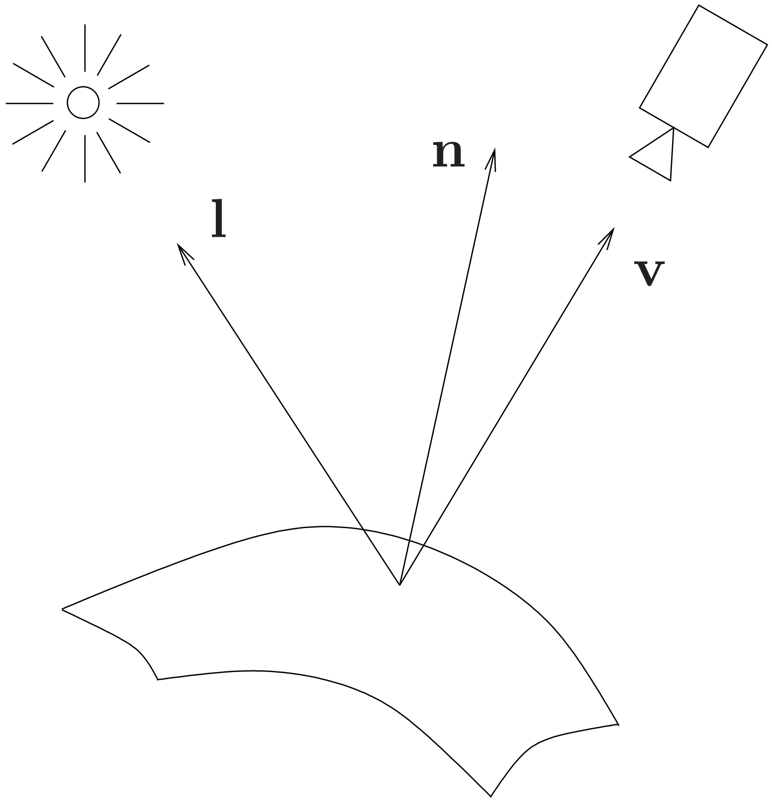
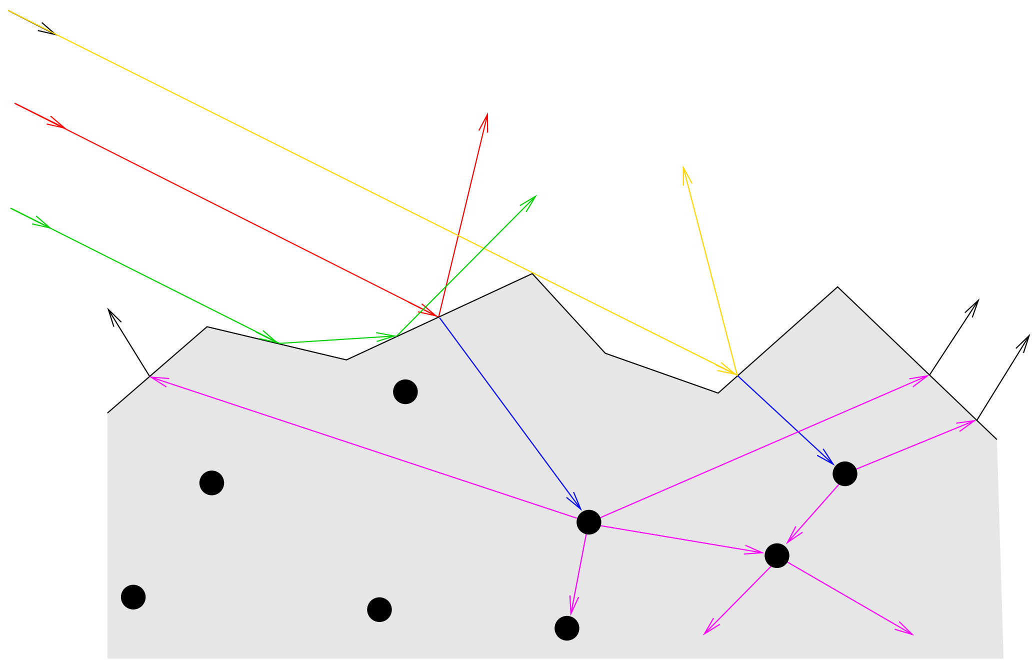
D. Shape from Reflectance



1. image formation
2. reflectance models
3. photometric stereo
4. shape from Lambertian shading
5. local shading analysis

