

# Frequency analysis in images

## Fourier pair of a rectangle function

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# Intro

- ◆ auxiliary introductory text
- ◆ excerpt from a longer [lecture](#)<sup>1</sup>

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<sup>1</sup><http://cmp.felk.cvut.cz/cmp/courses/ZS0/Prednasky/lininteg.pdf>

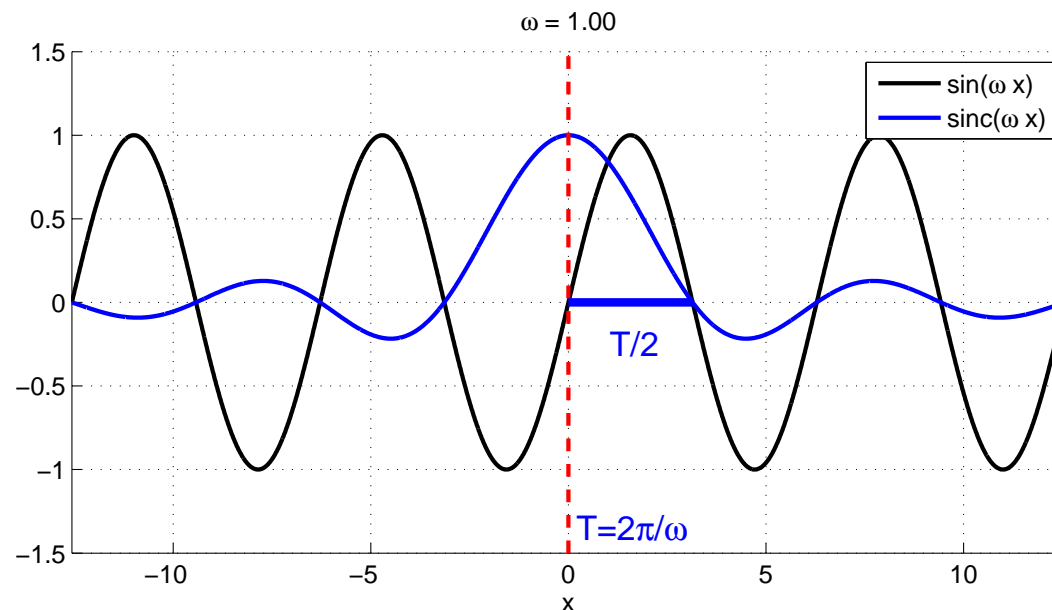
# Fourier pair of rectangle function

$f_A(x) = A$  for  $0 \leq x \leq X$  and  $f_A(x) = 0$  elsewhere.

For the derivation we will need:

