Tomas Pajdla

Scholar in
Computer Vision, Machine Learning, Robotics
Applied Algebra & Geometry

Czech Technical University in Prague
Czech Institute of Informatics, Robotics & Cybernetics
Faculty of Electrical Engineering

Czech Institute of Informatics, Robotics & Cybernetics
Distinguished Researcher
Head of Applied Algebra and Geometry Group

Faculty of Electrical Engineering
Associate Professor

National Institute of Informatics Tokyo
Visiting Associate Professor
3D Computer Vision
Camera & Robot Calibration
Mobile Robotics
Computer Vision for Industry
We will build on Robotics by V. Smutny and study more advanced robot kinematics problems, e.g.,

1. solving inverse kinematics of a general 6 DOF manipulator
2. identifying kinematic parameters of a manipulator
3. finding singular poses of a manipulator

with more advanced mathematical tools, such as

1. space rotation and motion and
2. solving algebraic equations
ROBOT = A GENERAL MANIPULATOR
Industrial Robotic Applications

Application areas

- Arc welding
- Assembly
- Foundry applications
- Gluing and Sealing
- Material handling and Machine Tending
- Packing
- Palletizing
- Picking
- Painting and coating
- Spot welding
- Waterjet cutting
Precision for robotic surgery

http://www.cts.usc.edu/rsi-article-robotputsuscatforefront.html
Precision for industry

Low (e.g. manipulation)

± 5 mm in the whole working space
± 0.5 mm locally

… often available

High (e.g. laser welding)

± 0.5 mm in the whole working space
± 0.05 mm locally

… often not available
Error ± 0.5 mm
Modeling kinematics – calibration – absolute accuracy ± 0.05 mm

MITSUBISHI robot
TRUMPF welding laser
NEOVISION vision guiding

Robot-Vision calibration (courtesy Neovision s.r.o.)
Two kinds of manipulators

1. Serial manipulators

2. Parallel manipulators
Serial manipulators

KUKA manipulator
Serial manipulators

1. Direct kinematic task – easy
2. Inverse kinematic task – difficult

Stäubli (courtesy Neovision s.r.o.)
Mitsubishi (courtesy Neovision s.r.o.)
Parallel manipulators

Stewart-Gough Platform
Parallel manipulators

1. Direct kinematic task – difficult
2. Inverse kinematic task – easy

Sliding Star (courtesy of Prof. Valášek, CTU Prague)
Kinematics in robotics

Three main problems

1. Direct kinematic task (přímá kinematická úloha)
2. Inverse kinematic task (inverzní kinematická úloha)
3. Kinematic calibration (kalibrace kinematiky)
Direct kinematic task

Position of the flange reference frame in the world reference frame

flange frame

world frame

joint angles
Inverse kinematic task

Position of the flange reference frame in the world reference frame

flange frame

world frame

joint angles
Kinematic calibration

Known position of a point on the flange in the world reference frame

calibration parameters (lengths, offsets)
Kinematic Calibration
Robot Calibration
Kinematic Calibration
Solving kinematic tasks

TASK
↓
GEOMETRY
↓
COORDINATE SYSTEMS
↓
KINEMATIC MODEL
↓
ALGEBRAIC EQUATIONS
↓
RESULT
Solving kinematic tasks

1968  Donald L. Pieper (Ph.D. thesis)

The inverse kinematics of any serial manipulator with six revolute joints, and with three consecutive joints intersecting, can be solved in closed-form, i.e., analytically.


A general technique for computing inverse kinematics for any serial manipulator with six revolute joints.

