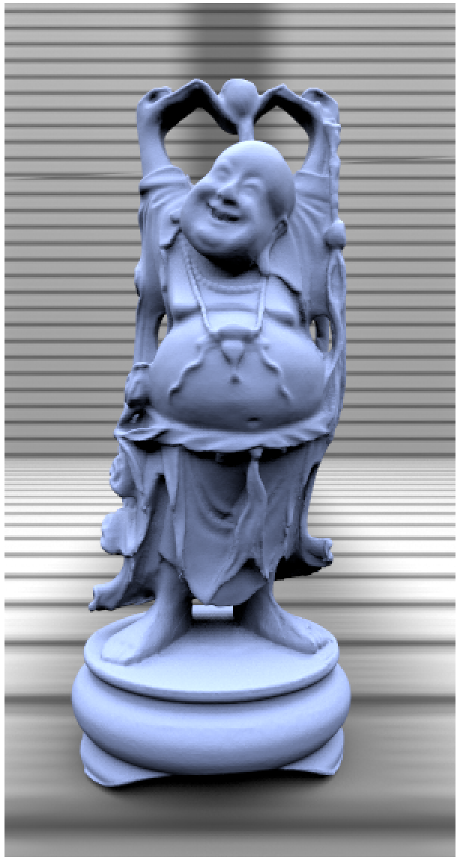
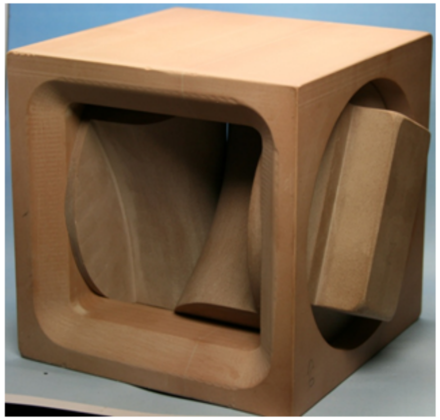
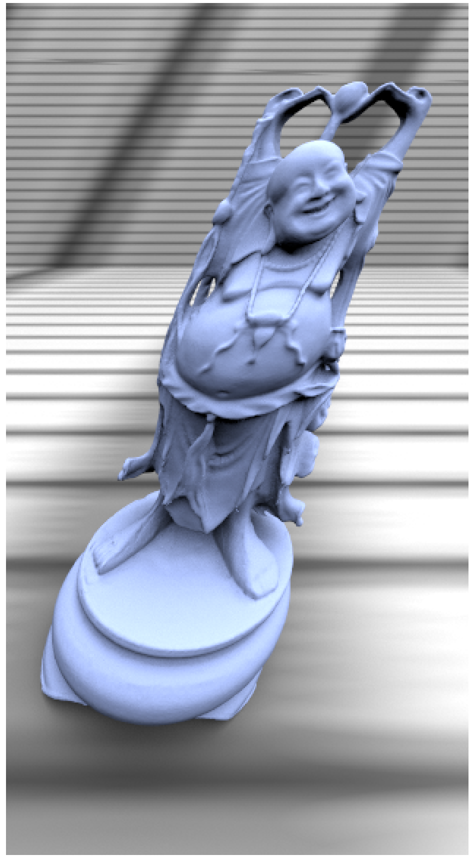
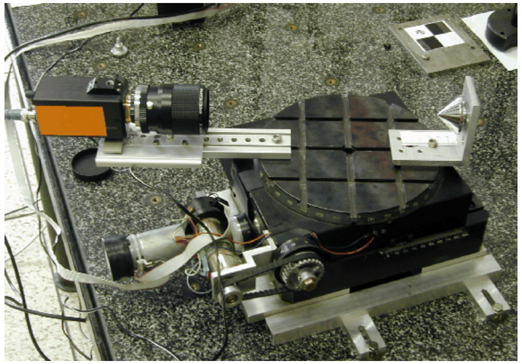
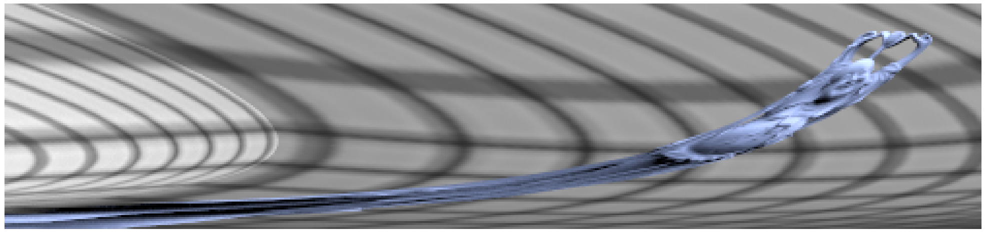
“Bag of lines” models for non-central linear cameras











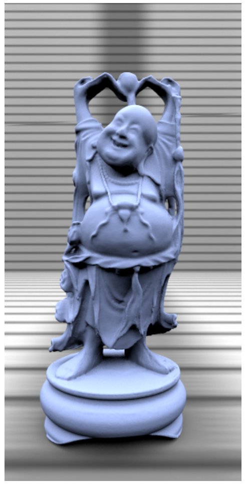
“Bag of lines” models for non-central linear cameras

Guillaume BATOG

(joint work with X. Goaoc and J. Ponce)

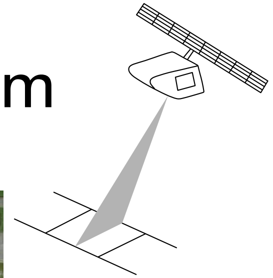
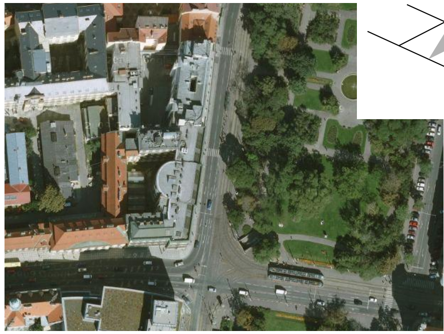
INRIA Nancy Grand-Est
project-team VEGAS

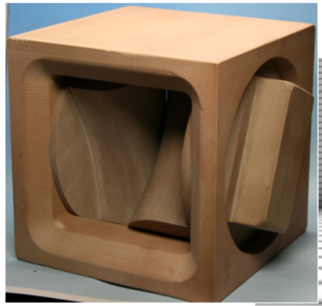
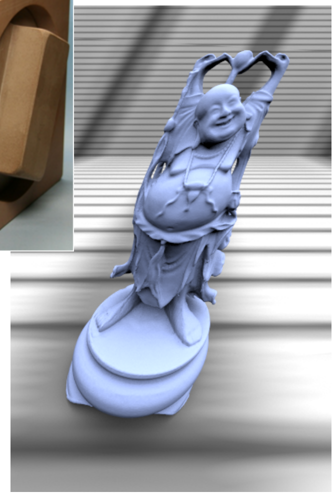
Linear cameras



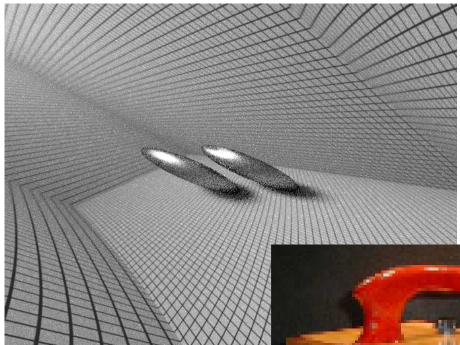

pinhole

pushbroom

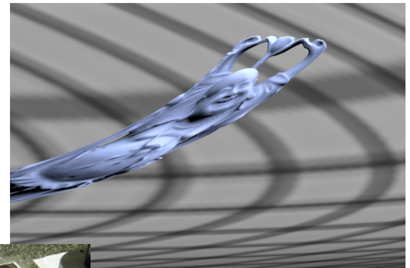
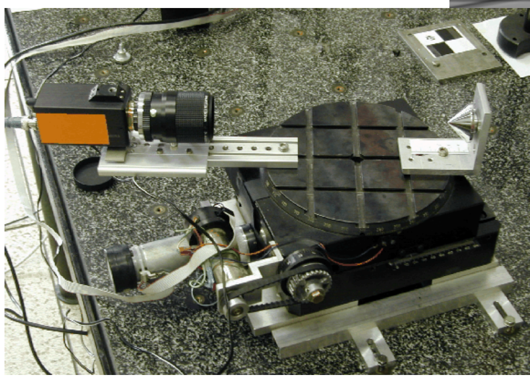



