

Computer Vision



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2006

Computer Vision

Lecture 1

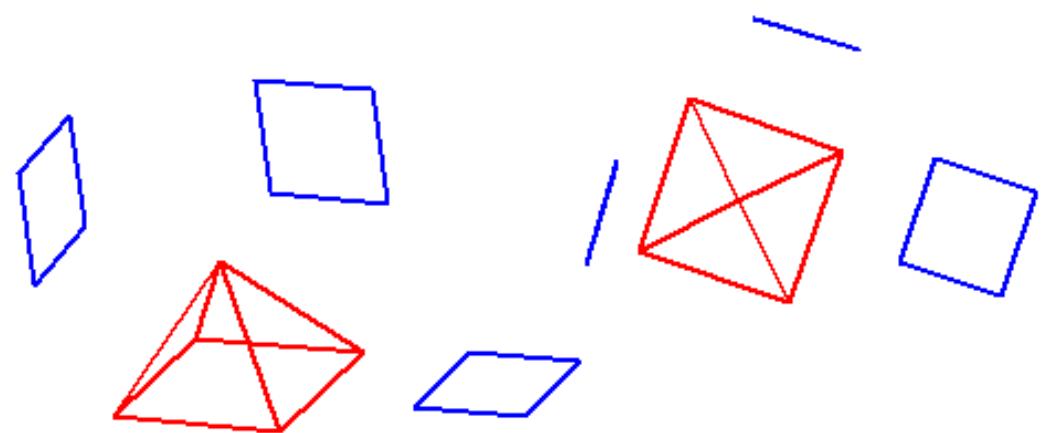
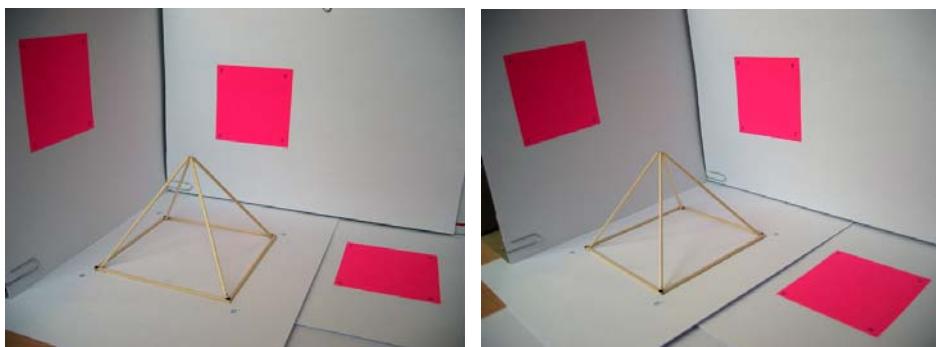
The goal

is to learn geometry of cameras in computer vision & use it to solve

1. Constructing panoramic images



2. Reconstructing a scene from its images



Literature

[PP] Linear algebra

P. Pták. *Introduction to Linear Algebra*. Vydavatelství ČVUT, Praha, 2006.

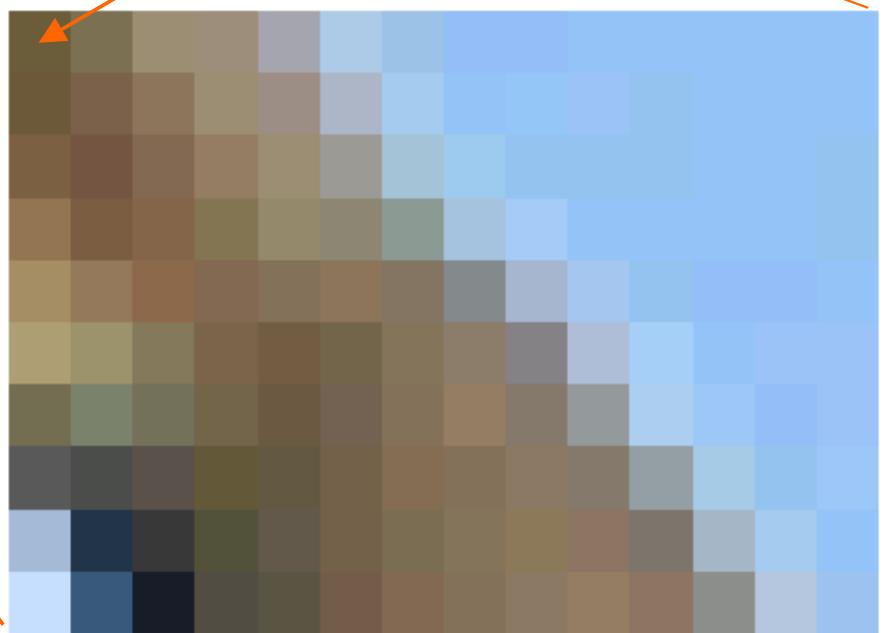
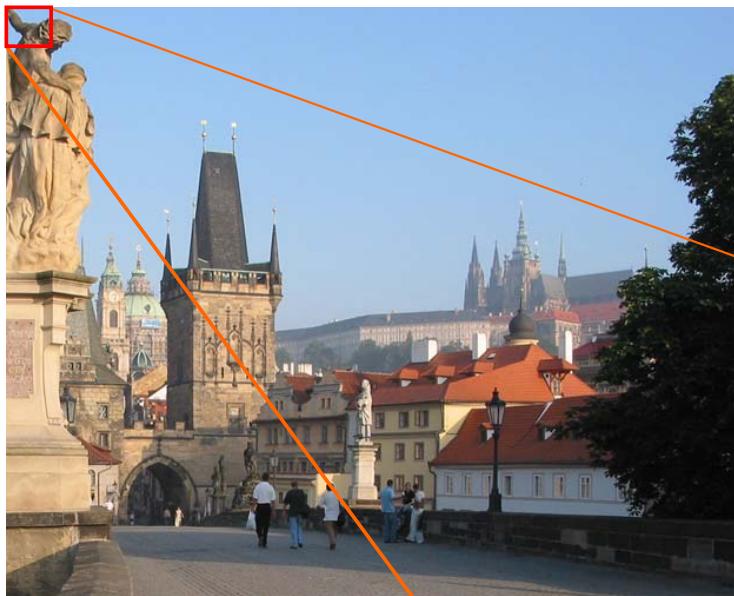
[EK] Numerical linear algebra

E. Krajiník. *Maticový počet*. Vydavatelství ČVUT, Praha, 2000.

[HZ] Computer Vision

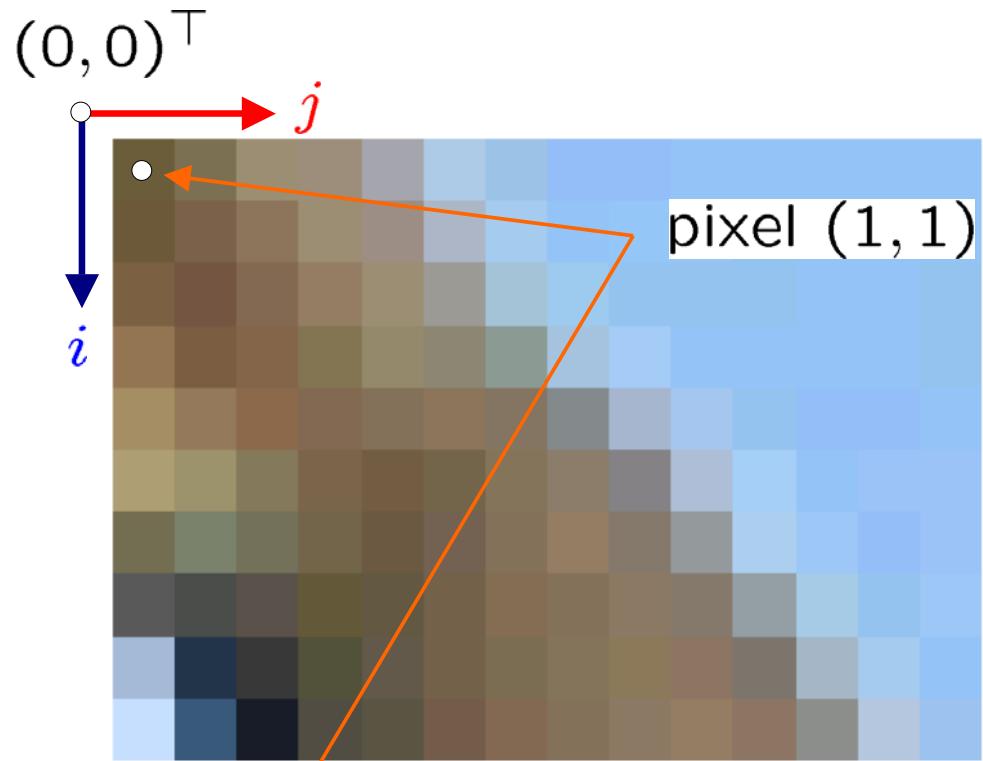
R. Hartley and A. Zisserman. *Multiple View Geometry*. 2nd ed. Cambridge Press 2003. <http://www.robots.ox.ac.uk/~vgg/hzbook/index.html>