D1. Surface Reflectance Models

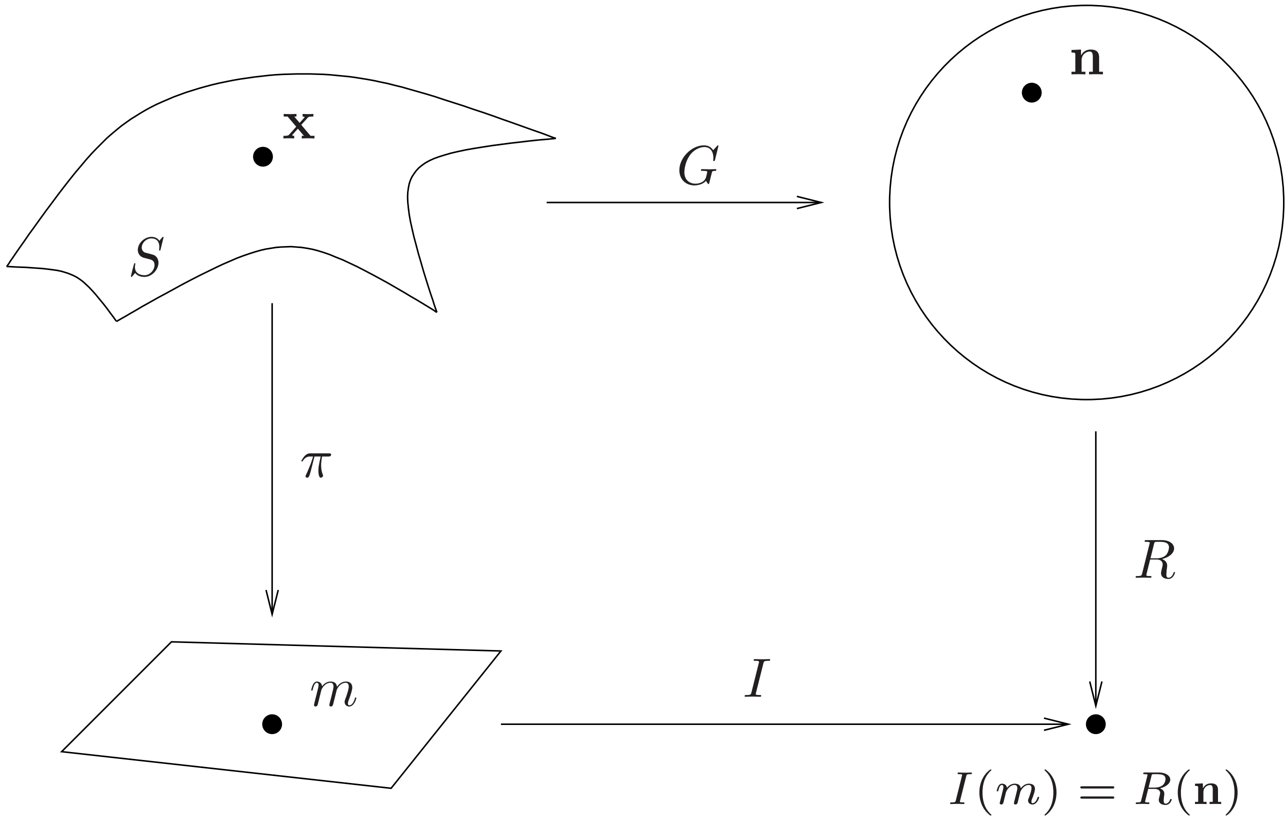
Basic reflectance mechanisms



- reflection on (rough) optical boundary
- masking and shadowing
- interreflection
- refraction into the body
- subsurface scatering
- refraction into the air

D2. Image Irradiance Equation

$$I \circ \pi(S) = R(\mathbf{l}, \mathbf{v}, \rho) \circ G(S)$$



S – surface $\quad (u, v) \in \mathbb{R}^2 \mapsto \mathbf{x} \in \mathbb{R}^3$

π – camera projection $\quad \mathbf{x} \in \mathbb{R}^3 \mapsto m \in \mathbb{R}^2$

I – image irradiance $\quad m \in \mathbb{R}^2 \mapsto \mathbb{R}$

R – surface reflectance function $\quad \mathbf{n} \in \mathbb{R}^2 \mapsto \mathbb{R}$

