A first-order primal-dual algorithm for convex problems with applications to imaging

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Variational methods have proven to be particularly useful to solve a number of ill-posed inverse imaging problems. In particular variational methods incorporating total variation regularization have become very popular for a number of applications. Unfortunately, these methods are difficult to minimize due to the non-smoothness of the total variation. The aim of this paper is therefore to provide a flexible algorithm which is particularly suitable for non-smooth convex optimization problems in imaging. In particular, we study a first-order primal-dual algorithm for non-smooth convex optimization problems with known saddle-point structure. We prove convergence to a saddle-point with rate $O(1/N)$ in finite dimensions, which is optimal for the complete class of non-smooth problems we are considering in this paper. We further show accelerations of the proposed algorithm to yield optimal rates on easier problems. In particular we show that we can achieve $O(1/N^2)$ convergence on problems, where the primal or the dual objective is uniformly convex, and we can show linear convergence, i.e. $O(1/e^N)$ on problems where both are uniformly convex. The wide applicability of the proposed algorithm is demonstrated on several imaging problems such as image denoising, image deconvolution, image inpainting, motion estimation and image segmentation. A pre-print of the paper is available from the webpage www.gpu4vision.org.