pencil

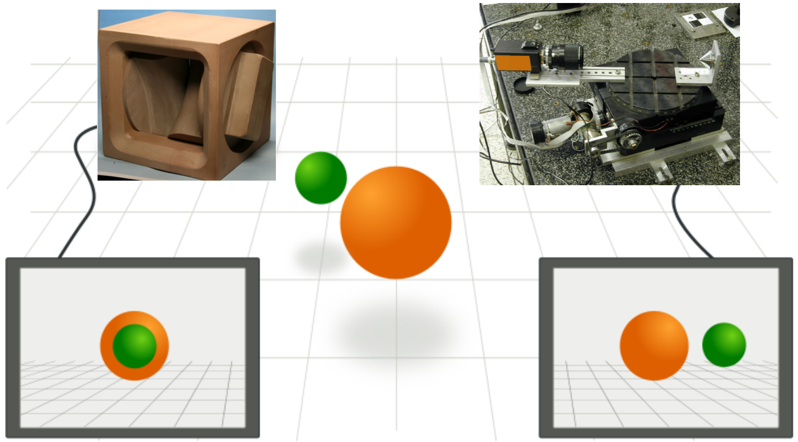



X-slit

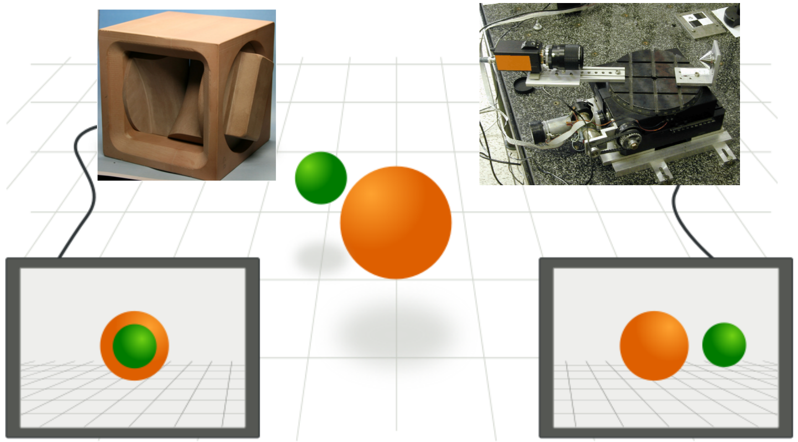



linear
oblique

Stereo-vision between *ANY* pair of linear cameras

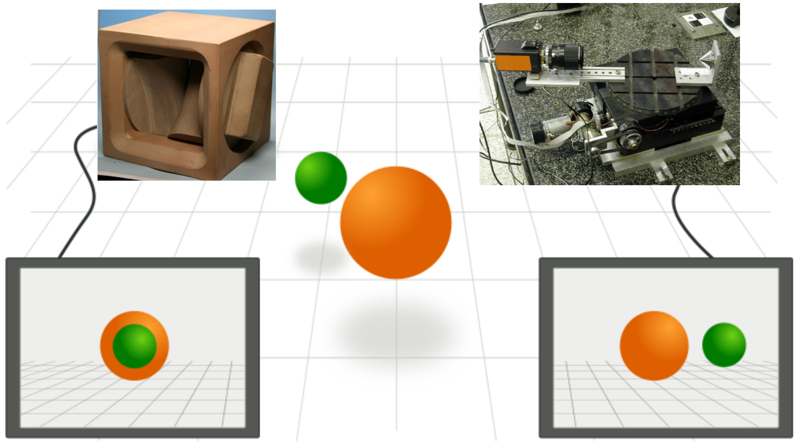


Stereo-vision between *ANY* pair of linear cameras

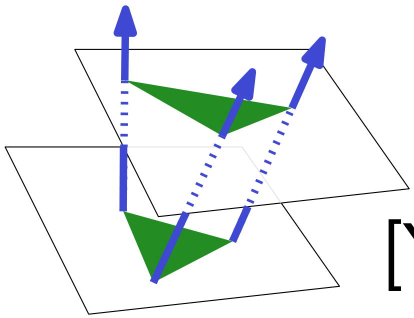


PROBLEMS { unified model, computations

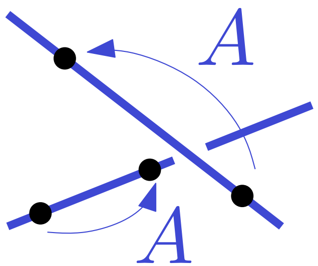
Stereo-vision between *ANY* pair of linear cameras



PROBLEMS { unified model / computations



General Linear Camera (GLC) [Yu & Mc-Millan,04]

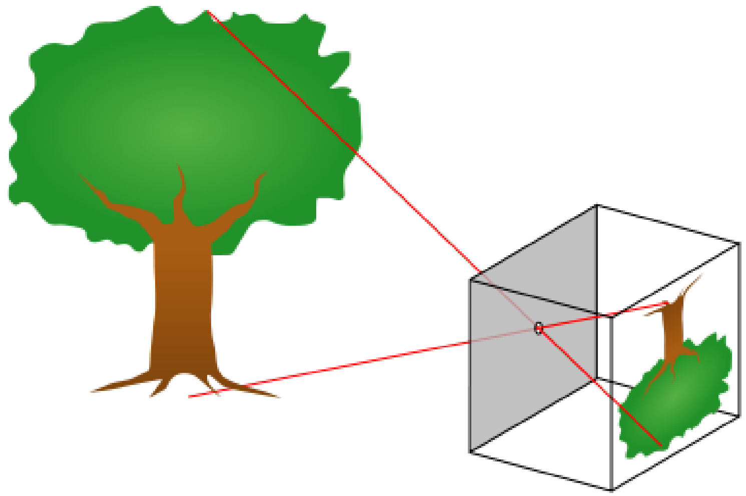


Admissible Map [Pajdla,02]