G – Gaussian map $\quad \mathbf{x} \in \mathbb{R}^3 \mapsto \mathbf{n} \in \mathbb{R}^2$

ρ – surface reflectivity (albedo)

Simple Reflectance Models

No optical boundary

Lambertian reflectance [Lambert 1760]

$$R = \rho \cos\theta_i$$

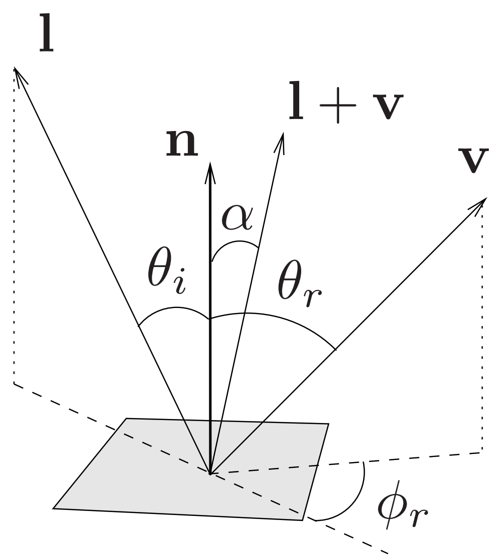
Rough-surface reflection [Torrance-Sparrow 1967]

$$R = \rho\, M(\theta_i, \theta_r, \phi_r)\, C(\alpha, \sigma_\alpha)$$

Subsurface scatering, smooth boundary [Wolff 1994]

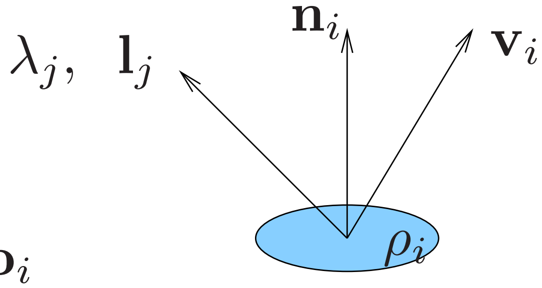
$$R = \rho\,(1 - F(\theta_i, n)) \left(1 - F\left(\sin^{-1}\left(\frac{\sin\theta_r}{n}\right), \frac{1}{n}\right)\right) \cos\theta_i$$

Subsurface scatering, rough boundary [Nayar 1995]



D3. Photometric Stereo

Lambertian model



$$I = (\lambda_j\, \mathbf{l}_j)^\top \rho_i\, \mathbf{n}_i = \mathbf{s}_j^\top\, \mathbf{b}_i$$

3 independent lights and n normals

$$\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3] \in \mathbb{R}^{3,3}$$

$$\mathbf{B} = [\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_n] \in \mathbb{R}^{3,n}$$

Problem solution

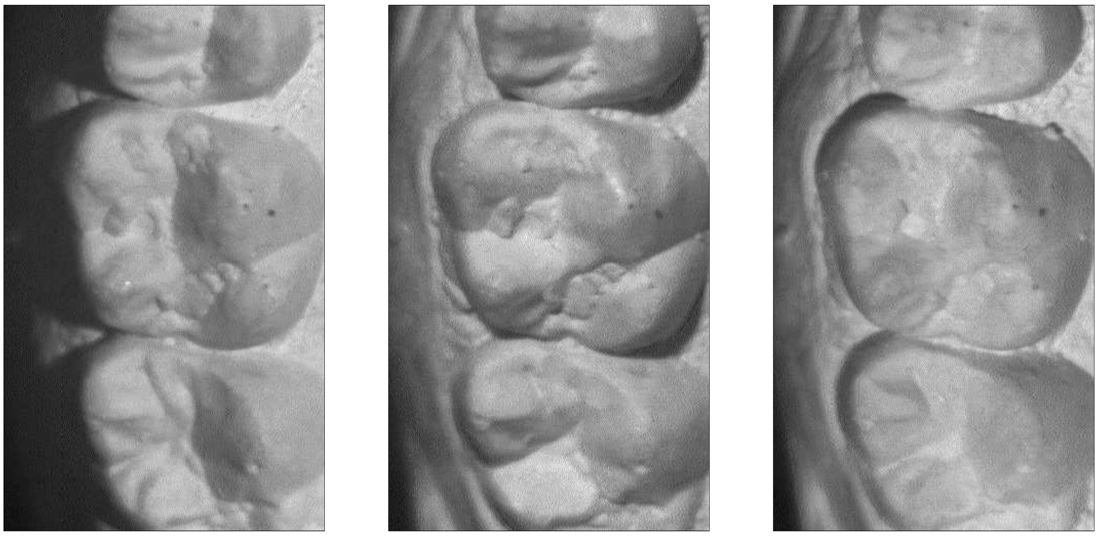
$$\mathbf{I} = \mathbf{S}^\top \mathbf{B} \quad \Rightarrow \quad \mathbf{B} = \mathbf{S}^{-\top} \mathbf{I}$$

