Optic Flow Computation with High Accuracy

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joint work with

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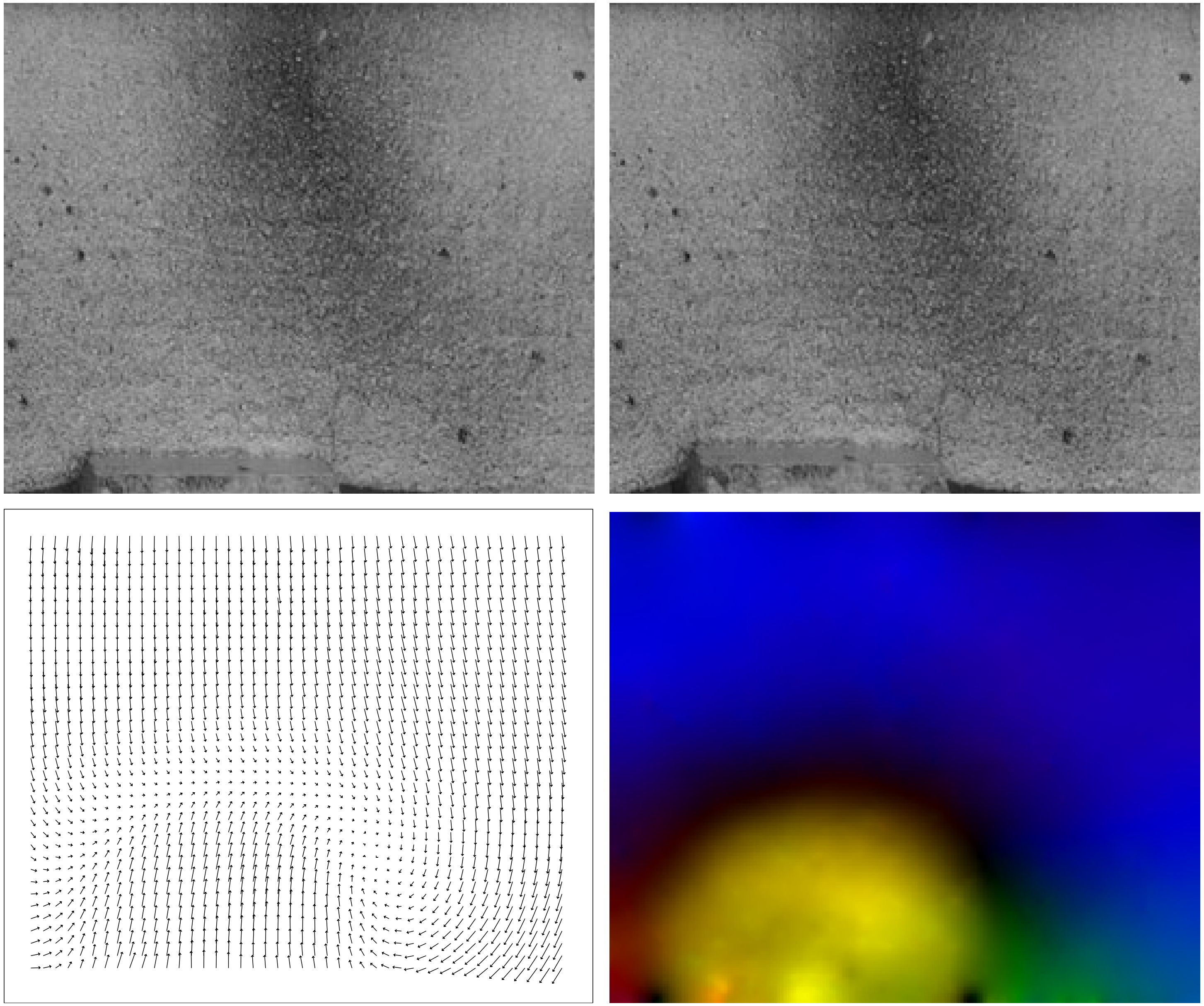
The Optic Flow Problem

- *given:* – image sequence $I(\mathbf{x})$ where $\mathbf{x} = (x, y, t)^\top$
  – can be Gaussian-smoothed: $I = K_\sigma * I_0$
- *wanted:* – displacement field *(optic flow)* $\mathbf{w} = (u, v, 1)^\top$
  – $\mathbf{w}$ matches object at location (x, y) at time t to its location $(x{+}u,\ y{+}v)$ at time $t+1$.

What is Optic Flow Good for?

- extracting motion information e.g. in robotics
- compact coding of image sequences
- related correspondence problems in computer vision: e.g. stereo reconstruction and medical image registration