Image



pixel

Image is a $m \times n \times 3$ matrix in Matlab



| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1 | 107 | 126 | 155 | 154 | 161 | 172 | 155 | 145 | 149 | 151 | 150 | 150 | 150 | 149 |
| 2 | 107 | 121 | 139 | 156 | 154 | 174 | 166 | 146 | 149 | 152 | 150 | 148 | 149 | 151 |
| 3 | 125 | 116 | 130 | 144 | 159 | 157 | 167 | 159 | 149 | 150 | 148 | 150 | 151 | 151 |
| 4 | 148 | 125 | 128 | 132 | 147 | 140 | 140 | 167 | 160 | 146 | 148 | 151 | 150 | 150 |
| 5 | 164 | 149 | 136 | 128 | 131 | 137 | 130 | 133 | 160 | 164 | 150 | 151 | 149 | 151 |
| 6 | 168 | 154 | 134 | 122 | 114 | 118 | 134 | 137 | 131 | 173 | 165 | 148 | 152 | 153 |
| 7 | 112 | 127 | 116 | 112 | 106 | 116 | 130 | 144 | 129 | 145 | 170 | 152 | 147 | 152 |
| 8 | 89 | 76 | 91 | 98 | 102 | 115 | 128 | 131 | 139 | 130 | 148 | 167 | 151 | 152 |
| 9 | 164 | 37 | 62 | 85 | 98 | 115 | 127 | 131 | 139 | 139 | 125 | 165 | 164 | 149 |
| 10 | 194 | 62 | 27 | 84 | 95 | 112 | 128 | 131 | 137 | 150 | 139 | 142 | 176 | 158 |

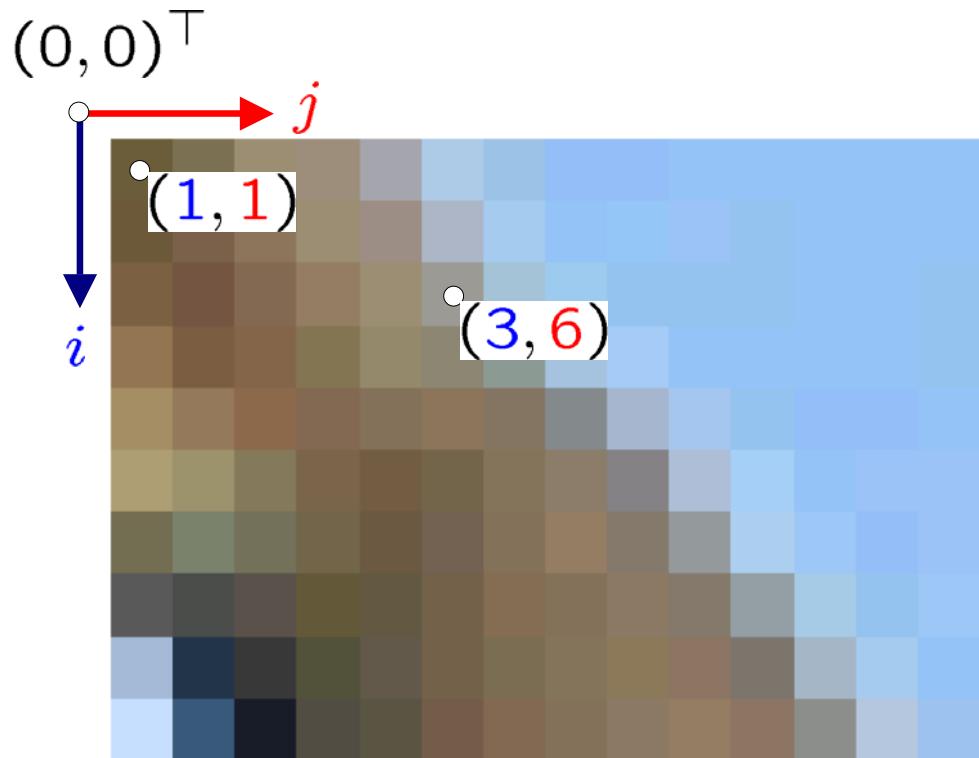
```
>>im = imread('karluv-most.jpg');  
>>imagesc(im(1:10,1:14,:));  
>>axis image
```

