Intersection of two circles in plane

Given centers A = [Ax,Ay], B=[Bx,By] and radiuses ra, rb. Intersections are C=[Cx,Cy], r is distance of A and B

```
$Assumptions = ({ra, rb, r, Ax, Ay, Bx, By} ∈ Reals);

A = {Ax, Ay};
B = {Bx, By};
```

- **Distance of A and B is r**

```
Sqrt[(Ax - Bx)^2 + (Ay - By)^2]
```

$$\sqrt{(Ax - Bx)^2 + (Ay - By)^2}$$

Analysis of singular cases

- **Identical circles (infinite number of solutions)**

```
Ax == Bx ⋀ Ay == By ⋀ ra == rb;
```

- **Concentric nonidentical circles (no intersection)**

```
Ax == Bx ⋀ Ay == By ⋀ ra ≠ rb;
```

- **One circle inside of the other (no intersection)**

```
r < Abs[ra - rb];
```

- **Circles too far to intersect (no intersection)**

```
ra + rb < r;
```

- **Circles touching, one inside the other (one intersection)**

```
r == Abs[ra - rb];
```

- **Circles touching, one outside the other (one intersection)**

```
r == ra + rb;
```

Alternative analysis of singular cases

- **Identical circles (infinite number of solutions)**

```
r == 0 ⋀ ra == rb;
```

- **No intersection**

```
ra + rb < r ⋁ r < Abs[ra - rb] ;
```

- **Circles touching, one intersection**

```
r == Abs[ra - rb] ⋁ r == ra + rb;
```

Analysis of regular case (two intersections)

- **Substitution**

```
condr = (Ax - Bx)^2 + (Ay - By)^2 → r^2;
```

- **Analytic equations of circles**

```
exp1 = (Cx - Ax)^2 + (Cy - Ay)^2 - ra^2;
exp2 = (Cx - Bx)^2 + (Cy - By)^2 - rb^2;
```

```
eq1 = exp1 == 0
eq2 = exp2 == 0
```

$$(-Ax + Cx)^2 + (-Ay + Cy)^2 - ra^2 == 0$$

$$(-Bx + Cx)^2 + (-By + Cy)^2 - rb^2 == 0$$

- **Using Solve**

```
solC2 = Solve[{eq2, eq1}, {Cx, Cy}] /. condr // FullSimplify /. condr
```

$$\left\{\left\{Cx \to \frac{1}{2\left((Ax-Bx)^2+(Ay-By)^2\right)}\left(Ax^3 - Ax^2\,Bx + Bx\left(Bx^2 + (Ay-By)^2 + ra^2 - rb^2\right) + Ax\left(-Bx^2 + (Ay-By)^2 - ra^2 + rb^2\right) - \sqrt{\left(-(Ay-By)^2\left(\left((Ax-Bx)^2+(Ay-By)^2-ra^2\right)^2 - 2\left((Ax-Bx)^2+(Ay-By)^2+ra^2\right)rb^2 + rb^4\right)\right)}\right),\right.$$

$$Cy \to \left(Ay^4 - 2\,Ay^3\,By + 2\,Ay\,By\left(By^2 + ra^2 - rb^2\right) - By^2\left((Ax-Bx)^2 + By^2 + ra^2 - rb^2\right) + Ay^2\left((Ax-Bx)^2 - ra^2 + rb^2\right) + Ax\sqrt{\left(-(Ay-By)^2\left(\left((Ax-Bx)^2+(Ay-By)^2-ra^2\right)^2 - 2\left((Ax-Bx)^2+(Ay-By)^2+ra^2\right)rb^2 + rb^4\right)\right)} - Bx\sqrt{\left(-(Ay-By)^2\left(\left((Ax-Bx)^2+(Ay-By)^2-ra^2\right)^2 - 2\left((Ax-Bx)^2+(Ay-By)^2+ra^2\right)rb^2 + rb^4\right)\right)}\right) \Big/ \left(2\left((Ax-Bx)^2+(Ay-By)^2\right)(Ay-By)\right)\Big\},$$