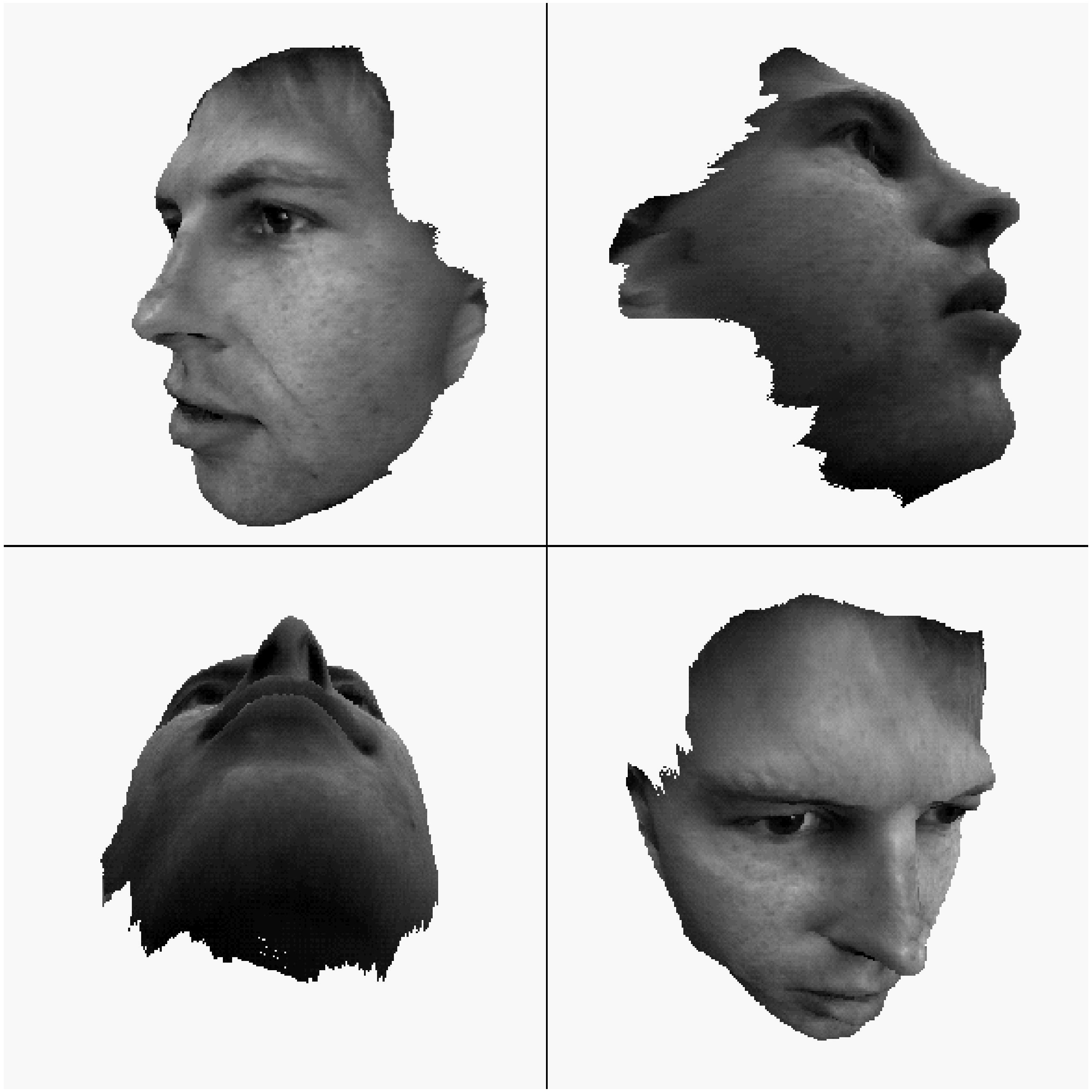
Introduction (2)



Deformation analysis of plastic foam using an optic flow method. (a) **Top left:** Frame 1 of a deformation sequence. (b) **Top right:** Frame 2. (c) **Bottom left:** Colour-coded displacement field. (d) **Bottom right:** Vector plot of the displacement field.

Introduction (3)

Pair of stereo images.



Four views of a stereo reconstruction algorithm based on optic flow ideas. **Authors:** Alvarez/Deriche/Sánchez/Weickert (2002)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

Variational Optic Flow Methods

- optic flow as minimiser of a suitable energy functional:
  data constraints plus smoothness constraints
- clear formalism without hidden model assumptions
- rotationally invariant continuous formulations possible
- create dense flow fields
- first model due to Horn and Schunck (1981),
  but many improvements in the meantime:
  - modified data and smoothness constraints
    (Nagel 1983, Cohen 1993, Alvarez et al. 1999, W./Schnörr 2000)
  - theoretical foundation (Snyder 1991, W./Schnörr 2000)
  - efficient numerical algorithms
    (Glazer 1984, Terzopoulos 1986, Ghosal/Vaněk 1996, Bruhn et al. 2003)
- competitive performance

Open Problems

- further improvements possible ?
- some very good methods use strategies that lack theoretical foundation

Goals

- presentation of an optic flow algorithm with very good performance
- theoretical justification of widely used warping technique

Some Related Work

- L. Alvarez, J. Weickert, and J. Sánchez, *IJCV* 2000.
- M. Lefébure and L. D. Cohen, *JMIV* 2001.
- E. Mémin and P. Pérez, *ICCV* 1998.

