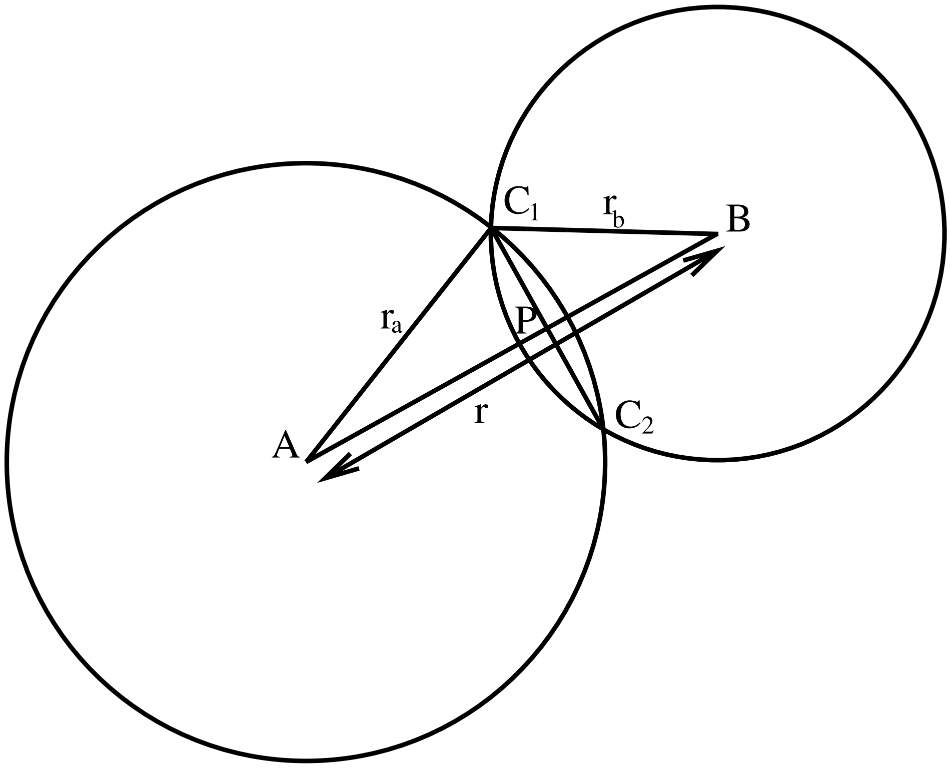
Intersection of two circles

- Given coordinates of centers of two circles and their radii $A = [A_x, A_y]^T$, r_a, $B = [B_x, B_y]^T$, r_b.
- Calculate intersection of the two circles $C = [C_x, C_y]^T$.



This document describes calculation of intersection of two circles in plane and explicit formulas to calculate their coordinates algebraically.

Good references are
http://mathworld.wolfram.com/Circle-CircleIntersection.html
http://paulbourke.net/geometry/2circle/

Let us label the distance of circle centers as

$$r = \sqrt{(A_x - B_x)^2 + (A_y - B_y)^2}$$

The analysis of cases is based on triangular inequality http://mathworld.wolfram.com/TriangleInequality.html which

- **Identical circles**

$$r = 0 \wedge r_a = r_b,$$

- **Nonintersecting circles**

$$|r_a - r_b| > r \vee r > r_a + r_b,$$

- **One point intersection** which in digital representation could happen only under rare circumstances

$$|r_a - r_b| = r \vee r = r_a + r_b,$$

In this case we often could neglect this case and handle it as regular case where two intersections are identical.

- **Two point intersection** happens when following condition is satisfied

$$|r_a - r_b| < r \wedge r < r_a + r_b.$$

Let us solve two point intersection. The intersection points have coordinates:

$$C_{1,2} = \begin{bmatrix} \frac{(A_x+B_x)r^2+(A_x-B_x)\left(rb^2-ra^2\right)\pm(A_y-B_y)\sqrt{4ra^2rb^2-(r^2-ra^2-rb^2)^2}}{2r^2} \\ \frac{(A_y+B_y)r^2+(A_y-B_y)\left(rb^2-ra^2\right)\mp(A_x-B_x)\sqrt{4ra^2rb^2-(r^2-ra^2-rb^2)^2}}{2r^2} \end{bmatrix} = \begin{bmatrix} \frac{A_x+B_x}{2} + \frac{(A_x-B_x)\left(rb^2-ra^2\right)}{2r^2} \pm \frac{(A_y-B_y)\sqrt{4ra^2rb^2-(r^2-ra^2-rb^2)^2}}{2r^2} \\ \frac{A_y+B_y}{2} + \frac{(A_y-B_y)\left(rb^2-ra^2\right)}{2r^2} \mp \frac{(A_x-B_x)\sqrt{4ra^2rb^2-(r^2-ra^2-rb^2)^2}}{2r^2} \end{bmatrix} = P \pm \overrightarrow{PC_1}$$