$$\left\{Cx \to \frac{1}{2\left((Ax-Bx)^2+(Ay-By)^2\right)}\left(Ax^3 - Ax^2\,Bx + Bx\left(Bx^2 + (Ay-By)^2 + ra^2 - rb^2\right) + Ax\left(-Bx^2 + (Ay-By)^2 - ra^2 + rb^2\right) + \sqrt{\left(-(Ay-By)^2\left(\left((Ax-Bx)^2+(Ay-By)^2-ra^2\right)^2 - 2\left((Ax-Bx)^2+(Ay-By)^2+ra^2\right)rb^2 + rb^4\right)\right)}\right),\right.$$

$$Cy \to \left(Ay^4 - 2\,Ay^3\,By + 2\,Ay\,By\left(By^2 + ra^2 - rb^2\right) - By^2\left((Ax-Bx)^2 + By^2 + ra^2 - rb^2\right) + Ay^2\left((Ax-Bx)^2 - ra^2 + rb^2\right) - Ax\sqrt{\left(-(Ay-By)^2\left(\left((Ax-Bx)^2+(Ay-By)^2-ra^2\right)^2 - 2\left((Ax-Bx)^2+(Ay-By)^2+ra^2\right)rb^2 + rb^4\right)\right)} + Bx\sqrt{\left(-(Ay-By)^2\left(\left((Ax-Bx)^2+(Ay-By)^2-ra^2\right)^2 - 2\left((Ax-Bx)^2+(Ay-By)^2+ra^2\right)rb^2 + rb^4\right)\right)}\right) \Big/ \left(2\left((Ax-Bx)^2+(Ay-By)^2\right)(Ay-By)\right)\Big\}\Big\}$$

Alternatively

- **Using substitution on the way, one can easily do it manually on the paper**

```
eq3 = exp1 - exp2 == 0 // Simplify
```

$$Ax^2 + Ay^2 + 2\,Bx\,Cx + 2\,By\,Cy + rb^2 == Bx^2 + By^2 + 2\,Ax\,Cx + 2\,Ay\,Cy + ra^2$$

- **Solving for Cx**

```
solCxeq3 = Solve[eq3, Cx]
```

$$\left\{\left\{Cx \to \frac{Ax^2 + Ay^2 - Bx^2 - By^2 - 2\,Ay\,Cy + 2\,By\,Cy - ra^2 + rb^2}{2\,(Ax - Bx)}\right\}\right\}$$

- **Inserting Cx into eq1 (note division by (Ax-Bx), this should not be zero)**

```
eq4 = eq1 /. solCxeq3[[1]]
```

$$(-Ay + Cy)^2 - ra^2 + \left(-Ax + \frac{Ax^2 + Ay^2 - Bx^2 - By^2 - 2\,Ay\,Cy + 2\,By\,Cy - ra^2 + rb^2}{2\,(Ax - Bx)}\right)^2 == 0$$

- **Solving for Cy (note division by r)**

```
solCy = Solve[eq4, Cy] // FullSimplify
```

$$\left\{\left\{Cy \to \frac{1}{2\left((Ax - Bx)^2 + (Ay - By)^2\right)}\left(Ax^2\,(Ay + By) - 2\,Ax\,Bx\,(Ay + By) + Bx^2\,(Ay + By) + (Ay - By)\left(Ay^2 - By^2 - ra^2 + rb^2\right) - \sqrt{-(Ax - Bx)^2\left(\left((Ax - Bx)^2 + (Ay - By)^2 - ra^2\right)^2 - 2\left((Ax - Bx)^2 + (Ay - By)^2 + ra^2\right)rb^2 + rb^4\right)}\right)\right\},\right.$$
$$\left\{Cy \to \frac{1}{2\left((Ax - Bx)^2 + (Ay - By)^2\right)}\left(Ax^2\,(Ay + By) - 2\,Ax\,Bx\,(Ay + By) + Bx^2\,(Ay + By) + (Ay - By)\left(Ay^2 - By^2 - ra^2 + rb^2\right) + \sqrt{-(Ax - Bx)^2\left(\left((Ax - Bx)^2 + (Ay - By)^2 - ra^2\right)^2 - 2\left((Ax - Bx)^2 + (Ay - By)^2 + ra^2\right)rb^2 + rb^4\right)}\right)\right\}\right\}$$

```
solCy /. condr // FullSimplify
```