Indexing

i j R \leftarrow RGB
1 ... R
2 ... G
3 ... B

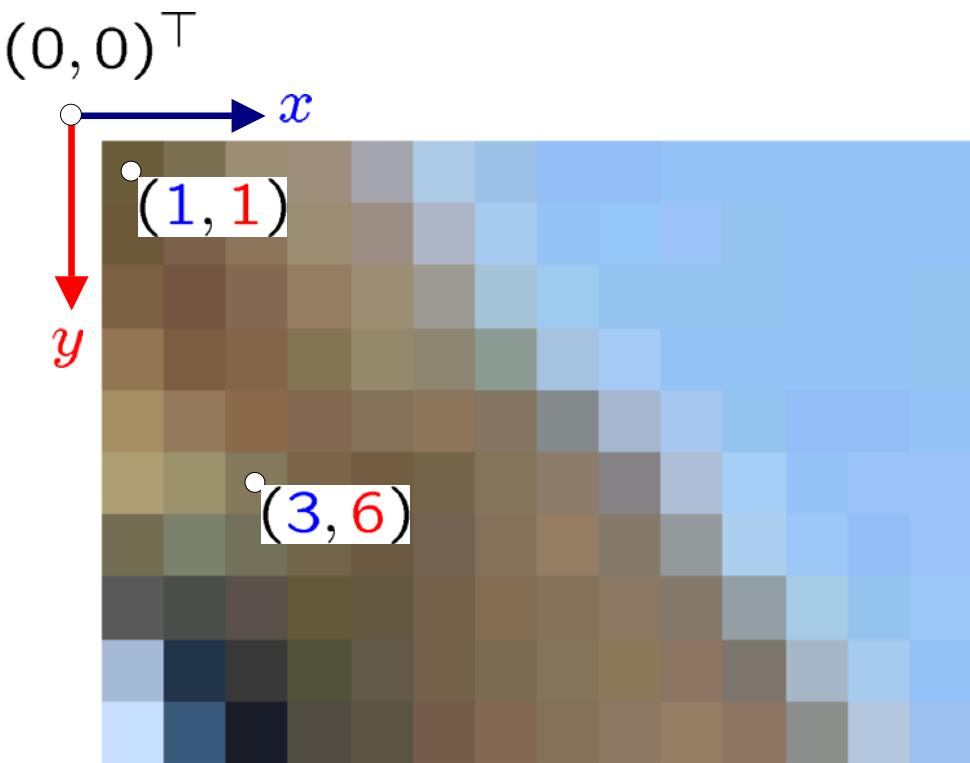
```
>>im(3,4,1)  
ans = 144
```

Image coordinate systems in Matlab



Indexing

```
>>im(3,6,1)
```



Plotting

```
>>axis image  
>>plot(3,6,'.');
```

$\text{im}(i, j, :) \leftrightarrow \text{plot}(j, i, '.');$

Camera



Digital cameras

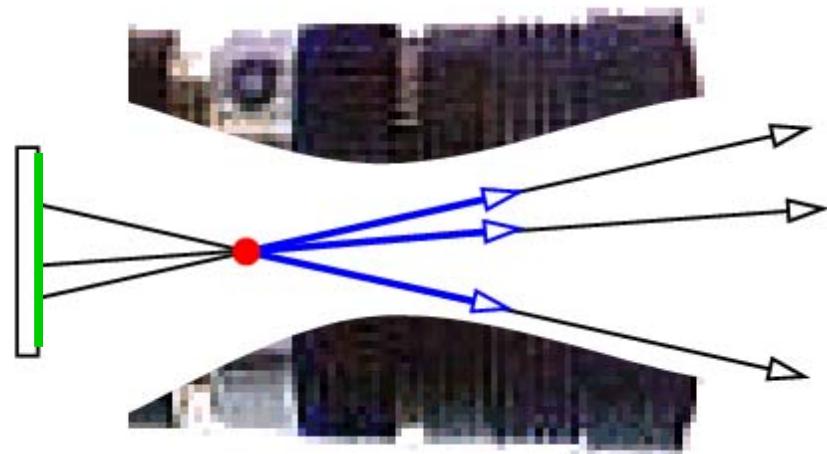
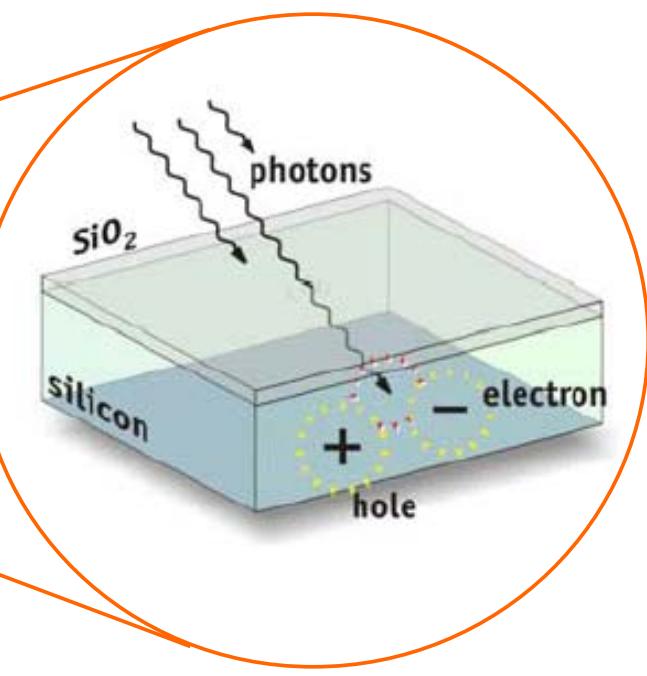
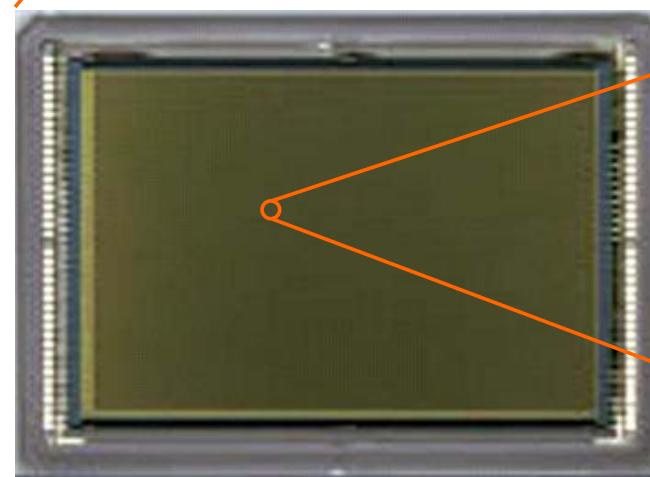
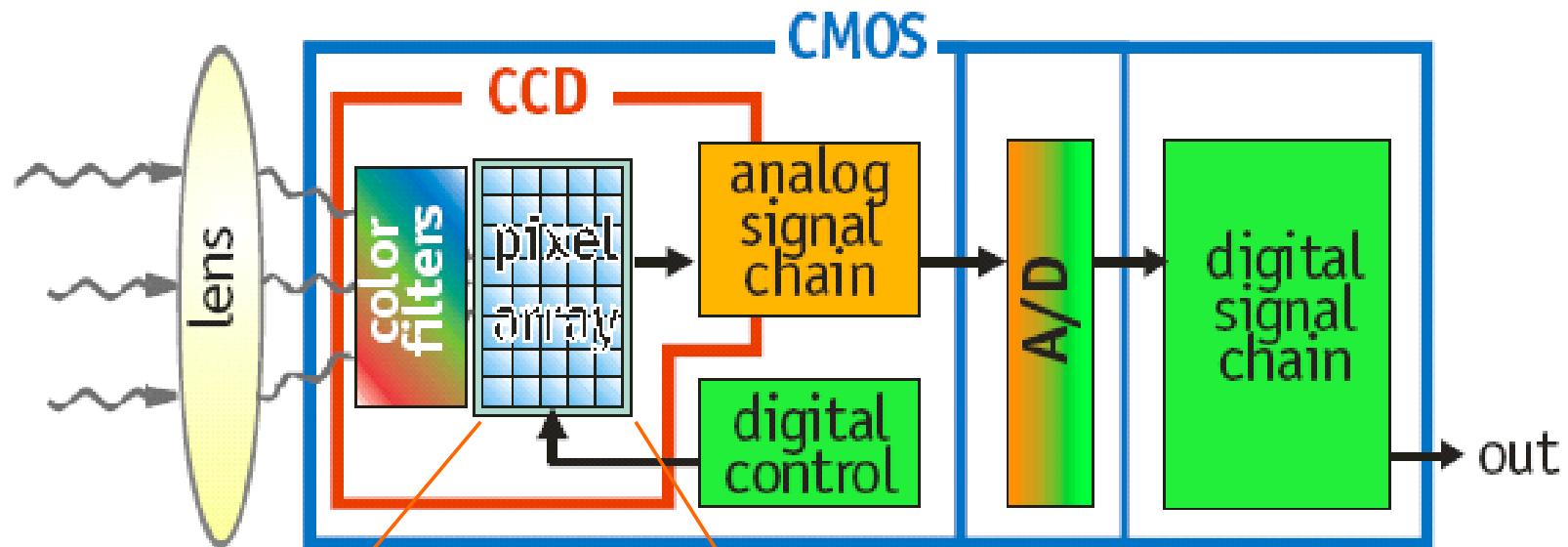


