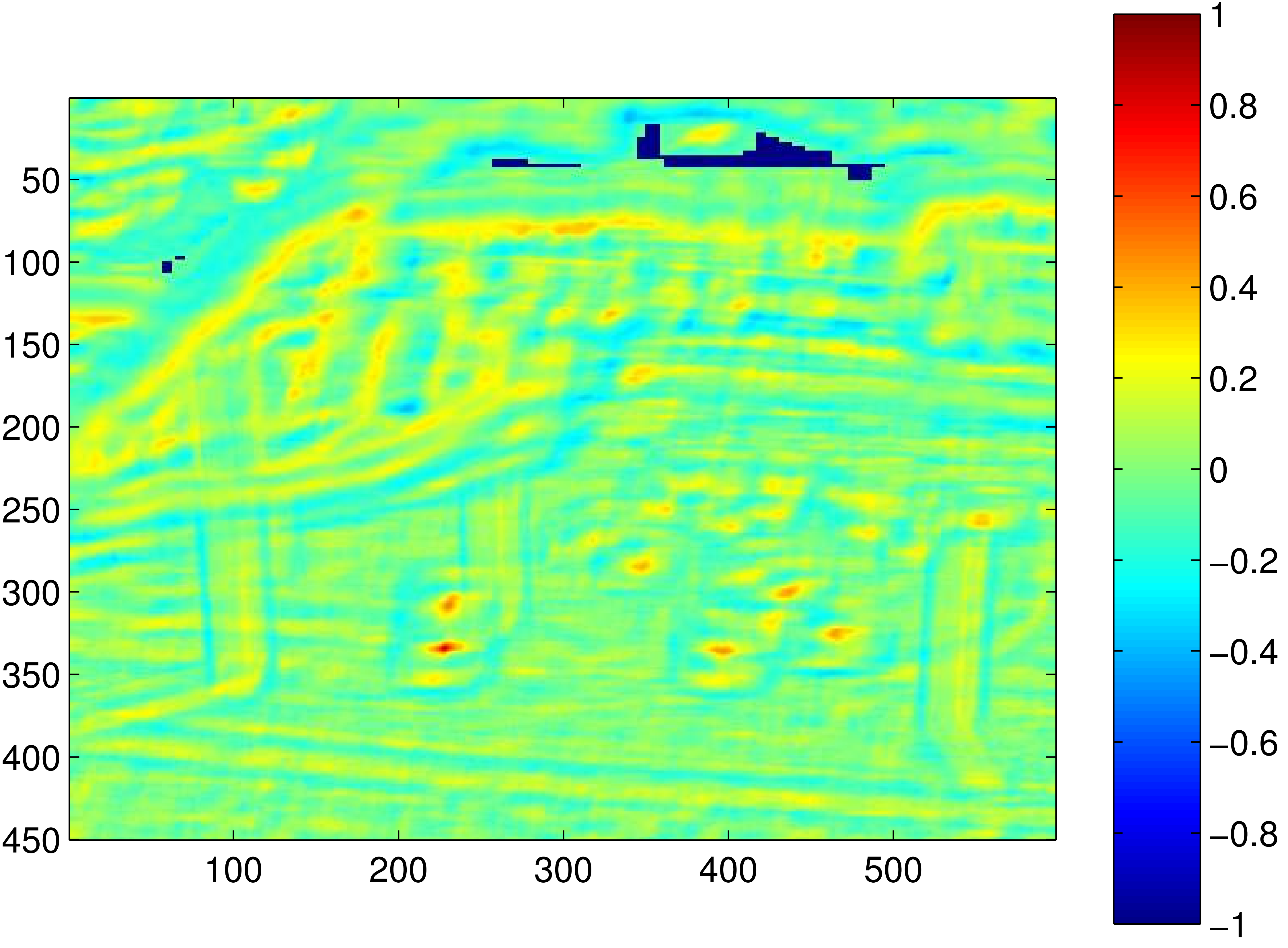

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