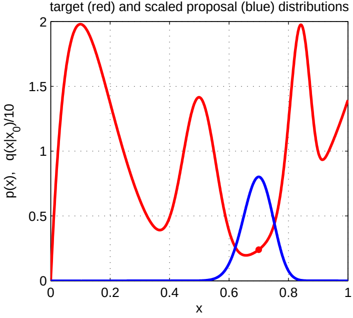
How To Generate Random Samples from a Complex Distribution?



- red: probability density function $\pi(x)$ of the toy distribution on the unit interval — target distribution

$$\pi(x) = \sum_{i=1}^{4} \gamma_i \, \mathrm{Be}(x; \alpha_i, \beta_i), \quad \sum_{i=1}^{4} \gamma_i = 1, \ \gamma_i \geq 0$$

$$\mathrm{Be}(x; \alpha, \beta) = \frac{1}{\mathrm{B}(\alpha, \beta)} \cdot x^{\alpha-1}(1-x)^{\beta-1}$$

- alg. for generating samples from $\mathrm{Be}(x; \alpha, \beta)$ is known
- $\Rightarrow$ we can generate samples from $\pi(x)$ — how?

- suppose we cannot sample from $\pi(x)$ but we can sample from some 'simple' distribution $q(x \mid x_0)$, given the last sample x_0 (blue) — proposal distribution

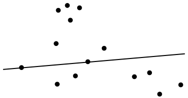
$$q(x \mid x_0) = \begin{cases} \mathrm{U}_{0,1}(x) & \text{(independent) uniform sampling} \\ \mathrm{Be}(x; \frac{x_0}{T}+1, \frac{1-x_0}{T}+1) & \text{'beta' diffusion (crawler)} \quad T - \text{temperature} \\ \pi(x) & \text{(independent) Gibbs sampler} \end{cases}$$

- note we have unified all the random sampling methods from the previous slide
- how to redistribute proposal samples $q(x \mid x_0)$ to target distribution $\pi(x)$ samples?

▶Putting Some Clothes Back: RANSAC [Fischler & Bolles 1981]

1. primitives = elementary measurements
   - points in line fitting
   - matches in epipolar geometry estimation

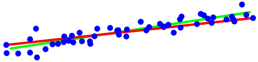
2. configuration = s-tuple of primitives   minimal subsets necessary for parameter estimate



the minimization will be over a discrete set:

- of point pairs in line fitting (left)
- of match 7-tuples in epipolar geometry estimation

3. proposal distribution $q(\cdot)$ is then given by the empirical distribution of s-tuples:
   a) propose s-tuple from data independently $q(S \mid C_t) = q(S)$
      i) q uniform $q(S) = \binom{mn}{s}^{-1}$ MAPSAC ($p(S)$ includes the prior)
      ii) q dependent on descriptor similarity   PROSAC (similar pairs are proposed more often)
   b) solve the minimal geometric problem $\mapsto$ parameter proposal



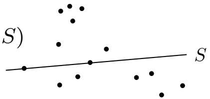
- pairs of points define line distribution from $p(\mathbf{n} \mid X)$ (left)
- random correspondence tuples drawn uniformly propose samples of $\mathbf{F}$ from a data-driven distribution $q(\mathbf{F} \mid M)$

4. local optimization from promising proposals

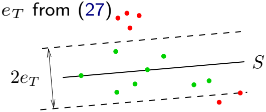
5. stopping based on the probability of mode-hitting   →123

▶RANSAC with Local Optimization and Early Stopping

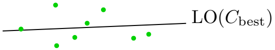
1. initialize the best sample as empty $C_{\text{best}} := \emptyset$ and time $t := 0$
2. estimate the number of needed proposals as $N := \binom{n}{s}$ n – No. of primitives, s – minimal sample size
3. while $t \leq N$:
   a) propose a minimal random sample S of size s from $q(S)$



   b) if $\pi(S) > \pi(C_{\text{best}})$ then
      i) update the best sample $C_{\text{best}} := S$ $\pi(S)$ marginalized as in (26); $\pi(S)$ includes a prior $\Rightarrow$ MAP
      ii) threshold-out inliers using e_T from (27)



      iii) start local optimization from the inliers of C_{best} LM optimization with robustified (→113) Sampson error possibly weighted by posterior $\pi(m_{ij})$ [Chum et al. 2003]



      iv) update C_{best}, update inliers using (27), re-estimate N from inlier counts →123 for derivation

$$N = \frac{\log(1-P)}{\log(1-\varepsilon^s)}, \quad \varepsilon = \frac{|\,\text{inliers}(C_{\text{best}})|}{m\,n},$$

   c) $t := t + 1$
4. output C_{best}

- see MPV course for RANSAC details see also [Fischler & Bolles 1981], [25 years of RANSAC]