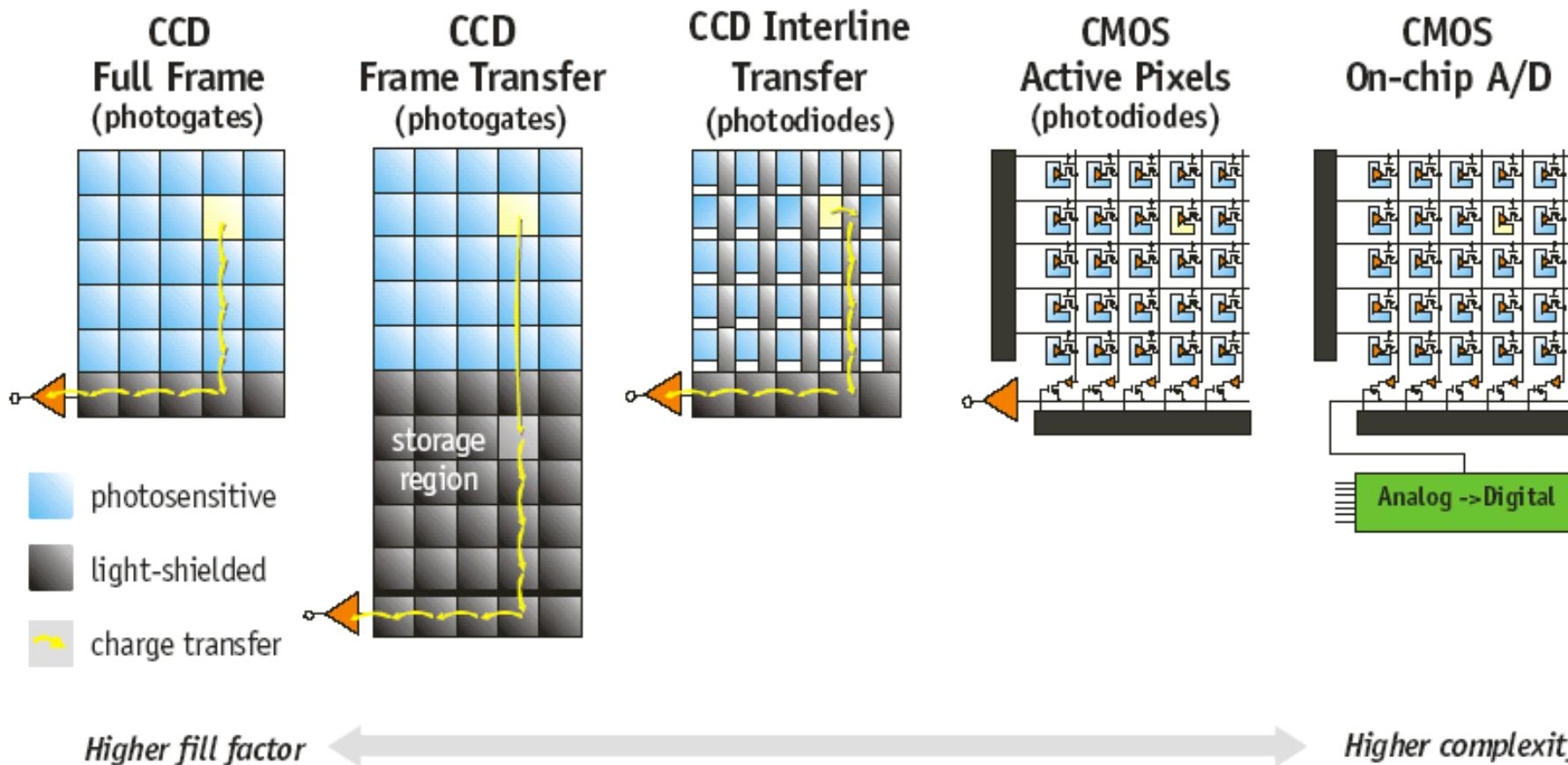
Image projection model

1. Light extends along straight **rays**
2. Projection **center**
3. Projection **plane**

Imager



Pixel indices are linear coordinates

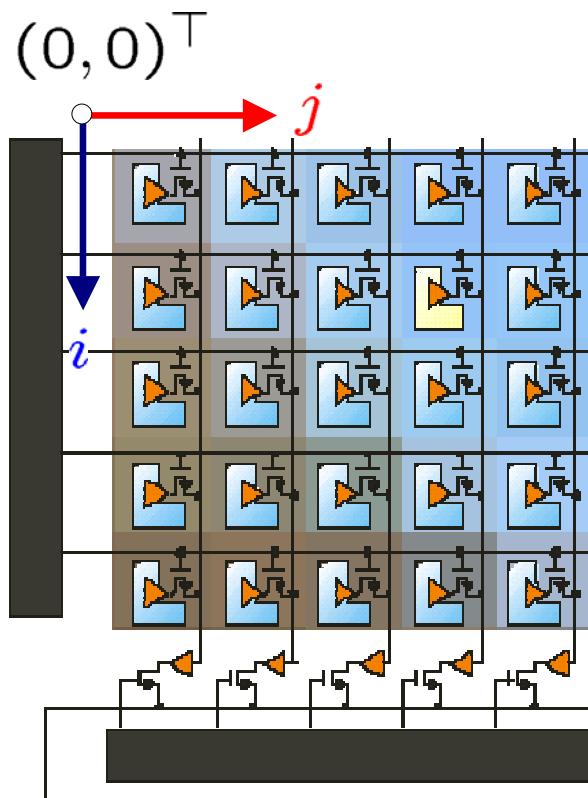


Pixels are arranged along straight lines in a rectangular grid



Pixel indices $(i, j)^\top$ are *linear coordinates*

Pixel indices are linear coordinates

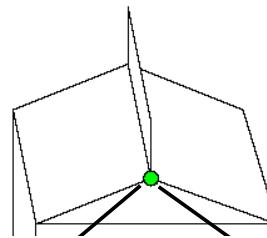


There is an affine coordinate system in the sensor plane such that pixel indices are the coordinates of pixels in that coordinate system.

Correspondences & matches

A correspondence

associates projections
of one 3D point in images



Projection

Image 1



correspondence

match

A match [meč]

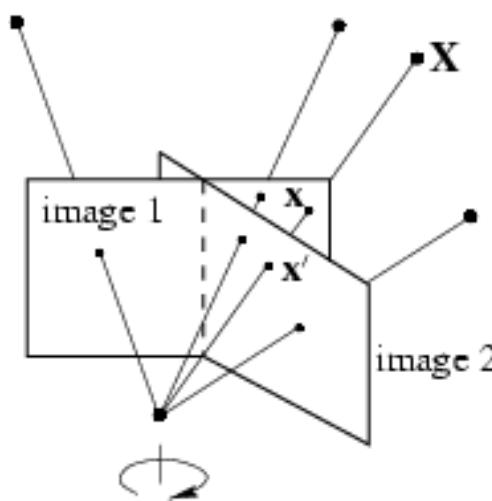
associates similar features
in images

Image 2

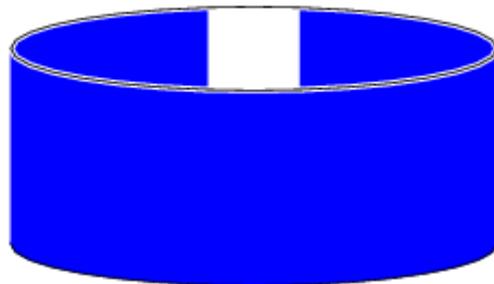


Matches are almost never
correspondences due to errors

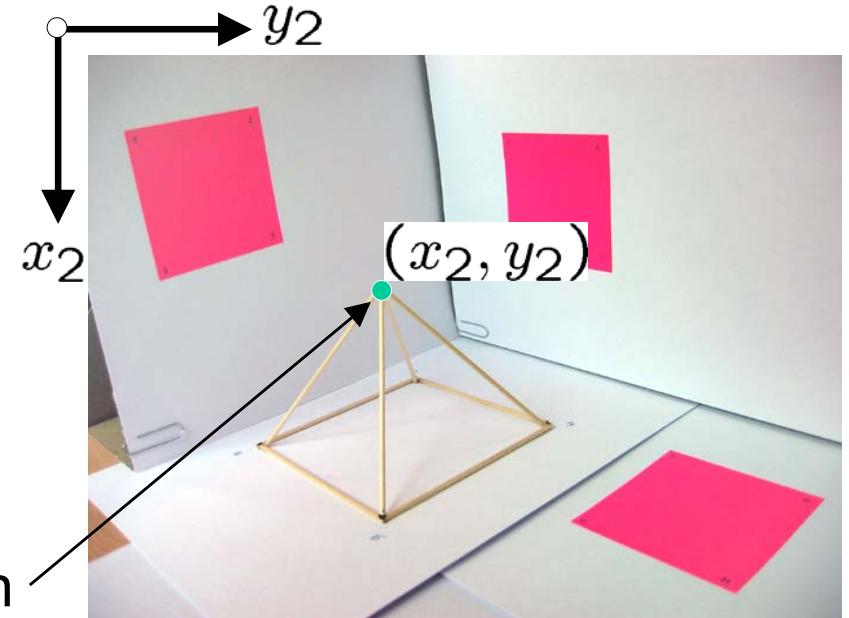
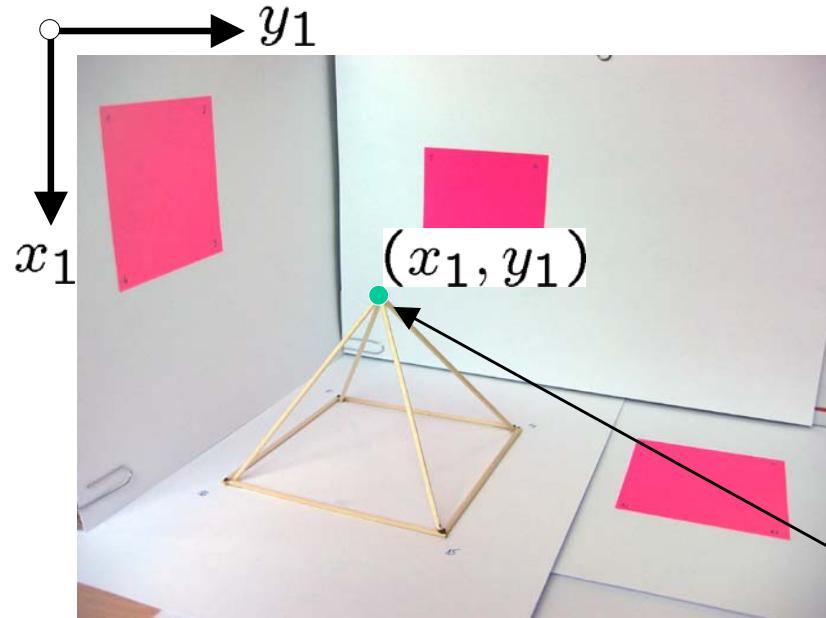
Panorama



Rotating camera acquires a sequence of images that are can be composed into a panoramic cylindrical image

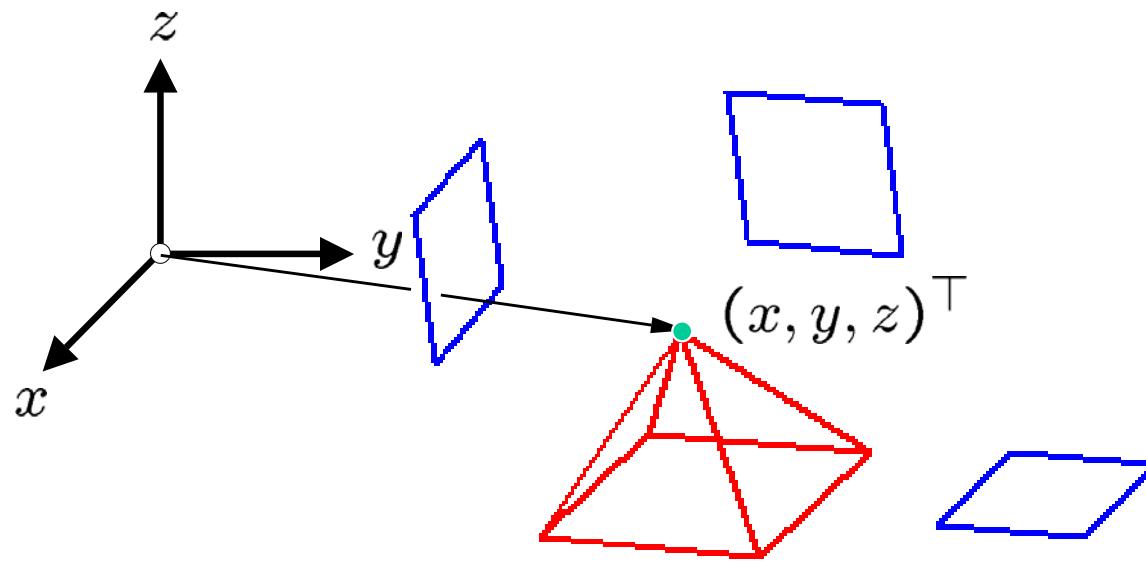


Reconstruction



match

Reconstruction: $(x_1, y_1, x_2, y_2)^\top \rightarrow (x, y, z)^\top$ in a cartesian coordinate system