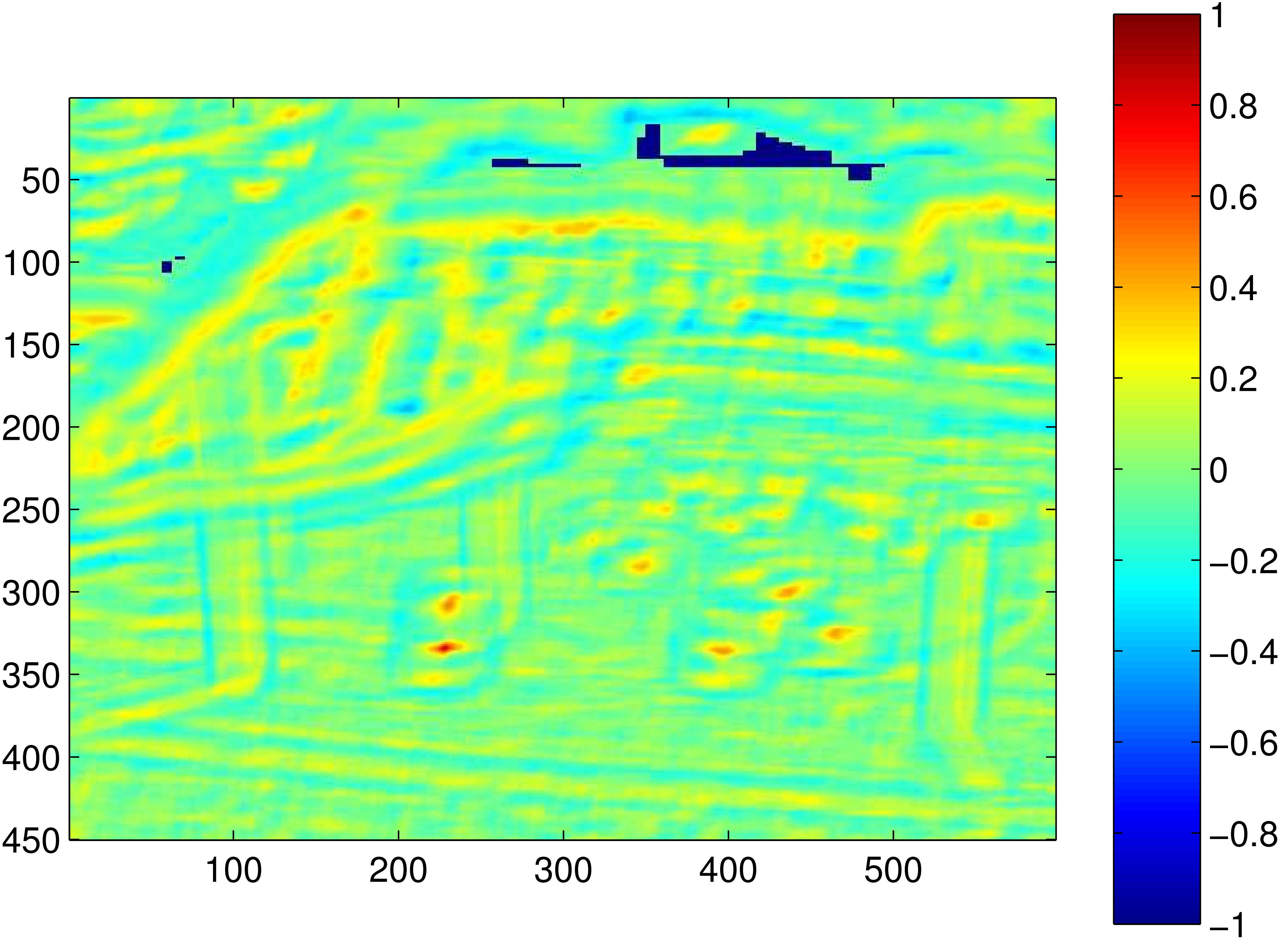

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