$$\sin(\omega x) = \frac{1}{2i} (e^{i\omega x} - e^{-i\omega x})$$

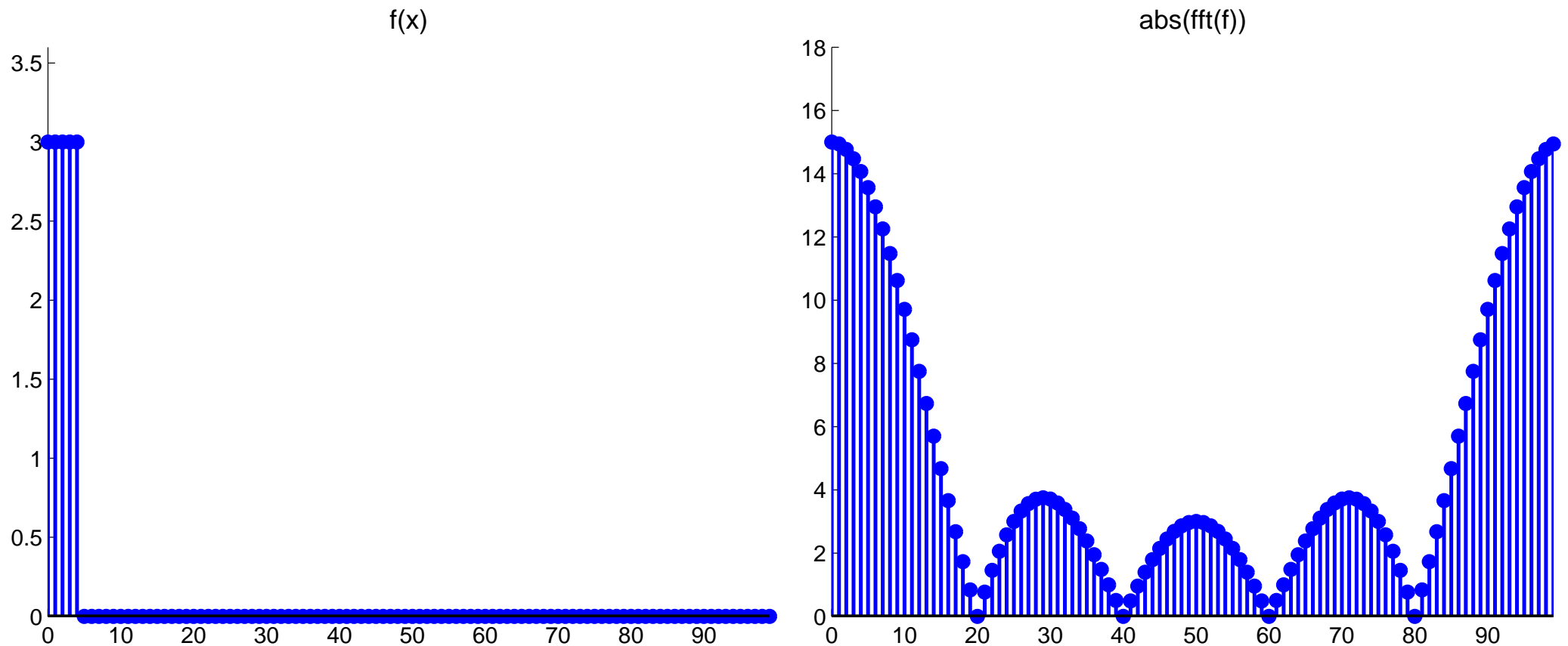
$$\text{sinc}(\omega x) = \frac{\sin(\omega x)}{\omega x}$$



# Fourier pair of rectangle function

$$\begin{aligned}
 F(u) &= \int_{-\infty}^{\infty} f(x) e^{-i2\pi ux} dx &&= \frac{A}{\pi u} e^{-i\pi Xu} \sin \pi Xu \\
 &= A \int_0^X e^{-i2\pi ux} dx &&= AX e^{-i\pi Xu} \frac{\sin \pi Xu}{\pi Xu} \\
 &= A \left[ \frac{e^{-i2\pi ux}}{-i2\pi u} \right]_0^X &&= AX e^{-i\pi Xu} \operatorname{sinc}(\pi Xu) \\
 &= \frac{A}{-i2\pi u} (e^{-i2\pi Xu} - 1) && \text{which implies} \\
 &= \frac{A}{-i2\pi u} (-e^{-i\pi Xu}) (e^{i\pi Xu} - e^{-i\pi Xu}) && |F(u)| = AX |\operatorname{sinc}(\pi Xu)| \\
 &= \frac{A}{\pi u} e^{-i\pi Xu} \left( \frac{e^{i\pi Xu} - e^{-i\pi Xu}}{2i} \right)
 \end{aligned}$$

# Fourier pair of discrete rectangle function



$A = 3, X = 5, M = 100$  makes  $T/2$  of sinc equal to 20.

$$\mathcal{F}\{f_A(x) = A \text{ for } 0 \leq x \leq X\} = AX e^{-i\pi Xu} \text{sinc}\left(\frac{\pi X}{M}u\right)$$

