B1. Linear Camera Calibration from a Known Point Set

$$\underline{m} = P\underline{X}, \qquad P = \begin{bmatrix} q_1^\top & q_{14} \\ q_2^\top & q_{24} \\ q_3^\top & q_{34} \end{bmatrix}$$

Known:

$$\underline{X}_i = \begin{bmatrix} x_i & y_i & z_i & 1 \end{bmatrix}^\top, \quad i = 1, 2, \dots, k,\ k \geq 6$$

$$\underline{m}_i = \alpha_i \begin{bmatrix} u_i & v_i & 1 \end{bmatrix}^\top, \quad \alpha_i \in \mathbb{R},\ \alpha_i \neq 0$$

Then

$$\begin{bmatrix} X_1^\top & 1 & 0^\top & 0 & -u_1 X_1^\top & -u_1 \\ 0^\top & 0 & X_1^\top & 1 & -v_1 X_1^\top & -v_1 \\ \vdots & & & & & \vdots \\ X_k^\top & 1 & 0^\top & 0 & -u_k X_k^\top & -u_k \\ 0^\top & 0 & X_k^\top & 1 & -v_k X_k^\top & -v_k \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_{14} \\ q_2 \\ q_{24} \\ q_3 \\ q_{34} \end{bmatrix} = 0$$

$$\boxed{A\, q = 0}$$

$A \sim 2k \times 12$

$q \sim 12 \times 1$, 11 free parameters

Calibration procedure

Problem:

$$A\,q = 0, \quad \|q\| = 1, \quad \operatorname{rank} A = 11, \quad q \sim 12 \times 1$$

- $\operatorname{rank} A = 12$ due to error: $\tilde{A}\,q = \varepsilon$

$$\|\varepsilon\| \to \min$$

- $\tilde{A} = U\,D\,V^\top$, (SVD)

$$\|\varepsilon\|^2 = q^\top V D^2 V^\top q = q^\top \left(\sum_{i=1}^{12} \sigma_i^2 \underline{v}_i \underline{v}_i^\top \right) q$$

$$\sigma_1^2 \geq \sigma_2^2 \geq \cdots \geq \sigma_{12}^2$$

- if $q = \underline{v}_{12}$ then $\|\varepsilon\| = \sigma_{12}$
- $\boxed{\text{the solution for } q \text{ is the last row in } V\text{, where } UDV^\top = \operatorname{SVD}(\tilde{A}).}$
- rank test: $\operatorname{rank} A = 11 \Leftrightarrow \sigma_{11} \gg \sigma_{12}$

Degenerate Configurations

In degenerate configurations $\operatorname{rank} A < 11$:

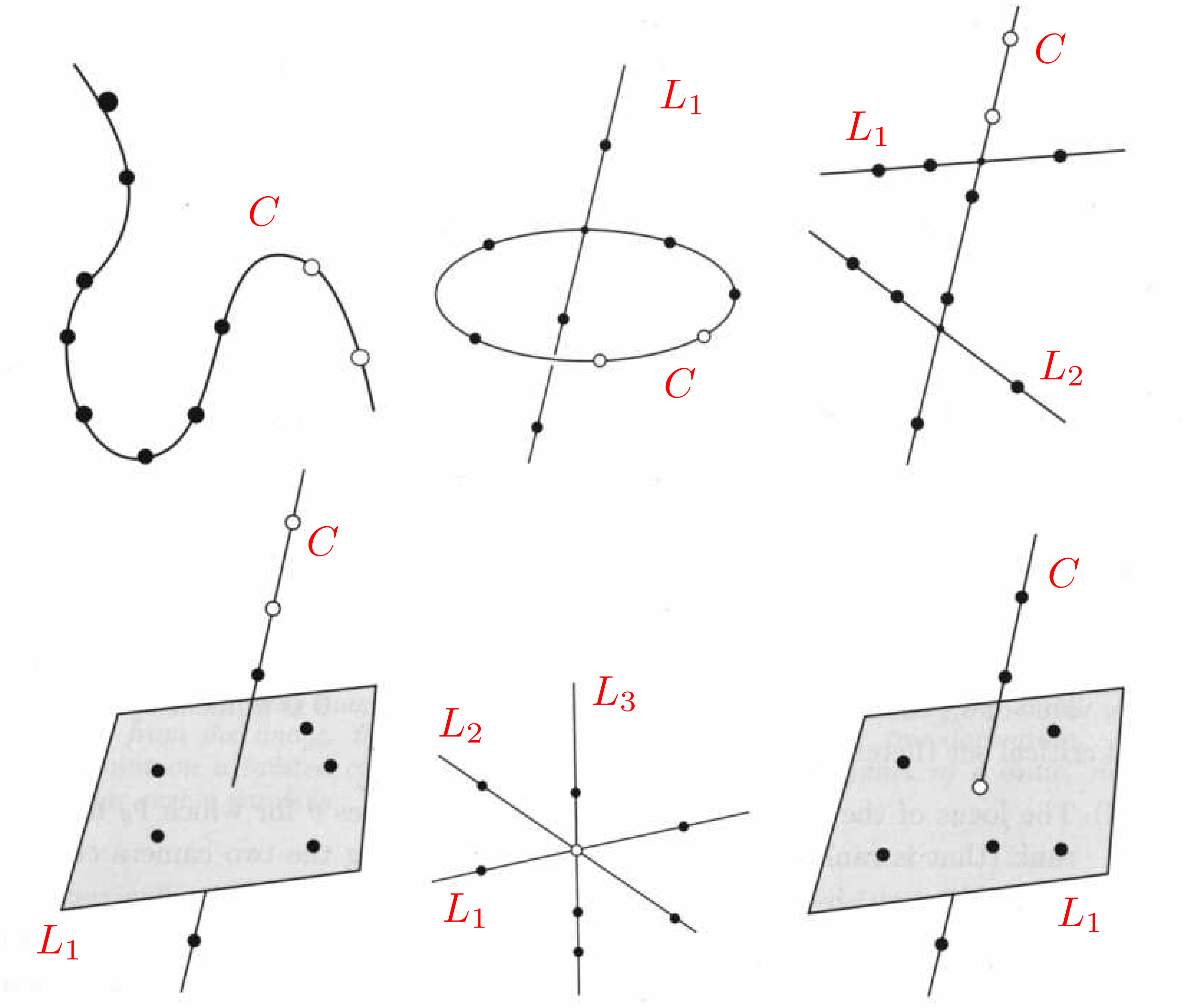


Fig. 21.3. *The different critical configurations for camera resectioning from a single view. The open circles represent centres of projection and the filled circles represent points. Each case consists of an algebraic curve (or single point) C containing the camera centres, plus a set of linear subspaces (lines or planes).*

[H&Z 2000, pp. 520-523]

Degenerate Configurations (cont'd)

Given: points X_i, $i = 1, \ldots, k$ and camera centre C

Critical point sets:

1. Algebraic curve $\mathcal{C}$ containing C and points X_i:
   a. a non-degenerate twisted cubic (degree 3), or
   b. a planar conic (degree 2), or
   c. a straight line (degree 1), or
   d. a point (degree 0).
2. A union of linear subspaces $\bigcup_j L_j$ containing X_i.