$$\rho_i = \|\mathbf{b}_i\|, \qquad \mathbf{n}_i = \frac{1}{\rho_i}\mathbf{b}_i$$

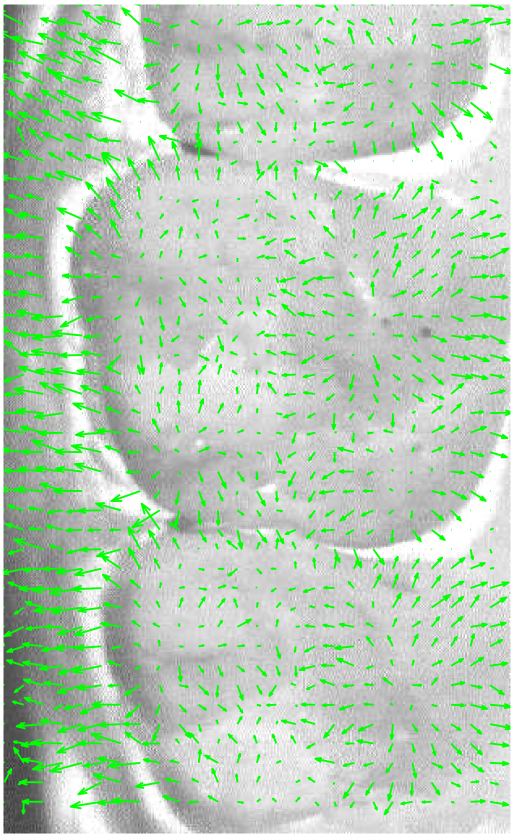
$\mathbf{n}_i$ – needle map

ρ_i – albedo map

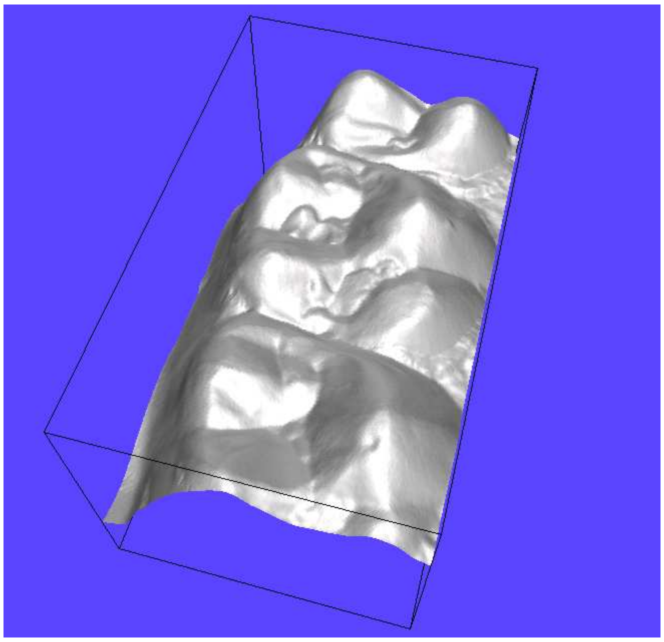
Photometric Stereo: Plaster Cast Example



input images (known lights)



needle & albedo maps



integrated surface

Uncalibrated Photometric Stereo

Factorization

[Hayakawa94]

$$\operatorname{rank}\mathbf{I} = \operatorname{rank}\mathbf{S}^\top\mathbf{B} = 3$$

decomposition via SVD $\rightarrow$ pseudonormals, pseudolights

Ambiguity

[Koenderink94]

$$\mathbf{I} = \mathbf{S}^\top\mathbf{B} = \underbrace{\mathbf{S}^\top\mathbf{A}^{-1}}_{\bar{\mathbf{S}}^\top}\ \underbrace{\mathbf{A}\mathbf{B}}_{\bar{\mathbf{B}}}, \qquad \mathbf{A} \in GL(3)$$

information	ambiguity	
3+ normals known	$\lambda\mathbf{E}$	
equal light intensity	$\lambda\mathbf{R}$	[Hayakawa94]
uniform albedo	$\lambda\mathbf{R}$	[Drew92]
integrable normals	$\begin{bmatrix} \lambda & 0 & -\mu \\ 0 & \lambda & -\nu \\ 0 & 0 & 1 \end{bmatrix}$	bas-relief ambiguity [Yuille99, Fan97] [Belhumeur99]

