"Division algorithm" for more than one divisor in $k[x_1, \dots, x_n]$

Input : $F = (f_1, \dots, f_s)$, f Output : $a_1, \dots, a_s, r \equiv \overline{f}^F$

$a_1 := a_2 := \dots a_s := r := 0$, $p := f$

WHILE $p \neq 0$ DO

{ $i := 1$

divisionoccured := FALSE

WHILE $i \leq s$ AND divisionoccured = FALSE DO

{IF $LT(f_i)$ divides $LT(p)$ THEN

{ $a_i := a_i + \dfrac{LT(p)}{LT(f_i)}$

$p := p - \dfrac{LT(p)}{LT(f_i)} \cdot f_i$

divisionoccured := TRUE }

ELSE { $i := i + 1$ } }

IF divisionoccured = FALSE THEN

{ $r := r + LT(p)$

$p := p - LT(p)$ }

}

Proof as for 1 variable

degree → multidegree

$r \to p$

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Example

$$x >_{lex} y \qquad f = \underset{(1,2)}{\underset{\downarrow}{xy^2}} + \underset{(1,0)}{\underset{\downarrow}{x}} + \underset{(0,0)}{\underset{\downarrow}{1}}, \quad f_1 = \underset{(1,1)}{\underset{\downarrow}{xy}} + \underset{(0,0)}{\underset{\downarrow}{1}}, \quad f_2 = \underset{(0,1)}{\underset{\downarrow}{y}} + \underset{(0,0)}{\underset{\downarrow}{1}}$$

$$f = y\,(xy+1) + \underset{(1,0)}{\underset{\downarrow}{x}} - \underset{(0,1)}{\underset{\downarrow}{y}} + \underset{(0,0)}{\underset{\downarrow}{1}} = \underset{a_1}{\underset{\downarrow}{y}}\,(xy+1) - \underset{a_2}{\underset{\downarrow}{1}}\,(y+1) + \underbrace{x+2}_{\substack{\downarrow \\ r}}$$

$$\begin{aligned}
f &= \overbrace{0}^{a_1} \cdot f_1 + \overbrace{0}^{a_2} \cdot f_2 + \overbrace{xy^2 + x + 1}^{p} + \overbrace{0}^{r} \\
&= y \cdot f_1 + 0 \cdot f_2 + x - y + 1 + 0 \\
&= y \cdot f_1 + 0 \cdot f_2 \quad - y + 1 + x \\
&= y \cdot f_1 - 1 \cdot f_2 \quad + 2 + x \\
&= y \cdot f_1 - 1 \cdot f_2 \quad + x + 2
\end{aligned}$$

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