Spring 2008 Pattern Recognition and Computer Vision Colloquium

Advanced Models for Brightness Changes and Confidence Measures in Motion Estimation

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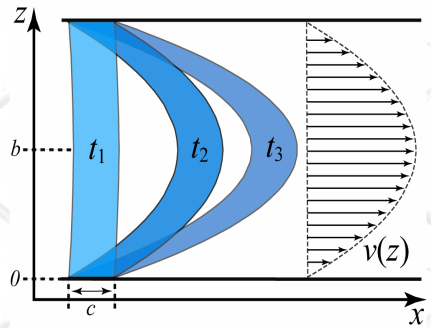
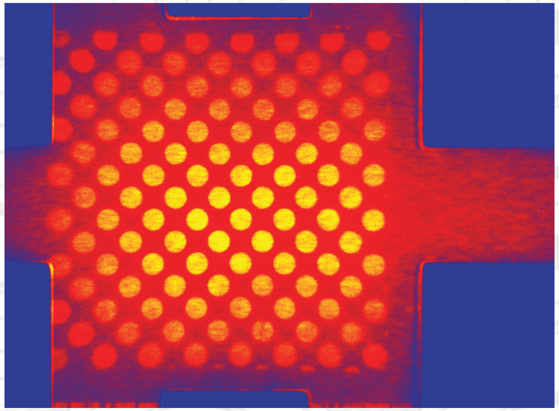
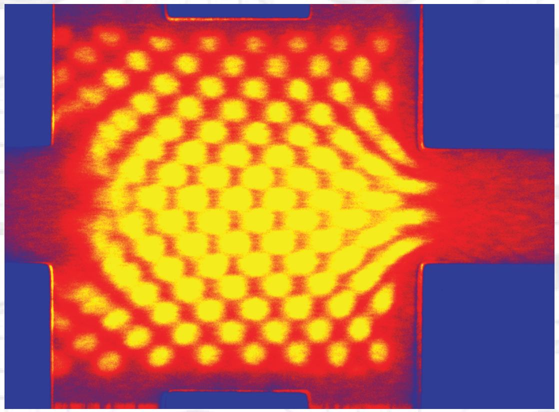
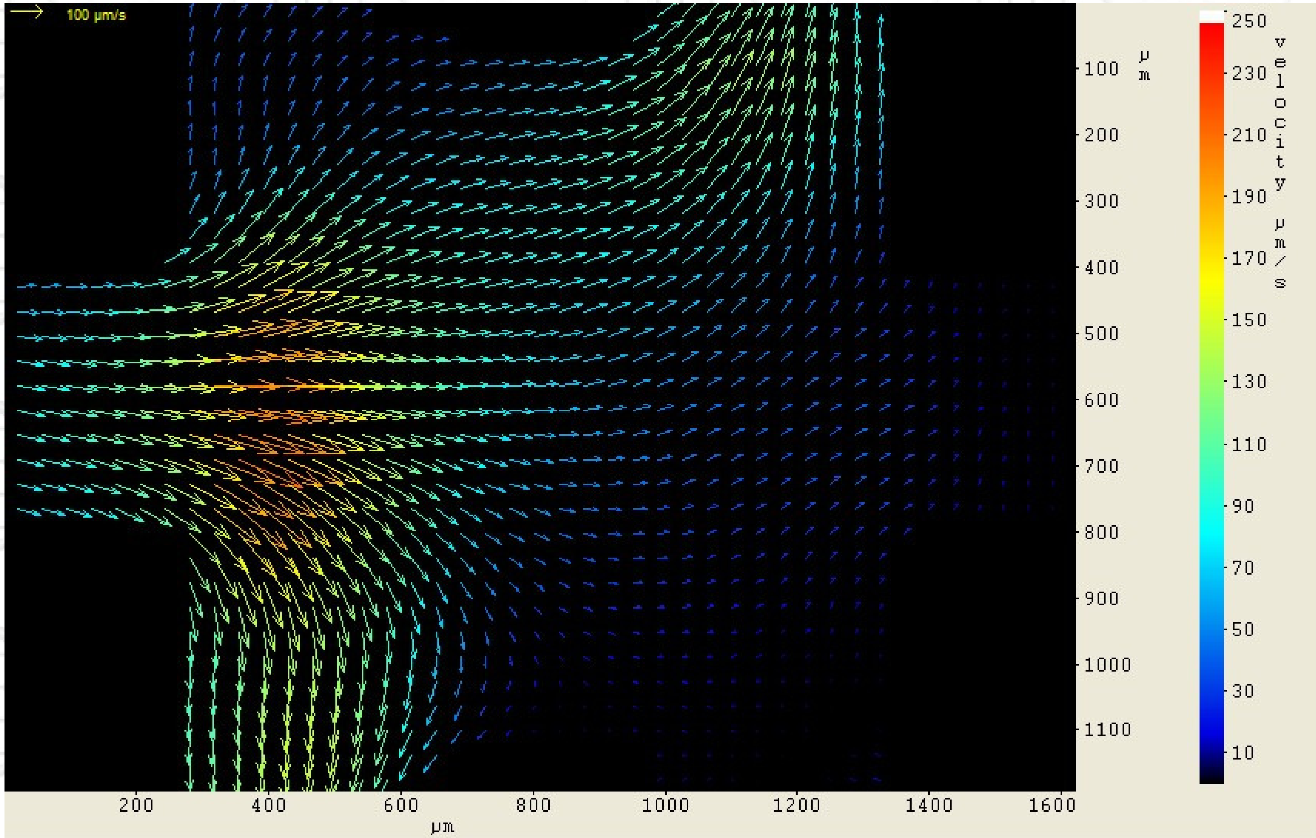
Outline: Differential Motion Estimation



- What is the problem?







Outline: Differential Motion Estimation

- What is **a (differential) optical flow algorithm**?

Image Aquisition → Problem Formulation → Optimization → Flow Postprocessing

- Spectrum/Modality (special cases: RGB, infrared, x-ray, MRI, …)
- Noise (Process, CCD/CMOS, Fixed Pattern, Quantization, …)
- Spatio-**Temporal** Image Resolution

- Not discussed here

Outline: Differential Motion Estimation

- What is **a (differential) optical flow algorithm**?

Image Aquisition → Problem Formulation → Optimization → Flow Postprocessing

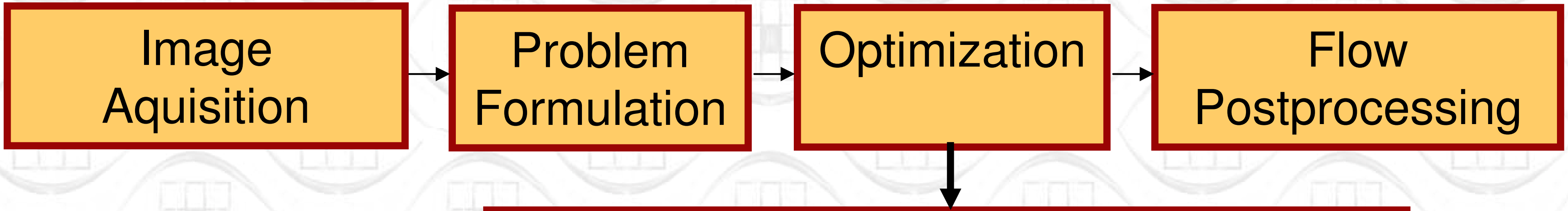
Problem Formulation ↓

- Energy/Objective Function/Probability consisting of:
- Brightness Variation Model
  - How do image intensities vary along the motion trajectory?
- Motion Model
  - How does the spatio-temporal flow distribution look like?

- **PART I**

Outline: Differential Motion Estimation

- What is **a (differential) optical flow algorithm**?



- Calculus of Variations/Graphical Models/…
- Relaxation/Multigrid Schemes (e.g. „Pyramids“)
- Linearization (Newton-type methods)
- Discretization (of continuous formulations)
- Eigenvalue Problems/Systems of Equations

- Not discussed here

Outline: Differential Motion Estimation

- What is **a (differential) optical flow algorithm**?

Image Aquisition → Problem Formulation → Optimization → Flow Postprocessing

- Confidence Measures
  - How well did optimization work?
- Flow Reconstruction
  - How to find motion in regions with unreliable flows?

- **PART II**

Brightness Variation Models - Basics

- Typical Assumption:
  - Brightness Constancy between each two frames:

$$I(\vec{x},t) - I(\vec{x}+\vec{u},t+1) = 0$$

- Typical Linearization (actually part optimization!)
  - First order Taylor Expansion:

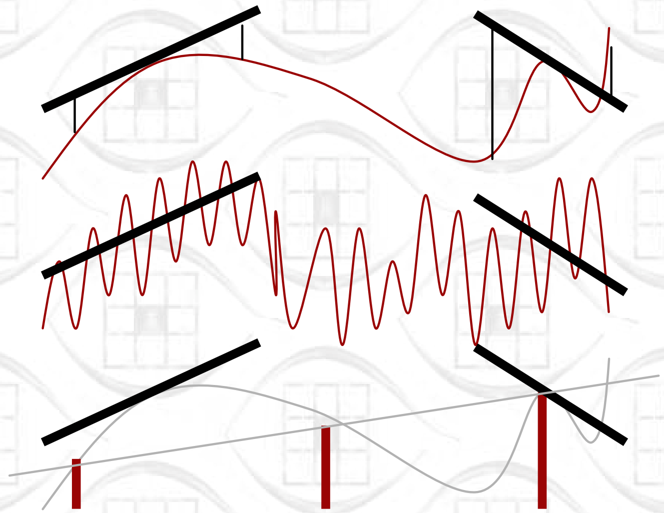
$$\nabla_{\vec{x}} I \cdot \vec{u} - \frac{\partial}{\partial t} I = 0$$

"Brightness Change Constraint Equation" (BCCE)

Brightness Variation Models - Problems

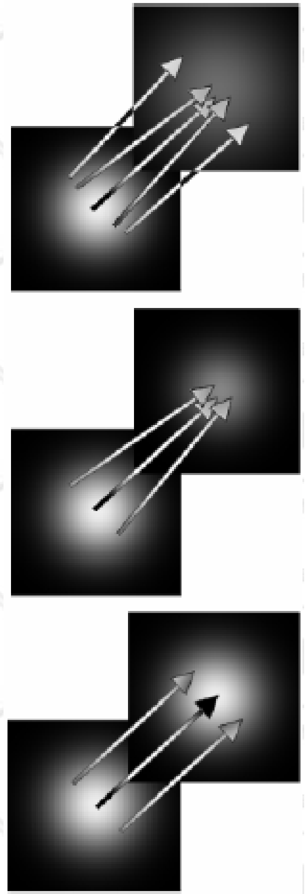
$$\nabla_{\vec{x}} I \cdot \vec{u} - \frac{\partial}{\partial t} I = 0$$

- Typical Problems:
  - Linearization
    - does not hold for large image gradients
  - Noise
    - increased by derivative
  - Image Derivative Computation
    - spatial resolution insufficient
  - **Or: Brightness constancy assumption does not hold!**



- Two fields of research could already help at this point (cf. previous talk):
  - Image Denoising
  - Image Interpolation/Reconstruction

Brightness Variation Models – Extensions I



- What if BCCE does not model the actual intensity variation?
  - Likely due to real world lighting changes, shadows, ...
  - Idea: Extend brightness variation models
- A simple extension: Linear brightness changes

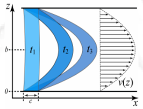
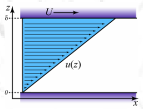
$$I(\vec{x},t) - I(\vec{x}+\vec{u},t+1) = ct$$

- BCCE:

$$\nabla_{\vec{x}} I \cdot \vec{u} - \frac{\partial}{\partial t} I - c = 0$$

- Basically, any model can be inserted on the right hand side
  - Linearization is common, but not necessary!
  - (Papenberg, Bruhn, Brox et al. have found a simple relation between iterative linearization and nonlinear solving via warping)

Brightness Variation Models – Extensions II

- More BCCE-Extensions developed at our group:
  - Source Terms: $$g(\vec{x}(t),t) = q \cdot t$$
  - Diffusion: $$\frac{dg}{dt} = g_x u + g_y v + g_t = D\left(g_{xx} + g_{yy}\right)$$
  - Exponential Decay: $$g(\vec{x}(t),t) = g_0 \exp(-\kappa \cdot t)$$
  -  Taylor Dispersion: $$\frac{dI}{dt} = \frac{d}{dt}\left(\sqrt{\frac{2}{t}}\left(\sqrt{\frac{c+x}{a}} - \sqrt{\frac{x}{a}}\right)\right) = -\frac{1}{2t} I$$
  -  Couette Flow: $$\frac{dI}{dt} = u\frac{\partial I}{\partial x} + v\frac{\partial I}{\partial x} + \frac{\partial I}{\partial t} = \frac{d}{dt}\left(\frac{c\delta}{tU}\right) = -\frac{1}{t} I$$

„physical BCCE's“

H. Haussecker, D.J. Fleet: Computing Optical Flow with Physical Models of Brightness Variation (PAMI 2001)
C. S. Garbe: Fluid Flow Estimation Through Integration of Physical Flow Configurations (DAGM 2007)
M. Jehle, B. Jähne: A novel method for three-dimensional three-component analysis of flows close to free water surfaces (EIF 2008)
A. Berthe, D. Kondermann et al.: Using single particles for the validation of a 3D-3C near wall measurement technique (EIF 2008, to appear)

Motion Models

- Another Problem with the BCCE ($\nabla_{\vec{x}} I \cdot \vec{u} - \frac{\partial}{\partial t} I = 0$):
  - It is underconstrained (2+ unknowns, one equation)
- Typical assumption:
  - Motion (and other parameters) **constant** for more than one pixel
  - Ideal case:
    - **2+ pixels solve flow problem** (system of equations)
  - But:
    - BCCE-problems (linearization, noise, resolution) corrupt results

Motion Models – Two Approaches

- Method A: Lucas & Kanade (1981)
  - Take all pixels *with the same flow* and solve overdetermined system of equations in a least-squares sense
  - Energy: $\forall \vec{x} \in \Omega : \arg\min_{\vec{u}(\vec{x})} \sum_{\vec{x}' \in N(\vec{x})} (\nabla_{\vec{x}'} I(\vec{x}', t) \cdot \vec{u} - \frac{\partial}{\partial t} I(\vec{x}', t))^2$
  - Methods based on this paper are referred to as **local methods**
- Method B: Horn & Schunck (1981)
  - Take all pixels and locally punish deviations from motion constancy
  - Energy: $\arg\min_{\vec{u}(\vec{x})} \int_\Omega (\nabla_{\vec{x}} I(\vec{x}, t) \cdot \vec{u}(\vec{x}) - \frac{\partial}{\partial t} I(\vec{x}, t))^2 + \lambda \|\nabla \vec{u}(\vec{x})\|_2^2 \, d\vec{x}$
  - Methods based on this idea are referred to as **global methods**
    - (sometimes also **variational methods**, referring to a commonly used optimization technique)

Motion Models - Problems

- Additional problems:
  - Missing information (e.g. homogenous regions, and many more)
  - **Or: Motion is not constant for more than one pixel**
- Solution:
  - Model spatial (and sometimes temporal) flow vector constellations
  - Can be considered as additional constraint (prior knowledge)

Motion Models - Local

- Approach for **local methods**:
  - Solve for model parameters instead of flow:
    - 2d/3d-Translation, -Rotation, -Divergence, -Shear, …
  - Insert motion model into BCCE:

$$\forall \vec{x} \in \Omega : \arg\min_{\vec{p}(\vec{x})} \sum_{\vec{x}' \in N(\vec{x})} \left(BCCE(M(\vec{p}(\vec{x}-\vec{x}')))\right)^2$$

  - The motion model *M* maps from a parameter vector to a flow vector
  - The flow vector is inserted into the BCCE (**<u>any</u>** BCCE!)