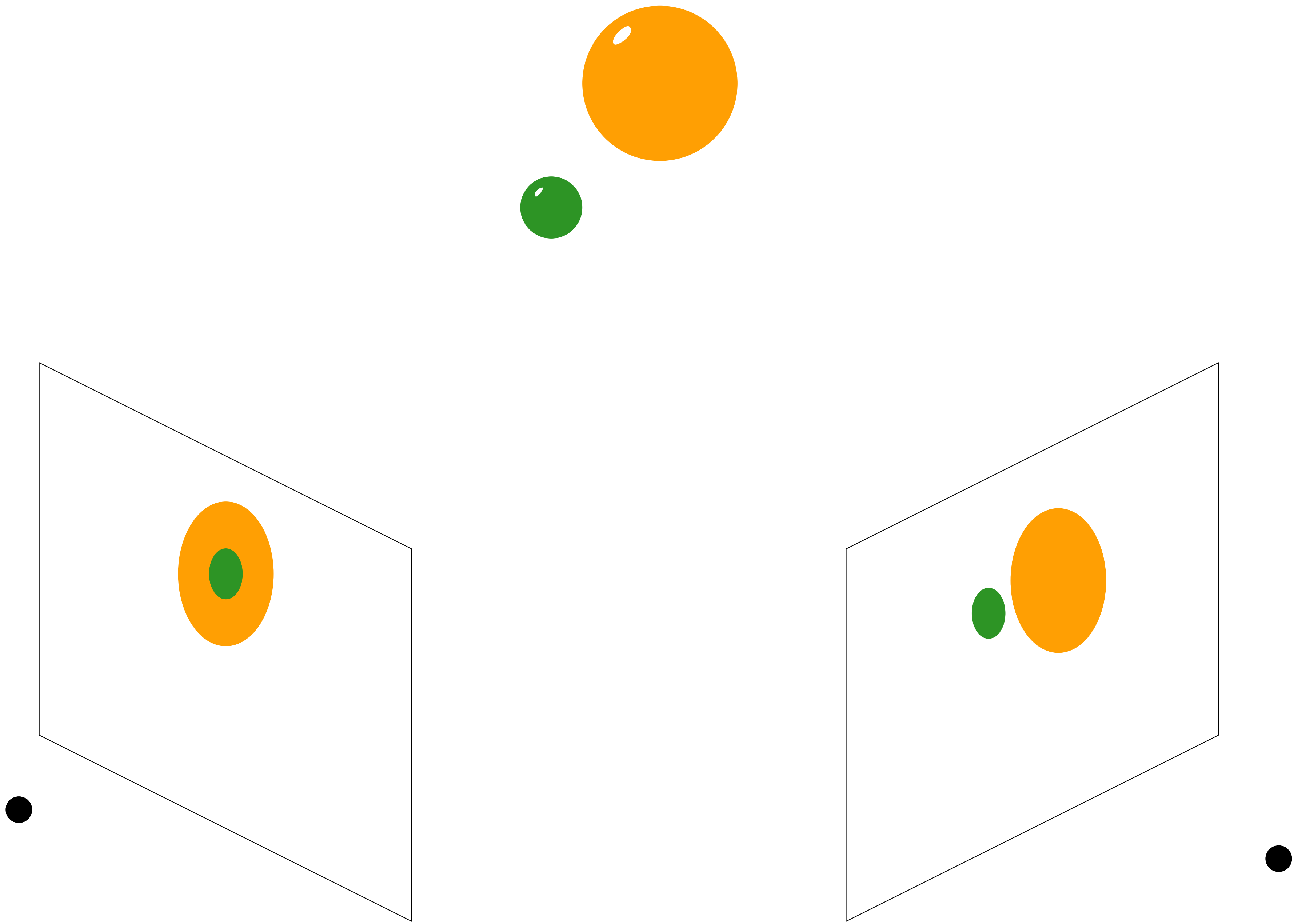
OUTLINE

1. Stereo-vision with pinhole cameras
2. The admissible map model
3. Stereo-vision with linear cameras

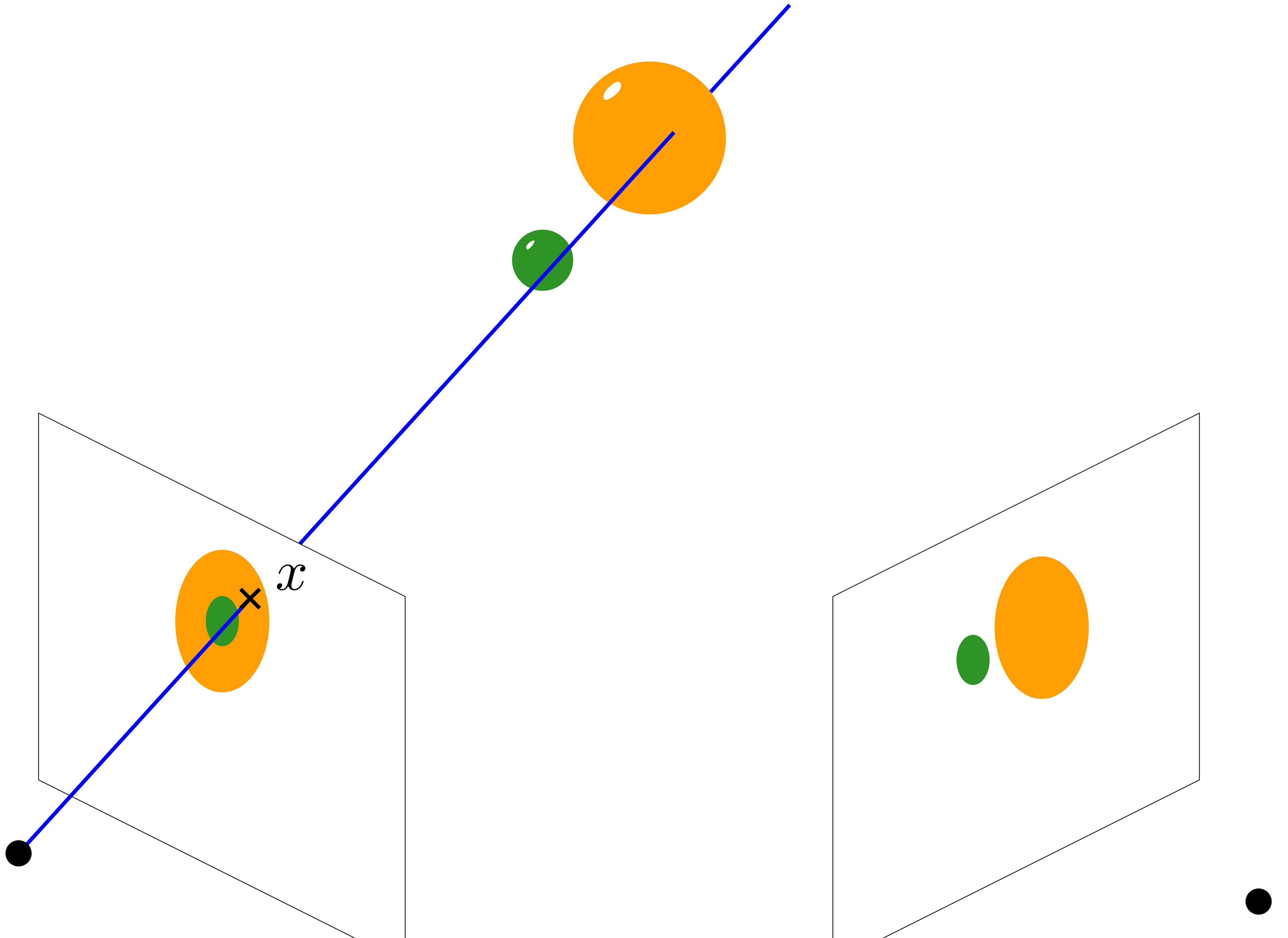
$\sim$



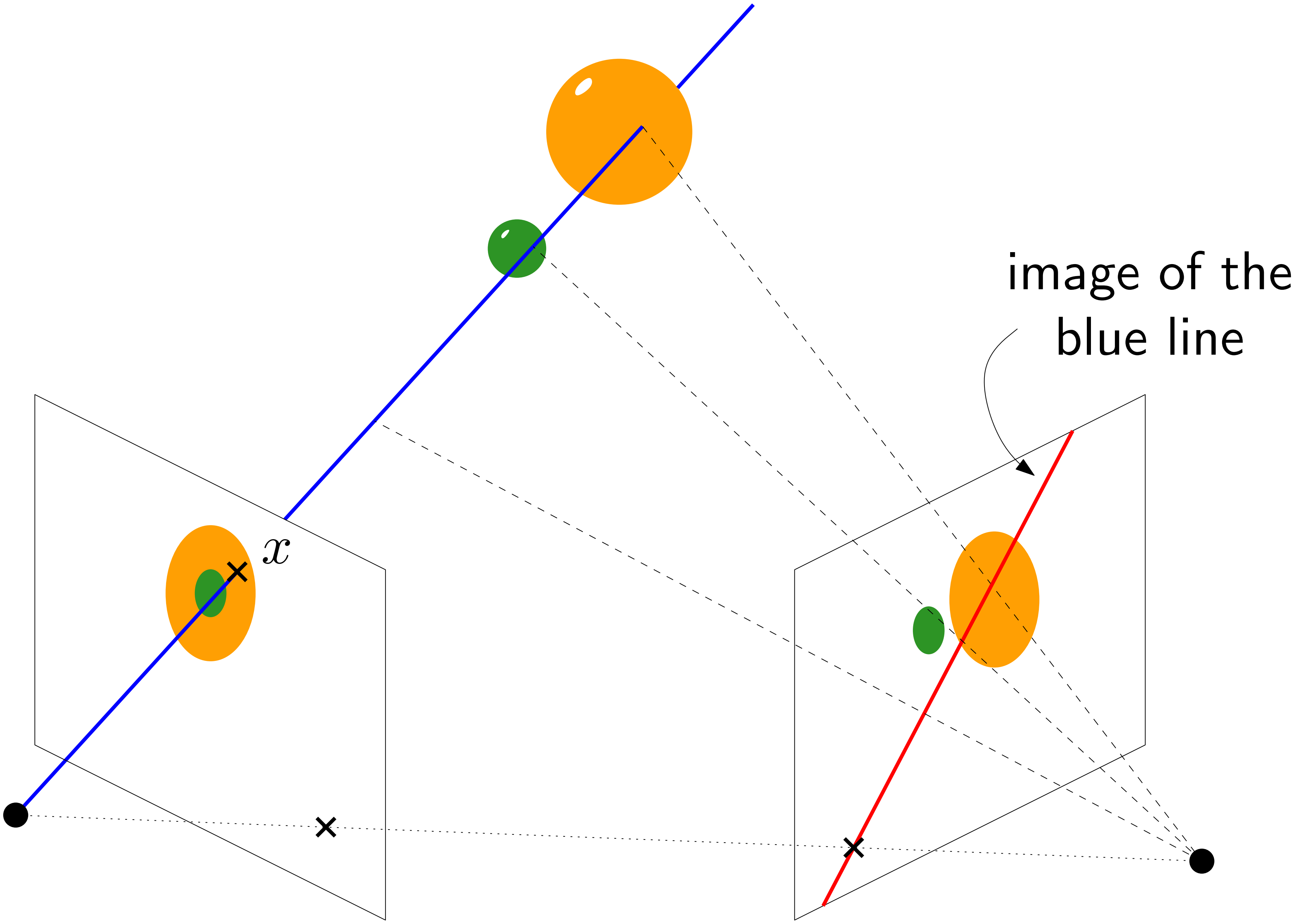
Inverse projection



Inverse projection



Inverse projection



Inverse projection

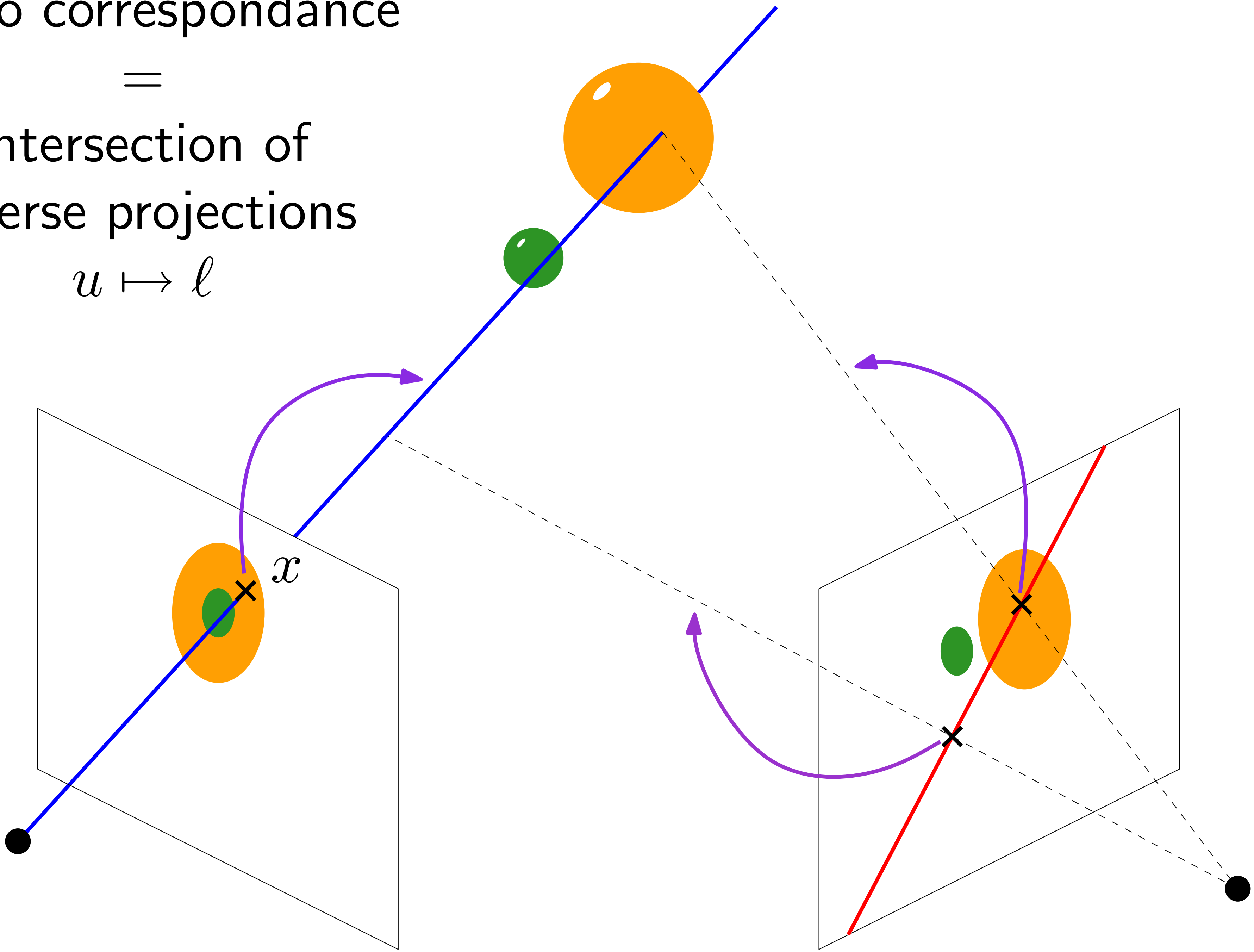
stereo correspondance

=

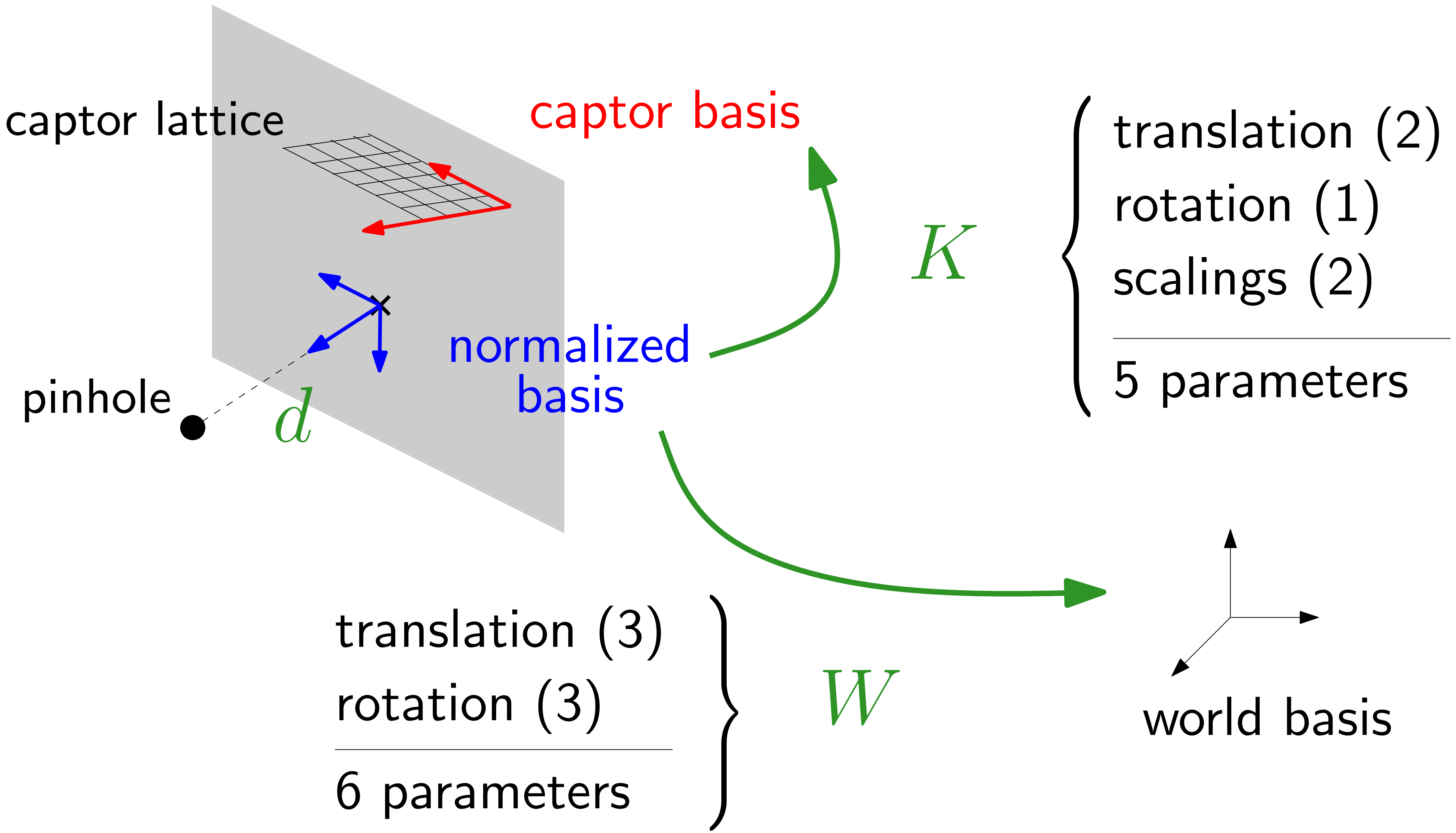
intersection of

inverse projections

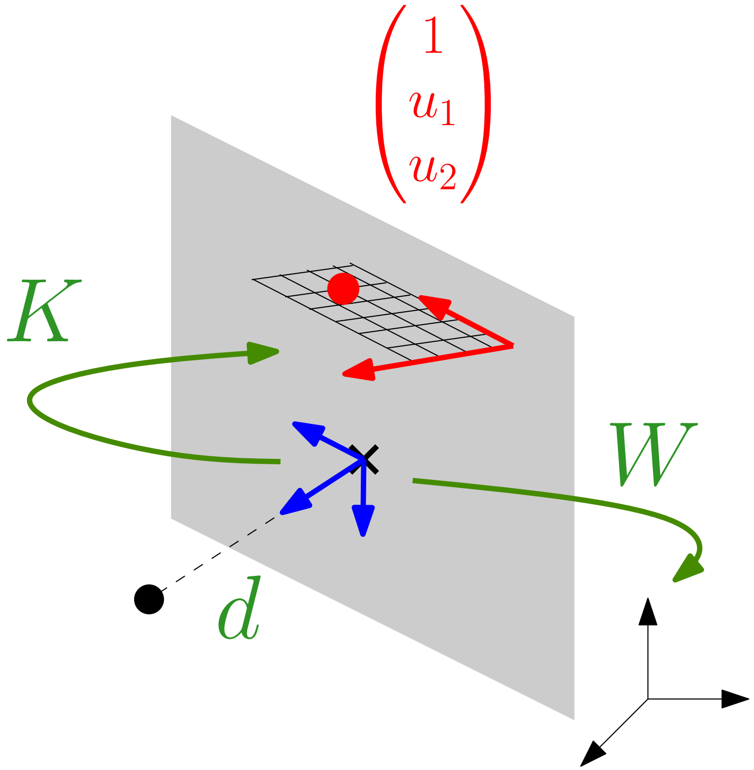
$u \mapsto \ell$



Normalized coordinates



Fundamental & Essential matrix



Fundamental matrix

$$u^T \; \mathcal{F} \; u' = 0$$

3×3 matrix

8 correspondances needed to determine $\mathcal{F}$

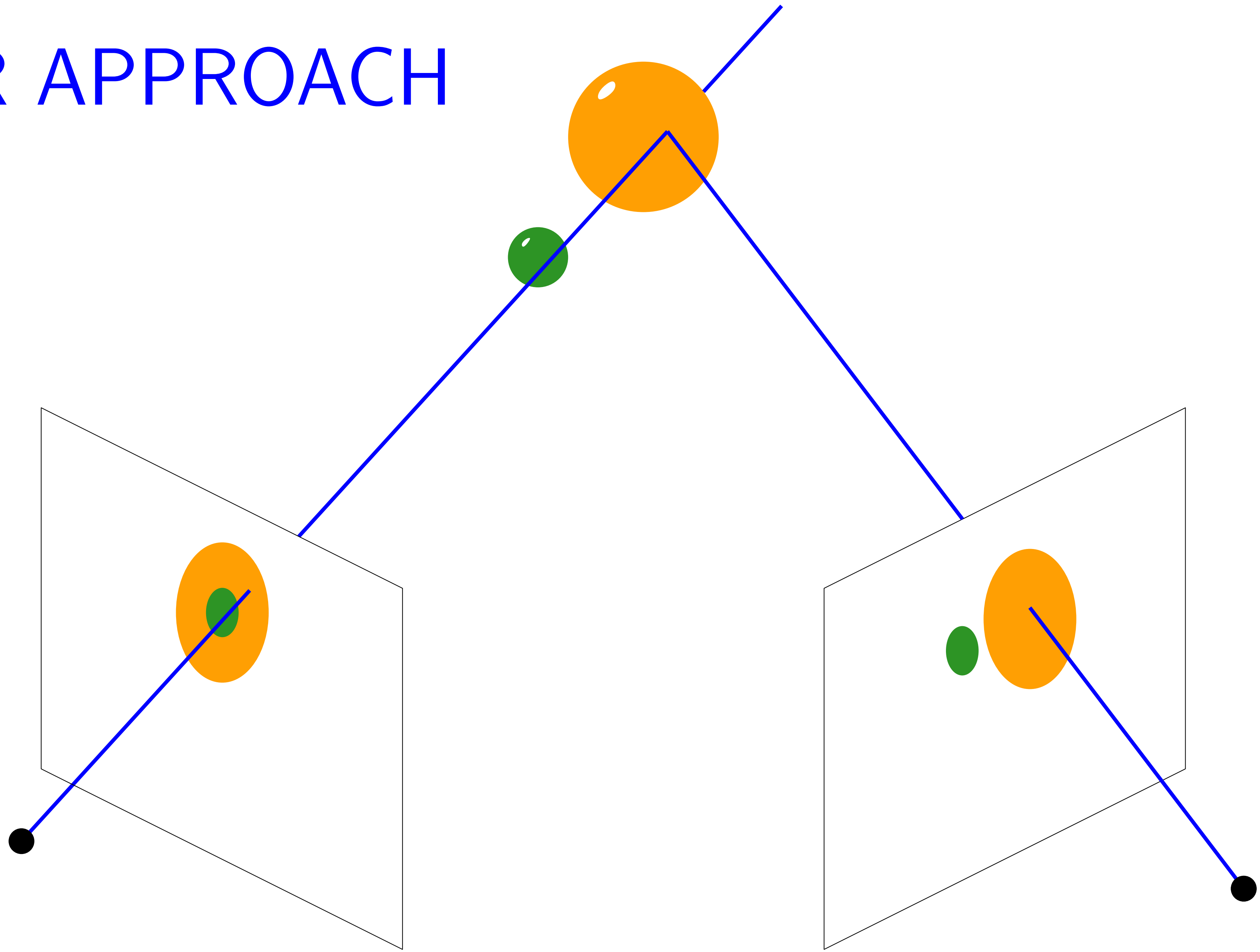
Essential matrix

$$u^T \; \mathcal{E} \; u' = 0$$

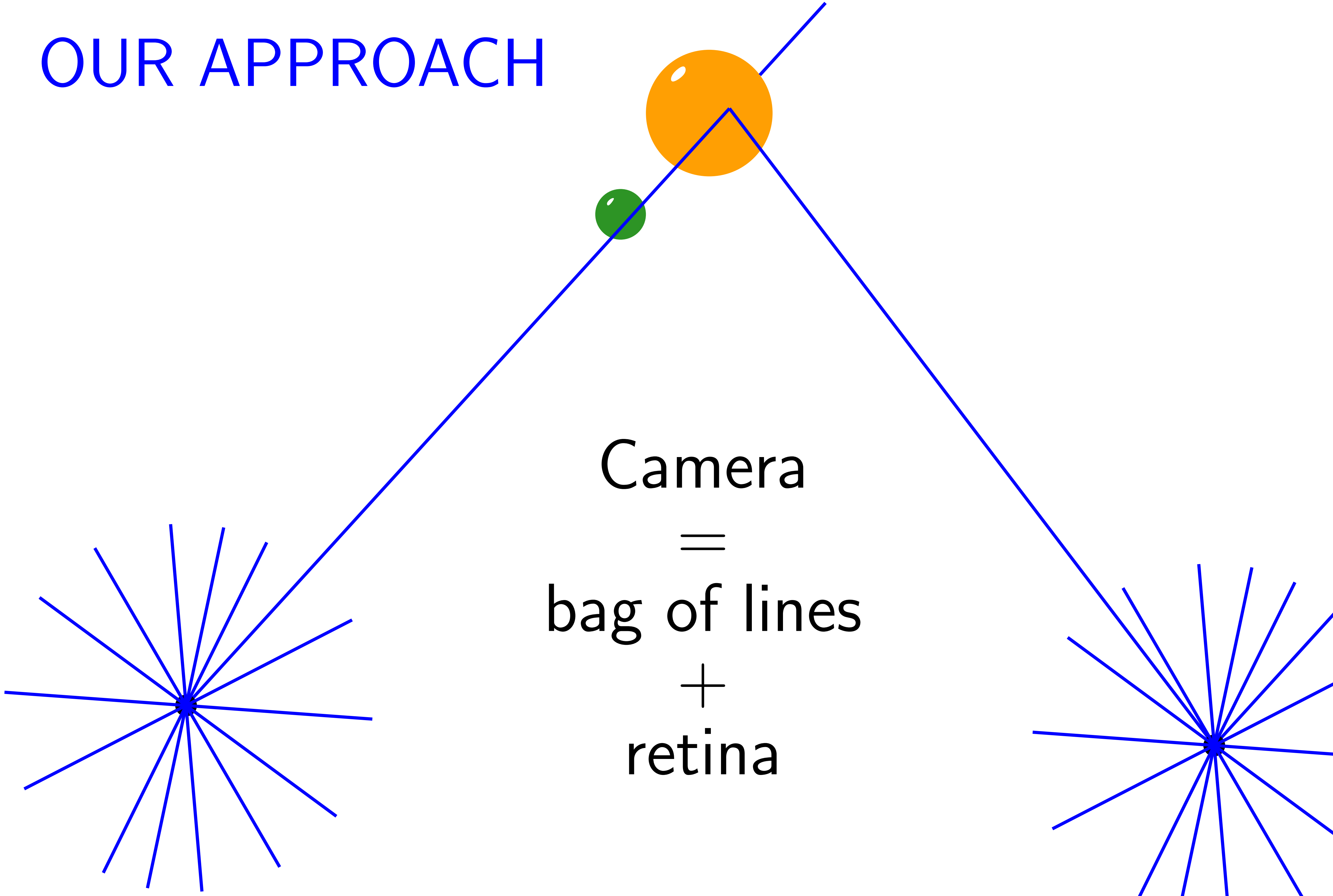
$\mathcal{E}$ depends only on W

6 correspondances needed to determine $\mathcal{E}$