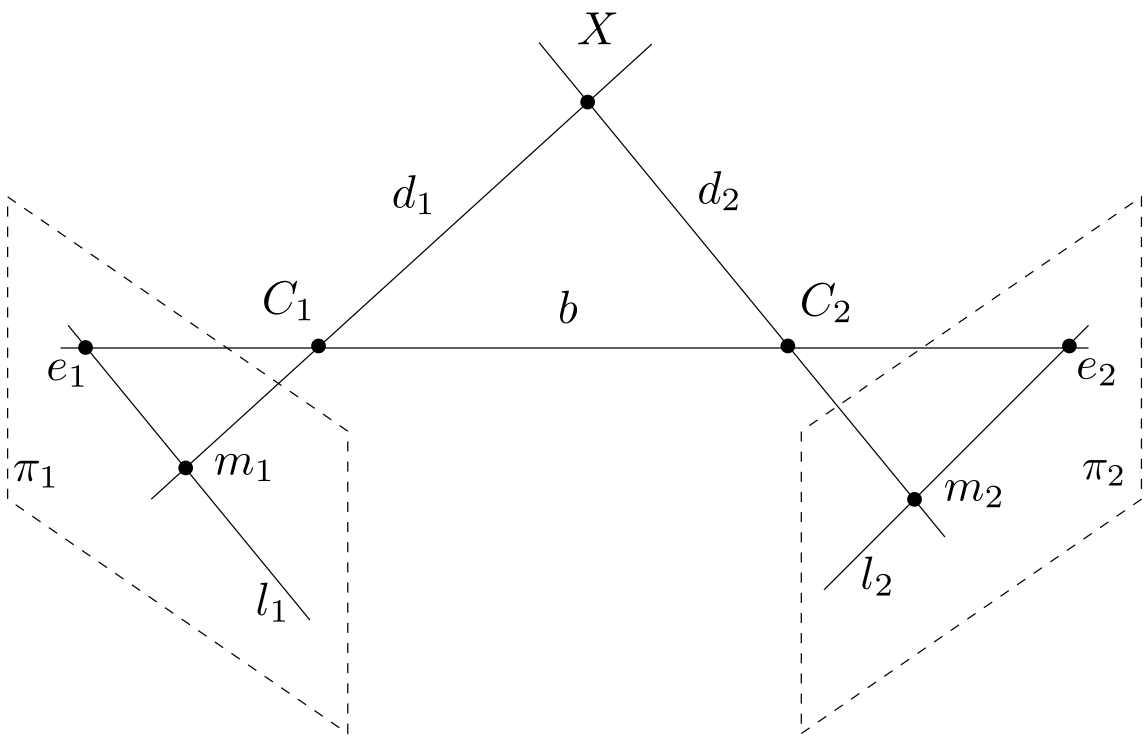
Such that

- $L_j \cap \mathcal{C} \neq \emptyset$
- $\deg(\mathcal{C}) + \sum_j \dim L_j \leq 3$
- if $\deg(\mathcal{C}) > 0$ then $C \notin \bigcap_j L_j \cap \mathcal{C}$

Twisted cubic (12 DOF) $A \sim 4 \times 4$:

$$\underline{X} = A \begin{bmatrix} 1 & \theta & \theta^2 & \theta^3 \end{bmatrix}^\top$$

B3. Epipolar Constraint



- baseline b joins projection centers C_1, C_2
- epipole $e_i \in \pi_i$ is the image of C_j
- $l_i \in \pi_i$ is the image of plane (C_2, X, C_1)
- $l_j \in \pi_j$ is the epipolar line of $m_i \in \pi_i$

Epipolar constraint

$$\begin{vmatrix} d_2 & b & d_1 \end{vmatrix} = 0$$

Epipole and Epipolar Line

$$0 = \begin{vmatrix} d_2 & b & d_1 \end{vmatrix} = (d_2,\, b \times d_1) = \left(Q_2^{-1}\underline{m}_2,\, Q_1^{\top}(\underline{e}_1 \times \underline{m}_1)\right)$$

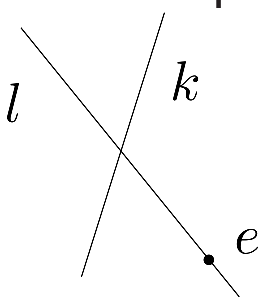
$$= \Big(\underline{m}_2,\, \underbrace{Q_2^{-\top}\overbrace{Q_1^{\top}(\underline{e}_1 \times \underline{m}_1)}^{\text{plane } (C_2, X, C_1)}}_{\text{its image in } \pi_2}\Big) = (\underline{m}_2,\, F\,\underline{m}_1) = (\underline{m}_2,\, \underline{l}_2)$$

$$[e_1]_\times = \begin{bmatrix} 0 & -e_{13} & e_{12} \\ e_{13} & 0 & -e_{11} \\ -e_{12} & e_{11} & 0 \end{bmatrix}$$

- Point-line incidence constraint
- For $\underline{e}_1$ and $\underline{e}_2$ the constraint holds trivially: they are left and right null spaces of F:

$$F\,\underline{e}_1 = 0, \qquad \underline{e}_2^{\top}\,F = 0.$$

- Image point e_i is a common *epipole* of its *epipolar lines* which are the images of the pencil of planes (C_2, X, C_1) for different X.
- Corresponding epipolar lines are

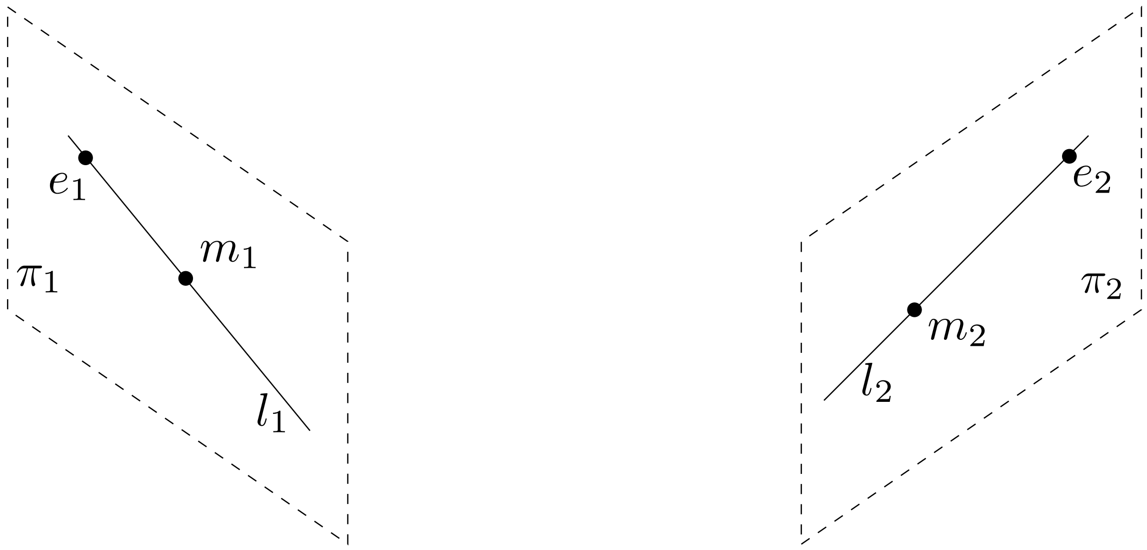


$$\underline{l}_2 = F(\underline{k} \times \underline{l}_1), \qquad \text{choose} \quad \underline{k} = \underline{e}_1$$

$$\underline{l}_2 = F\,(\underline{e}_1 \times \underline{l}_1) \qquad = Q_2^{-\top}Q_1^{\top}\,\underline{l}_1$$

$$\underline{l}_1 = F^{\top}\,(\underline{e}_2 \times \underline{l}_2) \qquad = Q_1^{-\top}Q_2^{\top}\,\underline{l}_2$$