Linear Space

P. Pták. *Introduction to Linear Algebra*. Vydavatelství ČVUT, Praha, 2006. pp. 8–9

1.1.1. Definition. Let L be a nonvoid set. Let L be equipped with the operations $\sharp: L \times L \rightarrow L$ and $\circ: R \times L \rightarrow L$ such that the following 8 conditions are satisfied:

Axiom 1. $\vec{x} \sharp \vec{y} = \vec{y} \sharp \vec{x}$ (the commutativity of \sharp), $\forall \vec{x}, \vec{y} \in L$,

Axiom 2. $\vec{x} \sharp (\vec{y} \sharp \vec{z}) = (\vec{x} \sharp \vec{y}) \sharp \vec{z}$ (the associativity of \sharp), $\forall \vec{x}, \vec{y}, \vec{z} \in L$,

Axiom 3. $\exists \vec{o} \in L$ such that $\vec{x} \sharp \vec{o} = \vec{x}$ (the existence of the zero vector), $\forall \vec{x} \in L$,

Axiom 4. $\forall \vec{x} \exists \vec{y}$ such that $\vec{x} \sharp \vec{y} = \vec{o}$ (the existence of the \sharp -inverse),

Axiom 5. $1 \circ \vec{x} = \vec{x}$ (the neutrality of 1 with respect to \circ), $\forall \vec{x} \in L$,

Axiom 6. $(\lambda\mu) \circ \vec{x} = \lambda \circ (\mu \circ \vec{x})$ (the interplay of \circ with multiplication in R),
 $\forall \vec{x} \in L, \forall \lambda, \mu \in R$,

Axiom 7. $(\lambda + \mu) \circ \vec{x} = \lambda \circ \vec{x} \sharp \mu \circ \vec{x}$ (the distributivity of \circ with addition in R),
 $\forall \vec{x} \in L, \forall \lambda, \mu \in R$,

Axiom 8. $\lambda \circ (\vec{x} \sharp \vec{y}) = \lambda \circ \vec{x} \sharp \lambda \circ \vec{y}$ (distributivity of \sharp and \circ), $\forall \vec{x}, \vec{y} \in L, \forall \lambda \in R$.

The triple (L, \sharp, \circ) is then called a *linear space*.

Linear Space

1. Linear space [PP, p. 8]
2. Linear independence – 2 definitions [PP, p.16: 1.1.18, 1.1.19]
3. Basis [PP, p. 19]
4. Dimension [PP, p. 22]
5. Coordinates w.r.t. an ordered basis [PP, p. 23]
6. Matrix [PP, p. 32]
7. Rank of a matrix [PP, p. 34]
8. Linear equations [PP, p. 56]
9. Frobenius theorem & linear independence [PP, p. 56]
10. Homogeneous & non-homog. systems of lin. eqns. [PP, p. 58]
11. Change of coordinates due to a change of the basis [PP, p. 94]
12. Eigenvalues and eigenvectors [PP, p. 180]

Affine Space

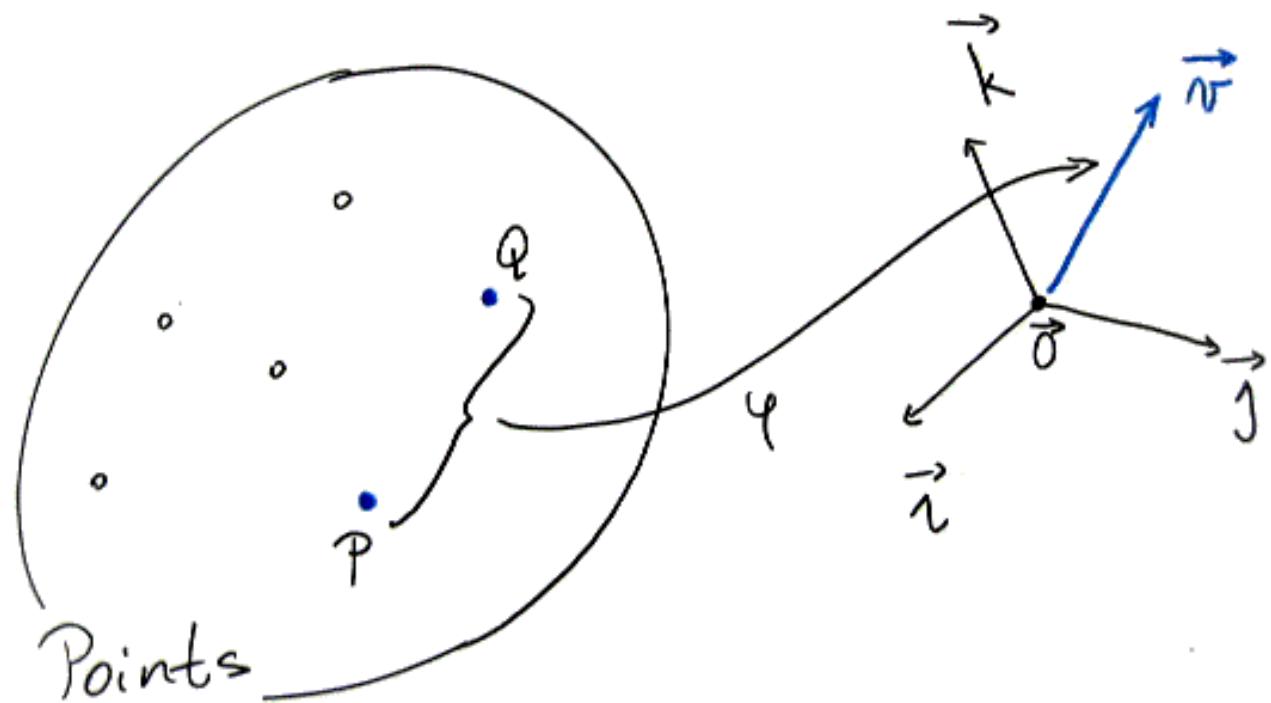
triple $\mathcal{A} = (\mathcal{P}, \mathcal{V}, \varphi)$