OUR APPROACH



OUR APPROACH



Camera

$=$

bag of lines

$+$

retina

OUTLINE

1. Stereo-vision with pinhole cameras
2. The admissible map model
3. Stereo-vision with linear cameras

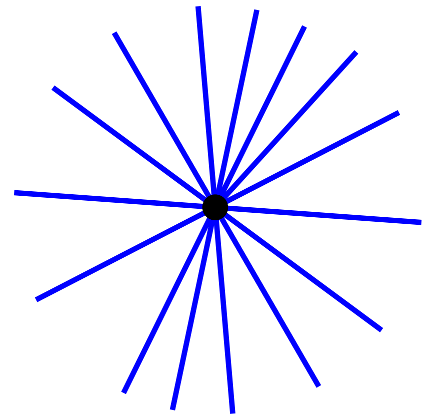
OUTLINE

"bag of lines" level

1. Stereo-vision with pinhole cameras
2. The admissible map model
3. Stereo-vision with linear cameras

adding the retina

Projective space



$\mathbb{P}^3(\mathbb{R}) \quad \sim \quad$ set of lines of $\mathbb{R}^4$
passing through the origin

Homogeneous coordinates

$$[x_0 : x_1 : x_2 : x_3] \sim [\lambda x_0 : \lambda x_1 : \lambda x_2 : \lambda x_3]$$

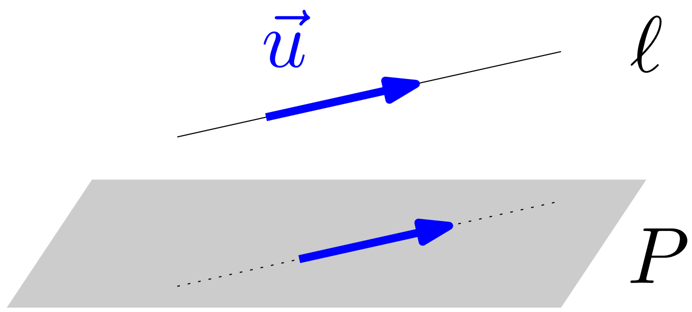
Intuition in the affine space $\mathbb{R}^3$

$\mathbb{P}^3(\mathbb{R}) \;=\;$ points $+$ directions

$[1 : x : y : z]$ $\quad$ $[0 : u : v : w]$

~~parallelism~~

~~angles~~

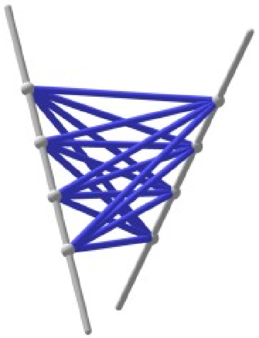


$$\ell \cap P = \{[\vec{u}]\}$$