… leads to solving an algebraic equation of degree 16.
Solving kinematic tasks

Algebraic equation of degree 16 … up to 16 solutions

4 typical solutions
Solving kinematic tasks

TASK
↓
GEOMETRY
↓
COORDINATE SYSTEMS
↓
KINEMATIC MODEL
↓
ALGEBRAIC EQUATIONS
↓
RESULT
Stäubli TX-90 – Geometry

Figure 1.3 - Standard arm
Kinematic model
\[ A_i^{i-1} = \begin{bmatrix}
  \cos \theta_i & -\sin \theta_i \cos \alpha_i & \sin \theta_i \sin \alpha_i & a_i \cos \theta_i \\
  \sin \theta_i & \cos \theta_i \cos \alpha_i & -\cos \theta_i \sin \alpha_i & a_i \sin \theta_i \\
  0 & \sin \alpha_i & \cos \alpha_i & d_i \\
  0 & 0 & 0 & 1
\end{bmatrix} \]

\[ G = A_1^0 A_2^1 A_3^2 A_4^3 A_5^4 A_6^5 \]

\[ A_1^0 = \begin{bmatrix}
  \cos \theta_1 & 0 & -\sin \theta_1 & a_1 \cos \theta_1 \\
  \sin \theta_1 & 0 & \cos \theta_1 & a_1 \sin \theta_1 \\
  0 & -1 & 0 & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix} \]

\[ A_2^1 = \begin{bmatrix}
  \cos \theta_2 & -\sin \theta_2 & 0 & a_2 \cos \theta_2 \\
  \sin \theta_2 & \cos \theta_2 & 0 & a_2 \sin \theta_2 \\
  0 & 0 & 1 & d_2 \\
  0 & 0 & 0 & 1
\end{bmatrix} \]

\[ A_3^2 = \begin{bmatrix}
  \cos \theta_3 & 0 & -\sin \theta_3 & 0 \\
  \sin \theta_3 & 0 & \cos \theta_3 & 0 \\
  0 & -1 & 0 & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix} \]

\[ A_4^3 = \begin{bmatrix}
  \cos \theta_4 & 0 & \sin \theta_4 & 0 \\
  \sin \theta_4 & 0 & -\cos \theta_4 & 0 \\
  0 & 1 & 0 & d_4 \\
  0 & 0 & 0 & 1
\end{bmatrix} \]

\[ A_5^4 = \begin{bmatrix}
  \cos \theta_5 & 0 & -\sin \theta_5 & 0 \\
  \sin \theta_5 & 0 & \cos \theta_5 & 0 \\
  0 & -1 & 0 & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix} \]

\[ A_6^5 = \begin{bmatrix}
  \cos \theta_6 & -\sin \theta_6 & 0 & 0 \\
  \sin \theta_6 & \cos \theta_6 & 0 & 0 \\
  0 & 0 & 1 & d_5 \\
  0 & 0 & 0 & 1
\end{bmatrix} \]

offset = \[0, -\frac{\pi}{2}, -\frac{\pi}{2}, 0, 0, -\pi\]
The Standard Kinematic model in Denavit-Hartenberg Convention

Stäubli TX 90

TX-90 (6 axis, RRRRRR) [Staubli]

\[
\begin{array}{cccc}
\alpha & a & \theta & d \\
-1.5708 & 50.0 & 0.0 & 350.0 \\
0.0 & 425.0 & 0.0 & 50.0 \\
-1.5708 & 0.0 & 0.0 & 0.0 \\
1.5708 & 0.0 & 0.0 & 425.0 \\
-1.5708 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 100.0 \\
\end{array}
\]

6 non-trivial parameters
The Standard Kinematic model in Denavit-Hartenberg Convention

ABB IBR 140
The Standard Kinematic model in Denavit-Hartenberg Convention

ABB IBR 140

IBR-140 (6 axis) [ABB]

<table>
<thead>
<tr>
<th>α</th>
<th>a</th>
<th>θ</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.5708</td>
<td>70.0</td>
<td>0.0</td>
<td>352.0</td>
</tr>
<tr>
<td>0.0</td>
<td>360.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-1.5708</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1.5708</td>
<td>0.0</td>
<td>0.0</td>
<td>380.0</td>
</tr>
<tr>
<td>-1.5708</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>65.0</td>
</tr>
</tbody>
</table>

5 non-trivial parameteres
The Standard Kinematic model in Denavit-Hartenberg Convention