Motion Models – Global

- Common approach for **global methods**:
  - Use different regularizers *R*
  - Insert "motion model" into global energy:

$$\arg\min_{\vec{u}(\vec{x})} \int_\Omega (BCCE(\vec{u}(\vec{x})))^2 + \lambda R(\vec{u}(\vec{x})) d\vec{x}$$

  - Intuition: High energies caused by *R* regularize solution ("tradeoff search")
    - Can also be understood as (less intuitive) motion model
    - Many regularizers exist (cf. Weickert & Schnörr 2001)

	isotropic	anisotropic
image-driven	$g(\lvert\nabla f\rvert^2) \sum_{i=1}^{2} \lvert\nabla u_i\rvert^2$	$\sum_{i=1}^{2} \nabla u_i^T D(\nabla f) \nabla u_i$
flow-driven	$\Psi\left(\sum_{i=1}^{2} \lvert\nabla u_i\rvert^2\right)$	$\operatorname{tr} \Psi\left(\sum_{i=1}^{2} \nabla u_i \nabla u_i^T\right)$

  - Alternative, more intuitive (and successful) approach (Nir 2007)

$$\arg\min_{\vec{p}(\vec{x})} \int_\Omega (BCCE(M(p(\vec{x}))))^2 + \lambda R(\vec{p}(\vec{x})) d\vec{x}$$

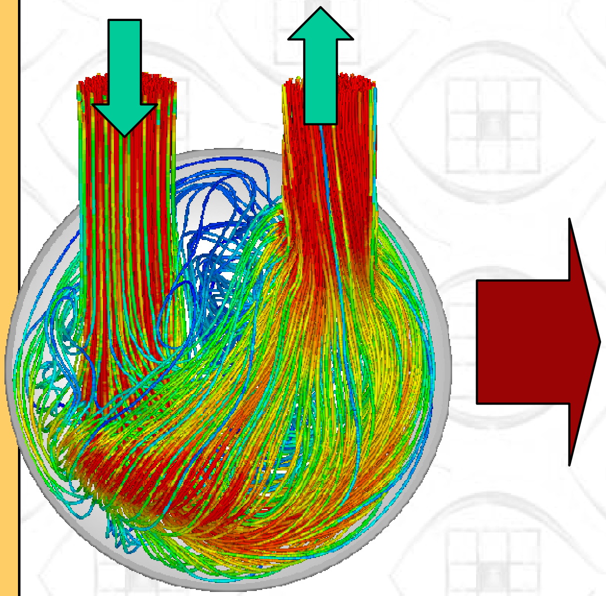
Motion Models – Recent Ideas

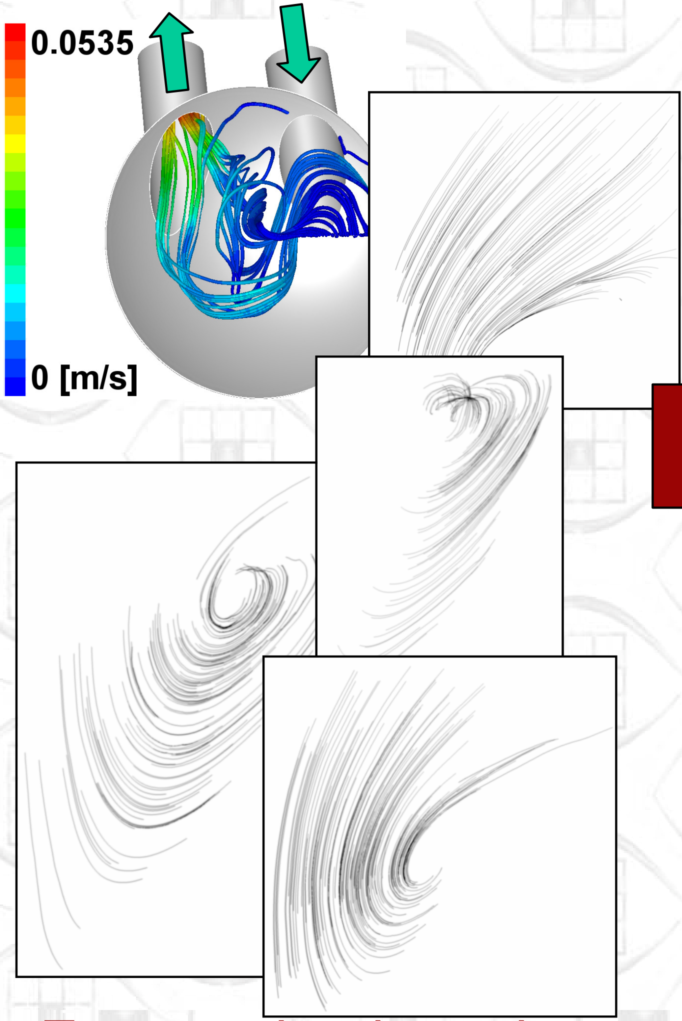
- Observation:
  - Optical Flow Algorithms are chosen application-dependent
  - Brightness and Motion model according to physics
    - Fluid Dynamics:
      - Motion: incompressible Navier-Stokes-Regularization
      - BCCE: Lambert-Beer's law of brightness changes of light in fluids
    - Driver Assistance/Robotics:
      - Motion: Affine transformations in 3d (Buildings, Street, Cars, …)
      - BCCE: Usually brightness constany (emerging: linear brightness changes)
  - Requires **adaptable** optical flow algorithms
- Idea (Black, Yacoob, Roth, …):
  - Learn **application-specific motion models** from examples, obtained by
    - Very slow, but highly accurate existing methods
    - Simulations (CFD, Computer Graphics, …)
    - Expensive laboratory measurements

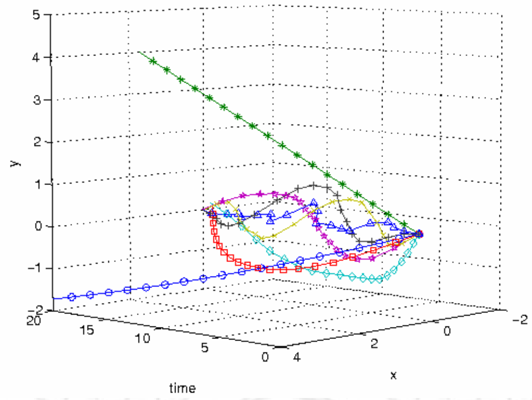
Motion Models – Trajectories

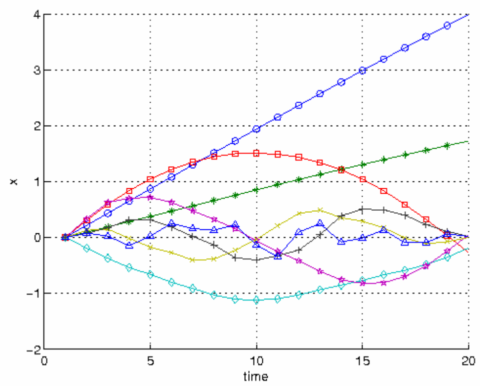
- Observations:
  - In many applications:
    - Motion is temporally consistent (e.g. due to inertia)
  - Regularization/Motion Models:
    - Essentially try to find pixels with the same flow
- Idea:
  - Learn a parameterization of application-specific **trajectories**
- Advantages:
  - A trajectory is physically meaningful and intuitive
  - One trajectory result comprises the flow of a whole sequence of images
  - No problems with motion discontinuities
  - Fewer parameters are needed to describe longer trajectories
  - Optimization can be carried out in a much smaller parameter space