Linear congruences

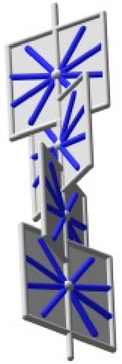
3D field of view $+$ order 1: $x \overset{!}{\longmapsto} \ell$ (for almost all x)

$\Rightarrow$ 2-parameter set of lines



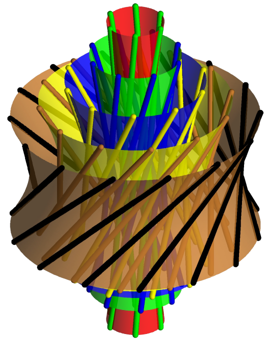
hyperbolic congruence

X-slit camera



parabolic congruence

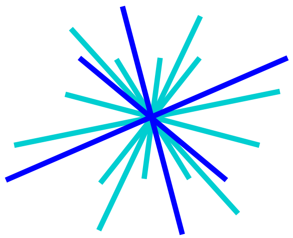
pencil camera



elliptic congruence

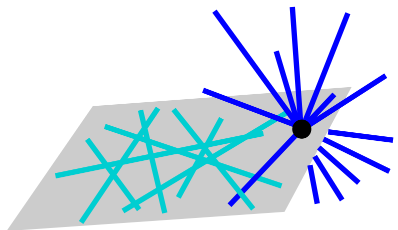
linear oblique camera

Projective classification



bundle

pinhole camera

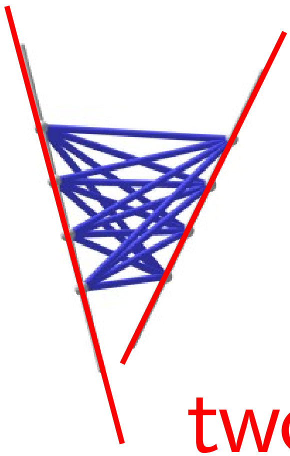


degenerate congruence

(no name)

3D field of view $+$ order 1: $x \stackrel{!}{\longmapsto} \ell$ (for almost all x)

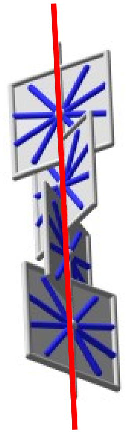
$\Rightarrow$ 2-parameter set of lines



hyperbolic congruence

X-slit camera

two lines

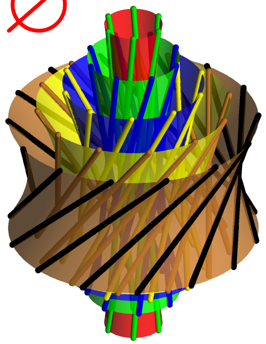


parabolic congruence

pencil camera

one line

$\emptyset$ empty

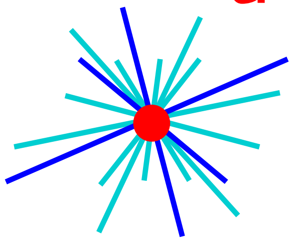


