

# 3D Computer Vision

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<https://cw.fel.cvut.cz/wiki/courses/tdv/start>

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Open Informatics Master's Course

## Content:

- Elements of projective geometry
- Perspective camera
- Geometric problems in 3D vision
- Epipolar geometry
- Optimization methods in 3D vision
- 3D structure and camera motion from (many) images
- Stereoscopic vision
- Shape from reflectance (optional)

## An Underlying Programme:

1. how to do things right in 3D vision      [cookbook of effective methods, pitfall avoidance](#)
2. things useful beyond CV      [task formulation exercise, powerful robust optimization methods](#)

## ► Background

### Absolutely Necessary Prior Knowledge

- basic geometry line in 2D and in 3D, plane in 3D, their intersections
- elementary linear algebra  
vectors: dot product, cross product, mixed product; matrices: bases, null space, linear systems of equations, eigensystem, matrix decompositions: QR, SVD, Choleski
- elementary optimization in continuous domain  
quadratic problems, constrained optimization, gradient descend, Newton method
- the basics of Bayesian modeling prior, posterior, likelihood
- Matlab<sup>®</sup>: At least elementary programming

### Important Material Covered Elsewhere

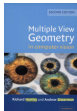
- Homography as a multiview model [H&Z Secs: 2.5, 2.3, 2.4, 4.1, 4.2, A6.1, A6.2]
- Sparse image matching using RANSAC [H&Z Sec. 4.7]

## ► Reading

**Annotated Slides** – this is the reference material make sure you have the latest version

- for deeper digs into geometry, see the GVG lecture notes at <https://cw.fel.cvut.cz/b172/courses/gvg/start>
- there is a Czech-English and English-Czech dictionary for this course <http://cmp.felk.cvut.cz/cmp/courses/TDV/2010W/lectures/3DV-slovník.pdf>

**The Book**, it will be referenced as [H&Z]



Hartley, R. and Zisserman, A. *Multiple View Geometry in Computer Vision*. Cambridge University Press, 2nd ed, 2003. Secs. 2, 4, 6, 7, 9, 10, 11, 12.

- you can borrow this book from the CMP library
- contact Ms. Hana Pokorná, room G102, <mailto:hana.pokorna@fel.cvut.cz>
- indicate you are a student of this course

**The Stereo Paper**, referenced as [SP]










Šára, R. *Stereoscopic Vision*. 2010

Czech: <http://cmp.felk.cvut.cz/~sara/Teaching/TDV/SP-cz.pdf>

English: <http://cmp.felk.cvut.cz/~sara/Teaching/TDV/SP-en.pdf>

# Optional Reading

(available from Google Scholar or CTU Library)

-  M. A. Fischler and R. C. Bolles. Random sample consensus: A paradigm for model fitting with applications to image analysis and automated cartography. *Communications of the ACM*, 24(6):381–395, 1981.
-  C. Harris and M. Stephens. A combined corner and edge detector. In *Proc ALVEY Vision Conf*, pp. 147–151, University of Manchester, England, 1988.
-  D. G. Lowe. Distinctive image features from scale-invariant keypoints. *International Journal of Computer Vision*, 60(2):91–110, 2004.
-  H. Li and R. Hartley. Five-point motion estimation made easy. In *Proc ICPR*, pp. 630–633, 2006.
-  B. Triggs, P. McLauchlan, R. Hartley, and A. Fitzgibbon. A comprehensive survey of bundle adjustment in computer vision. In *Proc Vision Algorithms: Theory and Practice*, LNCS 1883:298–372. Springer Verlag, 1999.
-  25 years of RANSAC. In *CVPR '06 Workshop and Tutorial [on-line]*, <http://cmp.felk.cvut.cz/ransac-cvpr2006/>. 2006.
-  G. H. Golub and C. F. Van Loan. *Matrix Computations*. Johns Hopkins University Press, Baltimore, USA, 4th edition, 2013.

## Some On-line Resources

1. OpenCV (Open Source Computer Vision): A library of programming functions for real time computer vision. [on-line]  
<http://opencv.org/>
2. T. Pajdla. Minimal problems in computer vision. [on-line]  
<http://cmp.felk.cvut.cz/mini/>  
Last update Jan 10, 2016.
3. Rob Hess. SIFT Feature Detector. [on-line]  
<http://robwhess.github.io/opensift/>.  
Last update Oct 24, 2013.
4. Marco Zuliani. RANSAC Toolbox for Matlab. [on-line]  
<http://vision.ece.ucsb.edu/~zuliani/Code/Code.html>.  
Last update Oct 18, 2009
5. Manolis Lourakis. A Generic Sparse Bundle Adjustment C/C++ Package based on the Levenberg-Marquardt Algorithm. [on-line]  
<http://www.ics.forth.gr/~lourakis/sba/>.  
Last update Jan 5, 2010.

