Lecture 10

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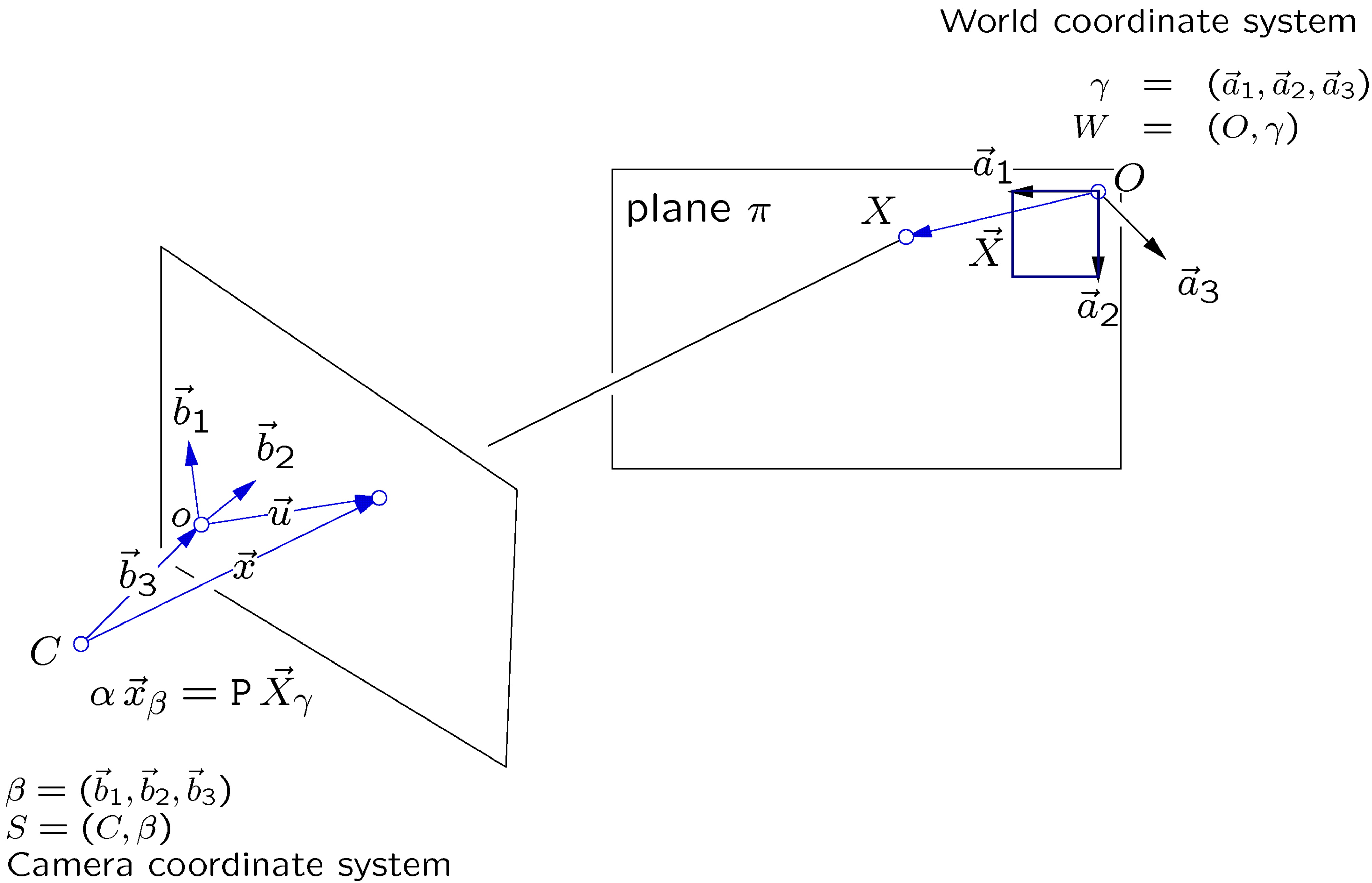
Camera calibration from homography to a “metric plane”



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Camera calibration from homography to a "metric plane"

Let us recall the relationship between the coordinates of points X, which all lie in a plane π and are measured in a coordinate system $(O, \vec{a}_1, \vec{a}_2)$ in π. The points X are projected by a perspective camera with projection matrix $\mathtt{P}$ into image coordinates (u, v), w.r.t. an image coordinate system $(o, \vec{b}_1, \vec{b}_2)$. The corresponding camera coordinate sysytem is (C, β) with $\beta = (\vec{b}_1, \vec{b}_2, \vec{b}_3)$.