$$\left\{\left\{Cy \to \frac{1}{2\,r^2}\left(Ax^2\,(Ay + By) - 2\,Ax\,Bx\,(Ay + By) + Bx^2\,(Ay + By) + (Ay - By)\left(Ay^2 - By^2 - ra^2 + rb^2\right) - \sqrt{-(r - ra - rb)\,(r + ra - rb)\,(r - ra + rb)\,(r + ra + rb)}\;\text{Abs}[Ax - Bx]\right)\right\},\right.$$
$$\left\{Cy \to \frac{1}{2\,r^2}\left(Ax^2\,(Ay + By) - 2\,Ax\,Bx\,(Ay + By) + Bx^2\,(Ay + By) + (Ay - By)\left(Ay^2 - By^2 - ra^2 + rb^2\right) + \sqrt{-(r - ra - rb)\,(r + ra - rb)\,(r - ra + rb)\,(r + ra + rb)}\;\text{Abs}[Ax - Bx]\right)\right\}\right\}$$

- **Finaly solving Cx**

```
solCx = Solve[eq3, Cx] /. solCy // FullSimplify
```

$$\left\{\left\{\left\{Cx \to \left(Ax^4 - 2\,Ax^3\,Bx + 2\,Ax\,Bx\left(Bx^2 + ra^2 - rb^2\right) - Bx^2\left(Bx^2 + (Ay - By)^2 + ra^2 - rb^2\right) + Ax^2\left((Ay - By)^2 - ra^2 + rb^2\right) + Ay\sqrt{-(Ax - Bx)^2\left(\left((Ax - Bx)^2 + (Ay - By)^2 - ra^2\right)^2 - 2\left((Ax - Bx)^2 + (Ay - By)^2 + ra^2\right)rb^2 + rb^4\right)} - By\sqrt{-(Ax - Bx)^2\left(\left((Ax - Bx)^2 + (Ay - By)^2 - ra^2\right)^2 - 2\left((Ax - Bx)^2 + (Ay - By)^2 + ra^2\right)rb^2 + rb^4\right)}\right) \Big/ \left(2\,(Ax - Bx)\left((Ax - Bx)^2 + (Ay - By)^2\right)\right)\right\}\right\},\right.$$
$$\left\{\left\{Cx \to \left(Ax^4 - 2\,Ax^3\,Bx + 2\,Ax\,Bx\left(Bx^2 + ra^2 - rb^2\right) - Bx^2\left(Bx^2 + (Ay - By)^2 + ra^2 - rb^2\right) + Ax^2\left((Ay - By)^2 - ra^2 + rb^2\right) - Ay\sqrt{-(Ax - Bx)^2\left(\left((Ax - Bx)^2 + (Ay - By)^2 - ra^2\right)^2 - 2\left((Ax - Bx)^2 + (Ay - By)^2 + ra^2\right)rb^2 + rb^4\right)} + By\sqrt{-(Ax - Bx)^2\left(\left((Ax - Bx)^2 + (Ay - By)^2 - ra^2\right)^2 - 2\left((Ax - Bx)^2 + (Ay - By)^2 + ra^2\right)rb^2 + rb^4\right)}\right) \Big/ \left(2\,(Ax - Bx)\left((Ax - Bx)^2 + (Ay - By)^2\right)\right)\right\}\right\}\right\}$$

```
solCx /. (Ax - Bx)^2 + (Ay - By)^2 → r^2 // FullSimplify
```