Outline

- Variational Model
- Algorithmic Aspects
- Relations to Warping
- Evaluation
- Conclusions

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- **Variational Model**
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Variational Model (1)

- Basic assumptions

• Greyvalue constancy	$I_{\mathbf{w}} \;:=\; I(\mathbf{x}+\mathbf{w}) - I(\mathbf{x}) \;=\; 0$
• Gradient constancy	$I_{x\mathbf{w}} \;:=\; \partial_x I(\mathbf{x}+\mathbf{w}) - \partial_x I(\mathbf{x}) \;=\; 0$ $I_{y\mathbf{w}} \;:=\; \partial_y I(\mathbf{x}+\mathbf{w}) - \partial_y I(\mathbf{x}) \;=\; 0$
• Spatio-temporal smoothness	$\lvert\nabla u\rvert^2 + \lvert\nabla v\rvert^2 \;=\; 0$ $\nabla = (\partial_x, \partial_y, \partial_t)^\top$
• Robustness	$\Psi\left(s^2\right) \;=\; \sqrt{s^2+\epsilon^2}$

- Energy to minimise:

$$E(u,v) \;=\; \int_\Omega \Psi\left(I_{\mathbf{w}}^2 + \gamma\cdot\left(I_{x\mathbf{w}}^2 + I_{y\mathbf{w}}^2\right)\right)\mathbf{dx} \;+\; \alpha\int_\Omega \Psi\left(\lvert\nabla u\rvert^2 + \lvert\nabla v\rvert^2\right)\mathbf{dx}$$

Variational Model (2a)

Minimiser has to fulfill the Euler-Lagrange equations

$$
\begin{aligned}
&\alpha \, \mathrm{div} \left(\Psi'(|\nabla u|^2 + |\nabla v|^2) \, \nabla u\right) \\
&\quad = \Psi'\big(I_{\mathbf{w}}^2 + \gamma(I_{x\mathbf{w}}^2 + I_{y\mathbf{w}}^2)\big) \cdot \big(I_x I_{\mathbf{w}} + \gamma\big(I_{xx} I_{x\mathbf{w}} + I_{xy} I_{y\mathbf{w}}\big)\big) \\
&\alpha \, \mathrm{div} \left(\Psi'(|\nabla u|^2 + |\nabla v|^2) \, \nabla v\right) \\
&\quad = \Psi'\big(I_{\mathbf{w}}^2 + \gamma(I_{x\mathbf{w}}^2 + I_{y\mathbf{w}}^2)\big) \cdot \big(I_y I_{\mathbf{w}} + \gamma\big(I_{xy} I_{x\mathbf{w}} + I_{yy} I_{y\mathbf{w}}\big)\big)
\end{aligned}
$$

where the indices denote differences or partial derivatives:

$$
\begin{array}{llllll}
I_{\mathbf{w}} := I(\mathbf{x}+\mathbf{w}) - I(\mathbf{x}) & & I_x := \partial_x I(\mathbf{x}+\mathbf{w}) & & I_y := \partial_y I(\mathbf{x}+\mathbf{w}) \\
I_{x\mathbf{w}} := \partial_x I(\mathbf{x}+\mathbf{w}) - \partial_x I(\mathbf{x}) & & I_{xx} := \partial_{xx} I(\mathbf{x}+\mathbf{w}) & & I_{yy} := \partial_{yy} I(\mathbf{x}+\mathbf{w}) \\
I_{y\mathbf{w}} := \partial_y I(\mathbf{x}+\mathbf{w}) - \partial_y I(\mathbf{x}) & & I_{xy} := \partial_{xy} I(\mathbf{x}+\mathbf{w}) & &
\end{array}
$$

Minimiser has to fulfill the Euler-Lagrange equations

$$\alpha\ \mathrm{div}\ \left(\Psi'(|\nabla u|^2 + |\nabla v|^2)\ \nabla u\right) = \Psi'\left(I_{\mathbf{w}}^2 + \gamma(I_{x\mathbf{w}}^2 + I_{y\mathbf{w}}^2)\right) \cdot \left(I_x I_{\mathbf{w}} + \gamma\left(I_{xx} I_{x\mathbf{w}} + I_{xy} I_{y\mathbf{w}}\right)\right)$$

$$\alpha\ \mathrm{div}\ \left(\Psi'(|\nabla u|^2 + |\nabla v|^2)\ \nabla v\right) = \Psi'\left(I_{\mathbf{w}}^2 + \gamma(I_{x\mathbf{w}}^2 + I_{y\mathbf{w}}^2)\right) \cdot \left(I_y I_{\mathbf{w}} + \gamma\left(I_{xy} I_{x\mathbf{w}} + I_{yy} I_{y\mathbf{w}}\right)\right)$$

where the indices denote differences or partial derivatives:

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$$S(\mathbf{w}) = D(\mathbf{w})$$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

Outline

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- **Algorithmic Aspects**
- Relations to Warping
- Evaluation
- Conclusions

Problem 1: Local Minima

- energy functional $E(u, v)$ is not convex
- reason: terms involving $I(\mathbf{x} + \mathbf{w})$
- should not be linearised for large displacements
- numerical algorithms may yield suboptimal local minima of $E(u, v)$, if initialisation is not chosen properly

Solution: Initialisation by Coarse-to-Fine Strategy

- downsample problem in a full pyramid
- start with zero displacement at coarsest scale
- solve Euler-Lagrange equations $S(w) = D(w)$
- use resulting **flow field** as initialisation at next finer scale

Problem 2: Solve Euler-Lagrange Equations

- discretise $S(w) = D(w)$ by finite differences
- yields large nonlinear system of equations
- nonlinearity caused by $I(\mathbf{x} + \mathbf{w})$ and nonlinear penaliser Ψ