\mathcal{P} ... set of points

\mathcal{V} ... linear space

φ ... function $\mathcal{P} \times \mathcal{P} \rightarrow \mathcal{V}$

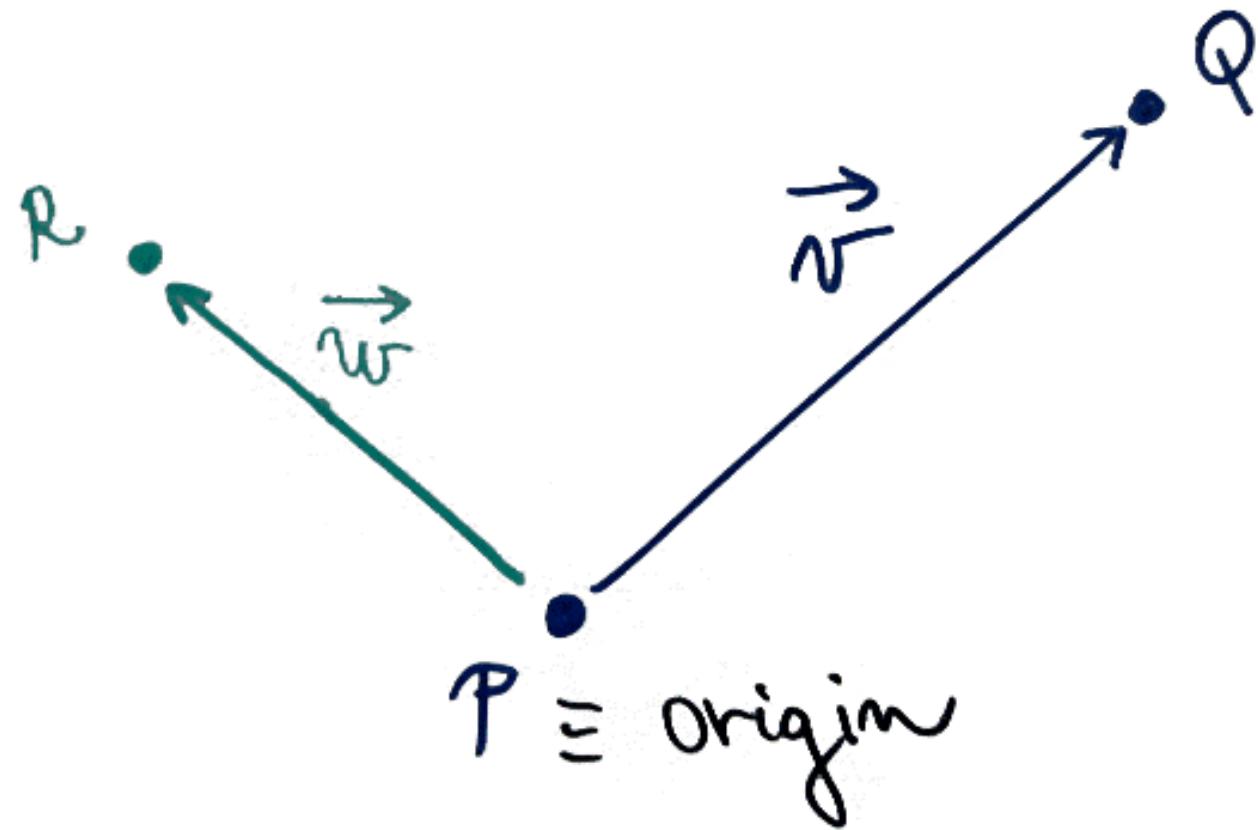
Linear Space



Affine Space

We always define φ when drawing vectors

$$\varphi : \mathcal{P} \times \mathcal{P} \rightarrow \mathcal{V}$$



Affine Space

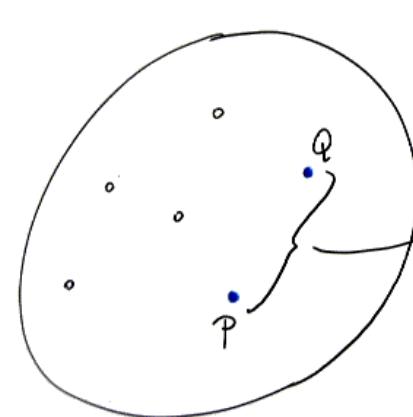
triple $\mathcal{A} = (\mathcal{P}, \mathcal{V}, \varphi)$

\mathcal{A} ... set of points

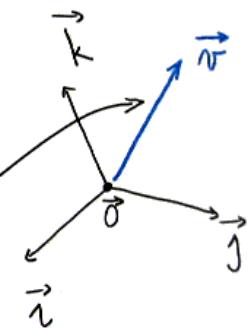
\mathcal{V} ... linear space

φ ... function $\mathcal{P} \times \mathcal{P} \rightarrow \mathcal{V}$

Points



Linear Space



1. $\forall P, Q \in \mathcal{P} \exists! \vec{v} \in \mathcal{V} : \varphi(P, Q) = \vec{v}$ (i.e. φ is a function)
2. $\forall P \in \mathcal{P} \forall \vec{v} \in \mathcal{V} \exists! Q \in \mathcal{P} : \varphi(P, Q) = \vec{v}$ ($\psi(P, \vec{v}) \rightarrow Q$ is a function)
3. $\forall P, Q, R \in \mathcal{P} : \varphi(P, Q) + \varphi(Q, R) - \varphi(P, R) = 0$ (\triangle -equality)

Affine Space

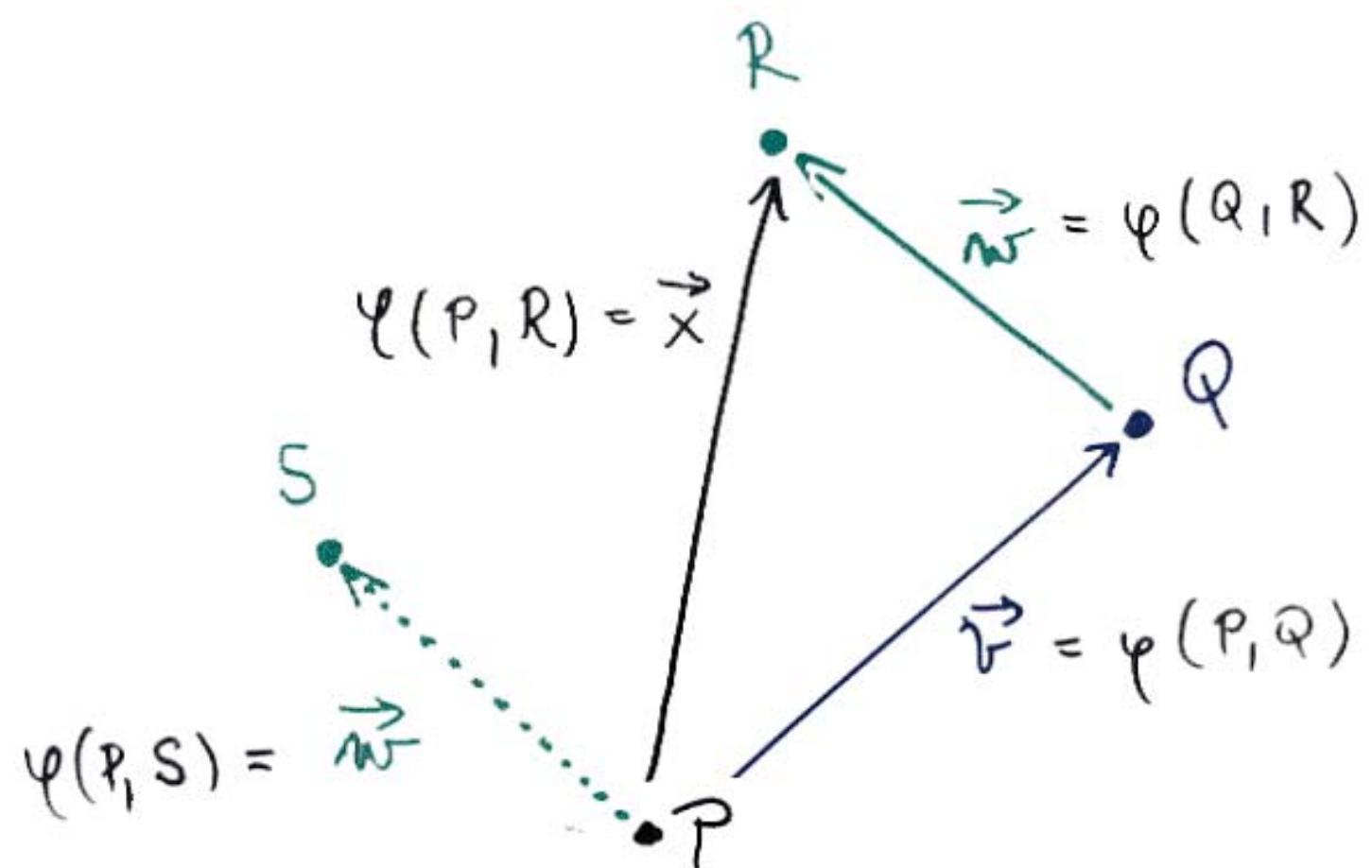
\triangle -equality in detail

$$\forall P, Q, R \in \mathcal{P} : \varphi(P, Q) + \varphi(Q, R) - \varphi(P, R) = 0$$