Trajectories – Training







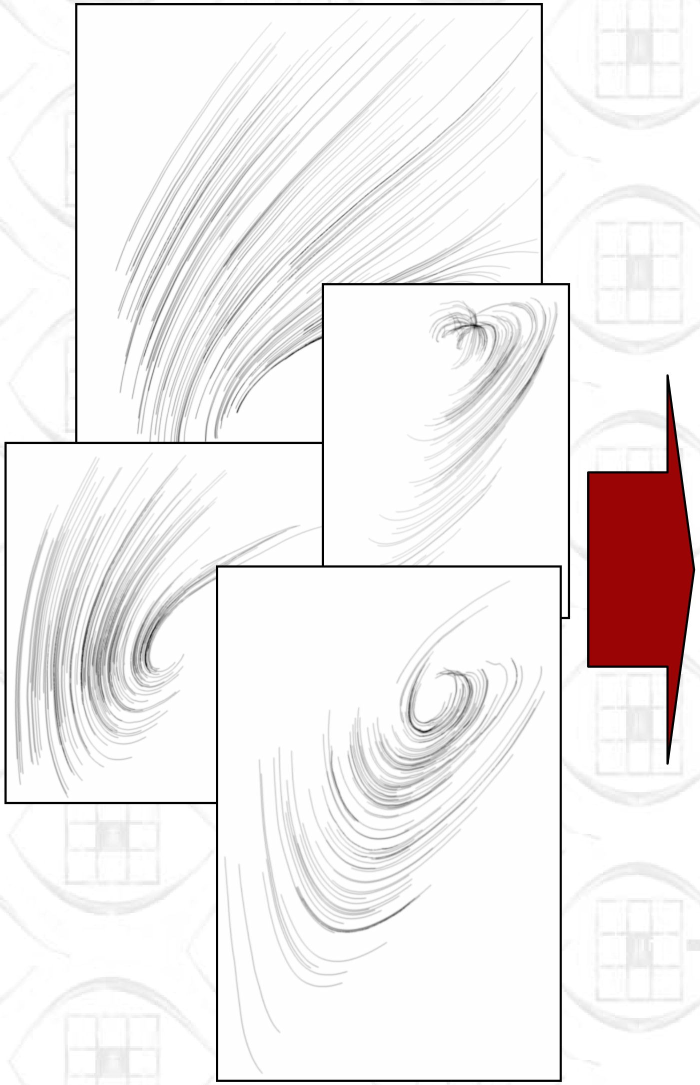


CFD Simulation
of real device

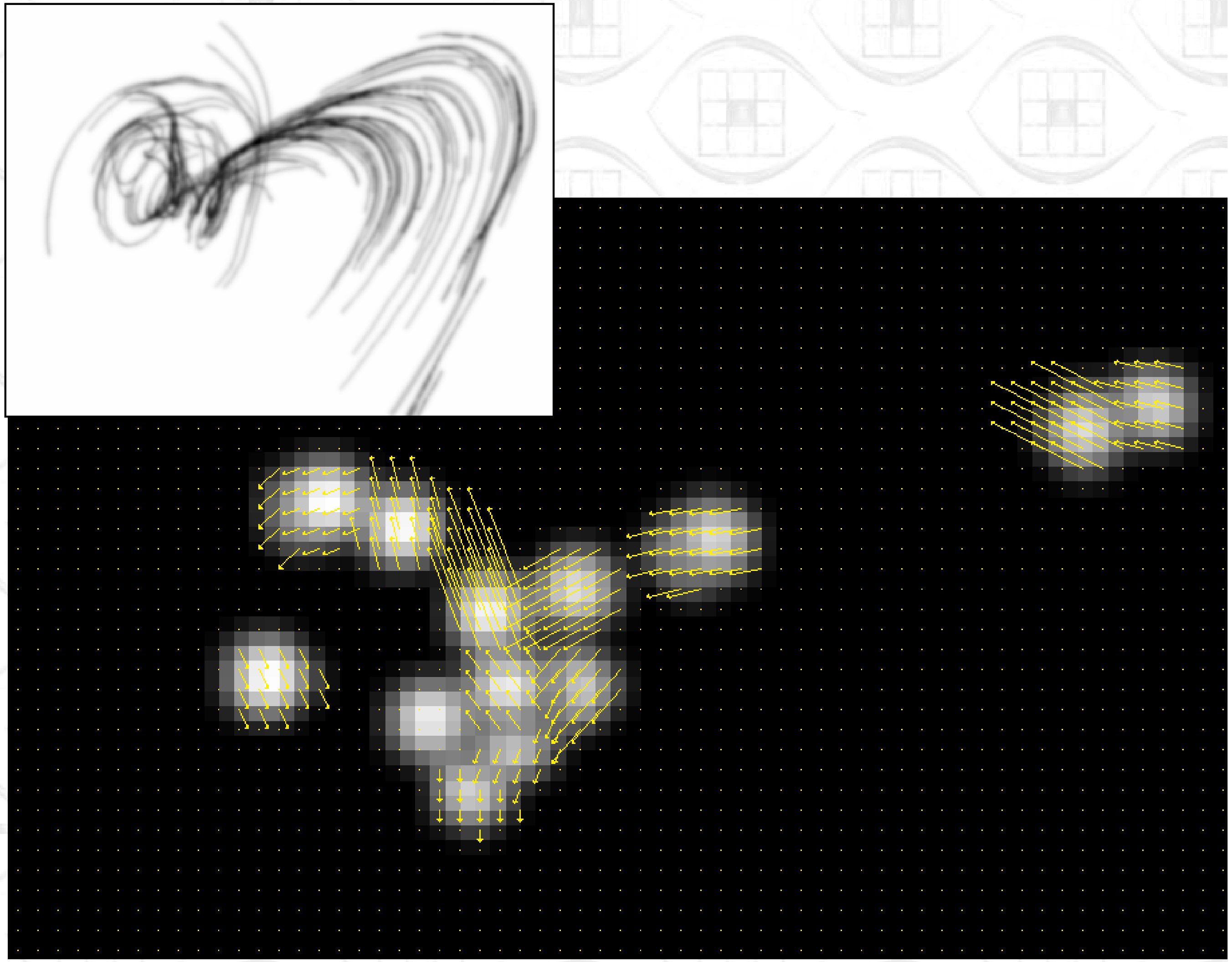
Extracted trajectories
for training

Basis trajectories
of PCA-Model

Trajectories – Experiments I



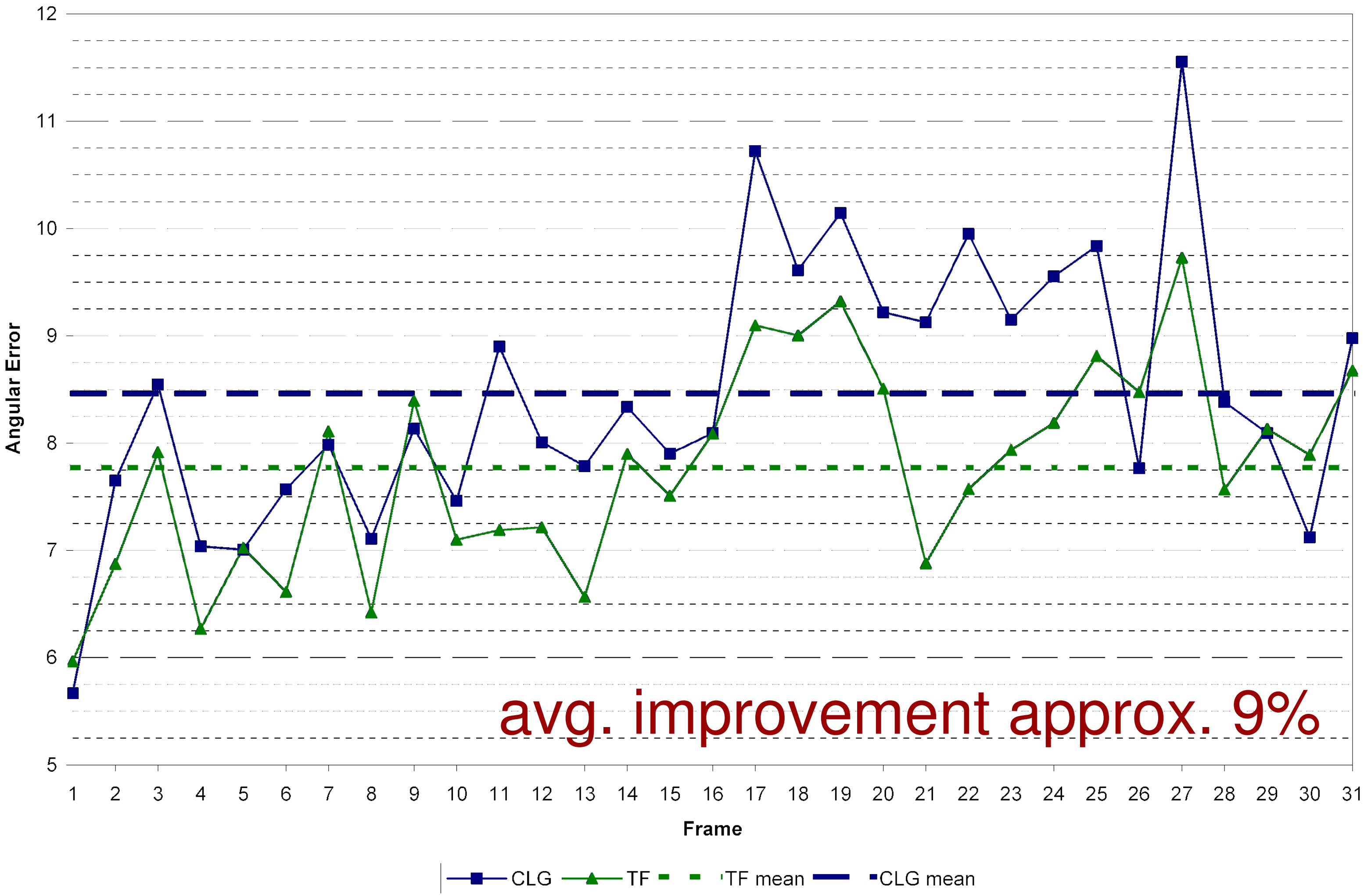
Training Data



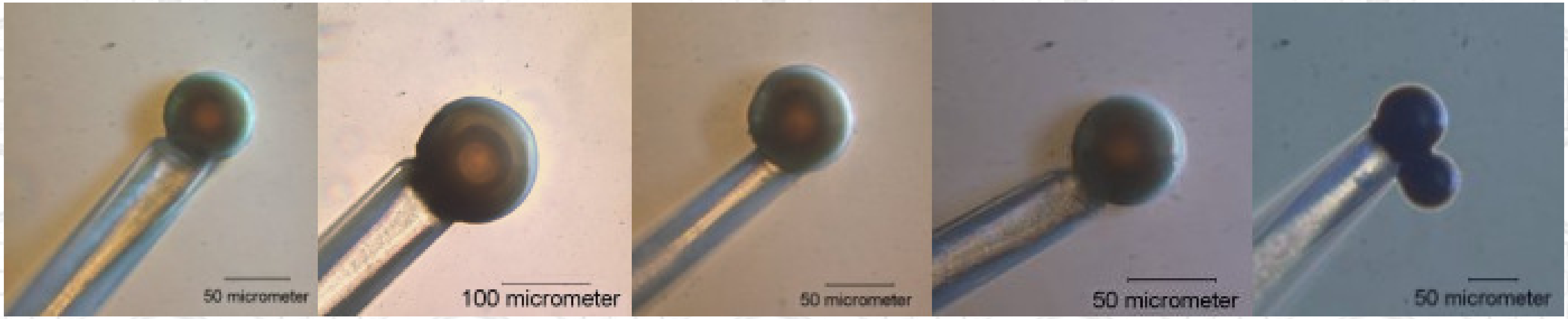
Test Data

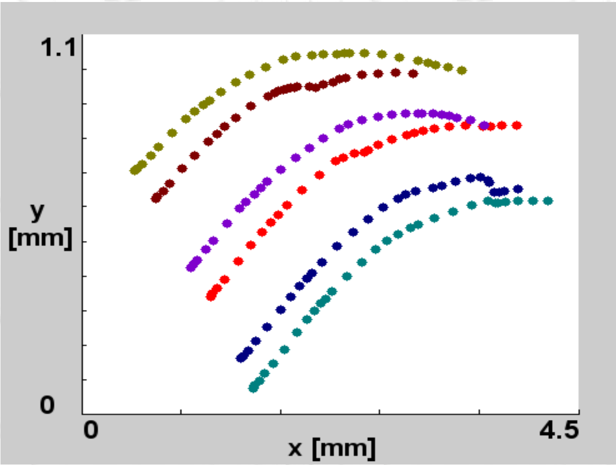
Trajectories – Results I

- Using Gradient Descent for the nonlinear optimization problem:



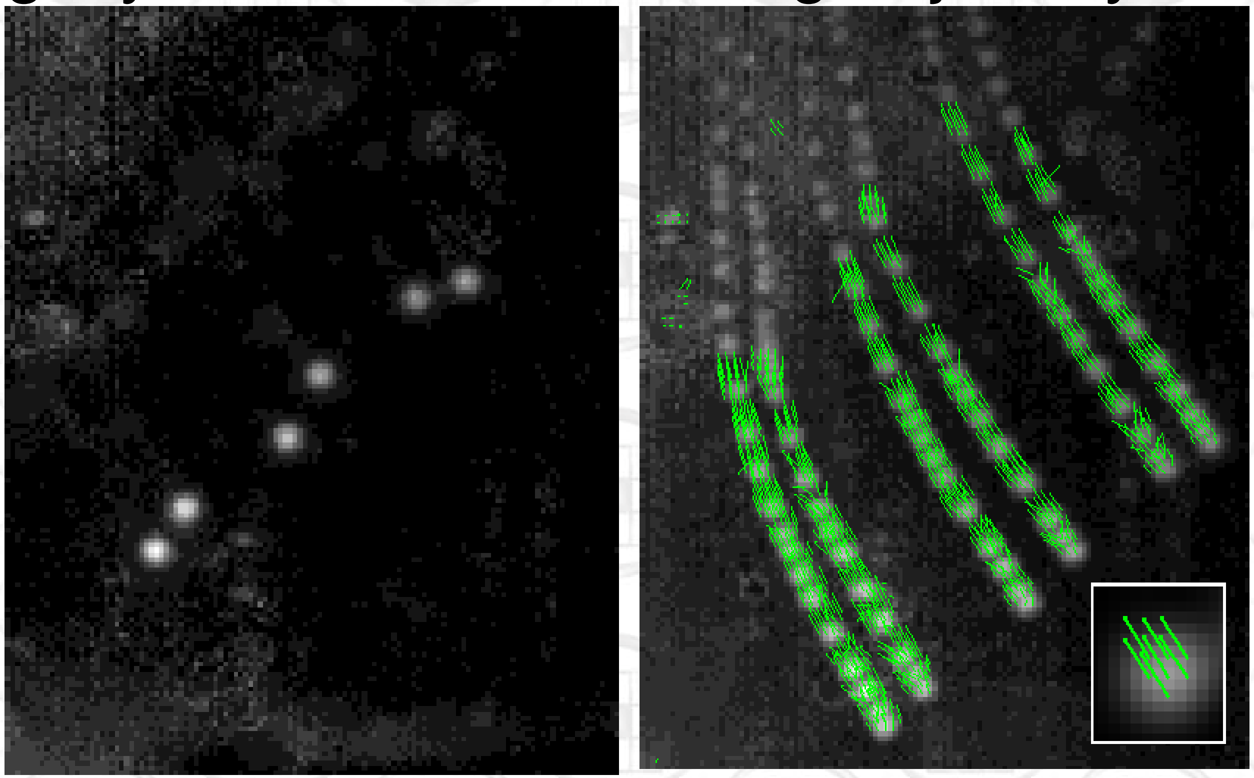
Trajectories – Experiments II





Trajectories – Results II

- Real world data without known accuracy
  - (slightly different model using trajectory ensembles)



D. Kondermann, A. Berthe et al.: Motion Estimation Based on a Temporal Model of Fluid Flows (ISFV 2008, to appear)
D. Kondermann, A. Berthe et al.: Trajectory Ensembles For Fluid Flow Estimation (submitted)

Summary – Part I

- Differential Optical Flow Estimation:

Local: $$\forall \vec{x} \in \Omega : \arg\min_{\vec{p}(\vec{x})} \sum_{\vec{x}' \in N(\vec{x})} (BCCE(M(\vec{p}(\vec{x}-\vec{x}'))))^2$$

Brightness Variation Model

Motion Model

Global: $$\arg\min_{\vec{p}(\vec{x})} \int_{\Omega} (BCCE(M(p(\vec{x}))))^2 + \lambda R(\vec{p}(\vec{x}))d\vec{x}$$

- PCA-Parameterized Trajectories
  - alternative motion model with interesting advantages