Solution

- nonlinear system is simplified by
  - two nested fixed point iterations
  - linearisation of $I(\mathbf{x} + \mathbf{w})$
- leads to large linear system of equations
- can be solved by iterative methods such as SOR

Detailed Structure on the Linear System

Resulting linear system for $du^{k,l+1}$, $dv^{k,l+1}$:

$$\begin{aligned} \alpha \quad & \operatorname{div}\left(({\Psi'})^{k,l}{}_{Smooth} \nabla(u^k + du^{k,l+1})\right) \\ = \quad & ({\Psi'})^{k,l}{}_{Data} \cdot \Big(I_x^k \left(I_z^k + I_x^k du^{k,l+1} + I_y^k dv^{k,l+1}\right) \\ + \quad & \gamma\big(I_{xx}^k(I_{xz}^k + I_{xx}^k du^{k,l+1} + I_{xy}^k dv^{k,l+1}) + I_{xy}^k(I_{yz}^k + I_{xy}^k du^{k,l+1} + I_{yy}^k dv^{k,l+1})\big)\Big) \end{aligned}$$

$$\begin{aligned} \alpha \quad & \operatorname{div}\left(({\Psi'})^{k,l}{}_{Smooth} \nabla(v^k + dv^{k,l+1})\right) \\ = \quad & ({\Psi'})^{k,l}{}_{Data} \cdot \Big(I_y^k \left(I_z^k + I_x^k du^{k,l+1} + I_y^k dv^{k,l+1}\right) \\ + \quad & \gamma\big(I_{xy}^k(I_{xz}^k + I_{xx}^k du^{k,l+1} + I_{xy}^k dv^{k,l+1}) + I_{yy}^k(I_{yz}^k + I_{xy}^k du^{k,l+1} + I_{yy}^k dv^{k,l+1})\big)\Big) \end{aligned}$$

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Warping

- widely used for optic flow computation with large displacements (e.g. Anandan 1989, Black/Anandan 1996, Mémin/Pérez 1998)
- downsample image data
- solve problem at coarse scale
- use this flow field at next finer scale:
  warp **image** in order to compensate for this estimated motion
- solve modified problem (with other image data) at finer scale
- continue until finest scale reached
- sum up optic flow contributions from all scales
- successful in practice, but no theoretical justification!

We have proven equivalence between

- our numerical method for minimising a simplified energy $E(u, v)$ by coarse-to-fine flow initialisations and nested fixed point iterations
- warping method (nested problems with motion-compensated image data) of Mémin / Pérez

They lead to the same linear system of equations.

This explains the success of warping:

Warping has a sound theory as a numerical algorithm for minimising a single energy functional !

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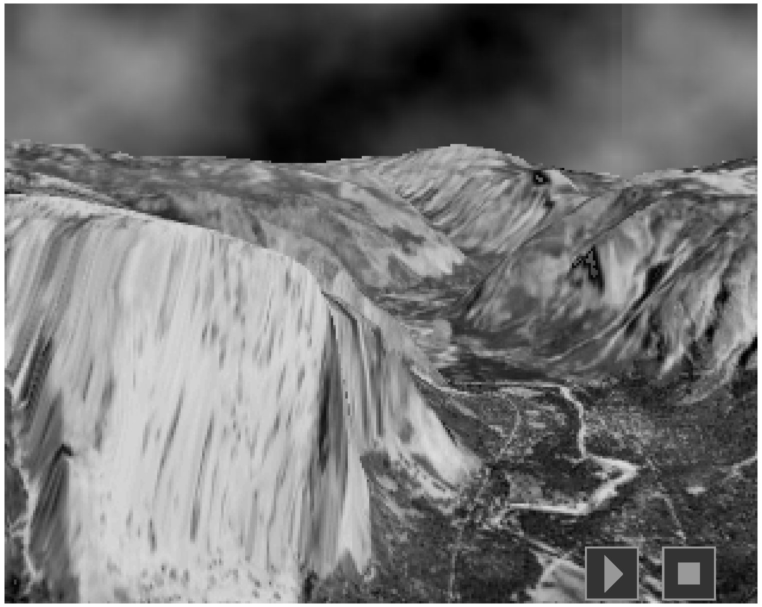
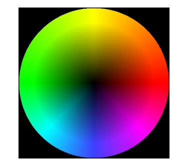
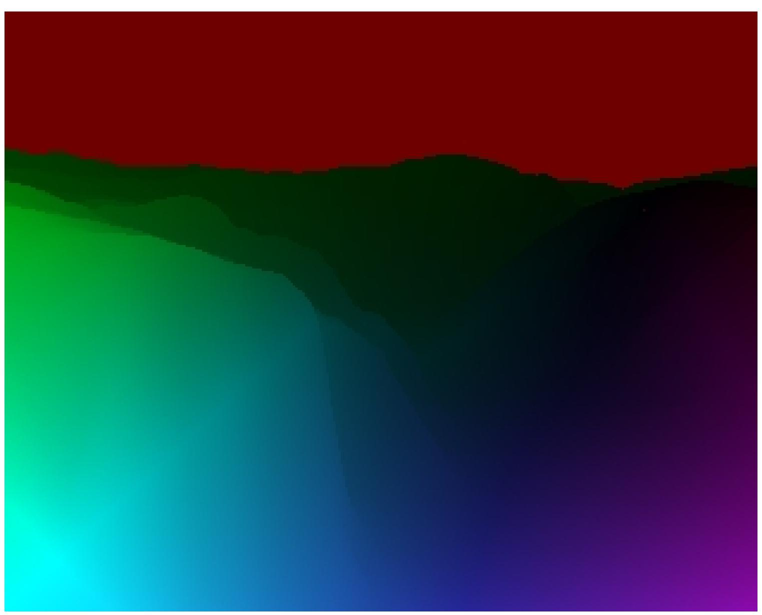