$$\left\{\left\{\left\{Cx \to \frac{1}{2\,r^2}\left(Ax^3 - Ax^2\,Bx + Bx\left(Bx^2 + (Ay - By)^2 + ra^2 - rb^2\right) + Ax\left(-Bx^2 + (Ay - By)^2 - ra^2 + rb^2\right) + 1 / \text{Sign}[Ax - Bx]\,(Ay - By)\sqrt{-(r - ra - rb)\,(r + ra - rb)\,(r - ra + rb)\,(r + ra + rb)}\right)\right\}\right\},$$

$$\left\{\left\{Cx \to \frac{1}{2\,r^2}\left(Ax^3 - Ax^2\,Bx + Bx\left(Bx^2 + (Ay - By)^2 + ra^2 - rb^2\right) + Ax\left(-Bx^2 + (Ay - By)^2 - ra^2 + rb^2\right) + 1 / \text{Sign}[Ax - Bx]\,(-Ay + By)\sqrt{-(r - ra - rb)\,(r + ra - rb)\,(r - ra + rb)\,(r + ra + rb)}\right)\right\}\right\}$$

Another alternative solution, this time solves all cases correctly without manual analysis

```
solC3 = Reduce[{eq2, eq1}, {Cx, Cy}]
```

$$\left(Ay == By \;\&\&\; Ax - Bx \neq 0 \;\&\&\; Cx == \frac{Ax^2 - Bx^2 - ra^2 + rb^2}{2\,(Ax - Bx)} \;\&\&\; \left(Cy == By - \sqrt{-Bx^2 + 2\,Bx\,Cx - Cx^2 + rb^2} \;||\; Cy == By + \sqrt{-Bx^2 + 2\,Bx\,Cx - Cx^2 + rb^2}\right)\right) ||$$