Part II: Confidence Measures and Postprocessing of Optical Flow Fields

Claudia & Daniel Kondermann

University of Heidelberg
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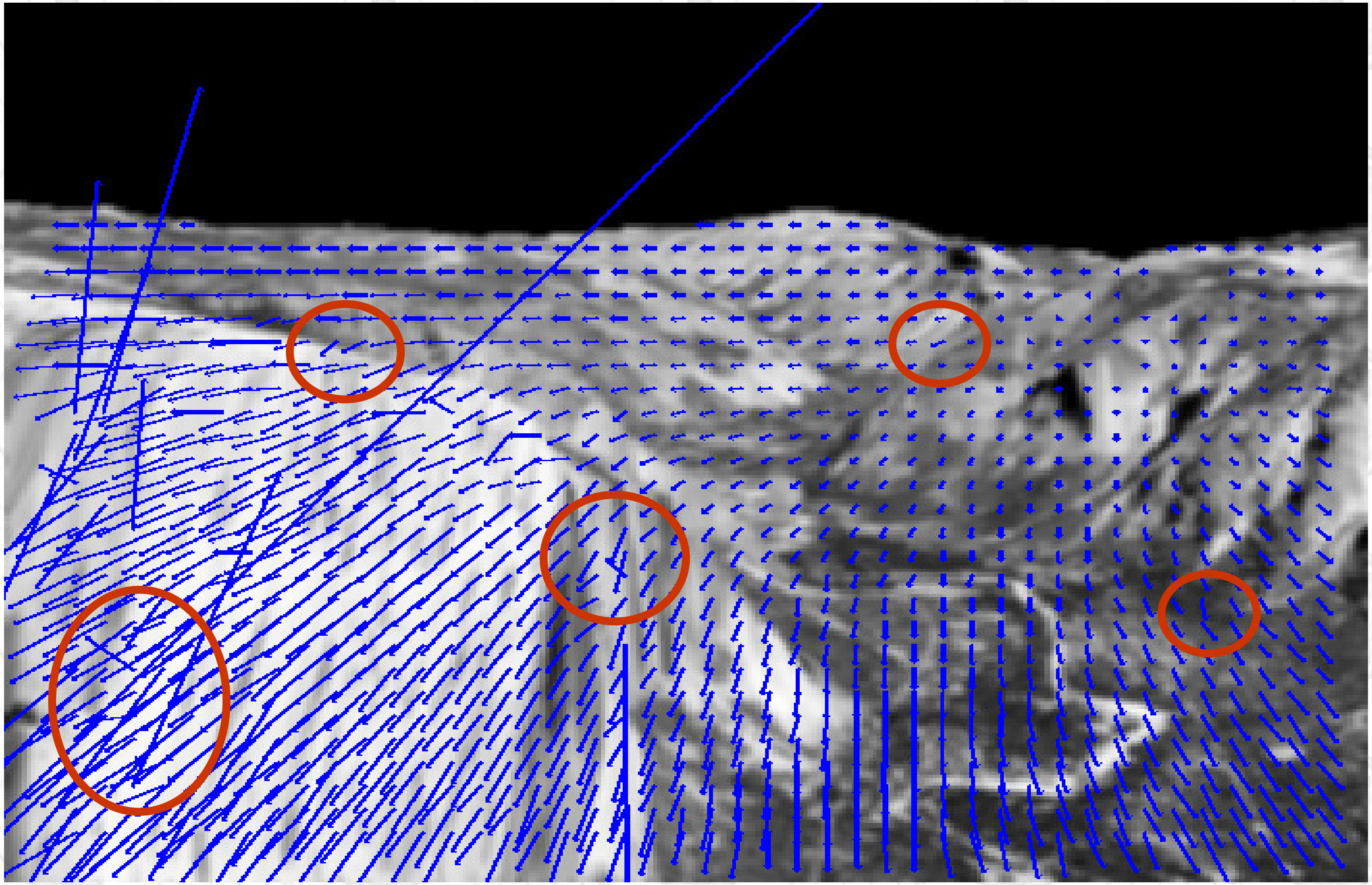
What we want to achieve…



Estimate accuracy of flow vectors

Confidence Measures

- Confidence measure $c: R^3 \times I \times F \to [0,1]$

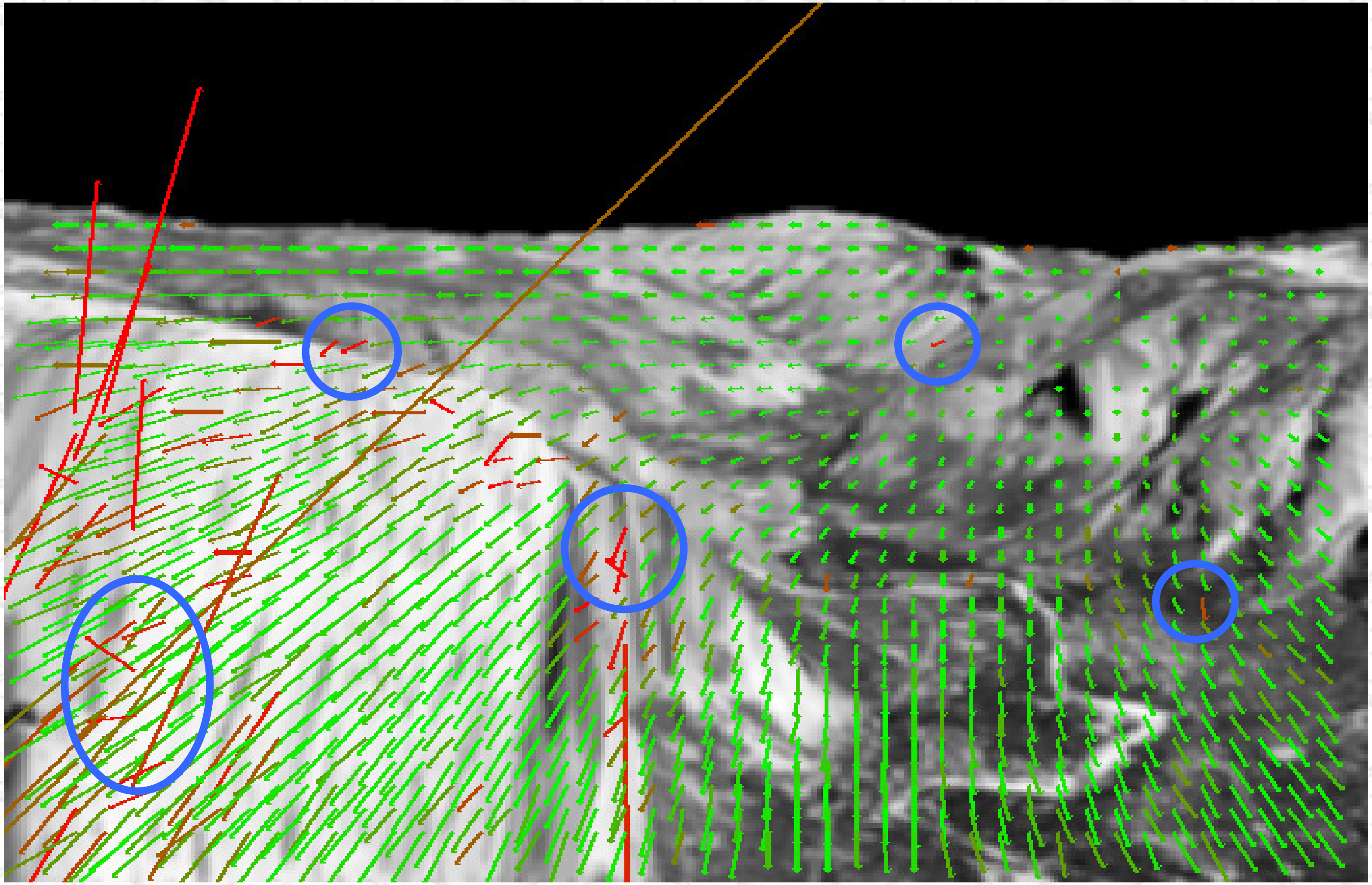
Image domain (R^3), Image sequence (I), Flow field (F)

- Global methods: Inverse of variational energy

$$c_{\text{ener}} = \frac{1}{D(u, v, \nabla_3 f) + \alpha\, S(\nabla_3 u, \nabla_3 v, \nabla_3 f) + \epsilon^2}$$

A. Bruhn, J. Weickert :"Confidence Measures for Variational Optic Flow Methods" (DAGM 2004)

Optimal Confidence for Yosemite Sequence

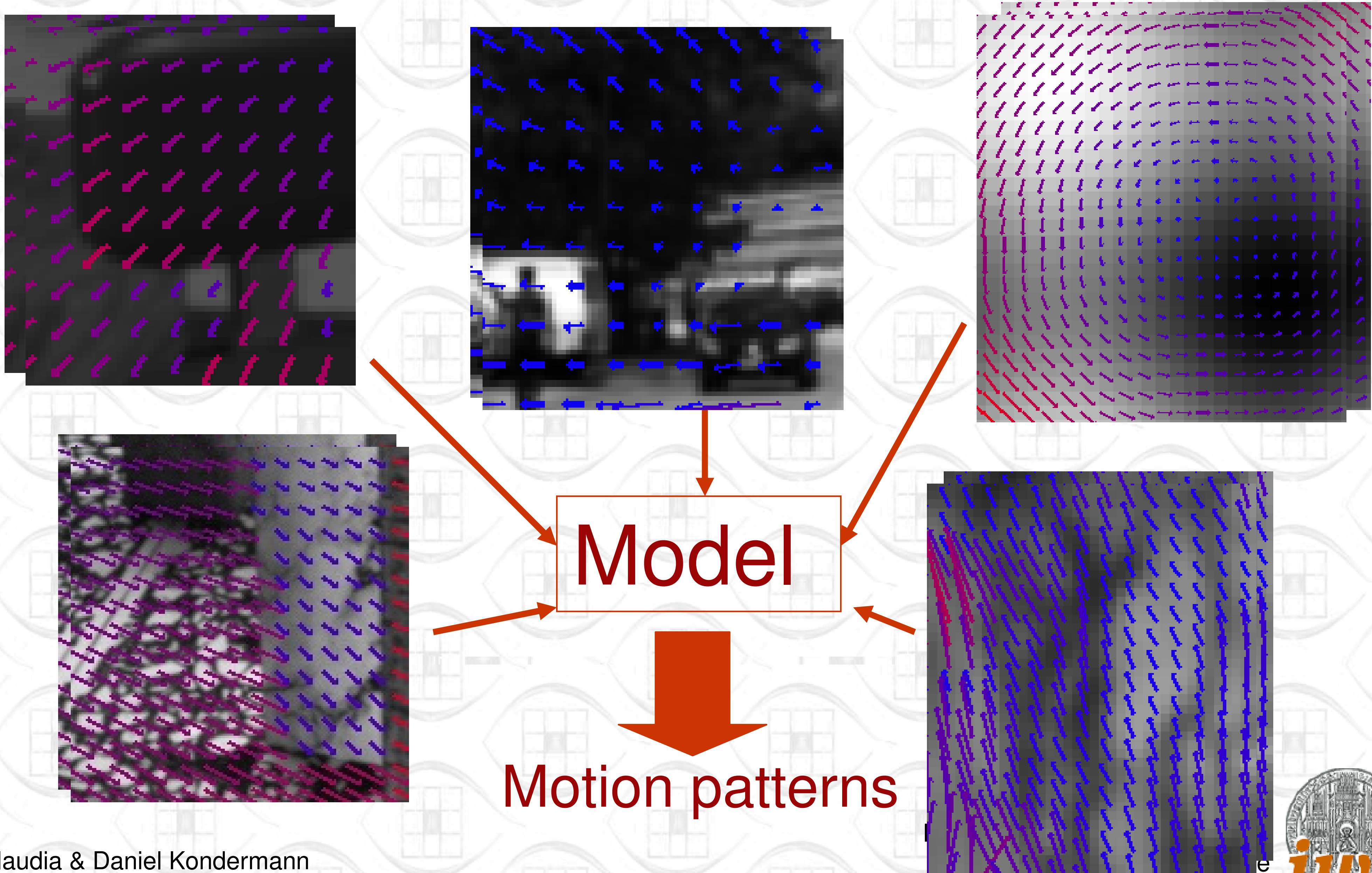


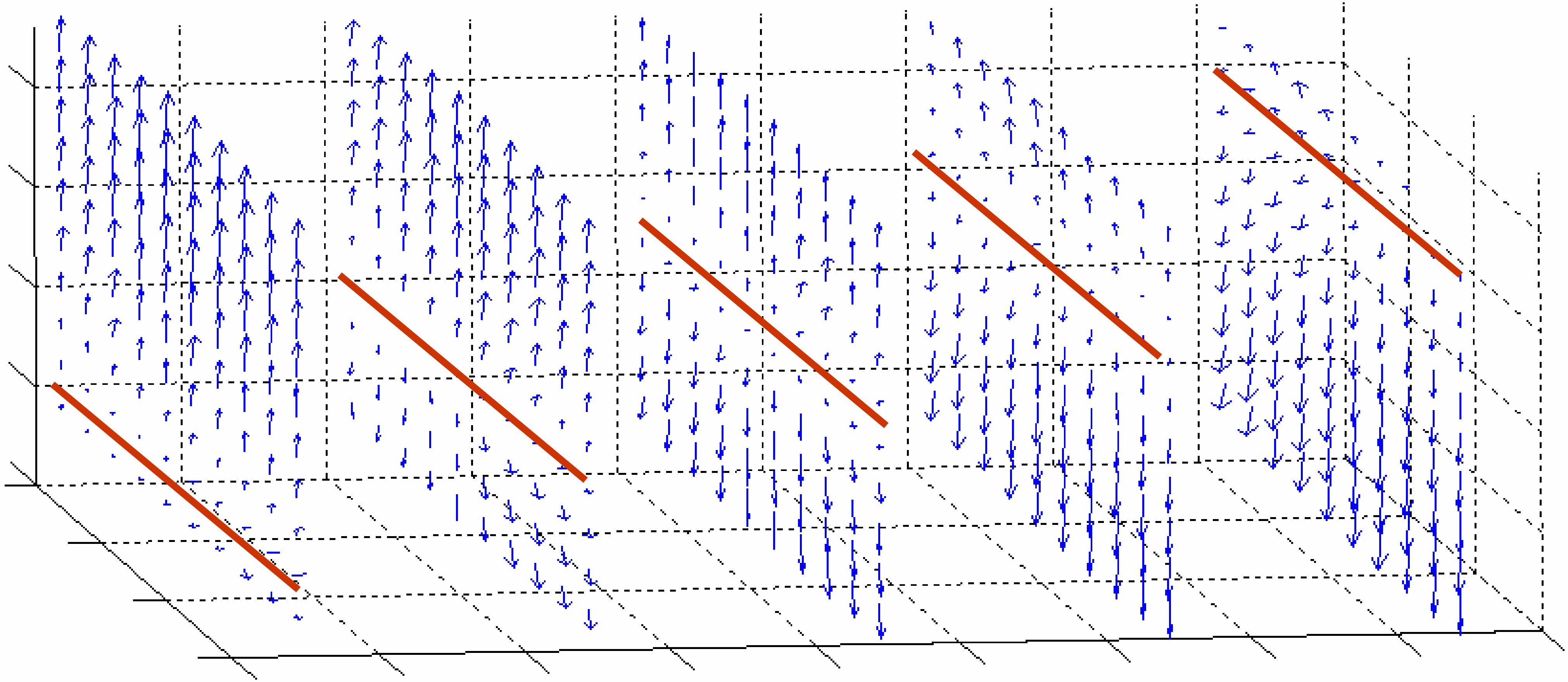
A new confidence measure...(1)