Sequence

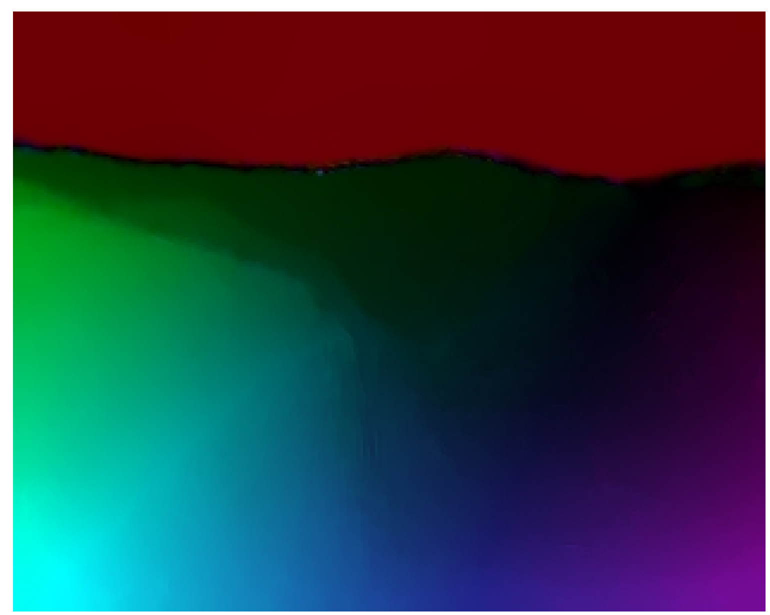
Yosemite Sequence

- Synthetic sequence $(316 \times 252 \times 15)$
- Known ground truth between frame 8 and frame 9