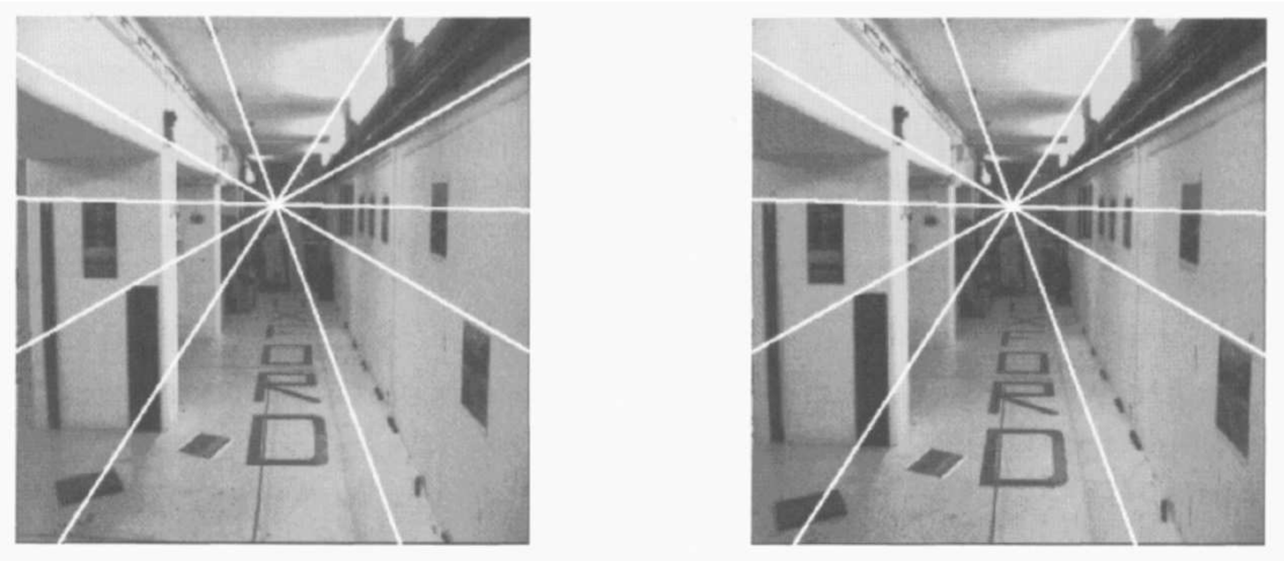
Two-View Relations



$$(\underline{m}_2, F\underline{m}_1) = 0$$

$$\begin{tikzcd}
m_1 \arrow[d, "F"'] & & m_2 \arrow[d, "F^\top"'] \\
l_2 \arrow[rr, bend right, "Q_1^{-\top}Q_2^\top = F^\top[e_2]_\times"'] & & l_1 \arrow[ll, bend right, "Q_2^{-\top}Q_1^\top = F\,[e_1]_\times"']
\end{tikzcd}$$

Epipole is the Image of the Other Camera



How high is the camera above floor?

epi- or **ep-** *pref.* **1. a.** On; upon: *epiphyte.* **b.** Over; above: *epicenter.* **c.** Around: *epicarp.* **d.** Near; close to: *epicalyx.* **e.** Besides: *epiphenomenon.* **f.** After: *epicrisis.* **2.** A chemical substance related to a specified chemical substance: *epicholesterol.* [Gk. < *epi*, upon].
The American Heritage Dictionary, Houghton Mifflin, 1985

B4. Fundamental and Essential Matrices

Two cameras:

$$
\begin{aligned}
P_1 &= K_1\,[E,\ -C_1] \\
P_2 &= K_2\,R\,[E,\ -C_1 - b] \\
b &= \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix}^\top
\end{aligned}
$$

Epipolar constraint:

$$
\boxed{\begin{vmatrix} d_2 & b & d_1 \end{vmatrix} = \underline{m}_2^\top F\,\underline{m}_1 = \underline{m}_2^\top K_2^{-\top}\mathcal{E}K_1^{-1}\,\underline{m}_1 = 0}
$$

F – fundamental matrix, $F = K_2^{-\top}\mathcal{E}K_1^{-1}$

$\mathcal{E}$ – essential matrix, $\mathcal{E} = R\,S(b)$

$$
S(b) = \begin{bmatrix} 0 & -b_3 & b_2 \\ b_3 & 0 & -b_1 \\ -b_2 & b_1 & 0 \end{bmatrix},
$$

$S^\top(b) = -S(b), \quad \operatorname{rank} S(b) = 2 \Leftrightarrow b_i \neq 0$

Observation: $\mathcal{E}$ and F invariant to camera pair pose

Essential Matrix Properties

1. Essential matrix (Longuet–Higgins 1981) captures relative camera position.

$$\mathcal{E} = R\,S(b)$$

2. $\operatorname{rank} \mathcal{E} \le 2$

3. $\mathcal{E} = UWV^\top, \quad W = \|\,b\,\| \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Proof:

1. $\mathcal{E}\mathcal{E}^\top = RS\,(RS)^\top = R(SS^\top)R^\top \sim SS^\top = \ldots = \|b\|^2 - b\,b^\top = \|b\|^2\,A$
2. A is square, $A^2 = A$, $A = A^\top$
3. $\Rightarrow$ A is a symmetric "projection matrix"
4. $\Rightarrow$ A has eigenvalues $\lambda_{1,2} = 1$ and $\lambda_3 = 0$
5. $\sigma_i(\mathcal{E}) = \sqrt{\lambda_i(\mathcal{E}\mathcal{E}^\top)} = \sqrt{\|b\|^2\lambda_i(A)}, \quad i = 1, 2, 3$

B5. Egomotion Estimation

Problem:

Estimate b and R from a set of $k \geq 8$ corresponding image points $\{(m_i, m'_i)\}_{i=1}^{k}$.

Hartley 1992:

Let $\hat{\mathcal{E}}$ be an essential matrix estimate. If $\hat{\mathcal{E}} = UDV^\top$ then

$$R = U \begin{bmatrix} 0 & \alpha & 0 \\ -\alpha & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} V^\top, \quad b = V \begin{bmatrix} 0 \\ 0 \\ \beta \end{bmatrix}$$

$$|\alpha| = 1, \quad \beta \neq 0$$

- 4 solution sets for 4 sign combinations of α, β
- b recovered up to scale β

---

1. Find matrix $\mathcal{E}$ closest to $\hat{\mathcal{E}}$ such that $\mathcal{E}$ is an essential matrix
2. Decompose $\mathcal{E} = RS$

B2. Singular Value Decomposition

If $A \in \mathbb{R}^{m,n}$ then there are orthogonal matrices

$$U = [u_1, \dots, u_m] \in \mathbb{R}^{m,m} \qquad U^\top U = E$$
$$V^\top = [v_1, \dots, v_n] \in \mathbb{R}^{n,n} \qquad V^\top V = E$$

such that

$$U^\top A V = \operatorname{diag}(\sigma_1, \dots, \sigma_p) \in \mathbb{R}^{m,n}, \quad p = \min(m, n)$$

where

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_p \geq 0$$

Proof: [Golub & Van Loan, p. 71]

σ_i – singular values of A,

u_i – left singular vectors, $u_i \in \mathbb{R}^m$

v_j – right singular vectors, $v_j \in \mathbb{R}^n$

$A = UDV^\top$ – singular value decomposition

SVD Properties

r – last non-zero singular value

$p = \min(m, n)$

Then

$$\operatorname{rank} A = r$$

$$A = \sum_{i=1}^{r} \sigma_i \, u_i \, v_i^\top$$

$$\|A\|_F^2 = \sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2 = \sigma_1^2 + \sigma_2^2 + \cdots + \sigma_p^2$$

$$\|A\|_2 = \sup_{x \neq 0} \frac{\|Ax\|_2}{\|x\|_2} = \sigma_1$$

Closest Singular Matrix

Let $A = UDV^\top$ be SVD of $A \in \mathbb{R}^{m,n}$. Let $k < r = \operatorname{rank} A$ and

$$A_k = UD_kV^\top, \quad D_k = \operatorname{diag}(\sigma_1, \ldots, \sigma_k, 0, \ldots, 0)$$

(one sets the last $r - k$ singular values to zero)

Then

$$\min_{B,\,\operatorname{rank}(B)=k} \|A - B\|_2 = \sigma_{k+1}$$

$$\arg\min_{B,\,\operatorname{rank}(B)=k} \|A - B\|_2 = A_k$$

(Proof: [Golub & Van Loan 1989, p. 73])

- A_k of rank k is closest to given A in the sense of the 2-norm.