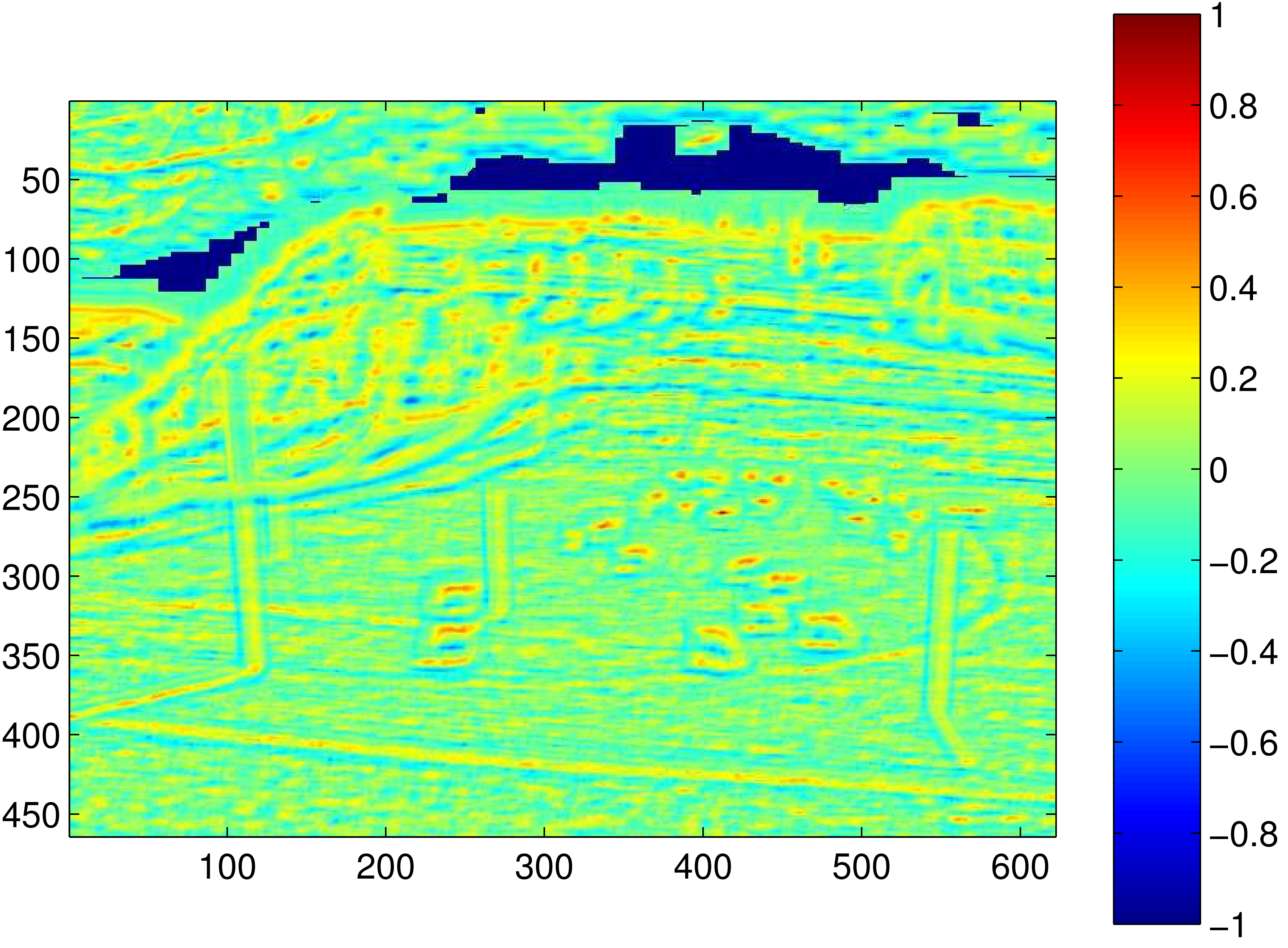
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