- “Learn“ correct/typical motion patterns from flow field neighborhood constellations
- Linear subspace of typical flow patches
  - e.g. Principal Component Analysis
- Spatio-temporal flow patches

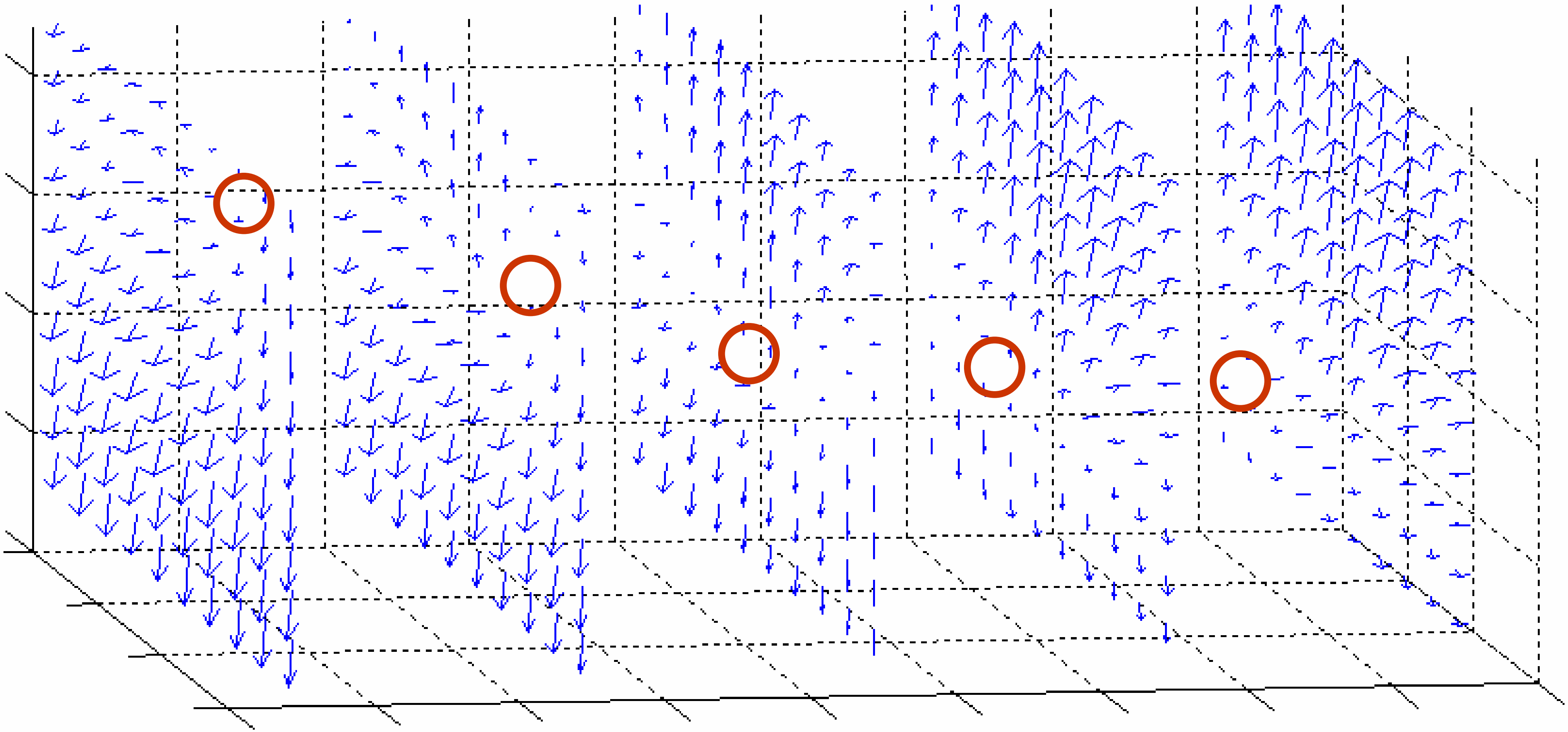
C. Kondermann, D. Kondermann, B. Jähne, C. Garbe: “An Adaptive Confidence Measure for Optical Flows Based on Linear Subspace Projections” (DAGM 2007)

A new confidence measure…(2)





Moving flow discontinuity



Moving divergence

pcaReconstruction Measure (1)

- 1. Projection of flow field patch into eigenspace

$$V = \begin{pmatrix} v_{11} & \cdots & v_{k1} \\ \vdots & \ddots & \vdots \\ v_{1n} & \cdots & v_{kn} \end{pmatrix} \qquad \vec{m} = \begin{pmatrix} m_1 \\ \vdots \\ m_n \end{pmatrix}$$

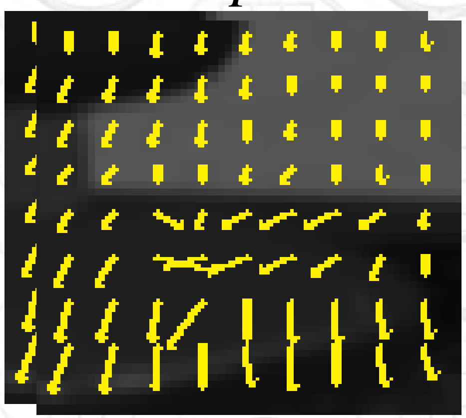
k eigenflows

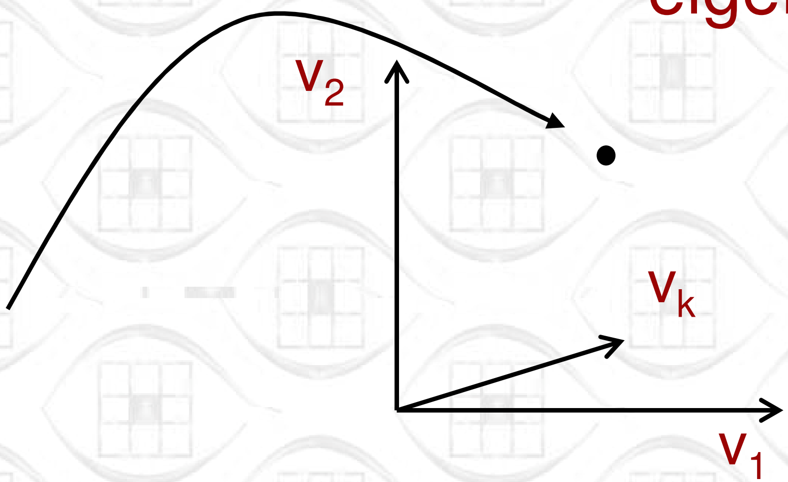
mean motion pattern

$$\vec{\beta}(\vec{u}_p) = V^T(\vec{u}_p - \vec{m})$$

eigenspace

u_p





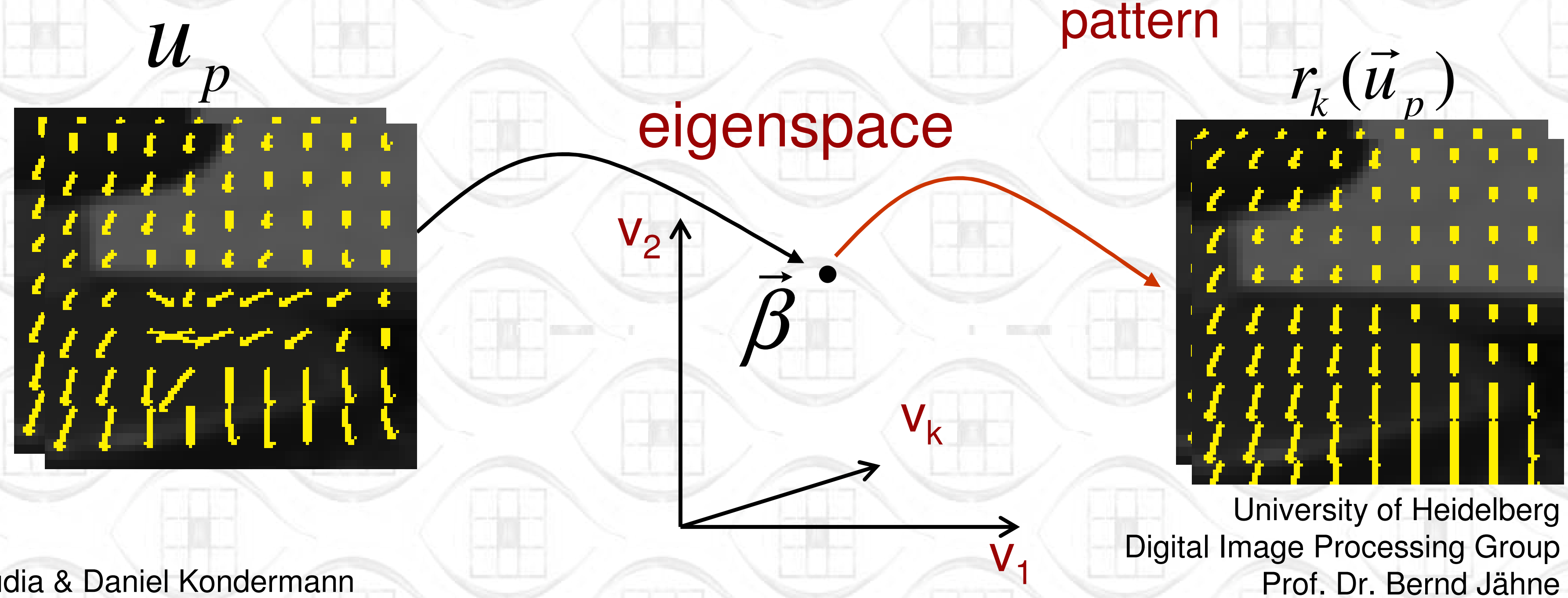
pcaReconstruction Measure (2)

- 2. Reconstruction of patch from linear subspace of correct flow patches

$$r_k(\vec{u}_p) = \sum_{i=1}^{k} \beta_i \vec{v}_i + \vec{m}$$

i-th eigenflow

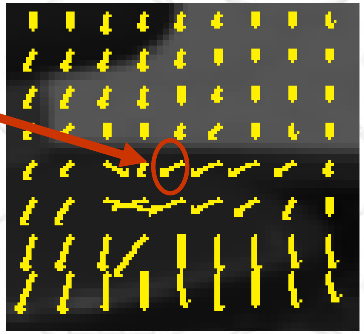
mean motion pattern



pcaReconstruction Measure (3)