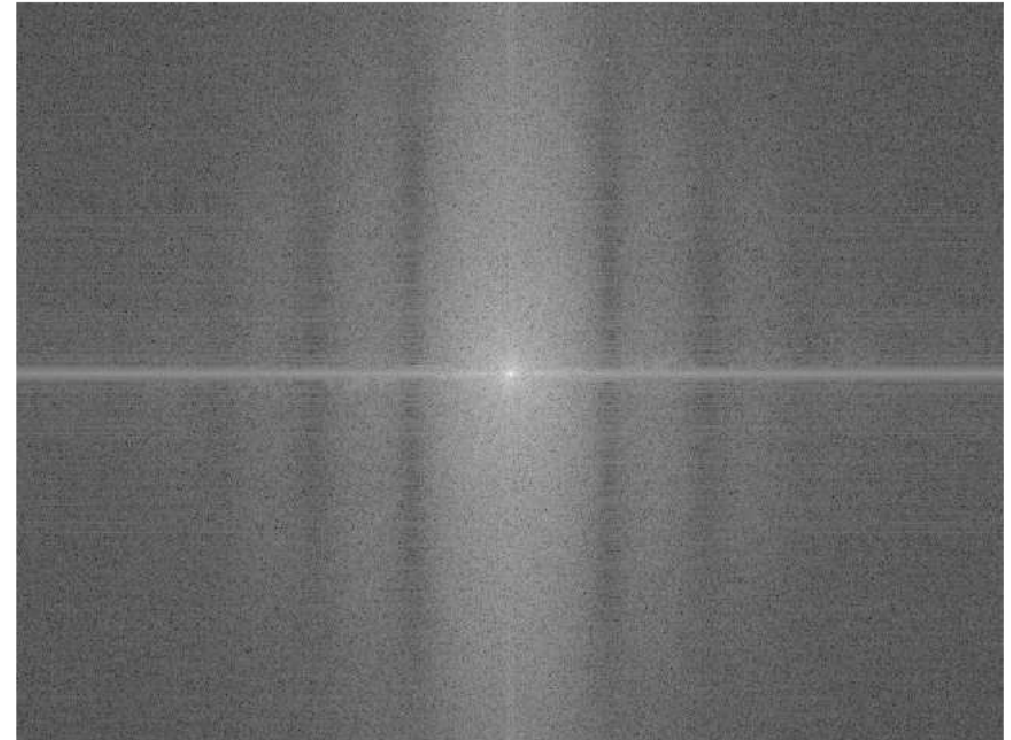
Remember that  $T = 2\pi/\omega$ , hence

$$X = \frac{2M}{T}$$

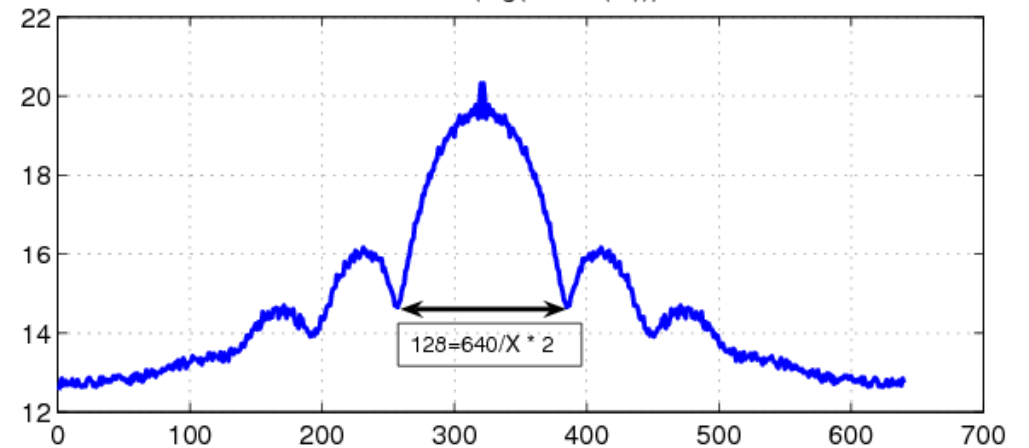
# Use of Rectangle function — Image Restoration



spectrum of blurred image



median(log(1+abs(fft)))



# Use of Rectangle function — Image Restoration



Blurred image.



Restored image.