Decomposition of $\mathcal{E}$

1. Let $\hat{\mathcal{E}}$ be an (inconsistent) essential matrix estimate.

2. Let $\mathrm{SVD}(\hat{\mathcal{E}}) = U \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} V^\top$.

3. Then $\mathcal{E} = \dfrac{\sigma_1 + \sigma_2}{2} U \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} V^\top$

   is an essential matrix, closest to $\hat{\mathcal{E}}$ in 2-norm
   (Hartley 1992)

4. and if $\mathcal{E} = R\,S$ then

$$R = U \begin{bmatrix} 0 & \alpha & 0 \\ -\alpha & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} V^\top = U Y V^\top$$

$$S = V \begin{bmatrix} 0 & -\alpha & 0 \\ \alpha & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} V^\top = V Z V^\top$$

$$b = V \begin{bmatrix} 0 & 0 & \beta \end{bmatrix}^\top, \quad |\alpha| = 1,\ |\beta| = 1$$

5. Four solutions for $\alpha = \pm 1$, $\beta = \pm 1$

8-Point Algorithm

Find F:

$$\underline{m}_i^\top F \, \underline{m}'_i = 0, \quad i = 1, 2, \ldots, k \geq 8,$$

known $\underline{m}_i = \begin{bmatrix} u_i & v_i & 1 \end{bmatrix}^\top, \; \underline{m}'_i = \begin{bmatrix} u'_i & v'_i & 1 \end{bmatrix}^\top$

Solution:

1. $A\,f = 0$

$$A = \begin{bmatrix} u_1 u'_1 & u_1 v'_1 & u_1 & v_1 u'_1 & v_1 v'_1 & v_1 & u'_1 & v'_1 & 1 \\ u_2 u'_2 & u_2 v'_2 & u_2 & v_2 u'_2 & v_2 v'_2 & v_2 & u'_2 & v'_2 & 1 \\ \vdots & & & & & & & & \vdots \\ u_k u'_k & u_k v'_k & u_k & v_k u'_k & v_k v'_k & v_k & u'_k & v'_k & 1 \end{bmatrix}$$

$$A \in \mathbb{R}^{k,9} \quad \operatorname{rank} A = 8$$

$$f = \begin{bmatrix} f_{11} & f_{12} & f_{13} & \ldots & f_{33} \end{bmatrix}^\top$$

$$f \in \mathbb{R}^9$$

2. Hartley decomposition on $\hat{\mathcal{E}} = K_2^\top F K_1$

Problems

- mismatches (. . . in a moment)
- numerical stability
  - data matrix normalization (conditioning)

$$
\begin{aligned}
& A D D^{-1} f = \bar{A}\bar{f} = 0, \\
& f = D\bar{f}, \\
& D = \operatorname{diag}\left(\tfrac{1}{N^2}, \tfrac{1}{N^2}, \tfrac{1}{N}, \tfrac{1}{N^2}, \tfrac{1}{N^2}, \tfrac{1}{N}, \tfrac{1}{N}, 1\right)
\end{aligned}
$$

  - min distance to epipolar line (the golden standard)

$$
\begin{aligned}
& F^* = \arg\min_F \frac{\left(\underline{m}_2^\top F \underline{m}_1\right)^2}{l_{11}^2 + l_{12}^2} + \frac{\left(\underline{m}_1^\top F^\top \underline{m}_2\right)^2}{l_{21}^2 + l_{22}^2} \\
& \underline{l}_1 = F\underline{m}_1 = \begin{bmatrix} l_{11} & l_{12} & l_{13} \end{bmatrix}^\top \\
& \underline{l}_2 = F^\top \underline{m}_2 = \begin{bmatrix} l_{21} & l_{22} & l_{23} \end{bmatrix}^\top
\end{aligned}
$$

  - minimum distance to $\underline{m}^\top F \underline{m}$ [Luong 92]

$$
F^* = \arg\min_F \frac{\left(\underline{m}_2^\top F \underline{m}_1\right)^2}{l_{11}^2 + l_{12}^2 + l_{21}^2 + l_{22}^2}
$$

  - bundle adjustment

[Triggs et al. Proc Vision Alg, LNCS 1883, Springer 1999]

B6. Fundamental Matrix Estimation

Given two sets of points $P_1 = \{m_{1i}\}$ and $P_2 = \{m_{2j}\}$,

find:

1. matching $M : P_1 \to P_2$ and
2. the best estimate for fundamental matrix F

under constraint: for all $m_{2j} = M(m_{1i})$, $m_{1i} \in P_1$

$$\underline{m}_{2j}^{\top} F \, \underline{m}_{1i} = 0$$

Two approaches:

1. At least 50% of matches comes from a single model, the rest comes from different models or are random.

   LMedS: Least median of squared residuals

2. Non-random matches can always be explained by a single model.

   RANSAC

RANSAC(ε)

Given: a set of tentative matches $C \subseteq P_1 \times P_2$

Fishler, Bolles 1981

1. repeat steps 2–5 for $j = 1, 2, \ldots, n$:
2. select s pairs from the candidate set C, $s \geq p = 8$.
3. compute model parameters F_j
   (such that $\operatorname{rank} F_j = 2$).
4. for F_j compute residuals of *all* pairs in C,
   $r_j = [r_1, r_2, \ldots, r_k]$
5. find the *consensus set* S_j

$$S_j = \left\{ (\underline{m}_{1i}, \underline{m}_{2i}) \;\middle|\; |r_i| < \varepsilon,\; i = 1, 2, \ldots, k \right\}$$

6. compute the best unbiased efficient estimate of F
   from the largest S_j.

The number of trials

$$n \geq \frac{\log(1-P)}{\log(1-(1-w)^s)}$$

P – probability that at least one selection is correct

w – the fraction of mismatches in C

s – sample size

$s = 8$

	P	
w	0.8	0.99
0.5	410	1200
0.8	$6.3 \cdot 10^5$	$1.8 \cdot 10^6$
0.9	$1.6 \cdot 10^8$	$4.6 \cdot 10^8$

Least Median of Squares (LMedS)