## ► Notes on Slide Style

### I am using a consistent slide style:

- the main material is in black (like this)
- remarks and notes are in small blue font typically flushed to the right like this
- papers or books are referenced like this [H&Z, p. 100] or [Golub & van Loan 2013] except H&Z or SP, references are pointers to the list on Slide 13
- most references are linked (clickable) in the PDF, the triple of icons ↻↻↻ on the bottom helps you navigate back and forth, they are: back, find, forward check the references above
- linked references to slides: A reference to Slide 21 looks like this: →21
- each major module starts with a slide listing the equivalent written material
- slides containing examined material have a **bold title** with a marker ► like this slide
- mandatory homework exercises are in small red font, starting with a circled asterisk  
⊛ H1; 10pt: syntax: <problem ID in submission system>; <points>: explanation  
deadline: Lecture Day + 2 weeks; unit penalty per 7 days; see the Submission System
- optional homework exercises are a source of bonus points, they are in green  
⊛ P1; 1pt: same syntax; deadline: end of semester

Thank You



SECOND EDITION

# Multiple View Geometry

in computer vision



Richard **Hartley** and Andrew **Zisserman**

CAMBRIDGE

SECOND EDITION

# Multiple View Geometry

in computer vision



Richard **Hartley** and Andrew **Zisserman**

CAMBRIDGE

# How To Teach Stereoscopic Matching?

(Invited Paper)

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**Abstract**—This paper describes a simple but non-trivial semi-dense stereoscopic matching algorithm that could be taught in Computer Vision courses. The description is meant to be instructive and accessible to the student. The level of detail is sufficient for a student to understand all aspects of the algorithm design and to make his/her own modifications. The paper includes the core parts of the algorithm in C code. We make the point of explaining the algorithm so that all steps are derived from first principles in a clear and lucid way. A simple method is described that helps encourage the student to benchmark his/her own improvements of the algorithm.

## I. INTRODUCTION

Teaching stereovision in computer vision classes is a difficult task. There are very many algorithms described in the literature that address various aspects of the problem. A good up-to-date comprehensive review is missing. It is thus very difficult for a student to acquire a balanced overview of the area in a short time. For a non-expert researcher who wants to design and experiment with his/her own algorithm, reading the literature is often a frustrating exercise.

This paper tries to introduce a well selected, simple, and reasonably efficient algorithm. My selection is based on a long-term experience with various matching algorithms and on an experience with teaching a 3D computer vision class. I believe that by studying this algorithm the student is gradually educated about the way researchers think about stereoscopic matching. I tried to address all elementary components of the algorithm design, from occlusion modeling to matching task formulation and benchmarking.

An algorithm suitable for teaching stereo should meet a number of requirements:

**Simplicity:** The algorithm must be non-trivial but easy to implement.

**Accessibility:** The algorithm should be described in a complete and accessible form.

**Performance:** Performance must be reasonably good for the basic implementation to be useful in practice.

**Education:** The student should exercise formal problem formulation. Every part of the algorithm design must be well justified and derivable from first principles.

**Development potential:** The algorithm must possess potential for encouraging the student to do further development.

**Learnability:** There should be a simple recommended method for choosing the algorithm parameters and/or selecting their most suitable values for a given application.

Besides the basic knowledge in computer science and algorithms, basics of probability theory and statistics, and introductory-level computer vision or image processing, it is assumed that the reader is familiar with the concept of epipolar geometry and plane homography. The limited extent of this paper required shortening some explanations.

The purpose of this paper is not to give a balanced state-of-the-art overview. The presentation, however, does give references to relevant literature during the exposition of the problem.

## II. THE BASIC STRUCTURE OF STEREOSCOPIC MATCHING

Stereoscopic matching is a method for computing disparity maps from a set of images. In this paper we will only consider pairs of cameras that are not co-located. We will often call them the left and the right camera, respectively. A disparity map codes the shift in location of the corresponding local image features induced by camera displacement. The direction of the shift is given by *epipolar geometry* of the two cameras [1] and the magnitude of the shift, called *disparity*, is approximately inversely proportional to the camera-object distance. Some disparity map examples are shown in Fig. 10. In these maps, disparity is coded by color, close objects are reddish, distant objects bluish, and pixels without disparity are black. Such color-coding makes various kinds of errors clearly visible.

This section formalizes object occlusion so that we could derive useful necessary constraints on disparity map correctness. We will first work with spatial points. The 3D space in front of the two cameras is spanned by two pencils of optical rays, one system per camera. The spatial points can be thought of as the intersections of the optical rays. They will be called *possible correspondences*. The task of matching is to accept some of the possible correspondences and reject the others. Fig. 1 shows a part of a surface in the scene (black line segment) and a point  $p$  on the surface. Suppose  $p$  is visible in both cameras (by optical ray  $r_1$  in the left camera and by optical ray  $t_1$  in the right camera, both cameras are located above the 'scene'). Then every point on ray  $r_1$  that lies in front of  $p$  must be transparent and each point of  $r_1$  behind  $p$  must be occluded. A similar observation holds for ray  $t_1$ . This shows that the decision on point acceptance and rejection are not independent: If the correspondence given by point  $p$  is accepted, then a set of correspondences  $X(p)$

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