- 3. Derive confidence

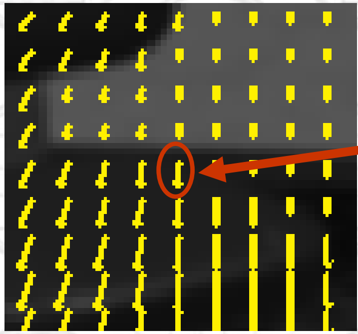
$\vec{u}(\vec{x})$



Computed patch

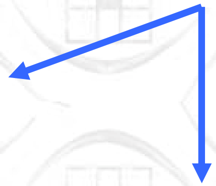
$\vec{u}_p$

$r_k(\vec{u}_p)(\vec{x})$



Reconstructed patch

$r_k(\vec{u}_p)$



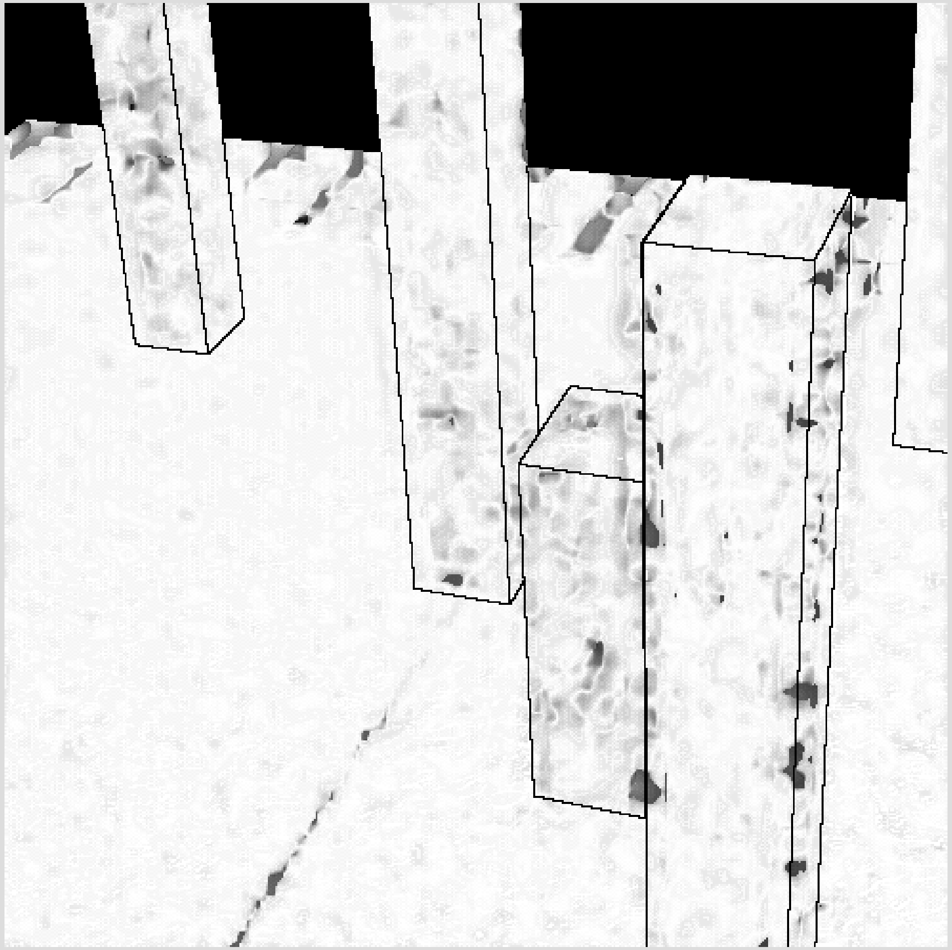
Angular error

$\alpha(\vec{u}(\vec{x}), r_k(\vec{u}_p)(\vec{x}))$

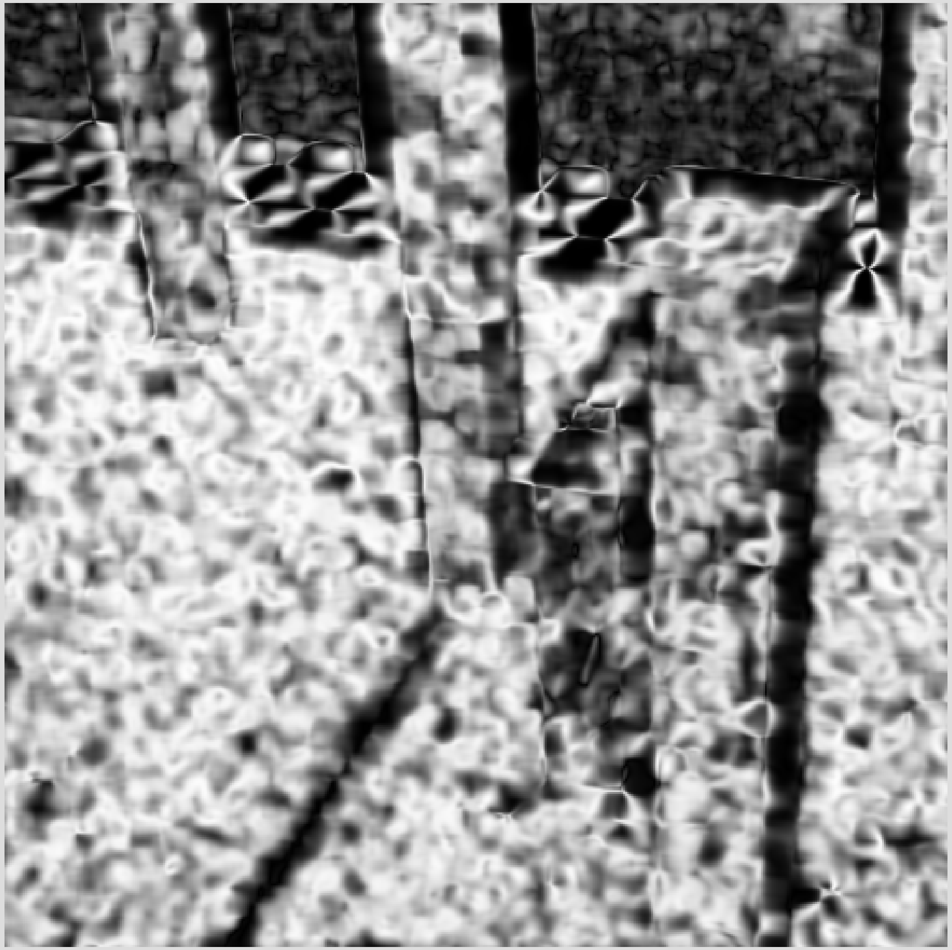
Confidence function:

$$c(\vec{x}, \vec{u}_p) = 1 - \frac{\alpha(\vec{u}(\vec{x}), r_k(\vec{u}_p)(\vec{x}))}{\pi}$$

Results (1)

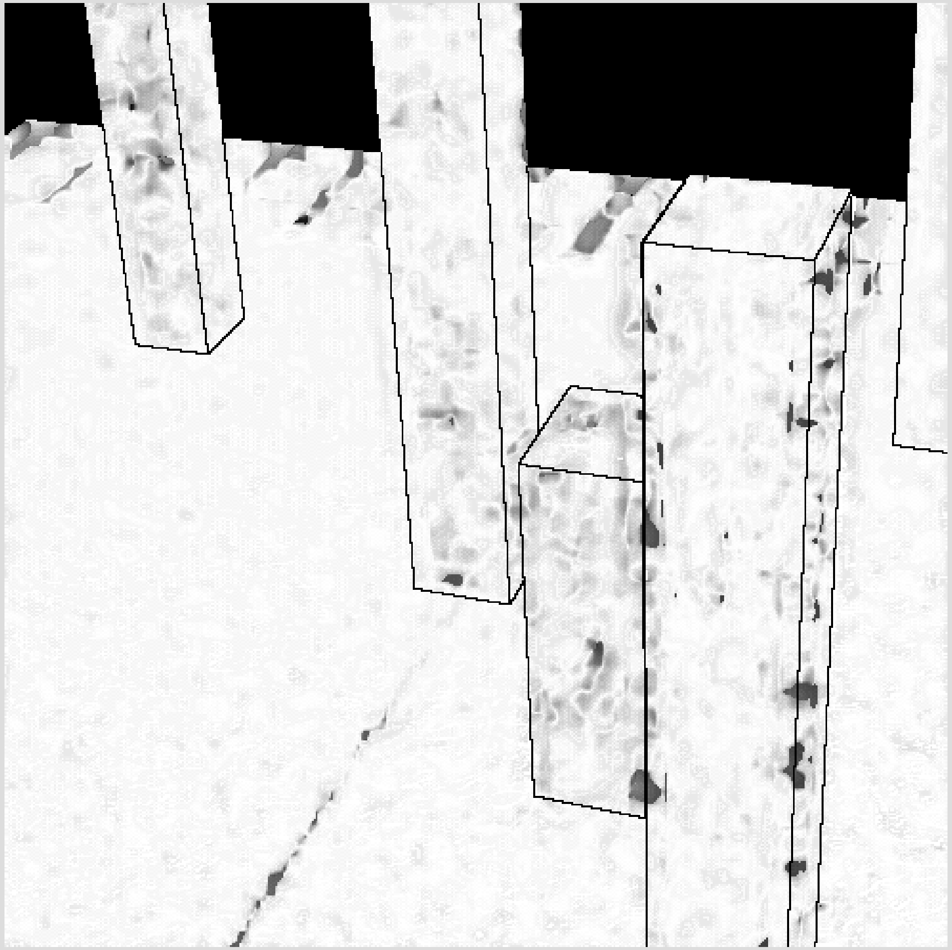


Optimal confidence

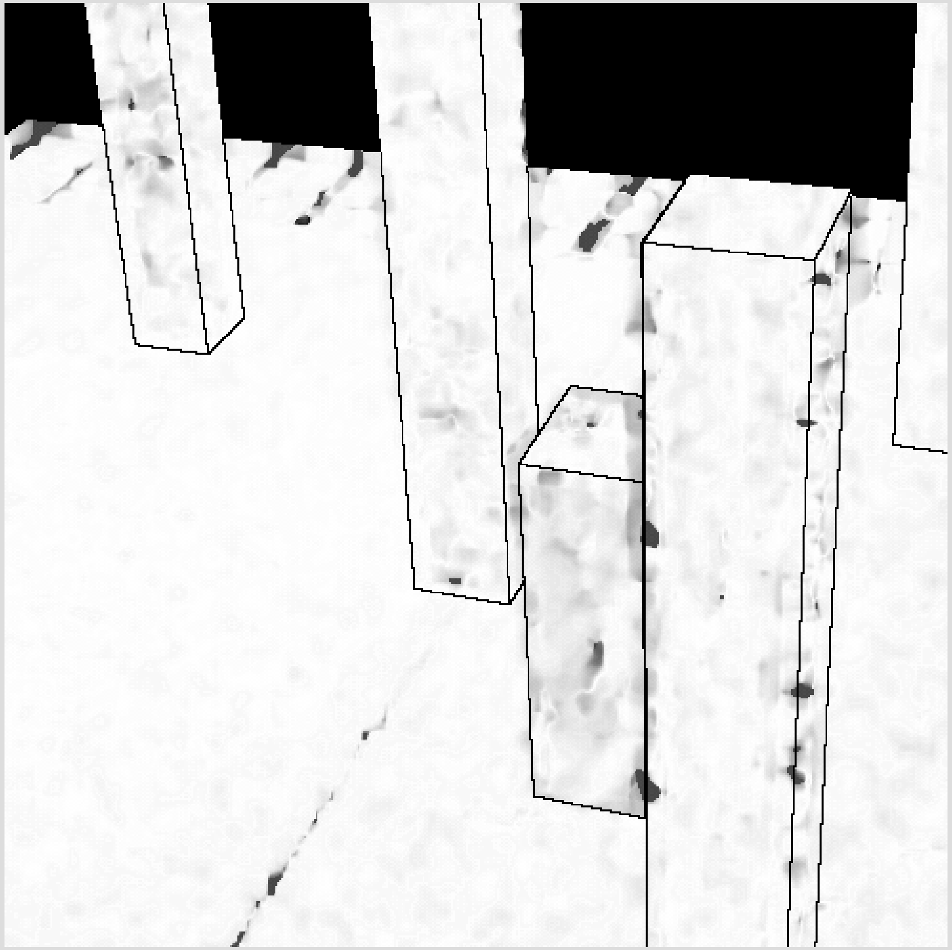


Corner measure of structure tensor

Results (2)



Optimal confidence



pcaReconstruction result

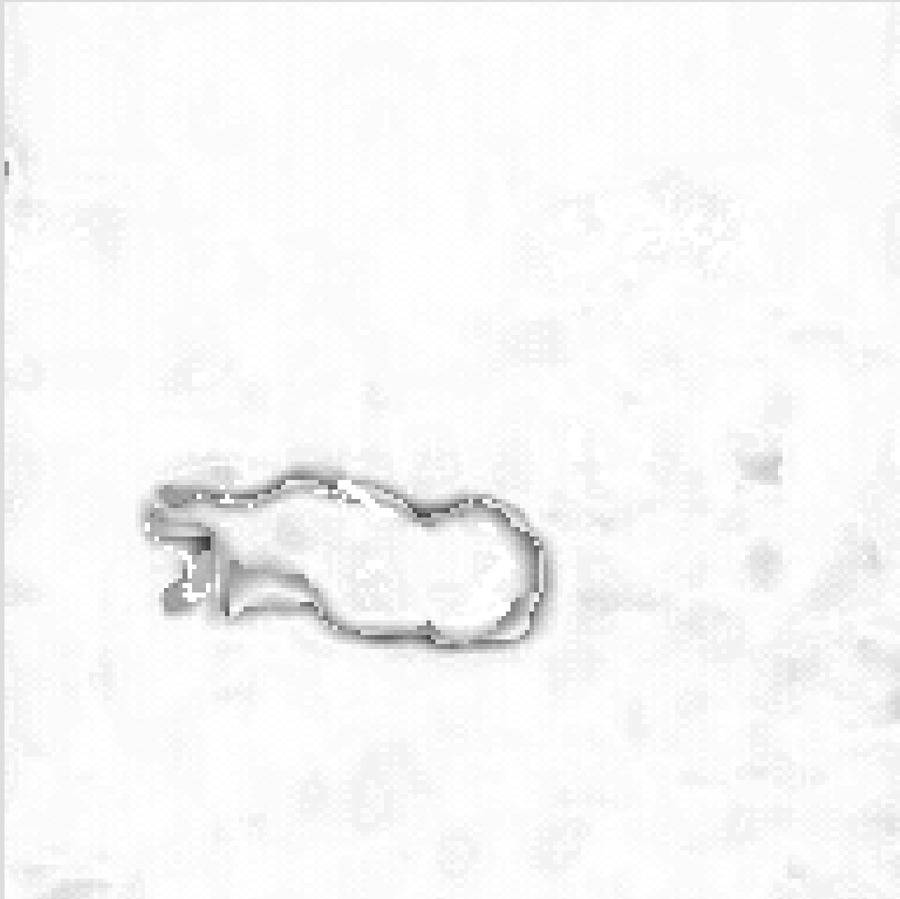
Results (3)

Optimal confidence

Gradient Measure

Results (4)

Optimal confidence



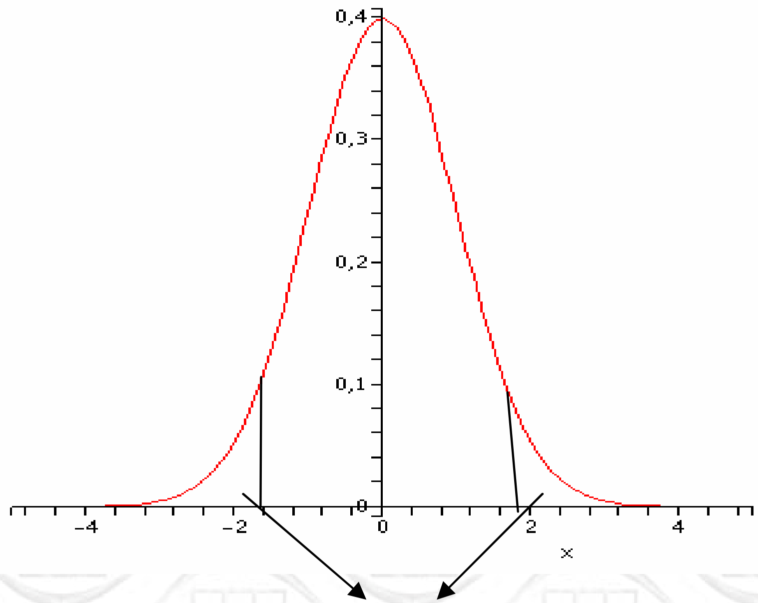
pcaReconstruction result

Evaluation

- Advantages
  - Method adaptable to all classes of flow fields
  - Temporal patterns (e.g. moving discontinuities) included
- Disadvantage
  - Incorrect flow fields following model not detected