$$\vec{v} = \varphi(P, Q)$$

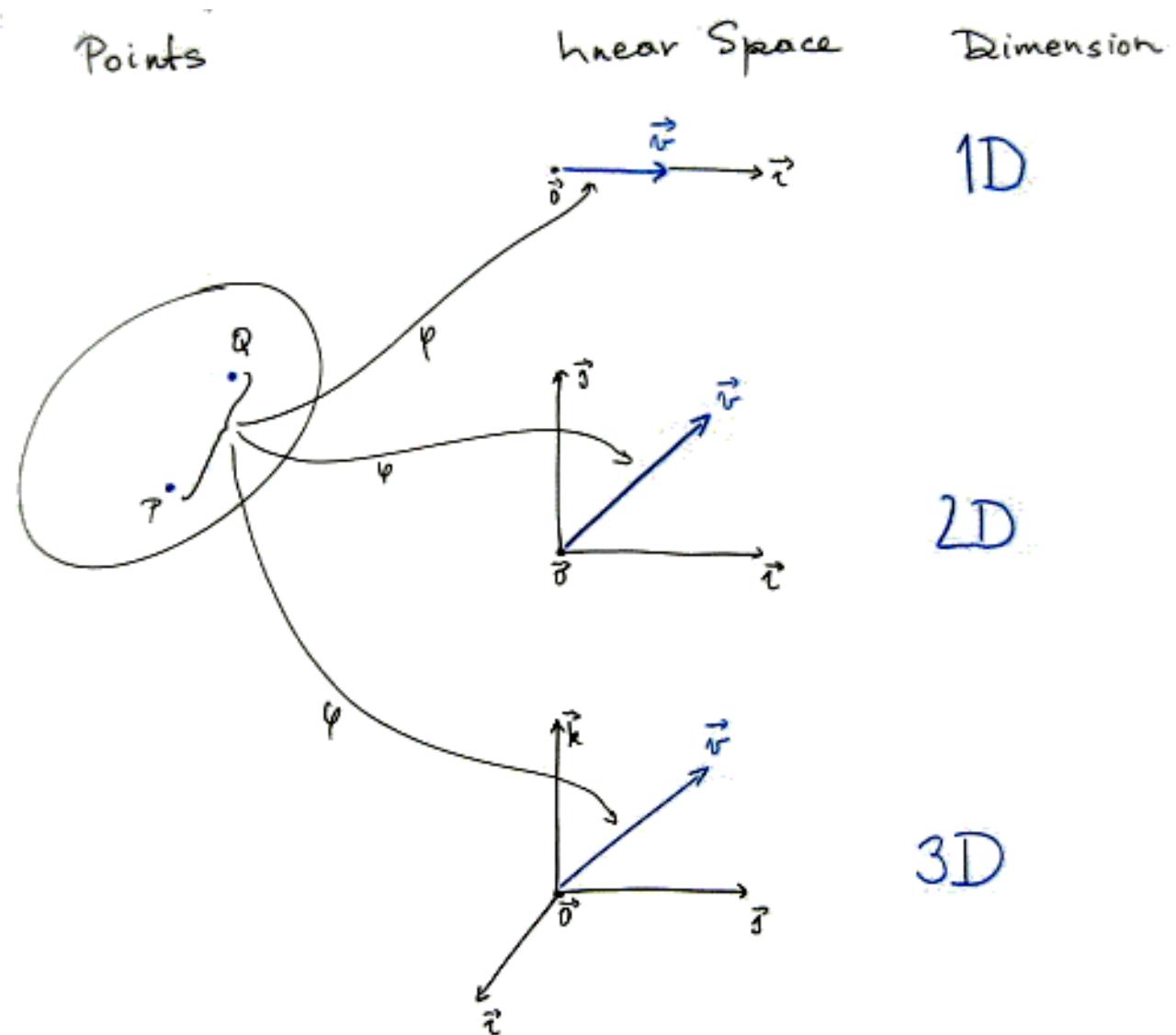
$$\vec{w} = \varphi(Q, R)$$

$$\vec{x} = \varphi(P, R)$$



Affine Space

Dimension of an affine space: $\dim \mathcal{A} \stackrel{\text{def}}{\equiv} \dim \mathcal{V}$



Affine Space

Affine coordinate system

1. ordered 4-tuple of points (O, A, B, C) and an
2. ordered basis $(\vec{b}_1, \vec{b}_2, \vec{b}_3)$

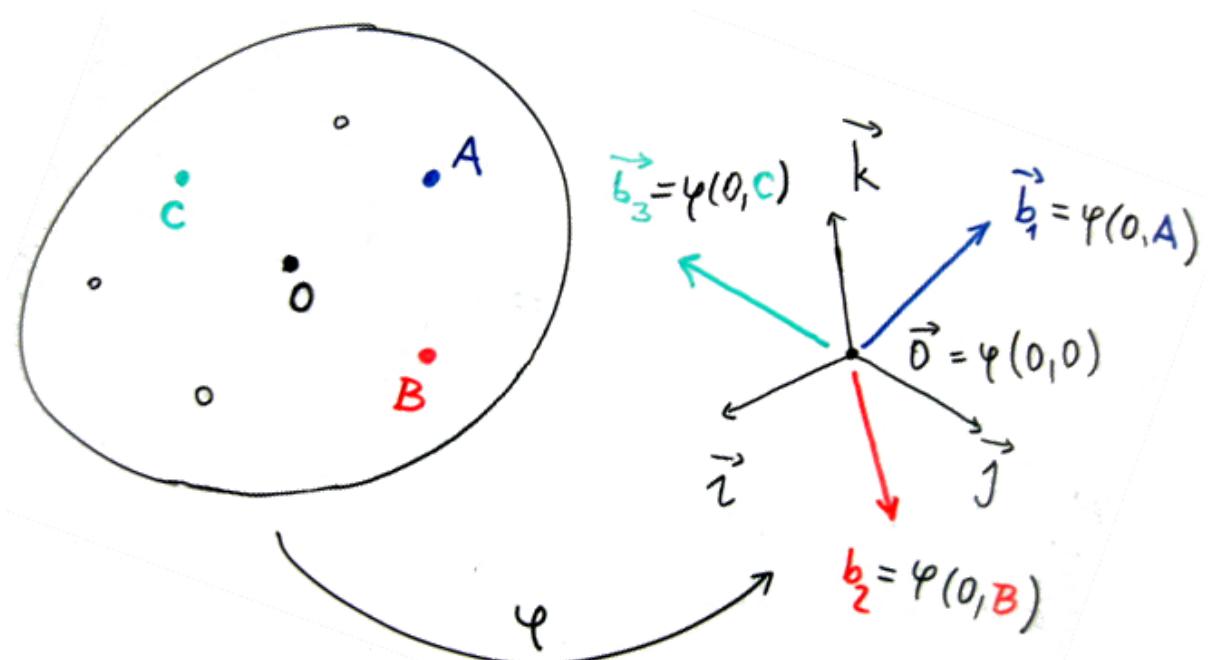
such that there is φ

1. satisfying conditions of the affine space

2. $\varphi(O, A) = \vec{b}_1$

$$\varphi(O, B) = \vec{b}_2$$

$$\varphi(O, C) = \vec{b}_3$$



1. $\vec{x} = \varphi(O, X)_{(\vec{b}_1, \vec{b}_2, \vec{b}_3)}$ is a 1:1 correspondence between P and V
2. $\varphi(P, Q) = \varphi(O, Q) - \varphi(O, P)$

Euclidean Space

$\stackrel{\text{def}}{\equiv}$ an affine space + the Euclidean distance d

quadruple $\mathcal{E} = (\mathcal{P}, \mathcal{V}, \varphi, d)$

\mathcal{P} ... set of points

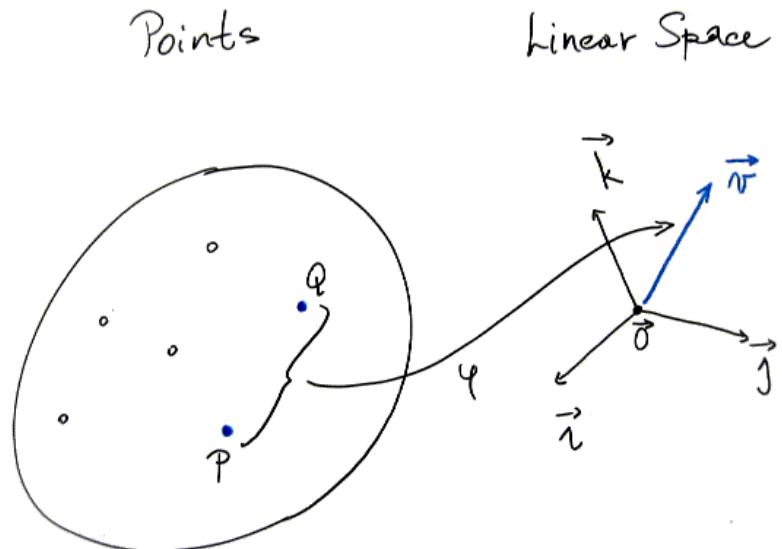
\mathcal{V} ... linear space

φ ... function $\mathcal{P} \times \mathcal{P} \rightarrow \mathcal{V}$

d ... distance $\mathcal{P} \times \mathcal{P} \rightarrow \mathbb{R}$ (the real numbers)

$$d(P, Q) = \sqrt{\vec{v} \cdot \vec{v}} = \sqrt{\varphi(P, Q) \cdot \varphi(P, Q))}$$

where $\cdot : \mathcal{V} \times \mathcal{V} \rightarrow \mathbb{R}$ is the “Euclidean” scalar product
(spheres are “spherical”)

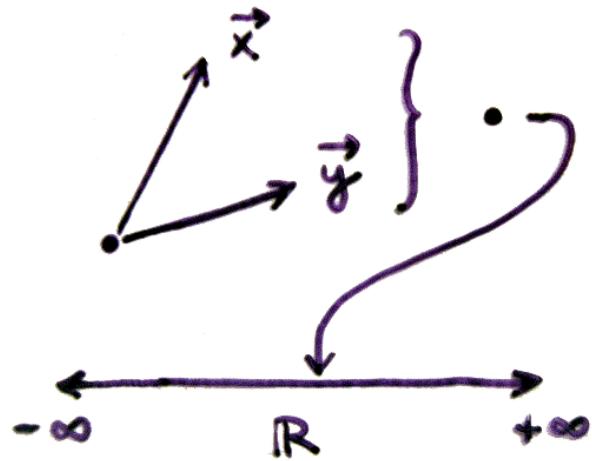


Euclidean Space

Scalar product $\cdot : \mathcal{V} \times \mathcal{V} \rightarrow \mathbb{R}$

For every $\vec{x}, \vec{y}, \vec{z} \in \mathcal{V}$

1. $\vec{x} \cdot \vec{y} = \vec{y} \cdot \vec{x}$
2. $(\vec{x} + \vec{y}) \cdot \vec{z} = \vec{x} \cdot \vec{z} + \vec{y} \cdot \vec{z}$
3. $\vec{x} \cdot \vec{x} \geq 0$
4. $\vec{x} \cdot \vec{x} = 0 \Leftrightarrow \vec{x} = \vec{0}$
5. $(\lambda \vec{x}) \cdot \vec{y} = \lambda (\vec{x} \cdot \vec{y})$ for all $\lambda \in \mathbb{R}$.