elliptic congruence

linear oblique camera

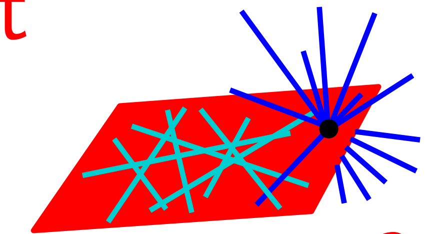
Ambiguity locus

a point



bundle

pinhole camera



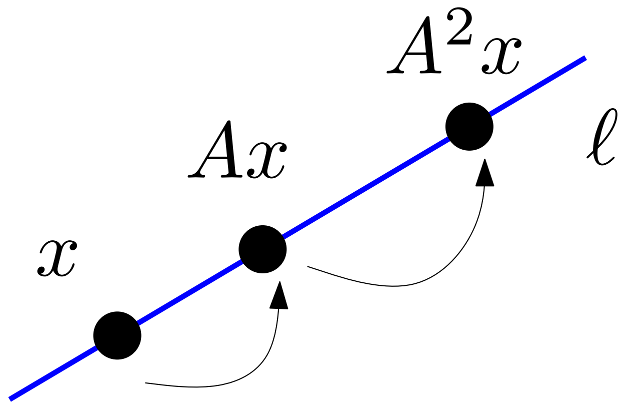
a plane

degenerate congruence

(no name)

Admissible maps

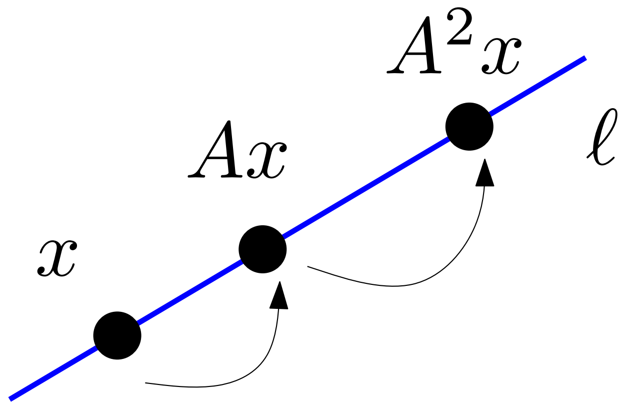
Idea: a linear map A that globally preserves each line of the bag



$$A = \begin{pmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{pmatrix} = \mathbb{R}^{4\times 4}$$

Admissible maps

<u>Idea:</u> a linear map A that globally preserves each line of the bag



$\Rightarrow \quad A^2x = \lambda_x Ax + \mu_x x$
(for almost all x)

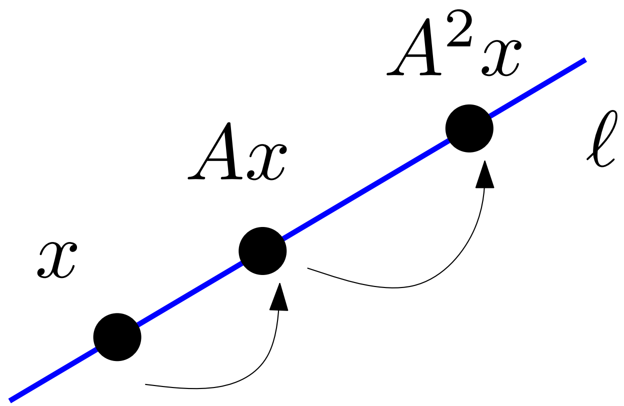
$\Rightarrow \quad A^2 = \lambda A + \mu \mathrm{Id}$ (linear algebra)

$$A = \begin{pmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{pmatrix} = \mathbb{R}^{4\times 4}$$

<u>Definition:</u> A has a minimal polynomial π_A of degree 2.