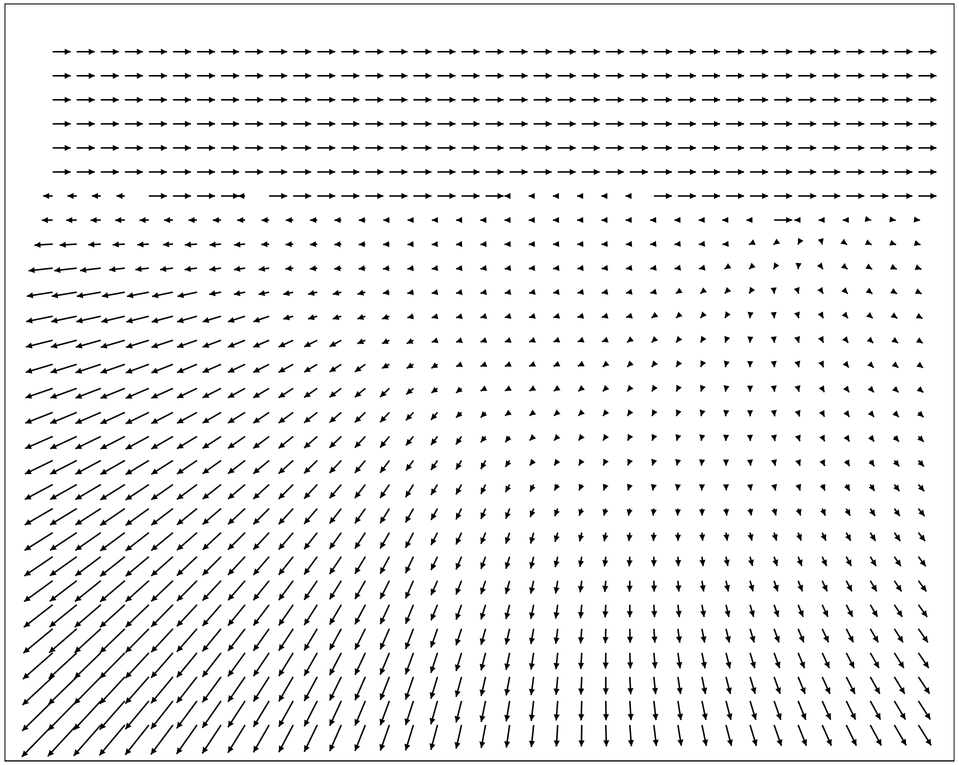


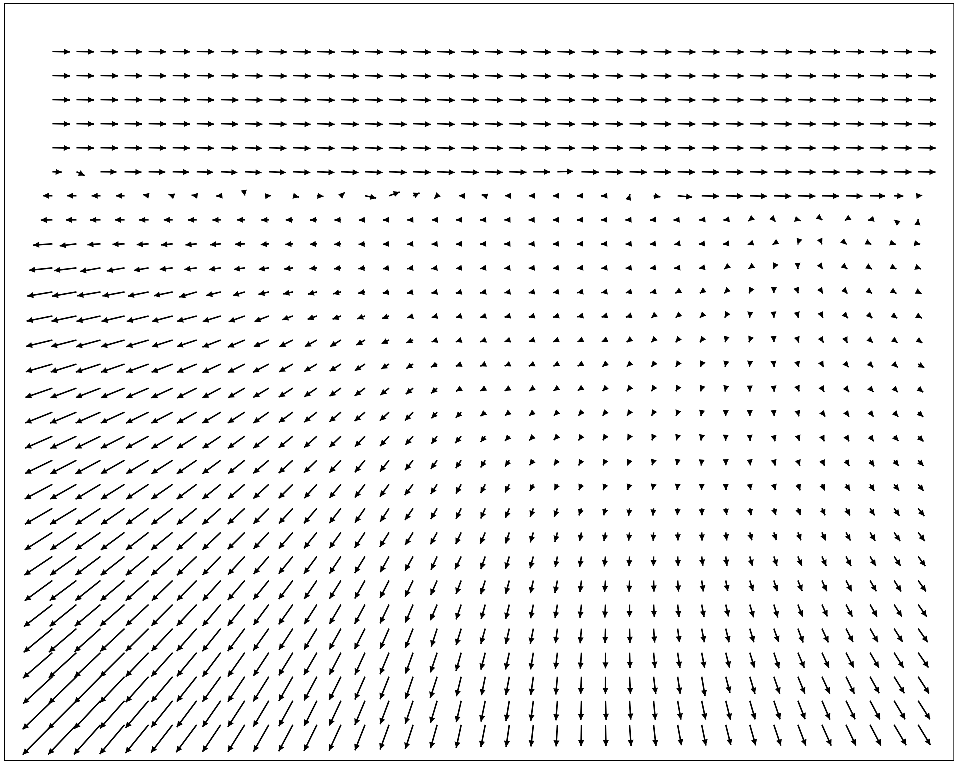
Ground Truth



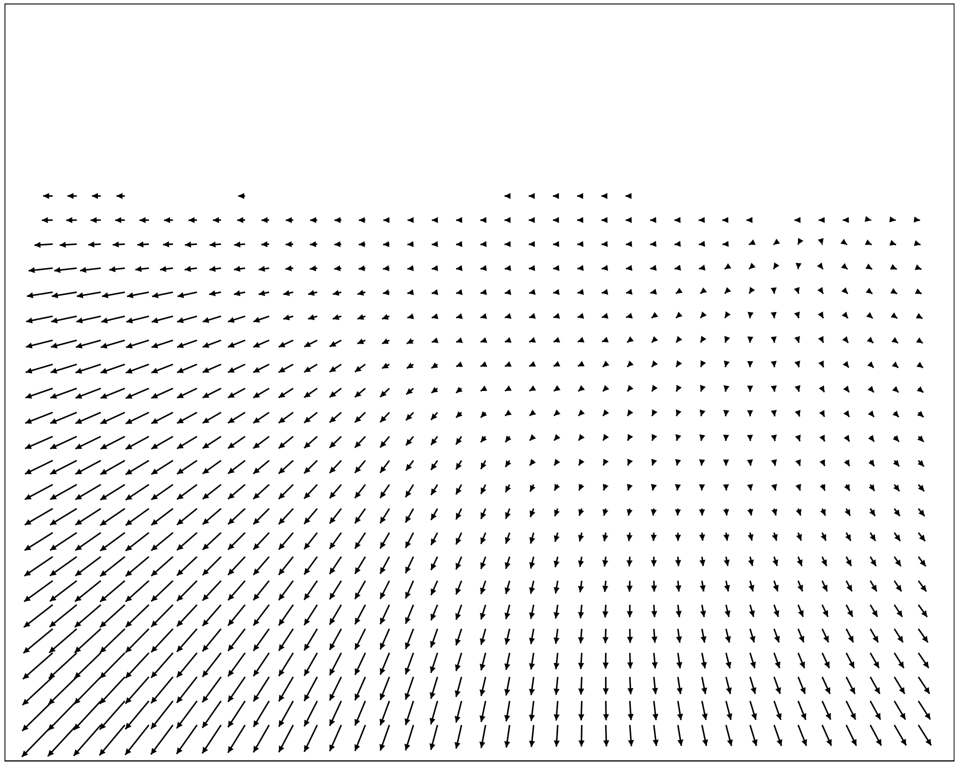
Computed Flow

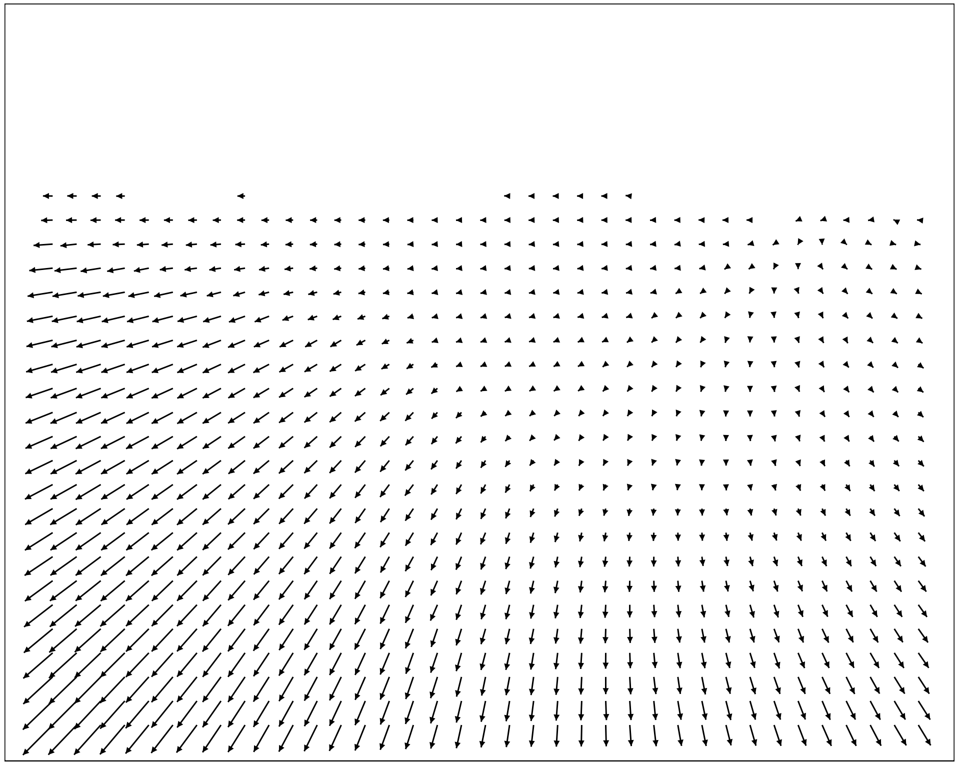
Qualitative Evaluation





Vector plot of the optic flow field for the Yosemite sequence **with** clouds. (a) **Left:** Ground truth. (b) **Right:** Computed flow.





Vector plot of the optic flow field for the Yosemite sequence **without** clouds. (a) **Left:** Ground truth. (b) **Right:** Computed flow.

Quantitative Evaluation

- Comparison to the best results from literature
- Average angular errors (AAE) and standard deviations (STD) for the Yosemite sequence **with** coluds:

Yosemite with clouds		
Technique	AAE	STD