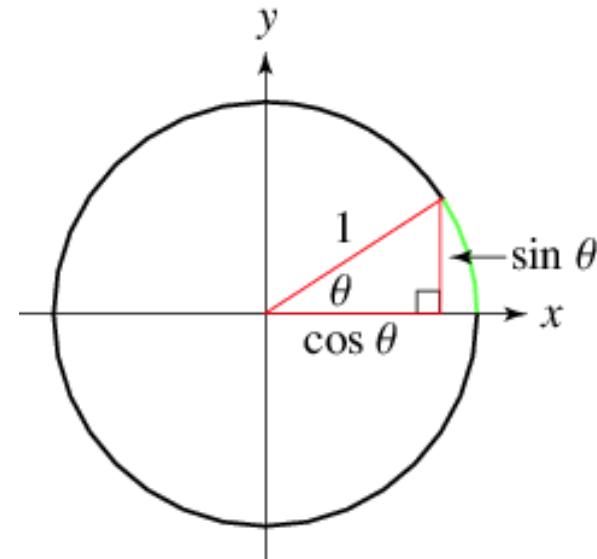
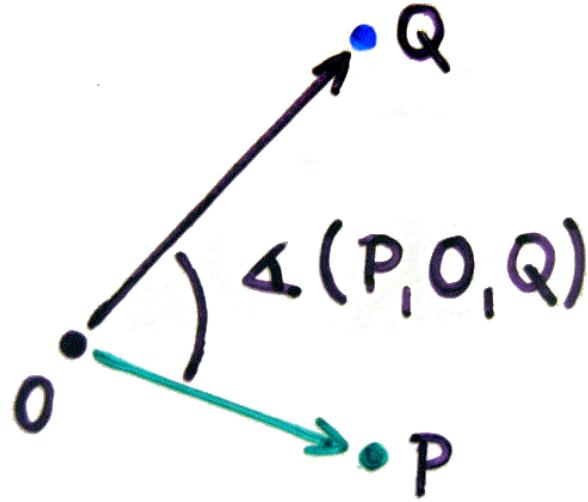


Euclidean Space

The “Euclidean” scalar product (\exists non-Euclidean s. p’s!)

$$\vec{x} \cdot \vec{y} = \sum_i^n x_i y_i$$

$\vec{x} = [x_1, x_2, \dots, x_n]^\top$, $\vec{y} = [y_1, y_2, \dots, y_n]^\top$ in the *standard basis*



Angle

$$\cos \angle(P, O, Q) = \frac{\varphi(O, P) \cdot \varphi(O, Q)}{\sqrt{\varphi(O, P) \cdot \varphi(O, P)} \sqrt{\varphi(O, Q) \cdot \varphi(O, Q)}}$$

Right-handed bases in a 3-dimensional linear space

An orthonormal basis $(\vec{b}_1, \vec{b}_2, \vec{b}_3)$ in a 3-dimensional linear space is right-handed if

$$\vec{b}_3 = \vec{b}_1 \times \vec{b}_2$$

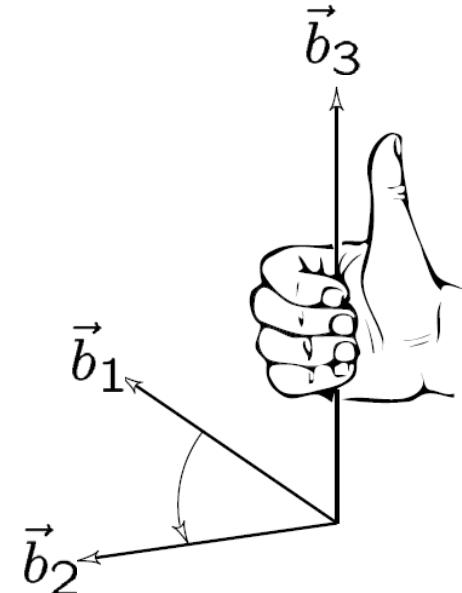
$\times \dots$ vector product of

$$\vec{a} = [a_1, a_2, a_3]^\top, \vec{b} = [b_1, b_2, b_3]^\top$$

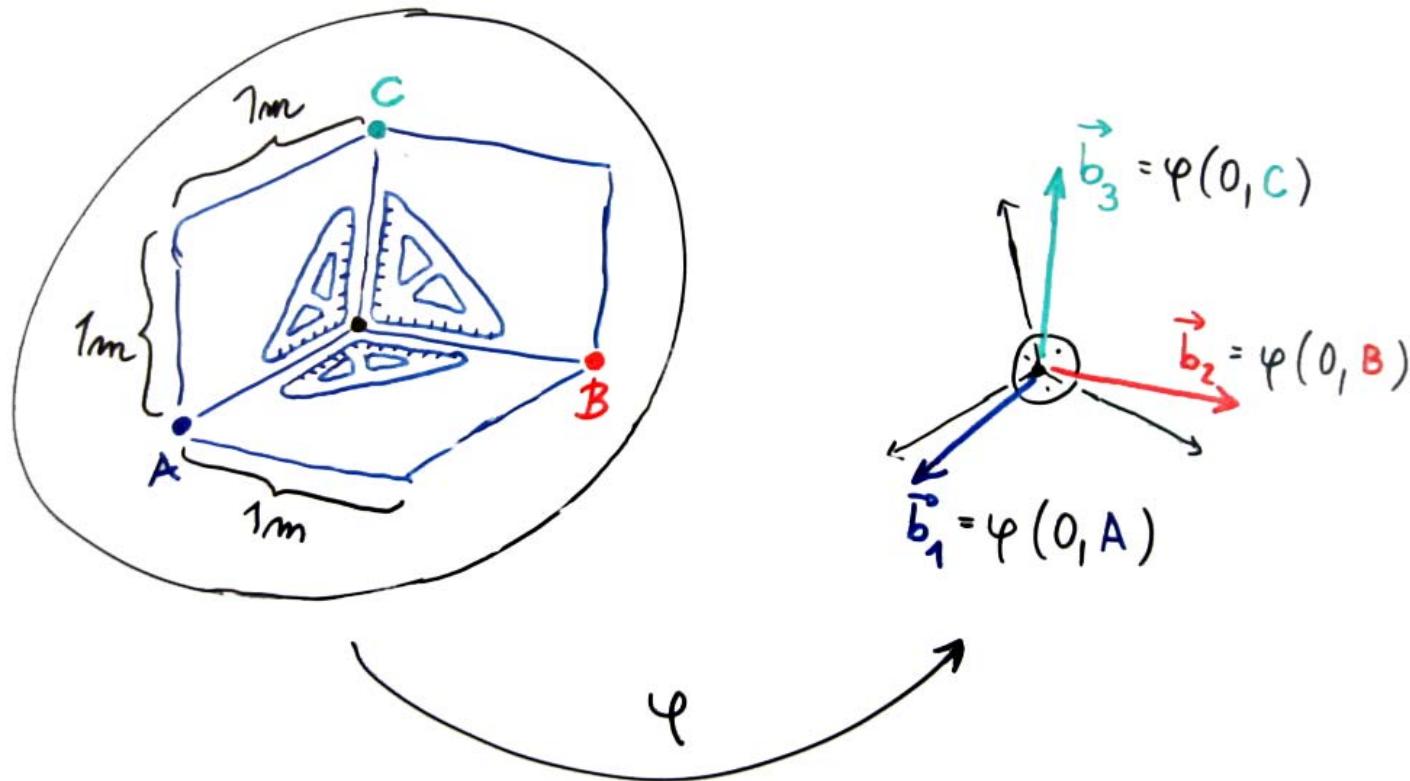
$$\vec{a} \times \vec{b} = \left[\det \begin{bmatrix} a_2 & a_3 \\ b_2 & b_3 \end{bmatrix}, -\det \begin{bmatrix} a_1 & a_3 \\ b_1 & b_3 \end{bmatrix}, \det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \right]$$

It holds

$$\begin{aligned}\vec{c} \cdot (\vec{a} \times \vec{b}) &= c_1 \det \begin{bmatrix} a_2 & a_3 \\ b_2 & b_3 \end{bmatrix} - c_2 \det \begin{bmatrix} a_1 & a_3 \\ b_1 & b_3 \end{bmatrix} + c_3 \det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \\ &= \det [\vec{a} \ \vec{b} \ \vec{c}]\end{aligned}$$



Euclidean Space



Cartesian coordinate system (O, A, B, C) & $(\vec{b}_1, \vec{b}_2, \vec{b}_3)$

1. $(O; A, B, C)$. . . positively oriented vertices of a unit cube
2. $(\vec{b}_1, \vec{b}_2, \vec{b}_3) = (\vec{i}, \vec{j}, \vec{k})$. . . the standard basis (is right-handed and orthonormal)