Admissible maps

<u>Idea:</u> a linear map A that globally preserves each line of the bag



$\Rightarrow \quad A^2x = \lambda_x Ax + \mu_x x$
(for almost all x)

$$A = \begin{pmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{pmatrix} = \mathbb{R}^{4\times 4}$$

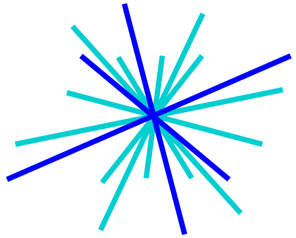
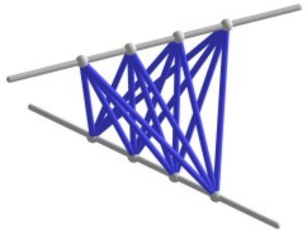
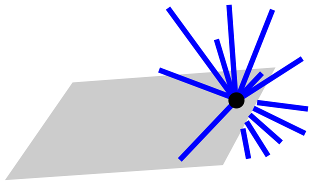
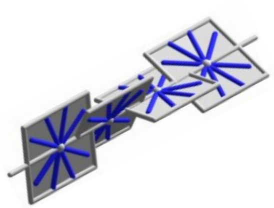
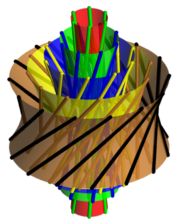
$\Rightarrow \quad A^2 = \lambda A + \mu \mathrm{Id}$ (linear algebra)

<u>Definition:</u> A has a minimal polynomial π_A of degree 2.

$\Rightarrow \quad \mathcal{L} = \{x \vee Ax$ for x not an eigenvector$\}$ has order 1.

Ambiguity locus of $\mathcal{L}$ $\subseteq$ Union of eigenspaces of A

Admissible maps

π_A	Eigenspaces	Reduced form of A	$\mathcal{L}$
$(X-\alpha)\,(X-\beta)$	plane + point	$\left[\begin{array}{c\|c} \alpha I_3 & 0 \\ \hline 0 & \beta \end{array}\right]$	
$(X-\alpha)\,(X-\beta)$	2 lines	$\left[\begin{array}{c\|c} \alpha I_2 & 0 \\ \hline 0 & \beta I_2 \end{array}\right]$	
$(X-\alpha)^2$	plane	$\left[\begin{array}{c\|cc} \alpha I_2 & \multicolumn{2}{c}{0} \\ \hline 0 & \alpha & 0 \\ & \lambda & \alpha \end{array}\right]$	
$(X-\alpha)^2$	1 line	$\left[\begin{array}{cc\|cc} \alpha & 0 & \multicolumn{2}{c}{0} \\ \lambda & \alpha & & \\ \hline \multicolumn{2}{c\|}{0} & \alpha & 0 \\ & & \mu & \alpha \end{array}\right]$	
$\Delta < 0$	$\emptyset$	$\left[\begin{array}{cc\|cc} \alpha & -\beta & \multicolumn{2}{c}{0} \\ \beta & \alpha & & \\ \hline \multicolumn{2}{c\|}{0} & \alpha & -\beta \\ & & \beta & \alpha \end{array}\right]$	

Geometric model

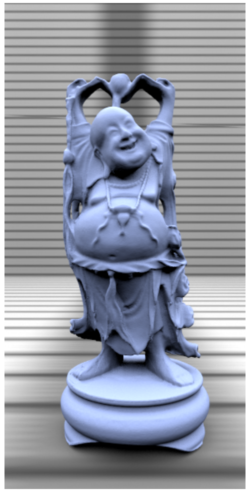
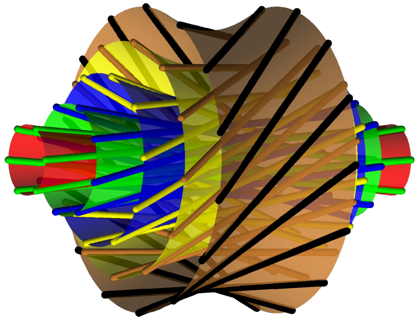
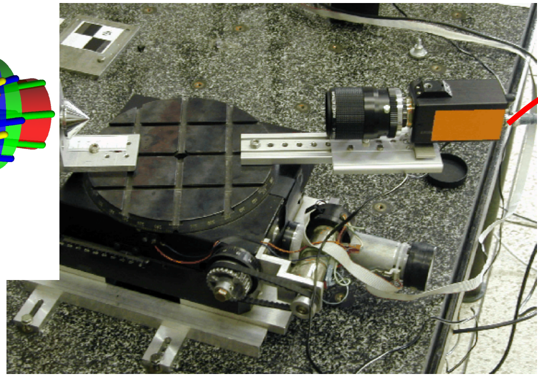
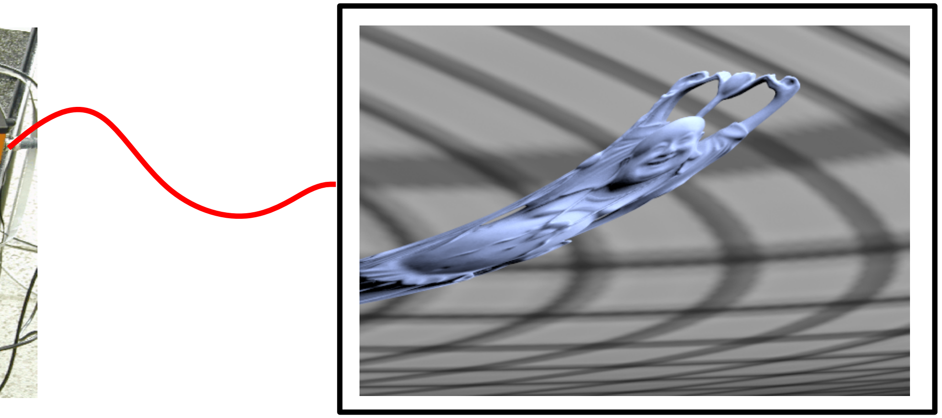
(linear congruences)

$\Updownarrow$

Analytical model

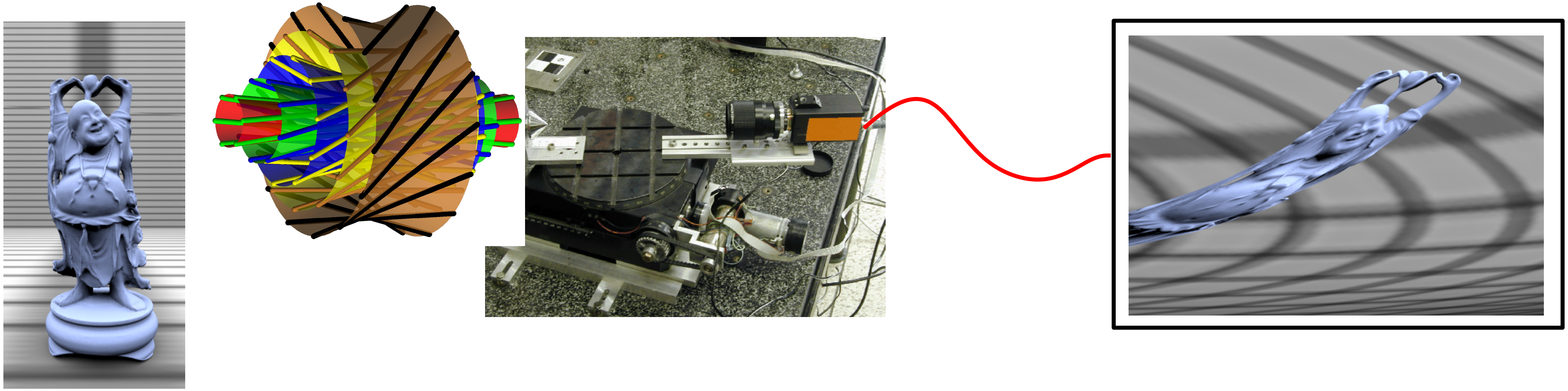
(admissible maps)