Points X are projected by a perspective camera with a projection matrix $\mathtt{P}$ into projections $\vec{x}_\beta$ as

$$\alpha\, \vec{x}_\beta = \mathtt{P}\, \mathbf{X}_\gamma = \begin{bmatrix} \mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3 & \mathbf{p}_4 \end{bmatrix} \begin{bmatrix} x \\ y \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_4 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \mathtt{H} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

where $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4$ are columns of $\mathtt{P}$. Recall that the columns of $\mathtt{P}$ can be writen as

$$\begin{aligned} \mathtt{P} &= \begin{bmatrix} \mathtt{A} \,|\, -\mathtt{A}\, \vec{C}_\gamma \end{bmatrix} \\ &= \begin{bmatrix} \vec{a}_{1\beta} & \vec{a}_{2\beta} & \vec{a}_{3\beta} & -\vec{C}_\beta \end{bmatrix} \end{aligned}$$

and therefore we get

$$\begin{aligned} \mathbf{h}_1 = \mathbf{p}_1 &= \vec{a}_{1\beta} \\ \mathbf{h}_2 = \mathbf{p}_2 &= \vec{a}_{2\beta} \\ \mathbf{h}_3 = \mathbf{p}_4 &= -\vec{C}_\beta \end{aligned}$$

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Now imagine, that we are observing a suqare with 4 corner points X_1, X_2, X_3 and X_4 in the plane π and we construct the coordinate system in π by assigning coordinates to the corners as

$$\begin{aligned} \vec{X}_{1\gamma} &= \begin{bmatrix}0 & 0 & 0\end{bmatrix} \\ \vec{a}_{1\gamma} = \vec{X}_{2\gamma} &= \begin{bmatrix}1 & 0 & 0\end{bmatrix} \\ \vec{a}_{2\gamma} = \vec{X}_{3\gamma} &= \begin{bmatrix}0 & 1 & 0\end{bmatrix} \\ \vec{X}_{4\gamma} &= \begin{bmatrix}1 & 1 & 0\end{bmatrix} \end{aligned}$$

By this construction, the angle measured by the formula

$$\cos\measuredangle(\vec{X}_1, \vec{X}_2) = \frac{\vec{X}_{1\gamma}^\top \vec{X}_{2\gamma}}{(\vec{X}_{1\gamma}^\top \vec{X}_{1\gamma})^{\frac{1}{2}} (\vec{X}_{2\gamma}^\top \vec{X}_{2\gamma})^{\frac{1}{2}}}$$

corresponds to the angle measured by a ruler and a compass.

We see that we get two constraints on $\vec{a}_{1\gamma}$, $\vec{a}_{2\gamma}$

$$\begin{aligned} \vec{a}_{1\gamma}^\top \vec{a}_{2\gamma} &= 0 \\ \vec{a}_{1\gamma}^\top \vec{a}_{1\gamma} - \vec{a}_{2\gamma}^\top \vec{a}_{2\gamma} &= 0 \end{aligned}$$

which lead to

$$\begin{aligned} \vec{a}_{1\beta}^\top \mathtt{K}^{-\top} \mathtt{K}^{-1} \vec{a}_{2\beta} &= 0 \\ \vec{a}_{1\beta}^\top \mathtt{K}^{-\top} \mathtt{K}^{-1} \vec{a}_{1\beta} - \vec{a}_{2\beta}^\top \mathtt{K}^{-\top} \mathtt{K}^{-1} \vec{a}_{2\beta} &= 0 \end{aligned}$$

by using $\vec{a}_{i\beta} = \mathtt{K}\,\mathtt{R}\,\vec{a}_{i\gamma}$ for $i = 1, 2$, and $\mathtt{R}^\top \mathtt{R} = \mathtt{I}$.