Probabilistic Approach (1)

- Covariance matrix and mean motion model learned from sample data
- Mahalanobis distance is optimal test statistic in case of normally distributed data
- Estimate critical values from ground truth data
- Hypothesis test: reject sample if outside confidence region

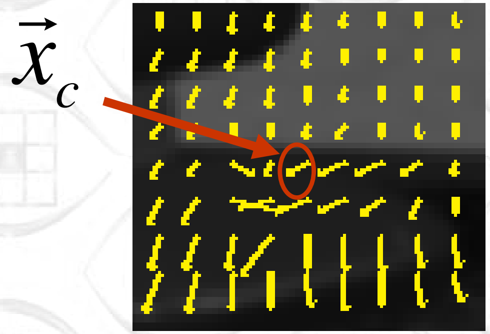


Rejection regions

C. Kondermann, R. Mester, C. Garbe:
„A statistical confidence measure for optical flows“ (Submitted)

Probabilistic Approach (2)

- Condition on central vector to minimize influence of neighbors

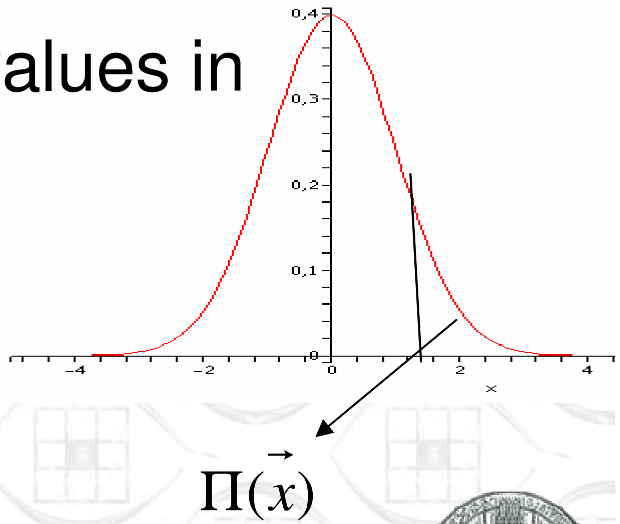


$$p(\vec{x}_c \mid \vec{x}_r) = \frac{p(\vec{x})}{p(\vec{x}_r)} \sim N(C', \mu')$$

- Compute p-value to obtain confidence values in range [0,1]

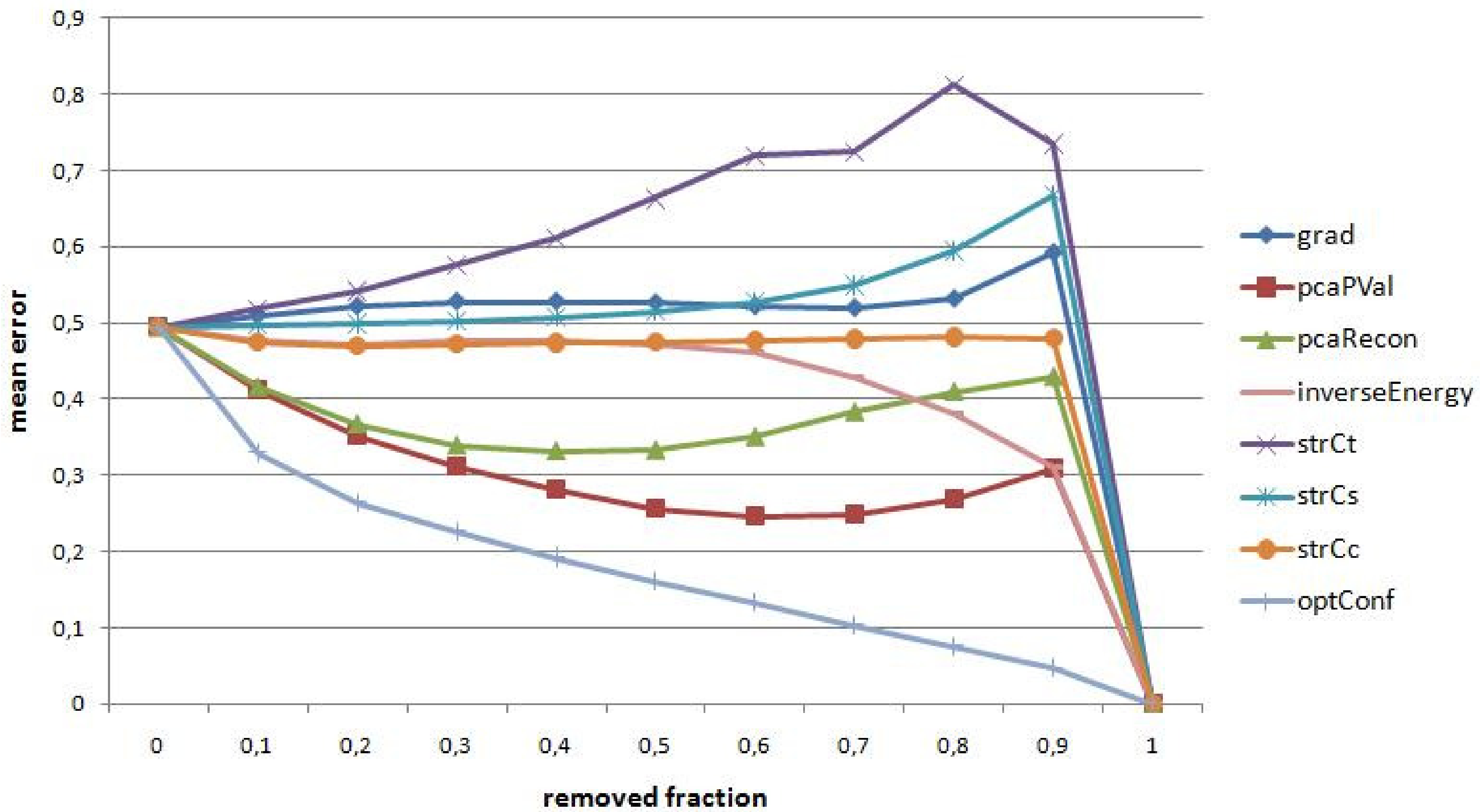
$$\Pi : R^n \rightarrow [0,1]$$

$$\Pi(\vec{x}) = \inf(\alpha \mid \varphi_\alpha(\vec{x}) = 1)$$



Results

Empirical error quantile function



Reconstruction of Optical Flow Fields

- Confidence measures to recognize difficult locations
- Remove difficult flow vectors
- Use inpainting to reconstruct flow

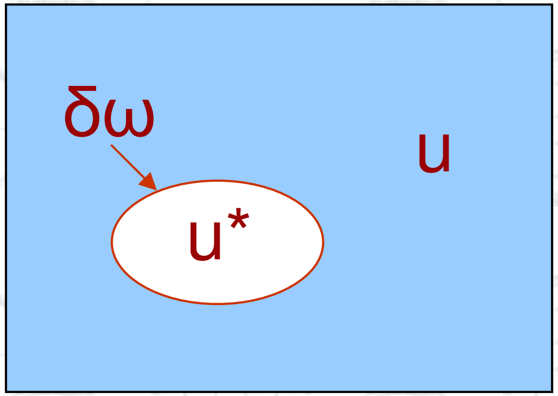
C. Kondermann, D. Kondermann, C. Garbe:
"Postprocessing of Optical Flows via Surface Measures and Motion Inpainting" (DAGM 2008)

Flow Inpainting

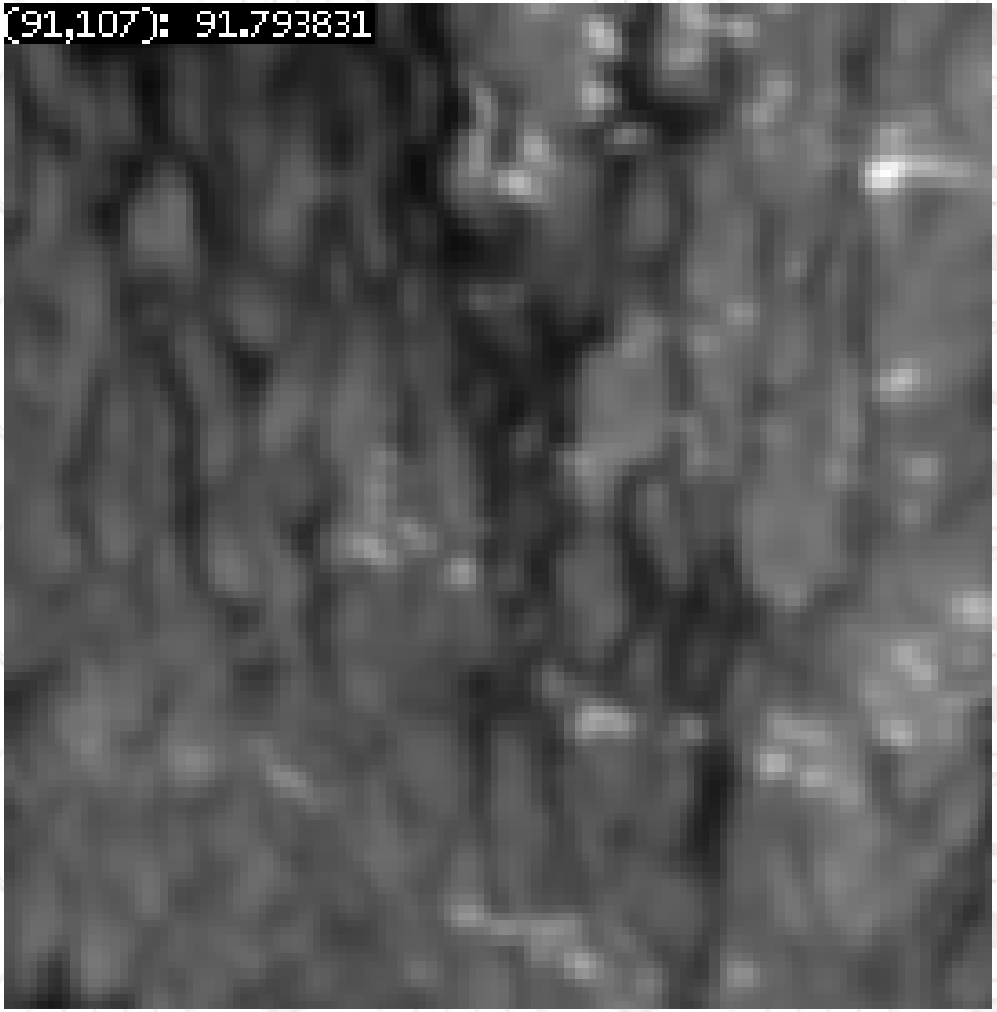
- Idea:
  - Reconstruct flow at unreliable locations indicated by confidence measure

$$\min \int_{\omega} \| \nabla_3 u^* \|_2^2 \, dx \, dy \qquad \text{with} \quad u^* |_{\partial\omega} = u |_{\partial\omega}$$

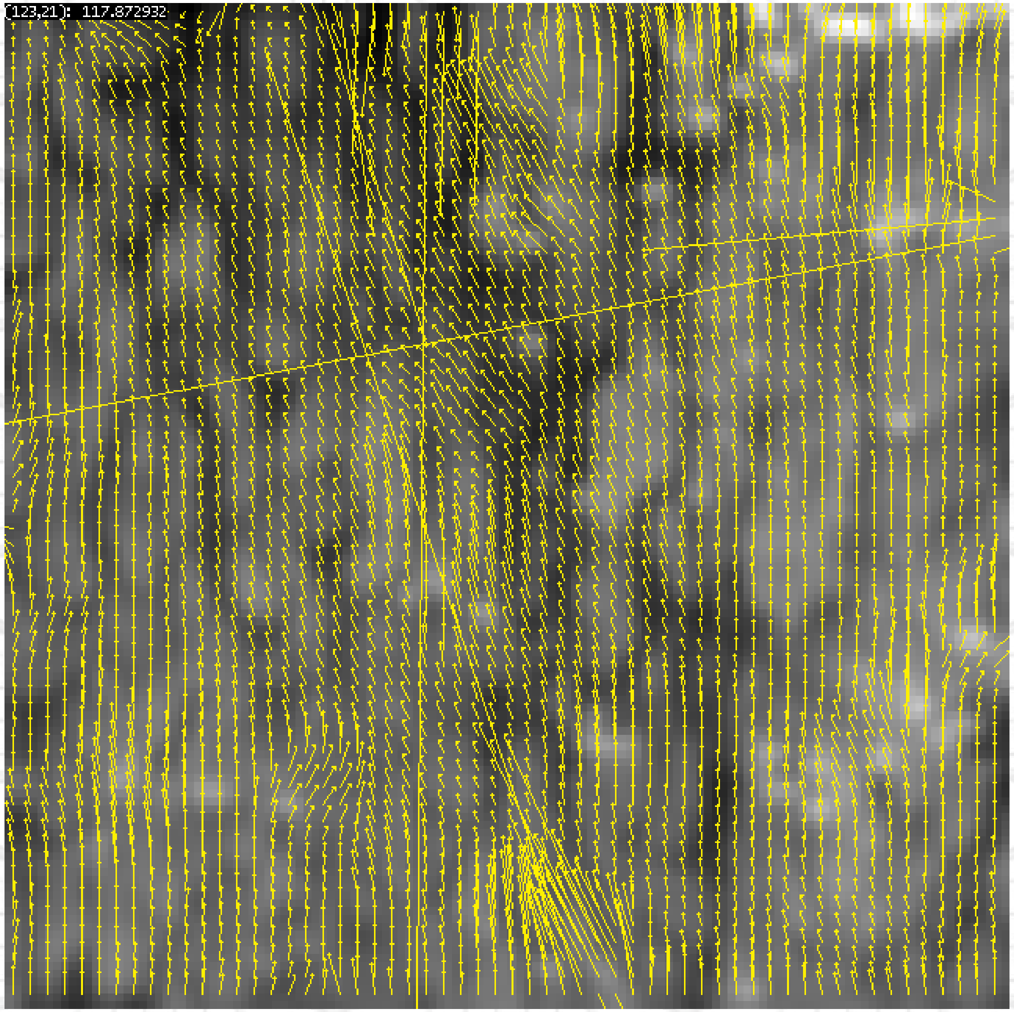
u: given flow field
u*: flow region to be reconstructed
δω: edge of flow region