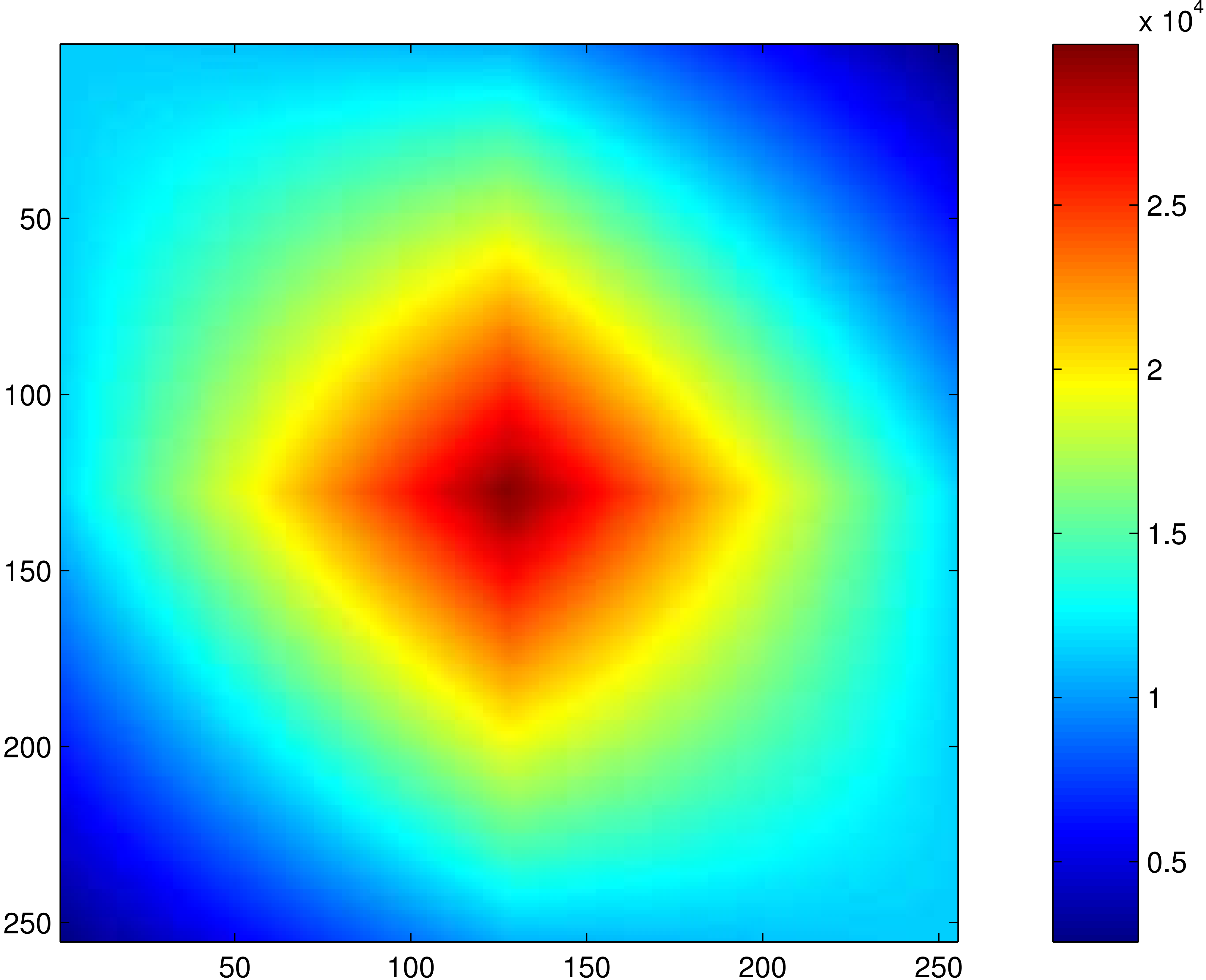

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