Rousseeuw 1984

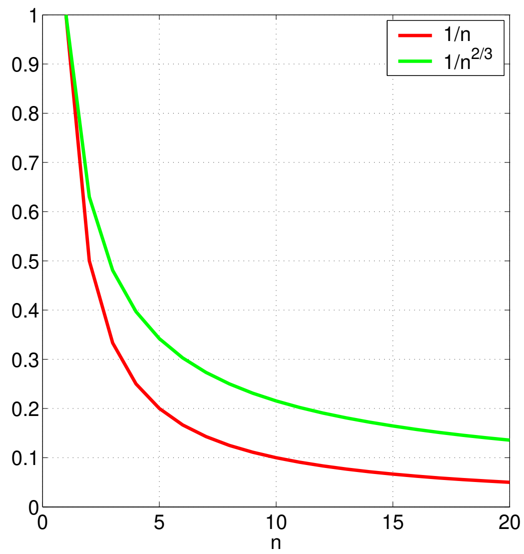
1. repeat steps 2–5 for $j = 1, 2, \ldots, n$:
2. select s pairs from the candidate set C, $s \geq p = 8$.
3. compute model parameters F_j
   (such that $\operatorname{rank} F_j = 2$).
4. for F_j compute residuals of *all* pairs in C,
   $r_j = [r_1, r_2, \ldots, r_k]$.
5. save F_j if it minimizes

$$\operatorname*{med}_j r_j^2.$$

---

low efficiency $\operatorname{var} T \sim \frac{1}{n^{\frac{2}{3}}}$

Least squares: $\operatorname{var} T \sim \frac{1}{n}$



Fundamental Matrix Estimation: Notes

1. 7 points are used

   - fewer random selections required
   - up to 3 real solutions for F from $A\,f = 0$:

$$F = \alpha F_1 + (1 - \alpha) F_2$$

F_1, F_2 span the space of all solutions

$$\det F = \det(\alpha F_1 + (1 - \alpha) F_2) = 0 \qquad \text{cubic in } \alpha$$

see [Hartley & Zisserman 2002, p.264]

2. F with the largest number of inliers is chosen of the three solutions

$s = 8$

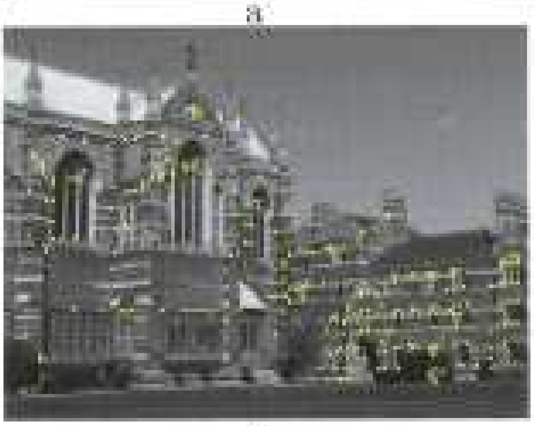
	P	
w	0.8	0.99
0.5	410	1200
0.8	$6.3 \cdot 10^5$	$1.8 \cdot 10^6$
0.9	$1.6 \cdot 10^8$	$4.6 \cdot 10^8$

$s = 7$

	P	
w	0.8	0.99
0.5	205	590
0.8	$1.3 \cdot 10^5$	$3.5 \cdot 10^5$
0.9	$1.6 \cdot 10^7$	$4.6 \cdot 10^7$

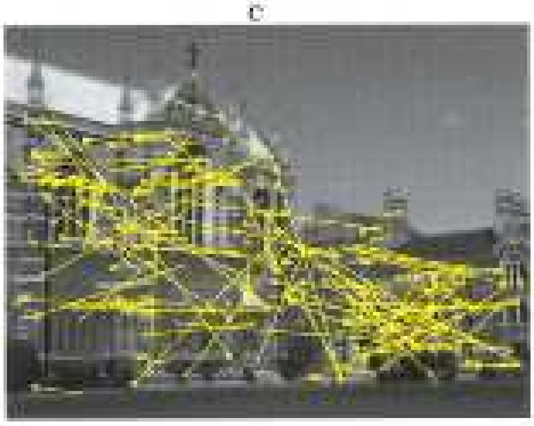
a

b

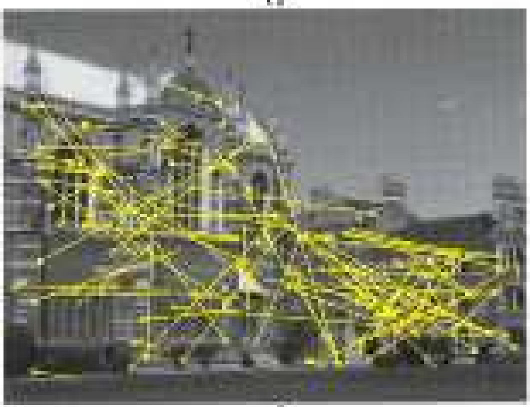


c

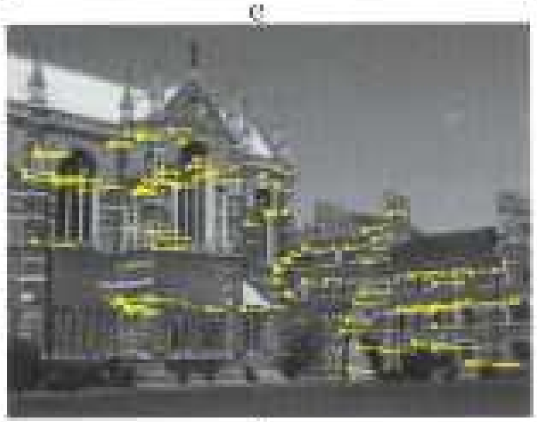
d



e



f



g

h

B7. The Correspondence Problem

Idea: Match similar image neighborhoods consistent with the epipolar geometry model.

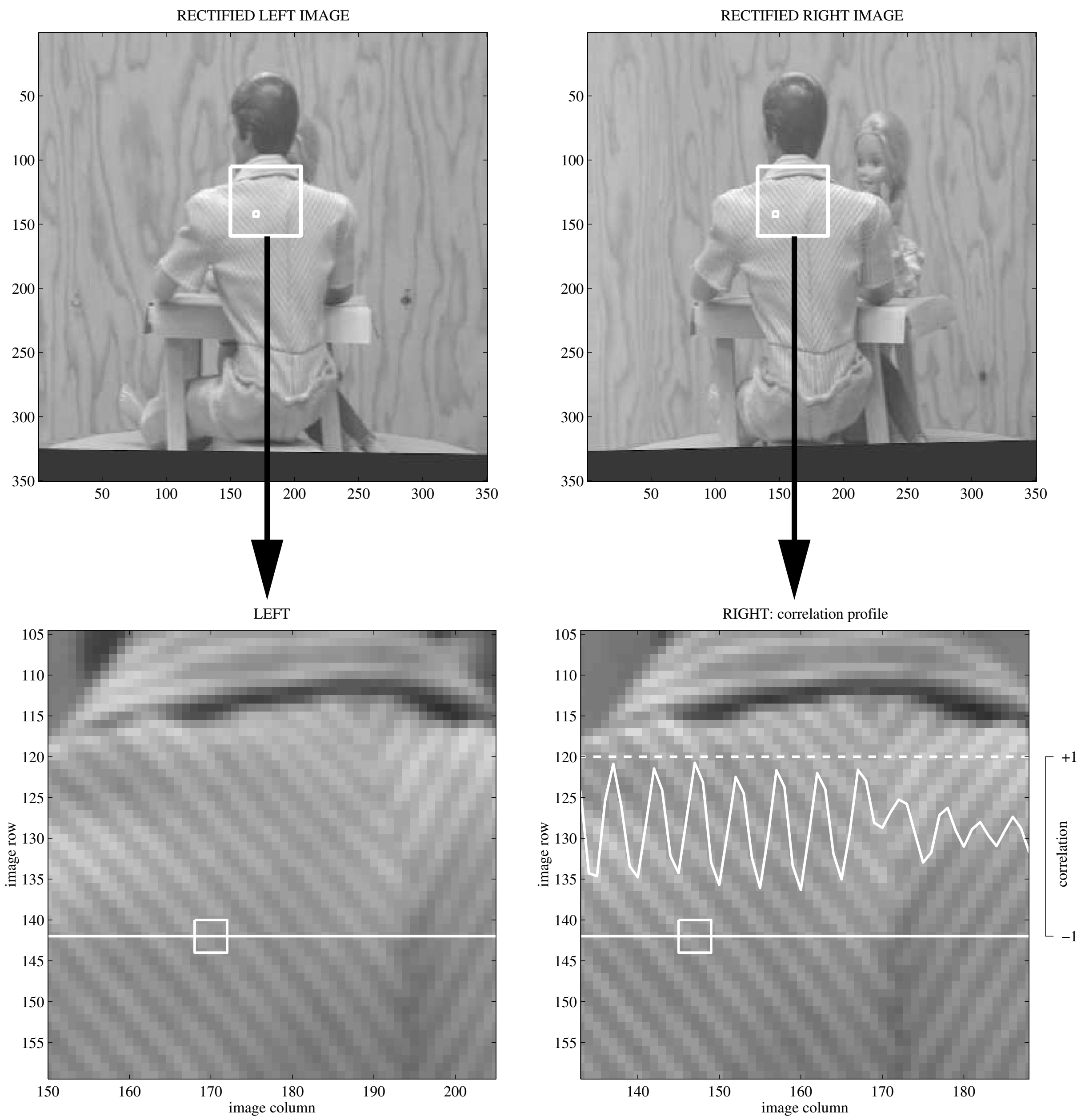
New problems:

1. **Nonexistent matching point**
   - occlusion,
   - occlusion boundary,
   - undetected matching point.
2. **Ambiguous image of a point**
   - repetitive texture,
   - transparent/semitransparent surfaces,
   - low SNR.
3. **Dissimilar images of the same space point**
   - non-Lambertian surfaces,
   - non-surfaces.
4. **Relative perspective distortion**

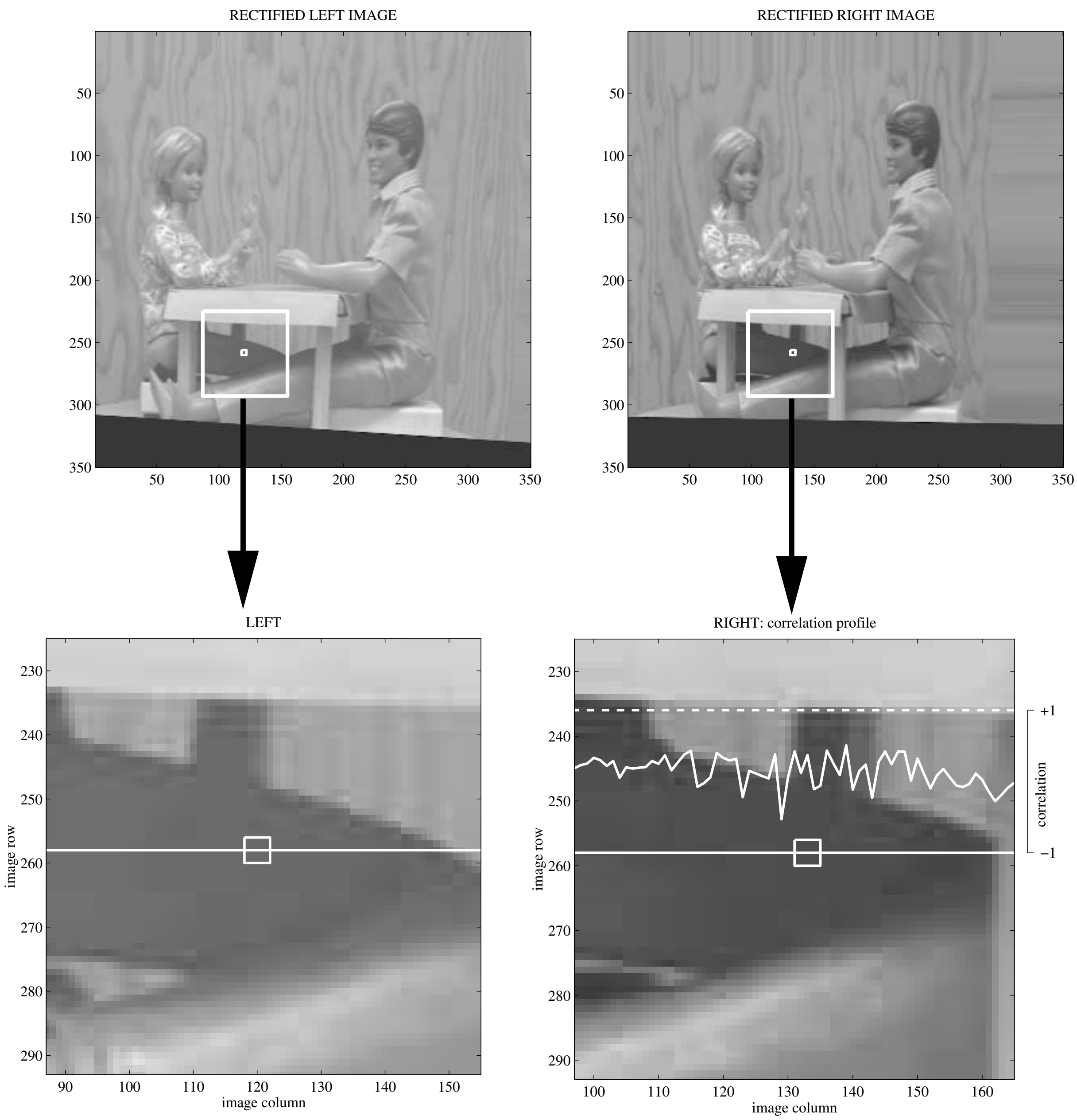
Finite Field of View and Self-Occlusion

Transparent Objects and Non-Surfaces

Ambiguity: Repetitive patterns

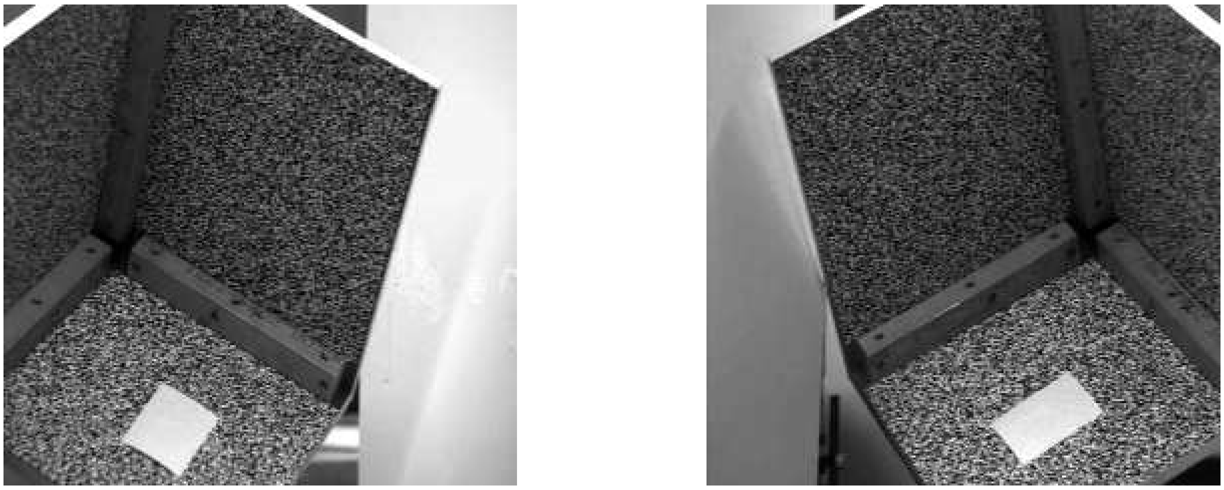


Uncertainty: Low SNR