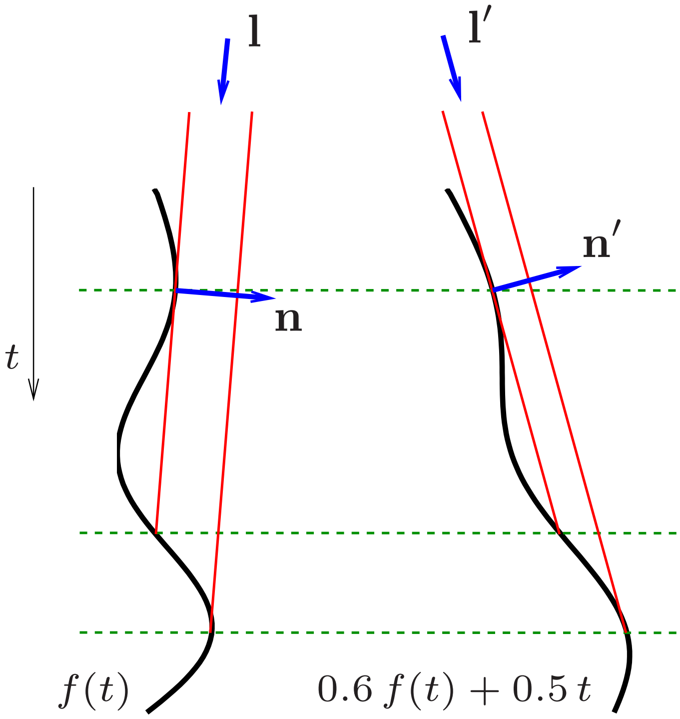
Bas Relief Ambiguity

Reprinted from [Belhumeur, Kriegman, Yuille, CVPR–97]

Shadow boundaries of surface $\mathcal{S}$ illuminated by light $\mathbf{l}$ are identical to those of surface $\mathcal{S}'$ transformed by BR G and illuminated by transformed light $\mathbf{l}'$.

(mild assumptions)

[Belhumeur97]



D4. Shape from Lambertian Shading

Assumptions:

1. Lambertian surface reflectance
2. uniform albedo
3. orthographic projection
4. ideal radiometric camera
5. illuminant at infinity in direction $\mathbf{l}$

Image irradiance equation

$$I(u,v) = \rho \, \frac{\lambda_1 \, p + \lambda_2 \, q + \lambda_3}{\sqrt{1 + p^2 + q^2}}$$

where

$$\begin{aligned}
\mathbf{x} &= [u,\ v,\ z(u,v)]^\top \\
p &= z_u \\
q &= z_v \\
\mathbf{n} &\sim [-p,\ -q,\ 1]^\top \\
\mathbf{l} &= [\lambda_1,\ \lambda_2,\ \lambda_3]^\top, \quad \|\mathbf{l}\| = 1
\end{aligned}$$