$$\left(Ax^2 + Ay^2 - 2\,Ax\,Bx + Bx^2 - 2\,Ay\,By + By^2 \neq 0 \;\&\&\; \left(Cx == \left(4\,Ax^3 + 4\,Ax\,Ay^2 - 4\,Ax^2\,Bx + 4\,Ay^2\,Bx - 4\,Ax\,Bx^2 + 4\,Bx^3 - 8\,Ax\,Ay\,By - 8\,Ay\,Bx\,By + 4\,Ax\,By^2 + 4\,Bx\,By^2 - 4\,Ax\,ra^2 + 4\,Bx\,ra^2 + 4\,Ax\,rb^2 - 4\,Bx\,rb^2 - \sqrt{\left(\left(-4\,Ax^3 - 4\,Ax\,Ay^2 + 4\,Ax^2\,Bx - 4\,Ay^2\,Bx + 4\,Ax\,Bx^2 - 4\,Bx^3 + 8\,Ax\,Ay\,By + 8\,Ay\,Bx\,By - 4\,Ax\,By^2 - 4\,Bx\,By^2 + 4\,Ax\,ra^2 - 4\,Bx\,ra^2 - 4\,Ax\,rb^2 + 4\,Bx\,rb^2\right)^2 - 4\left(4\,Ax^2 + 4\,Ay^2 - 8\,Ax\,Bx + 4\,Bx^2 - 8\,Ay\,By + 4\,By^2\right)\left(Ax^4 + 2\,Ax^2\,Ay^2 + Ay^4 - 2\,Ax^2\,Bx^2 + 2\,Ay^2\,Bx^2 + Bx^4 - 4\,Ax^2\,Ay\,By - 4\,Ax^3\,By - 4\,Ay\,Bx^2\,By + 2\,Ax^2\,By^2 + 6\,Ay^2\,By^2 + 2\,Bx^2\,By^2 - 4\,Ay\,By^3 + By^4 - 2\,Ax^2\,ra^2 - 2\,Ay^2\,ra^2 + 2\,Bx^2\,ra^2 + 4\,Ay\,By\,ra^2 - 2\,By^2\,ra^2 + ra^4 + 2\,Ax^2\,rb^2 - 2\,Ay^2\,rb^2 - 2\,Bx^2\,rb^2 + 4\,Ay\,By\,rb^2 - 2\,By^2\,rb^2 - 2\,ra^2\,rb^2 + rb^4\right)\right)\Big/\left(2\left(4\,Ax^2 + 4\,Ay^2 - 8\,Ax\,Bx + 4\,Bx^2 - 8\,Ay\,By + 4\,By^2\right)\right)\right) ||\; Cx == \left(4\,Ax^3 + 4\,Ax\,Ay^2 - 4\,Ax^2\,Bx + 4\,Ay^2\,Bx - 4\,Ax\,Bx^2 + 4\,Bx^3 - 8\,Ax\,Ay\,By - 8\,Ay\,Bx\,By + 4\,Ax\,By^2 + 4\,Bx\,By^2 - 4\,Ax\,ra^2 + 4\,Bx\,ra^2 + 4\,Ax\,rb^2 - 4\,Bx\,rb^2 + \sqrt{\left(\left(-4\,Ax^3 - 4\,Ax\,Ay^2 + 4\,Ax^2\,Bx - 4\,Ay^2\,Bx + 4\,Ax\,Bx^2 - 4\,Bx^3 + 8\,Ax\,Ay\,By + 8\,Ay\,Bx\,By - 4\,Ax\,By^2 - 4\,Bx\,By^2 + 4\,Ax\,ra^2 - 4\,Bx\,ra^2 - 4\,Ax\,rb^2 + 4\,Bx\,rb^2\right)^2 - 4\left(4\,Ax^2 + 4\,Ay^2 - 8\,Ax\,Bx + 4\,Bx^2 - 8\,Ay\,By + 4\,By^2\right)\left(Ax^4 + 2\,Ax^2\,Ay^2 + Ay^4 - 2\,Ax^2\,Bx^2 + 2\,Ay^2\,Bx^2 + Bx^4 - 4\,Ax^2\,Ay\,By - 4\,Ax^3\,By - 4\,Ay\,Bx^2\,By + 2\,Ax^2\,By^2 + 6\,Ay^2\,By^2 + 2\,Bx^2\,By^2 - 4\,Ay\,By^3 + By^4 - 2\,Ax^2\,ra^2 - 2\,Ay^2\,ra^2 + 2\,Bx^2\,ra^2 + 4\,Ay\,By\,ra^2 - 2\,By^2\,ra^2 + ra^4 + 2\,Ax^2\,rb^2 - 2\,Ay^2\,rb^2 - 2\,Bx^2\,rb^2 + 4\,Ay\,By\,rb^2 - 2\,By^2\,rb^2 - 2\,ra^2\,rb^2 + rb^4\right)\right)\Big/\left(2\left(4\,Ax^2 + 4\,Ay^2 - 8\,Ax\,Bx + 4\,Bx^2 - 8\,Ay\,By + 4\,By^2\right)\right)\right) \;\&\&\; Ay - By \neq 0 \;\&\&\; Cy == \frac{Ax^2 + Ay^2 - Bx^2 - By^2 - 2\,Ax\,Cx + 2\,Bx\,Cx - ra^2 + rb^2}{2\,(Ay - By)}\right) ||$$