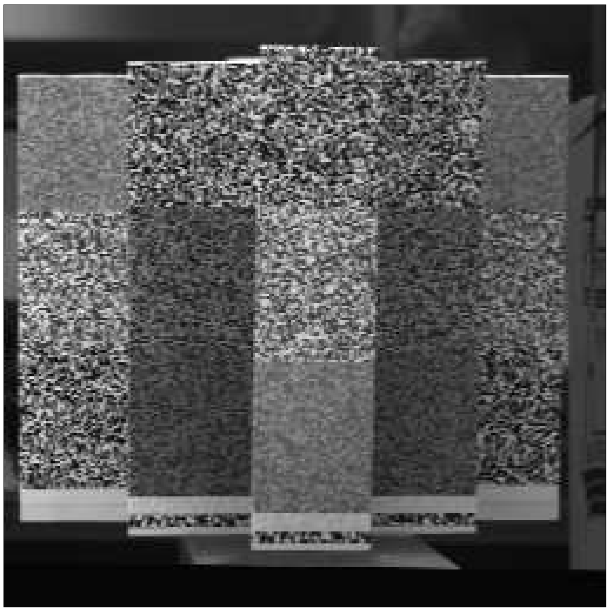


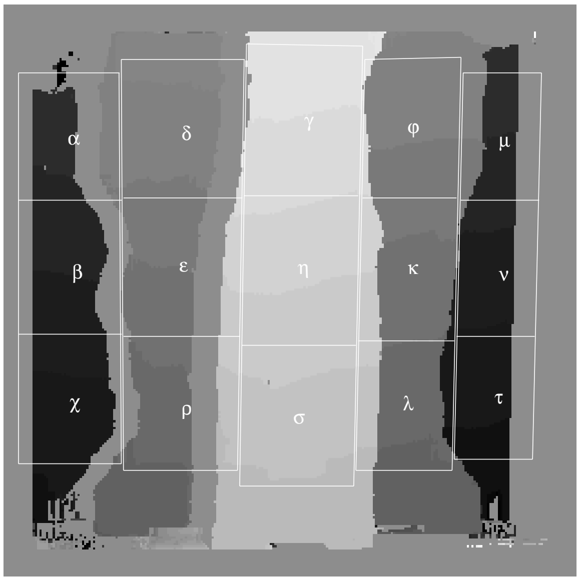
Specularities

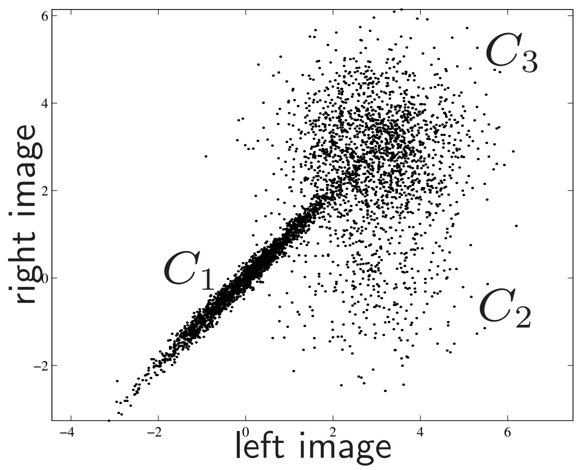
Relative Image Distortion



Occluding Boundary







matching window 25×25, normalized cross-correlation

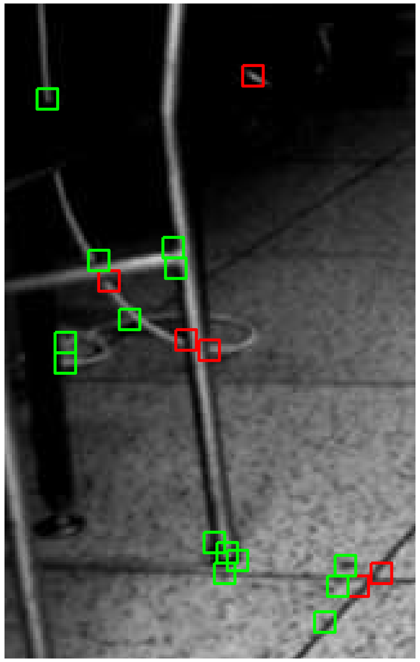
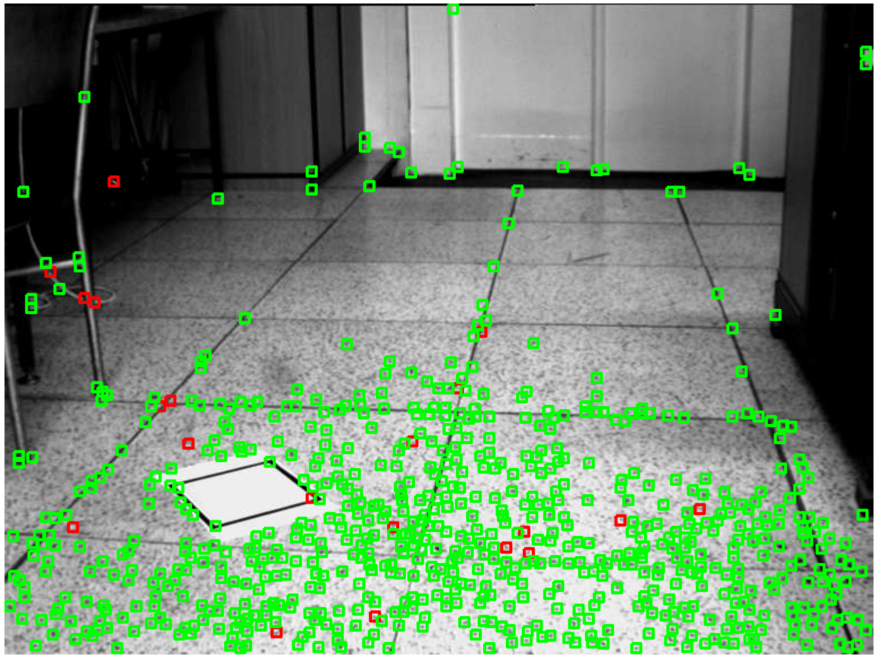
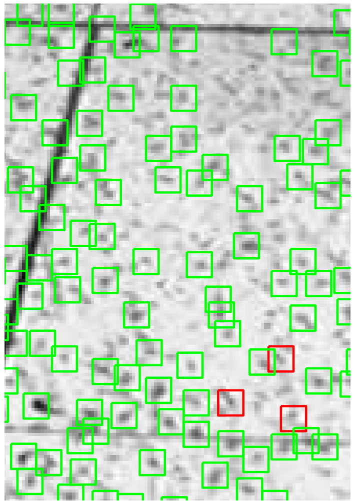
Matching Constraints

1. Matching points must be consistent with a *single* model $\underline{m}_2^\top F \underline{m}_1 = 0$.
2. The set of corresponding points must be a graph-theoretic *assignment* (matching).