Characteristic strip solution

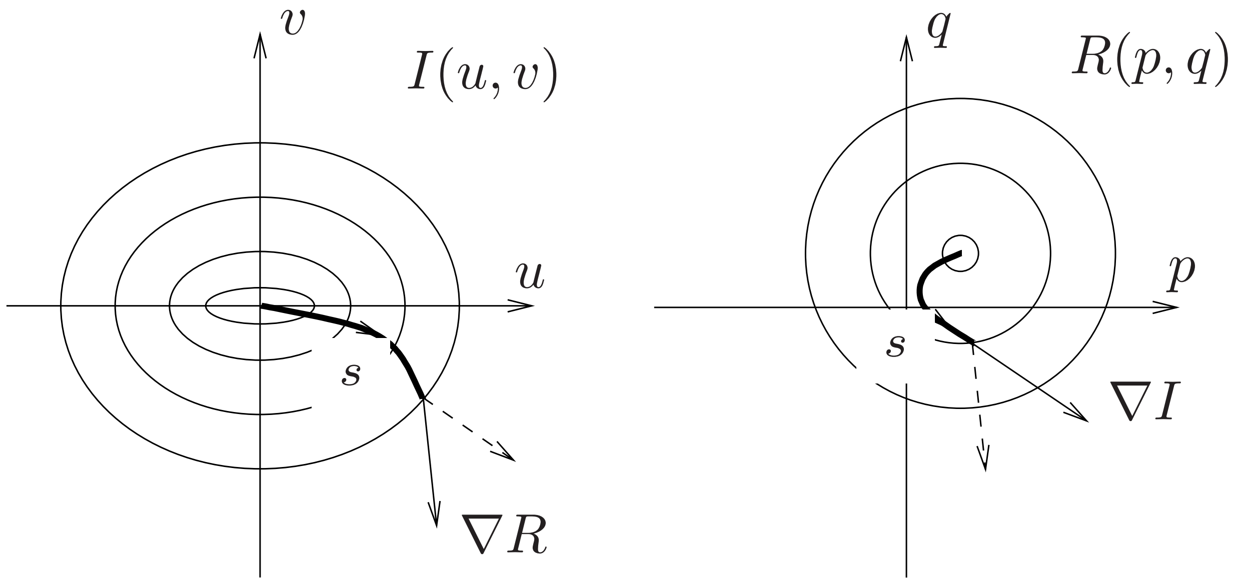
[Horn 1975]

$I(u,v) = R(p,q)$

Solution extension in a direction $(\dot{u}, \dot{v})$:

$$\frac{du}{ds} = R_p, \quad \frac{dv}{ds} = R_q, \quad \frac{dz}{ds} = p\,R_p + q\,R_q$$

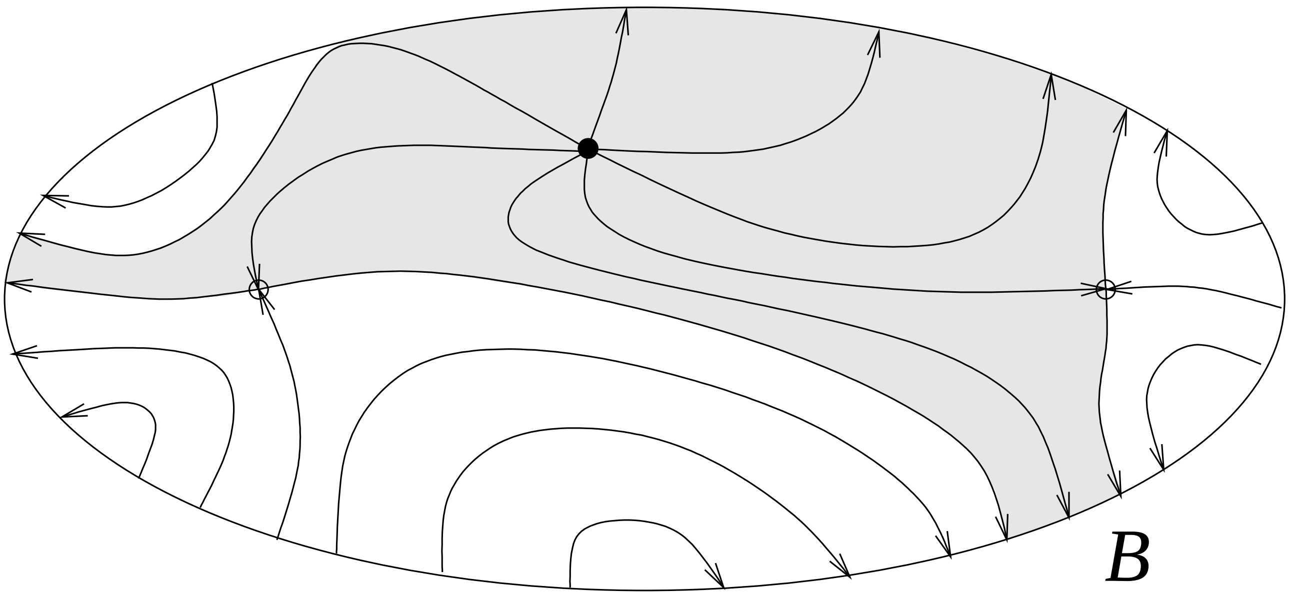
$$\frac{dp}{ds} = I_u, \quad \frac{dq}{ds} = I_v$$