Stäubli TX 90

RV-6S (6 axis, RRRRRR) [Mitsubishi]

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$a$</th>
<th>$\theta$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.5708</td>
<td>85.0</td>
<td>0.0</td>
<td>350.0</td>
</tr>
<tr>
<td>0.0</td>
<td>280.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-1.5708</td>
<td>100.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1.5708</td>
<td>0.0</td>
<td>0.0</td>
<td>315.0</td>
</tr>
<tr>
<td>-1.5708</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>85.0</td>
</tr>
</tbody>
</table>

6 non-trivial parameters
Special versus General Mechanisms

Special × General

simple & tractable complicated & hard

<table>
<thead>
<tr>
<th>α</th>
<th>a</th>
<th>θ</th>
<th>d</th>
<th>α</th>
<th>a</th>
<th>θ</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.5708</td>
<td>70.0</td>
<td>-</td>
<td>352.0</td>
<td>-1.42</td>
<td>70.1</td>
<td>- (+0.2)</td>
<td>352.0</td>
</tr>
<tr>
<td>0.0</td>
<td>360.0</td>
<td>-</td>
<td>0.0</td>
<td>0.10</td>
<td>360.0</td>
<td>- (+0.1)</td>
<td>0.2</td>
</tr>
<tr>
<td>-1.5708</td>
<td>0.0</td>
<td>-</td>
<td>0.0</td>
<td>-1.57</td>
<td>0.2</td>
<td>- (-0.3)</td>
<td>0.3</td>
</tr>
<tr>
<td>1.5708</td>
<td>0.0</td>
<td>-</td>
<td>380.0</td>
<td>1.58</td>
<td>0.1</td>
<td>- (+0.1)</td>
<td>380.2</td>
</tr>
<tr>
<td>-1.5708</td>
<td>0.0</td>
<td>-</td>
<td>0.0</td>
<td>-1.59</td>
<td>0.4</td>
<td>- (-0.1)</td>
<td>0.1</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>-</td>
<td>65.0</td>
<td>0.07</td>
<td>0.2</td>
<td>- (-0.2)</td>
<td>65.1</td>
</tr>
</tbody>
</table>

6 non-trivial parameters × 18 (+6) non-trivial parameters

High precision → Small misalignments important → General mechanisms
Literature

Linear algebra

Numerical linear algebra

The solution

The numerical solution

The pedagogical solution will be developed using
Software

Matlab: www.matworks.com

Maple: www.maplesoft.com
One algebraic equation in one variable
SOLVING 1 ALGEBRAIC EQUATION

1 equation, 1 variable → companion matrix → eigenvalues

\[ f(x) = x^3 + 4x^2 + x - 6 = -6 + 1x + 4x^2 + 1x^3 \]

\[ M_x = \begin{bmatrix} 0 & 0 & 6 \\ 1 & 0 & -1 \\ 0 & 1 & -4 \end{bmatrix} \]

... a simple rule

\[ e = \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix} \]

\[ x_1 = 1, \ x_2 = -2, \ x_3 = -3 \]

It works when eig works, i.e. order 100 in Matlab is often OK.
SOLVING 1 ALGEBRAIC EQUATION

Linear mapping \( M \in \mathbb{R}^{n \times n} \)

Eigenvalues \( Mx = \lambda x \)

\[ Mx - \lambda x = 0 \]

\[ (M - \lambda I)x = 0 \]

\( x \neq 0 \Rightarrow \)

\( \text{rank}(M - \lambda I) < n \)

\[ \det(M - \lambda I) = 0 \]
SOLVING 1 ALGEBRAIC EQUATION

algebraic equation

\[ f(x) = x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 = \det(-M + x I) \]

\[
-M + x I = \begin{bmatrix}
-1 & a_0 \\
-1 & a_1 \\
-1 & a_2 \\
-1 & x + a_3
\end{bmatrix}
\]

\[ f(x) = x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 \]

Numerical solution to \( f(x) \) is obtained by

\[
>> x = \mathrm{eig}(M);
\]