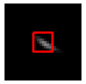
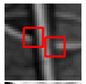
Additional heuristics:

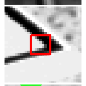
- Similarity (matched are similar-neighborhood points)
- Distinctiveness (matched are sets of mutually distinct pts.)
  1. localizability (local maxima of localization precision)
     Interest Operator [Moravec 1974, Harris 1988]
  2. uniqueness (mutual independence)
- Stability (viewpoint invariance)
  1. structural
     (T-corners: unstable, Y-corners: stable)
  2. radiometric
     (non-Lambertian surfaces: unstable, Helmholtz reciprocity)
- Consistency of close matches (too strong)

Interesting Points

Feature point:

  localized, distinctive, unstable

 localized, distinctive, stable

 localized, ambiguous, stable

B8. Harris Interest Operator

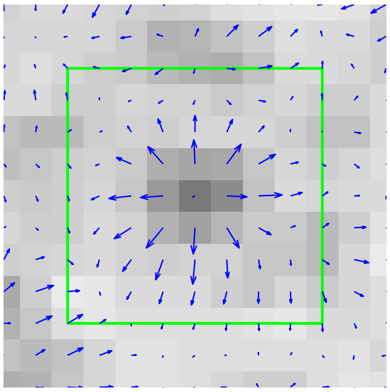
Criterion: localization precision

Local image autocorrelation function

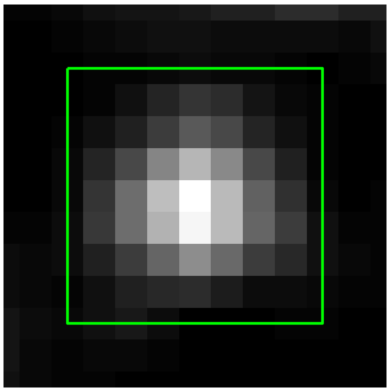
$$
\begin{aligned}
f(u,v) &\approx f(u_0,v_0) + f_u\,du + f_v\,dv \\
E[f](u_0,v_0) &\approx f(u_0,v_0) \\
A[f](u_0,v_0) &\approx (d\mathbf{u}, \nabla f\,\nabla f^\top\,d\mathbf{u})
\end{aligned}
$$

Mean autocorrelation function curvature

$$\bar{H}(u_0,v_0) = E(\nabla f\,\nabla f^\top)$$



gradient field



response L

Harris 1988:

$$
\begin{aligned}
L &= \det \bar{H} - k \operatorname{tr}^2 \bar{H} \\
&= \lambda_1 \lambda_2 (1 - 2k) - k\,(\lambda_1^2 + \lambda_2^2) \\
&\stackrel{\text{def}}{=} \lambda^2 \big((1-2k)\,\varepsilon - k\,(\varepsilon^2 + 1)\big)
\end{aligned}
$$

increasing for $k < 0.25$, usually $k \sim 0.04$

extremal for small ($\lambda \gg 0$) circular ($\varepsilon = 1$) features
Note: circular features are rarely structurally unstable

Haralick 1993:

1. mean localization error

$$
L = \frac{2}{\operatorname{tr}\Sigma} = \frac{2}{\operatorname{tr}\bar{H}^{-1}} = \frac{2\lambda_1\lambda_2}{\lambda_1 + \lambda_2} = 2\lambda\,\frac{\varepsilon}{1+\varepsilon}
$$

2. form factor

$$
q = 1 - \left(\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}\right)^2 = \frac{4\varepsilon}{(1+\varepsilon)^2} \in [0, 1]
$$

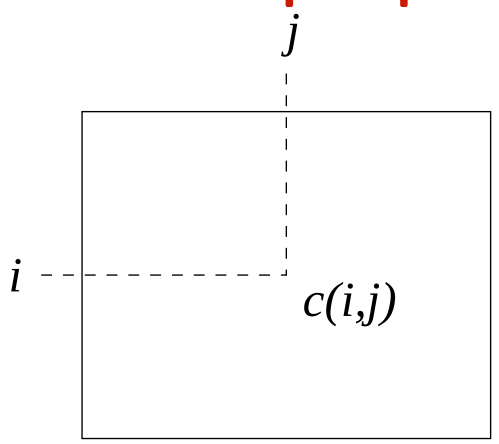
$q = 1$: circular feature, $q = 0$: linear feature

B9. A Sparse Matching Algorithm

1. **Feature point detection** (localizability, stability)

   a set of well-localized points

2. **Candidate pair pre-selection** (uniqueness, similarity)



A pair (i, j) is pre-selected if it has largest correlation in row i and column j with margin ϵ.

3. **Matching via consistent subset search**

$$\underline{m}_2^\top F \underline{m}_1 = 0, \quad \operatorname{rank} F = 2$$

LMedS – subset minimizing $\frac{1}{n} \operatorname*{med}_i r_i$

RANSAC – the largest consensus set

B10. 3D Structure Reconstruction

Problem: given a set of pairwise correspondences $m_{1i} \leftrightarrow m_{2i}$, reconstruct 3D positions X_i of the corresponding points up to similarity in 3D.

1. Estimate fundamental matrix F.
2. Construct a canonic camera pair

$$P_1 = \begin{bmatrix} E, & 0 \end{bmatrix},$$
$$P_2 = \begin{bmatrix} [\underline{e}_2]_\times F + \underline{e}_2 \, \underline{v}^\top, & \lambda \, \underline{e}_2 \end{bmatrix},$$

   where $\underline{v}$ is any 3-vector, $\lambda \neq 0$ such that $[\underline{e}_2]_\times F + \underline{e}_2 \, \underline{v}^\top$ is nonsingular, e.g. $\underline{v} \simeq \underline{e}_1$.
   ([H&Z 2000, p.238])
3. Reconstruct points by triangulation

$$\underline{m}_{1i} = P_1 \, \underline{X}_i,$$
$$\underline{m}_{2i} = P_2 \, \underline{X}_i,$$

   where P_1, P_2, m_{1i}, m_{2i} are known, X_i is unknown, problem is linear in X_i, can be solved by SVD.
4. Upgrade to metric reconstruction (comes next).

Metric Upgrade

Obs: projective ambiguity of the reconstruction

$$\underline{m}_i = P_i\,\underline{X} = \underbrace{P_i\,H}_{P_i'}\;\underbrace{H^{-1}\underline{X}}_{\underline{X}'} = P_i'\,\underline{X}'$$

for any number of cameras $i = 1, 2, \ldots, n$.

Special case: known center of projection, square pixel. Then

$$K_i = \begin{bmatrix} f_i & 0 & 0 \\ 0 & f_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$\omega_i^* = K_i K_i^\top = \begin{bmatrix} f_i^2 & 0 & 0 \\ 0 & f_i^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

We know that ([H & Z 2002, p. 191])

$$\lambda\,\omega_i^* = P_i\,Q_\infty^*\,P_i^\top, \quad \lambda \neq 0$$

is the image of the absolute dual quadric.

(its projection, its rim)

$$\lambda\,\omega_i^* = P_i\,Q_\infty^*\,P_i^\top \stackrel{\text{def}}{=} M^i = [m_{jk}^i]$$

$\lambda^{-1}Q_\infty^*$ unknown $\sim$ need 9 constraints (symmetric mtx)

under our choice of

$$\omega^* = \begin{bmatrix} f_i^2 & 0 & 0 \\ 0 & f_i^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

we have

$$m_{12}^i = 0, \quad m_{13}^i = 0, \quad m_{23}^i = 0, \quad m_{11}^i - m_{22}^i = 0.$$

- 4 linear constraints on Q_∞^* per camera i
- $n = 2$ cams: additional constraint: $\det Q_\infty^* = 0$
- use method similar to the 7-point algorithm
- $n \geq 3$ cameras give enough linear constraints
- find closest singular matrix
- decompose $Q_\infty^* = H I H^\top$, where $I = \mathrm{diag}(1,1,1,0)$ using SVD (canonical form of Q_∞^*)
- provides update collineation H:

$$P_i \mapsto P_i H, \qquad \underline{X} \mapsto H^{-1}\underline{X}$$

- gives reconstruction $\underline{X}$ up to similarity H_s. (since $H_s\,Q_\infty^* H_s \simeq Q_\infty^*$)