$$\left(rb == 0 \;\&\&\; ra == 0 \;\&\&\right.$$

$$\begin{aligned}
&Ay == By \;\&\&\; Ax == Bx \;\&\&\; \left(Cy == By - \sqrt{-Bx^2 + 2\,Bx\,Cx - Cx^2} \;||\; Cy == By + \sqrt{-Bx^2 + 2\,Bx\,Cx - Cx^2}\right) \;||\\
&\left(rb == 0 \;\&\&\; ra == 0 \;\&\&\; \left(Ax == Bx - \sqrt{-Ay^2 + 2\,Ay\,By - By^2} \;||\; Ax == Bx + \sqrt{-Ay^2 + 2\,Ay\,By - By^2}\right) \;\&\&\; Ay - By \neq 0 \;\&\&\; Cy == \frac{Ax\,Bx - Bx^2 + Ay\,By - By^2 - Ax\,Cx + Bx\,Cx}{Ay - By}\right) \;||\\
&\left(\left(Ax == Bx - \sqrt{-Ay^2 + 2\,Ay\,By - By^2} \;||\; Ax == Bx + \sqrt{-Ay^2 + 2\,Ay\,By - By^2}\right) \;\&\&\; (Ax - Bx)\left(ra^2 - rb^2\right) \neq 0 \;\&\&\; Cx == \left(4\,Ax\,Bx\,ra^2 - 4\,Bx^2\,ra^2 - ra^4 + 4\,Ay^2\,rb^2 - 4\,Ax\,Bx\,rb^2 + 4\,Bx^2\,rb^2 - 8\,Ay\,By\,rb^2 + 4\,By^2\,rb^2 + 2\,ra^2\,rb^2 - rb^4\right) \Big/ \left(4\,(Ax - Bx)\left(ra^2 - rb^2\right)\right) \;\&\&\; Ay - By \neq 0 \;\&\&\; Cy == \frac{2\,Ax\,Bx - 2\,Bx^2 + 2\,Ay\,By - 2\,By^2 - 2\,Ax\,Cx + 2\,Bx\,Cx - ra^2 + rb^2}{2\,(Ay - By)}\right) \;||\\
&\left((ra == -rb \;||\; ra == rb) \;\&\&\; Ay == By \;\&\&\; Ax == Bx \;\&\&\; \left(Cy == By - \sqrt{-Bx^2 + 2\,Bx\,Cx - Cx^2 + rb^2} \;||\; Cy == By + \sqrt{-Bx^2 + 2\,Bx\,Cx - Cx^2 + rb^2}\right) \;\&\&\; rb \neq 0\right)
\end{aligned}$$

solC3 /. condr // FullSimplify

$$\left(Ay == By \&\& Ax \neq Bx \&\& Cx == \frac{Ax^2 - Bx^2 - ra^2 + rb^2}{2\,Ax - 2\,Bx} \&\& \left(Cy + \sqrt{-(Bx - Cx)^2 + rb^2} == By \,||\, By + \sqrt{-(Bx - Cx)^2 + rb^2} == Cy\right)\right) ||$$

$$\left(\left(Ax + Bx == 2\,Cx + \left((Ax - Bx)\,(ra - rb)\,(ra + rb) + \sqrt{\left(-(Ay - By)^2 \left(\left((Ax - Bx)^2 + (Ay - By)^2 - ra^2\right)^2 - 2\left((Ax - Bx)^2 + (Ay - By)^2 + ra^2\right) rb^2 + rb^4\right)\right)}\right) \Big/ \left((Ax - Bx)^2 + (Ay - By)^2\right) \,||\, Ax + Bx + \left(\sqrt{\left(-(Ay - By)^2 \left(\left((Ax - Bx)^2 + (Ay - By)^2 - ra^2\right)^2 - 2\left((Ax - Bx)^2 + (Ay - By)^2 + ra^2\right) rb^2 + rb^4\right)\right)}\right) \Big/ \left((Ax - Bx)^2 + (Ay - By)^2\right) == 2\,Cx + \frac{(Ax - Bx)\,(ra - rb)\,(ra + rb)}{(Ax - Bx)^2 + (Ay - By)^2}\right) \&\& Ay \neq By \&\& Cy == \frac{Ax^2 + Ay^2 - Bx^2 - By^2 - 2\,Ax\,Cx + 2\,Bx\,Cx - ra^2 + rb^2}{2\,Ay - 2\,By}\right) || \left(rb == 0 \&\& ra == 0 \&\& Ay == By \&\& Ax == Bx \&\& \left(\sqrt{-(Bx - Cx)^2} + Cy == By \,||\, By + \sqrt{-(Bx - Cx)^2} == Cy\right)\right) ||$$

$$\left((ra + rb == 0 \,||\, ra == rb) \&\& Ay == By \&\& Ax == Bx \&\& \left(Cy + \sqrt{-(Bx - Cx)^2 + rb^2} == By \,||\, By + \sqrt{-(Bx - Cx)^2 + rb^2} == Cy\right) \&\& rb \neq 0\right)$$