Water Surface Example

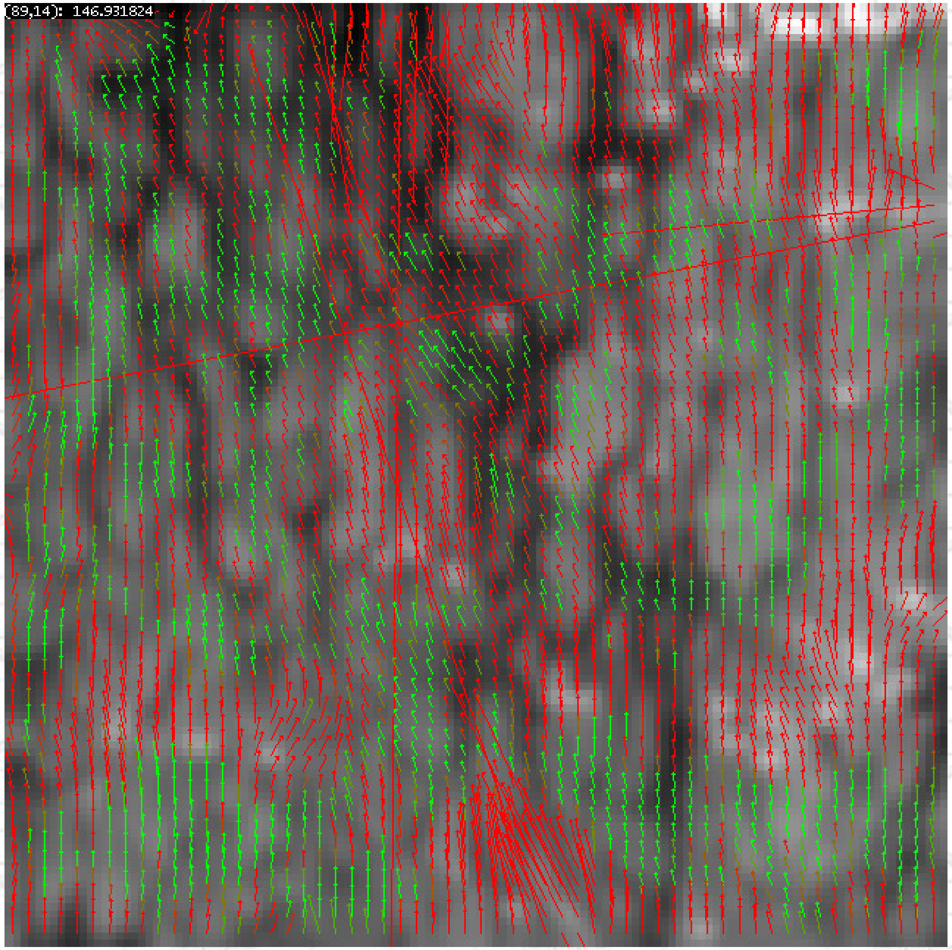


Sequence

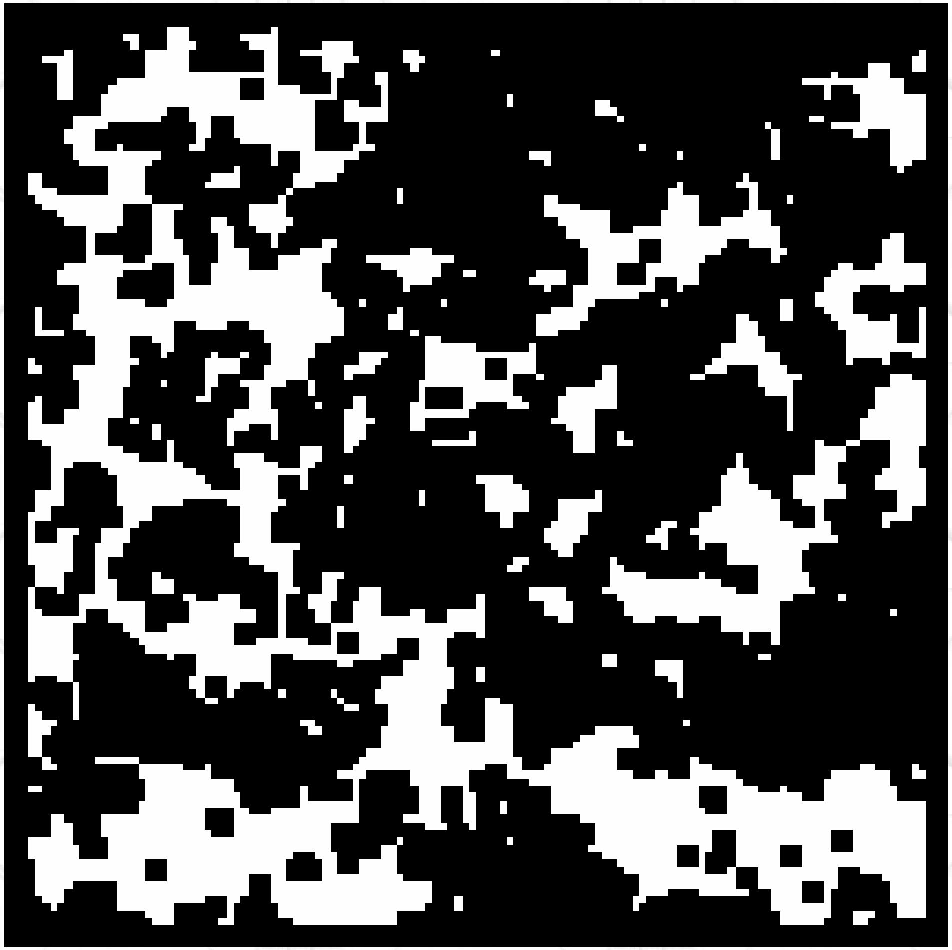


Computed flow field

Application of Confidence Measure

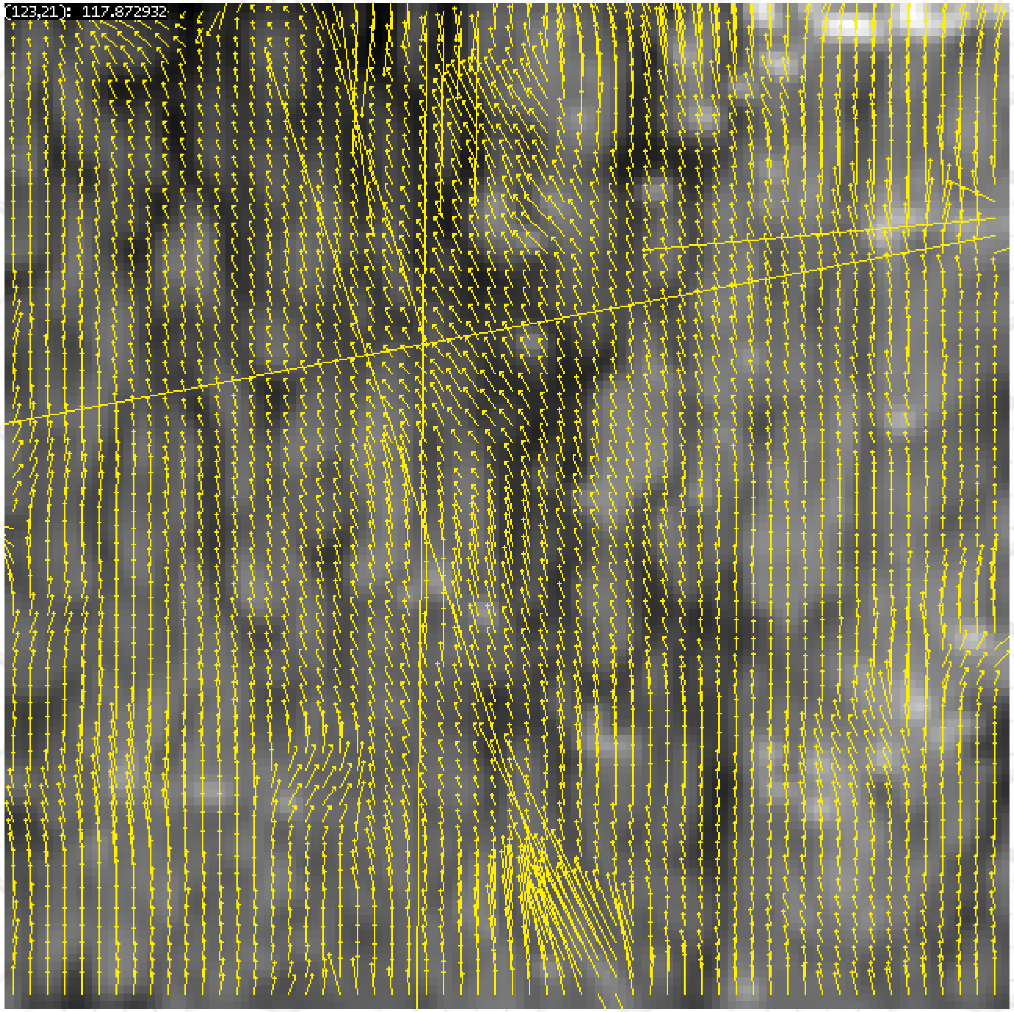


Confidence measure map

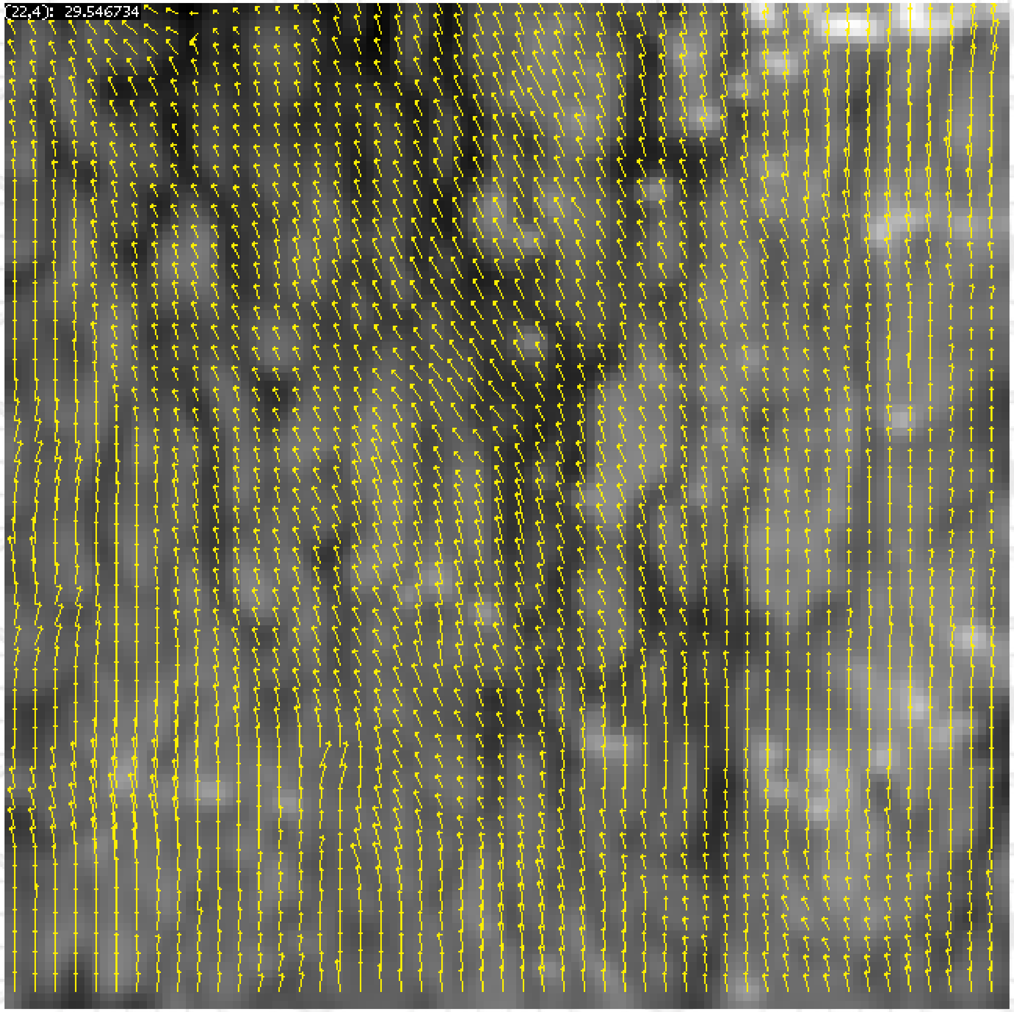


Binarized map

Results



Original field



Field after flow inpainting

Results

CLG	original	inpainting
Marble	3.88 ± 3.39	3.87 ± 3.38
Yosemite	4.13 ± 3.36	3.85 ± 3.00
Street	8.01 ± 15.47	7.73 ± 16.23
Office	3.74 ± 3.93	3.59 ± 3.93
ST	original	inpainting
Marble	4.49 ± 6.49	3.40 ± 3.56
Yosemite	4.52 ± 10.10	2.76 ± 3.94
Street	5.97 ± 16.92	4.95 ± 13.23
Office	7.21 ± 11.82	4.48 ± 4.49

Contrary to widely accepted opinion:
Local methods after postprocessing can perform better!

Summary – Part II

- New confidence measure
  - Computation of a flow model
  - Projection of flow patch into parameter space
  - Backprojection from parameter space
  - Confidence assignment
  - Statistical approach via hypothesis test
- Reconstruction of flow fields by means of inpainting

Thank you for your attention!