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These are two linear equations

$$\begin{aligned} \vec{a}_{1\beta}^{\top}\,\omega\,\vec{a}_{2\beta} &= 0 \\ \vec{a}_{1\beta}^{\top}\,\omega\,\vec{a}_{1\beta} - \vec{a}_{2\beta}^{\top}\,\omega\,\vec{a}_{2\beta} &= 0 \end{aligned}$$

on ω in terms of estimated $\lambda\,\mathtt{H}$

$$\begin{aligned} \mathbf{h}_1^{\top}\,\omega\,\mathbf{h}_2 &= 0 \\ \mathbf{h}_1^{\top}\,\omega\,\mathbf{h}_1 - \mathbf{h}_2^{\top}\,\omega\,\mathbf{h}_2 &= 0 \end{aligned}$$

Every square provides 2 equations and therefore 3 squares in planes in general positions suffice to calibrate full $\mathtt{K}$ matrix and two such squares suffice to calibrate $\mathtt{K}$ when pixels are square.

To calibrate the camera, we first assign coordinates to the corners of the square as above, then find the homography $\mathtt{H}$ from the plane to the image

$$\alpha_i\,\vec{x}_{i\beta} = \mathtt{H}\,\vec{X}_{i\gamma}$$

for $\alpha_i = 1, \ldots, 4$ and finally use columns of $\mathtt{H}$ the find ω.

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Line coordinates under homography

Let us now investigate the behaviour of homogeneous coordinates of lines in projective plane mapped by a homography.

Let us have two points represented by vectors $\vec{x}_\beta$, $\vec{y}_\beta$. We now map the points, represented by vectors $\vec{x}_\beta$, $\vec{y}_\beta$, by a homogprahy, represented by matrix $\mathtt{H}$, to points represented by vectors $\vec{x'}_\beta$, $\vec{y'}_\beta$ such that $\exists\, \lambda_1, \lambda_2 \in \mathbb{R}, \lambda_1, \lambda_2 \neq 0$

$$
\begin{aligned}
\lambda_1\, \vec{x'}_\beta &= \mathtt{H}\, \vec{x}_\beta \\
\lambda_2\, \vec{y'}_\beta &= \mathtt{H}\, \vec{y}_\beta
\end{aligned}
$$

Homogeneous coordinates $\vec{p}_\beta$ of the line passing through points represeted by $\vec{x}_\beta$, $\vec{y}_\beta$ and homogeneous coordinates $\vec{p'}_\beta$ of the line passing through points represeted by $\vec{x'}_\beta$, $\vec{y'}_\beta$ are obtained by solving the linear systems

$$
\begin{aligned}
\vec{p}_\beta^\top\, \vec{x}_\beta = 0 &\qquad \vec{p'}_\beta^\top\, \vec{x'}_\beta = 0 \\
\vec{p}_\beta^\top\, \vec{y}_\beta = 0 &\qquad \vec{p'}_\beta^\top\, \vec{y'}_\beta = 0
\end{aligned}
$$

Substituting into the equations above, we get

$$
\begin{aligned}
\lambda_1\, \vec{p}_\beta^\top\, \mathtt{H}^{-1}\, \vec{x'}_\beta &= 0 \quad \Leftrightarrow \quad \vec{p}_\beta^\top\, \mathtt{H}^{-1}\, \vec{x'}_\beta = 0 \\
\lambda_2\, \vec{p}_\beta^\top\, \mathtt{H}^{-1}\, \vec{y'}_\beta &= 0 \quad \Leftrightarrow \quad \vec{p}_\beta^\top\, \mathtt{H}^{-1}\, \vec{y'}_\beta = 0
\end{aligned}
$$

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We see that $\vec{p'}_\beta$ and $\mathtt{H}^{-\top}\vec{p}_\beta$ are solutions of the same set of homogeneous equations. If $\vec{x}_\beta$, $\vec{y}_\beta$ are independent, then there is $\lambda \in \mathbb{R}$ such that

$$\lambda\,\vec{p'}_\beta = \mathtt{H}^{-\top}\,\vec{p}_\beta$$

since the solution space is one-dimensional.

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