Anandan 1989	13.36°	15.64°
Nagel 1983	10.22°	16.51°
Horn/Schunck, mod. 1981	9.78°	16.19°
Uras *et al.* 1988	8.94°	15.61°
Alvarez *et al.* 2000	5.53°	7.40°
Weickert *et al.* 2003	5.18°	8.68°
Mémin/Pérez 1998	4.69°	6.89°
our method	1.94°	6.02°

- For the Yosemite sequence **without** clouds, even better results are possible:

Yosemite without clouds		
Technique	AAE	STD
Black/Anandan 1996	4.56°	4.21°
Ju *et al.* 1996	2.16°	2.00°
Bab-Hadiashar/Suter 1997	2.05°	2.92°
Lai/Vemuri 1998	1.99°	1.41°
Mémin/Pérez 1998	1.58°	1.21°
Weickert *et al.* 2003	1.46°	1.50°
Farnebäck 2003	1.14°	2.14°
our method	0.98°	1.17°

Robustness under Noise

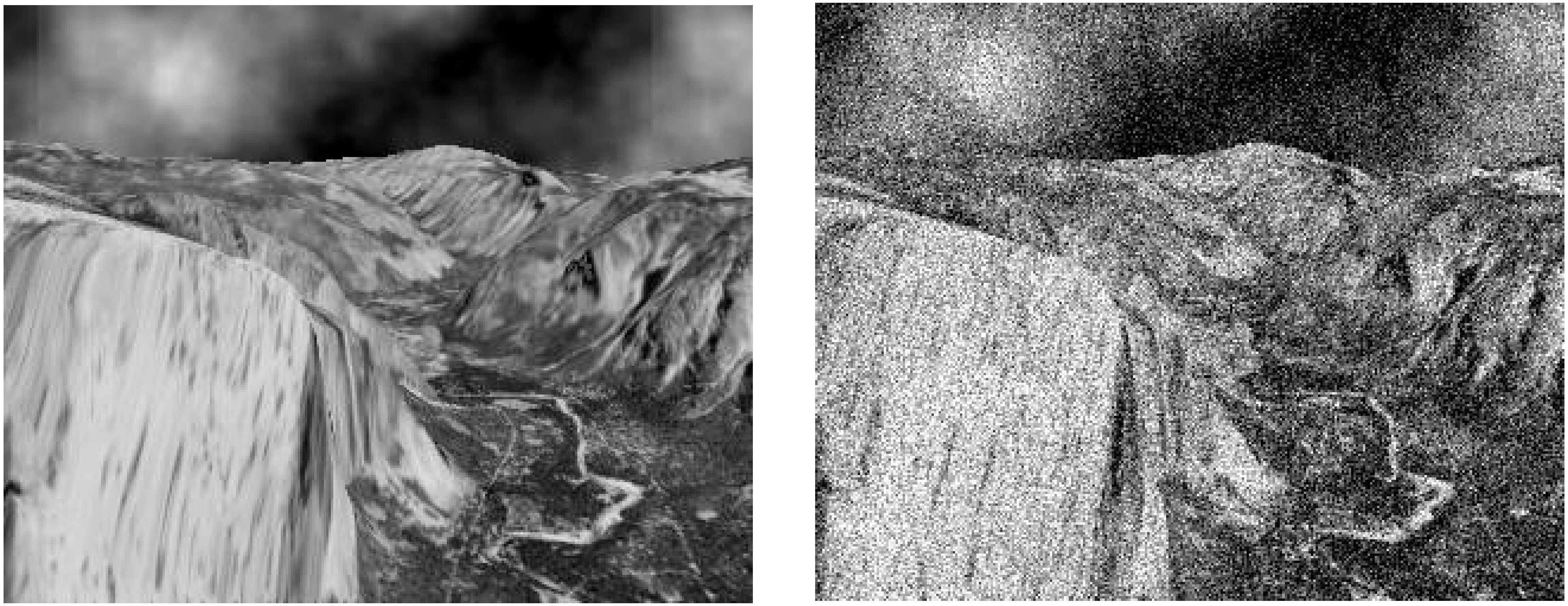
- Added Gaussian noise with zero mean and different standard deviations σ_n.