- **It is given to the reader to analyze all cases, what they mean geometrically.**

Resulting human readable formula

- **The above results could be after manual analysis rewritten as a sum of two parts. One key used is that we do not care, which solution is the first one and which is the second one until both of them are correct. This allows to ignore the signature of (Ax-Bx) or (Ay-By) and simplify the formulas.**
- **Coordinates of the point on the intersection of lines AB and C_1 C_2 (point P)**

```
CxHumanPart1 = ((Ax + Bx) r^2 + (Ax - Bx) (rb^2 - ra^2)) / 2 / r^2
CyHumanPart1 = ((Ay + By) r^2 + (Ay - By) (rb^2 - ra^2)) / 2 / r^2
```

$$\frac{(Ax + Bx)\,r^2 + (Ax - Bx)\left(-ra^2 + rb^2\right)}{2\,r^2}$$

$$\frac{(Ay + By)\,r^2 + (Ay - By)\left(-ra^2 + rb^2\right)}{2\,r^2}$$

Vector PC_1

```
CxHumanPart2 = ((Ay - By) Sqrt[4 ra^2 rb^2 - (r^2 - ra^2 - rb^2)^2]) / 2 / r^2
CyHumanPart2 = ((Ax - Bx) Sqrt[4 ra^2 rb^2 - (r^2 - ra^2 - rb^2)^2]) / 2 / r^2
```

$$\frac{(\text{Ay}-\text{By})\sqrt{4\,\text{ra}^2\,\text{rb}^2-\left(\text{r}^2-\text{ra}^2-\text{rb}^2\right)^2}}{2\,\text{r}^2}$$

$$\frac{(\text{Ax}-\text{Bx})\sqrt{4\,\text{ra}^2\,\text{rb}^2-\left(\text{r}^2-\text{ra}^2-\text{rb}^2\right)^2}}{2\,\text{r}^2}$$

Combining it into a result

```
CxHuman1 = CxHumanPart1 - CxHumanPart2 // TraditionalForm
CxHuman2 = CxHumanPart1 + CxHumanPart2 // TraditionalForm
CyHuman1 = CyHumanPart1 + CyHumanPart2 // TraditionalForm
CyHuman2 = CyHumanPart1 - CyHumanPart2 // TraditionalForm
```

$$\frac{r^2\,(\text{Ax}+\text{Bx})+(\text{Ax}-\text{Bx})\,(\text{rb}^2-\text{ra}^2)}{2\,r^2}-\frac{(\text{Ay}-\text{By})\sqrt{4\,\text{ra}^2\,\text{rb}^2-(r^2-\text{ra}^2-\text{rb}^2)^2}}{2\,r^2}$$

$$\frac{r^2\,(\text{Ax}+\text{Bx})+(\text{Ax}-\text{Bx})\,(\text{rb}^2-\text{ra}^2)}{2\,r^2}+\frac{(\text{Ay}-\text{By})\sqrt{4\,\text{ra}^2\,\text{rb}^2-(r^2-\text{ra}^2-\text{rb}^2)^2}}{2\,r^2}$$

$$\frac{(\text{Ax}-\text{Bx})\sqrt{4\,\text{ra}^2\,\text{rb}^2-(r^2-\text{ra}^2-\text{rb}^2)^2}}{2\,r^2}+\frac{r^2\,(\text{Ay}+\text{By})+(\text{Ay}-\text{By})\,(\text{rb}^2-\text{ra}^2)}{2\,r^2}$$

$$\frac{r^2\,(\text{Ay}+\text{By})+(\text{Ay}-\text{By})\,(\text{rb}^2-\text{ra}^2)}{2\,r^2}-\frac{(\text{Ax}-\text{Bx})\sqrt{4\,\text{ra}^2\,\text{rb}^2-(r^2-\text{ra}^2-\text{rb}^2)^2}}{2\,r^2}$$

Verification

```
({eq1, eq2} /. {Cx → CxHuman1, Cy → CyHuman1} // FullSimplify) /. condr
```

```
{True, True}
```