- Flow of characteristic strips in the image plane

Region of Influence

- boundary conditions
- initial conditions at extremal points do not suffice



J. Oliensis. Uniqueness in Shape from Shading, *IJCV* 6:2, 1991.

Regularization: Variational Formulation

Error functional:

$$E = \underbrace{(\lambda_1 E_R + \lambda_2 E_O)}_{\text{data term}} + \underbrace{(\lambda_3 E_M + \lambda_4 E_P)}_{\text{smoothness term}}$$

Solution: $z(u,v)$ minimizing E

$$E_R = \frac{1}{2} \iint \Big(I(u,v) - R(z_u, z_v) \Big)^2 \, du \, dv$$

$$E_O = \frac{1}{2} \iint \big(z_u - p(u,v) \big)^2 + \big(z_v - q(u,v) \big)^2 \, du \, dv$$

$$E_M = \frac{1}{2} \iint z_u^2 + z_v^2 \, du \, dv$$

$$E_P = \frac{1}{2} \iint z_{uu}^2 + 2 z_{uv}^2 + z_{vv}^2 \, du \, dv$$

$$z_u = z_u(u,v), \quad z_v = z_v(u,v), \quad \text{etc.}$$

Finite element solution:

$$\mathbf{z} = \mathbf{A}^{-1} \mathbf{b}, \quad \mathbf{A} \sim [\, N^2 \times N^2;\ \text{25–diagonal} \,]$$

D5. Local Shading Analysis

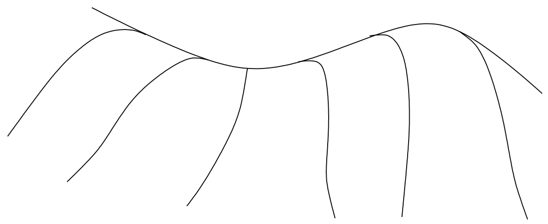
Image is a field of isophotes

$$I(u,v) = const.$$

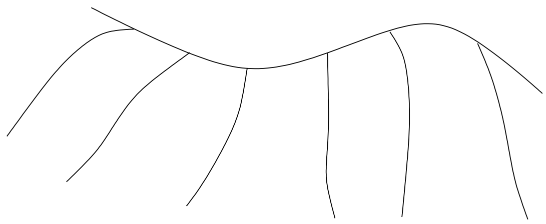
Topics

- Occluding contour structure
- Self-shadow contour
- Gaussian image structure, isophotes in GI
- Isophotes are parallel to zero-curvature principal direction
- Isophote orientation does not change along parabolic lines
- How and where singular image points are born
- Classes of singular image points

Occluding Contour Structure

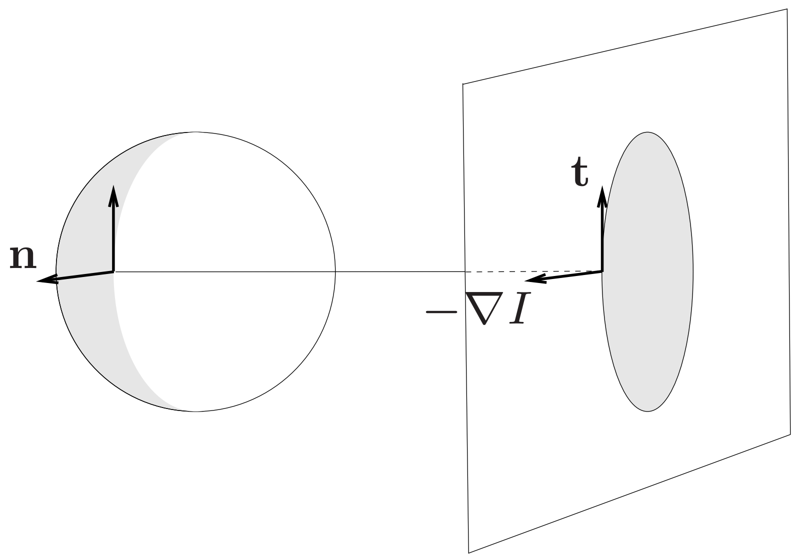


smooth self-occlusion boundary



not smooth boundary

Surface orientation



Surface quality

$$K = \kappa_s \, \kappa_t \quad \rightarrow \quad \mathrm{sign}(K) = \mathrm{sign}(\kappa_t)$$

$\kappa_s > 0$ curvature in the direction of sight

κ_t occluding contour curvature