

Robotics

Intersection of Two Circles

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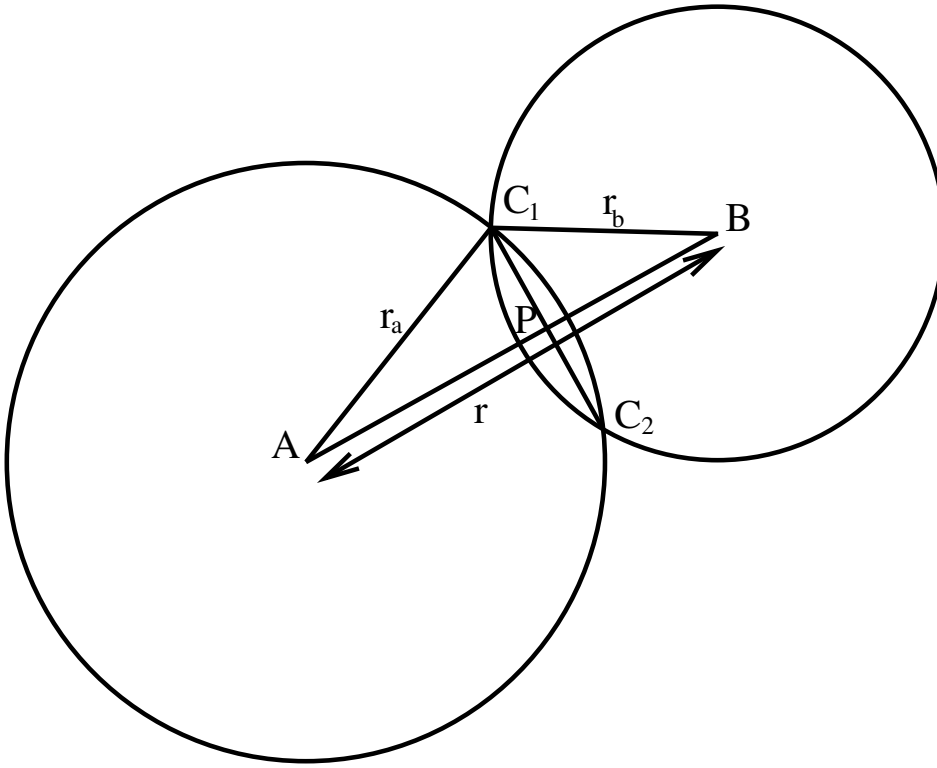
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- Given coordinates of centers of two circles and their radii
 $A = [A_x, A_y]^T, r_a, B = [B_x, B_y]^T, r_b$.
- Calculate intersection of the two circles $C = [C_x, C_y]^T$.



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This document describes calculation of intersection of two circles in plane and explicit formulas to calculate their coordinates algebraically.

Good references are

<http://mathworld.wolfram.com/Circle-CircleIntersection.html>

<http://paulbourke.net/geometry/2circle/>

Let us label the distance of circle centers as

$$r = \sqrt{(A_x - B_x)^2 + (A_y - B_y)^2}$$

The analysis of cases is based on triangular inequality <http://mathworld.wolfram.com/TriangleInequality.html> which

• Identical circles

$$r = 0 \wedge r_a = r_b,$$

• Nonintersecting circles

$$|r_a - r_b| > r \vee r > r_a + r_b,$$

- One point intersection which in digital representation could happen only under rare circumstances

$$|r_a - r_b| = r \vee r = r_a + r_b,$$

In this case we often could neglect this case and handle it as regular case where two intersections are identical.

- Two point intersection happens when following condition is satisfied

$$|r_a - r_b| < r \wedge r < r_a + r_b.$$

Let us solve two point intersection. The intersection points have coordinates:

$$C_{1,2} = \left[\begin{array}{c} \frac{(A_x + B_x)r^2 + (A_x - B_x)(rb^2 - ra^2) \pm (A_y - B_y)\sqrt{4ra^2rb^2 - (r^2 - ra^2 - rb^2)^2}}{2r^2} \\ \frac{(A_y + B_y)r^2 + (A_y - B_y)(rb^2 - ra^2) \mp (A_x - B_x)\sqrt{4ra^2rb^2 - (r^2 - ra^2 - rb^2)^2}}{2r^2} \end{array} \right] = \left[\begin{array}{c} \frac{A_x + B_x}{2} + \frac{(A_x - B_x)(rb^2 - ra^2)}{2r^2} \pm \frac{(A_y - B_y)\sqrt{4ra^2rb^2 - (r^2 - ra^2 - rb^2)^2}}{2r^2} \\ \frac{A_y + B_y}{2} + \frac{(A_y - B_y)(rb^2 - ra^2)}{2r^2} \mp \frac{(A_x - B_x)\sqrt{4ra^2rb^2 - (r^2 - ra^2 - rb^2)^2}}{2r^2} \end{array} \right] = P \pm \overrightarrow{PC_1}$$