- Results for Yosemite sequence with clouds:

σ_n	AAE	STD
0	1.94°	6.02°
10	2.50°	5.96°
20	3.12°	6.24°
30	3.77°	6.54°
40	4.37°	7.12°

- Average angular error for $\sigma_n = 40$ outperforms all other methods with $\sigma_n = 0$!

Evaluation (7)



Frame 8 of the Yosemite sequence with clouds. (a) **Left:** Original. (b) **Right:** Gaussian noise with standard deviation $\sigma_n = 40$ added.

Robustness under Parameter Variations

- Three intuitive parameters:
  σ: Gaussian presmoothing of the input data
  α: weight of smoothness term
  γ: weight of gradient constancy term
- Parameter variation for the Yosemite sequence with clouds:

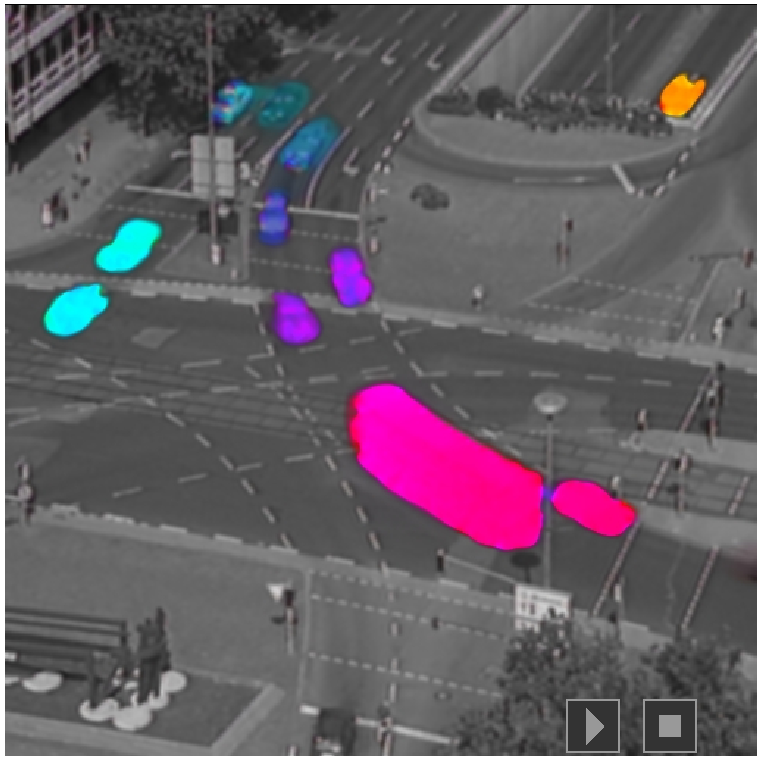
σ	α	γ	AAE
0.8	80	100	$1.94°$
0.4	"	"	$2.10°$
1.6	"	"	$2.04°$
0.8	80	100	$1.94°$
"	40	"	$2.67°$
"	160	"	$2.21°$
0.8	80	100	$1.94°$
"	"	50	$2.07°$
"	"	200	$2.03°$

- Deviations from the optimum by a factor 2 hardly influence the result.

Real-World Data

- Real-world image sequence "Ettlinger Tor" by Nagel $(512 \times 512 \times 50)$

Sequence



Computed Flow

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Summary

- **novel model**
  - gradient constancy assumption within energy functional
  - combines many successful features in a single functional
- **novel theory**
  - postpone all linearisations to the numerical scheme
  - numerical scheme based on two nested iterations
  - warping theoretically justified as a special numerical approximation
- **excellent results**
  - angular errors belong to smallest in the literature
  - robust under parameter variations
  - highly robust under noise

Ongoing Work

- alternative data terms
- correpondence between data and smoothness terms
- automatised selection of smoothing parameters σ, α
- more efficient numerics: PCG, multigrid, domain decomposition
- novel warpings inspired from suitable numerics ?

Message

- It is advantageous to combine transparent continuous modelling with consistent numerics.
- Good performance and deeper theoretical understanding are not contradictive: They are two sides of the same medal.

Thank you very much!

more informations:
www.mia.uni-saarland.de