Application: Ray-tracing (using pbrt)

```
class LinearCamera : public Camera {
    ⟨ LinearCamera Constructor ⟩
    float GenerateRay(Sample &s, Ray *r);
    Transform AdMap;
    Transform RasterToWorld;
    Vector vview;
    ⟨ Other Attributes ⟩ };
```

Application: Ray-tracing (using pbrt)



```
float GenerateRay(Sample &s,Ray *r) {
        Point pras = Point(sample.imageX,sample.imageY,0);
        Point pret = RasterToWorld(pras);
        ray->o = pret;
        ray->d = Normalize(AdMap(pret)-pret);
        if (Dot(ray->d,vview) < 0)  ray->d = -(ray->d);
        ⟨ Setting Ray Time and Endpoints ⟩ };
```

OUTLINE

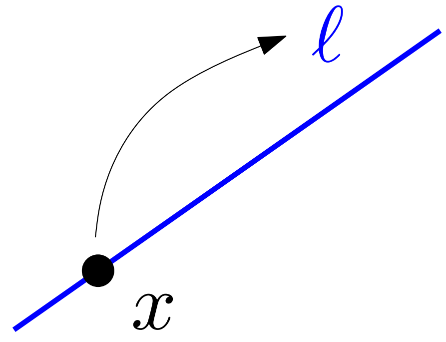
1. Stereo-vision with pinhole cameras
2. The admissible map model
3. Stereo-vision with linear cameras

   2 key ingredients:

   ⋆ inverse projection

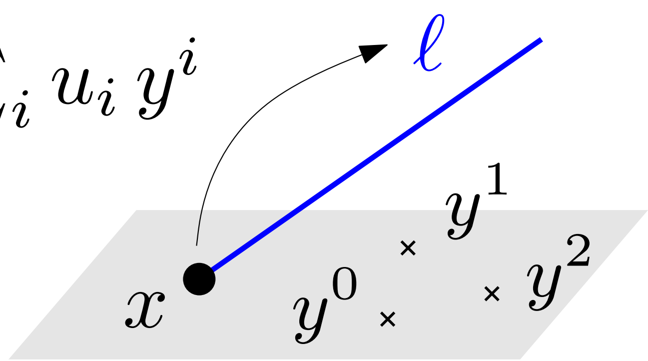
   ⋆ normalized coordinates

$\forall\, x$ non-ambiguous $\quad x \longmapsto x \vee Ax \in \mathcal{L}$
where A is an admissible map for $\mathcal{L}$



$$x \vee Ax = [\xi_0 : \xi_1 : \xi_2 : \xi_3 : \xi_4 : \xi_5]$$

$x = \sum_i u_i\, y^i$



Inverse projection

$$\pi_i : (u_i) \longmapsto (\xi_i)$$

Inverse projection

$$\pi_i(u) = \left(\sum_{i=0}^{2} u_i y^i\right) \vee \left(\sum_{i=0}^{2} u_i A y^i\right)$$

$$\pi_i(u) = \sum_{i=0}^{2} \frac{{u_i}^2}{2} \zeta^{ii} + \sum_{0 \le i < j \le 2} u_i u_j \, \zeta^{ij}$$

where $\zeta^{ij} = y^i \vee Ay^j + y^j \vee Ay^i$

$\widetilde{\mathcal{P}} = [\zeta^{00}, \zeta^{01}, \zeta^{02}, \zeta^{11}, \zeta^{12}, \zeta^{22}]$
(6×6 matrix)

$\mu(u) = ({u_0}^2, u_0 u_1, u_0 u_2, {u_1}^2, u_1 u_2, {u_2}^2)^T$
(6-vector)

$$\pi_i(u) = \widetilde{\mathcal{P}}\, \mu(u)$$

Fundamental matrix

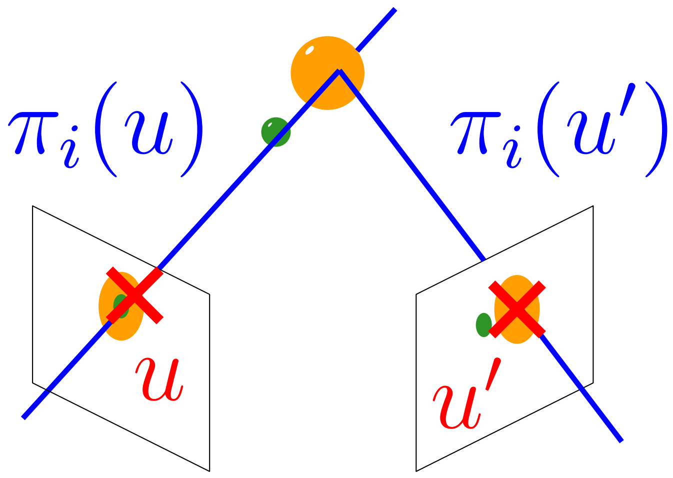


Image coordinates u and u' are in stereo correspondance

$$\Longleftrightarrow$$

$$\pi_i(u) \odot \pi_i(u') = 0$$

$$\xi = [\xi_0 : \xi_1 : \xi_2 : \xi_3 : \xi_4 : \xi_5]^T$$

$$\xi^* = [\xi_3 : \xi_4 : \xi_5 : \xi_0 : \xi_1 : \xi_2]^T$$

$$\xi \odot \xi' = \xi^T \cdot (\xi')^*$$

side-operator

$$\ell \cap \ell' \neq \varnothing \quad \Leftrightarrow \quad \xi \odot \xi' = 0$$

Fundamental matrix

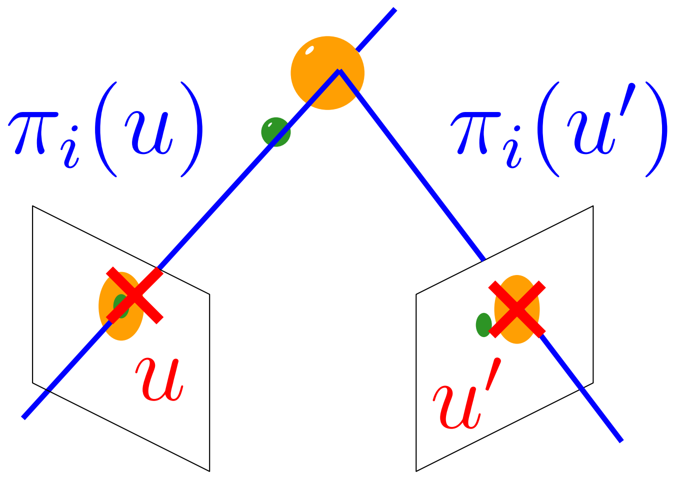


Image coordinates u and u' are in stereo correspondance

$$\Longleftrightarrow$$

$$\pi_i(u) \odot \pi_i(u') = 0$$

$$\mu(u)^T \; \widetilde{\mathcal{P}}^T \, (\widetilde{\mathcal{P}}')^* \; \mu(u') = 0$$

6×6 fundamental matrix

35 correspondances needed

Normalized coordinates

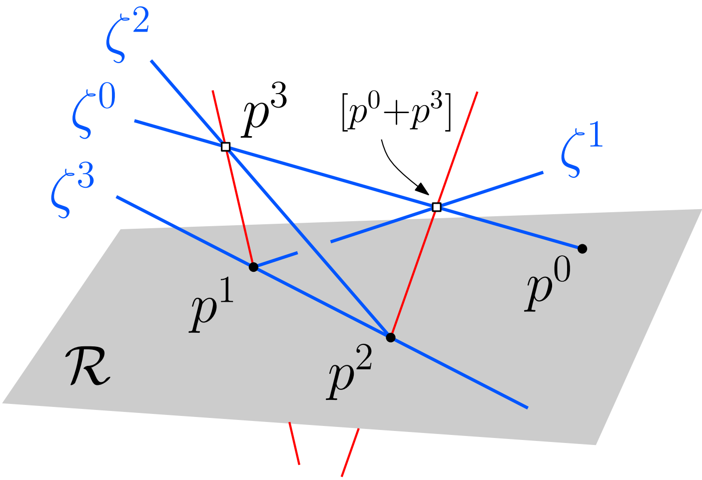
Idea: the bag of lines is spanned by at most 4 lines

⟶ express π_i in that “base” of lines

Normalized coordinates

Idea: the bag of lines is spanned by at most 4 lines

⟶ express π_i in that "base" of lines



X-slit camera

(p^0, p^1, p^2, p^3) = basis of the camera

(p^0, p^1, p^2) = basis of the retina

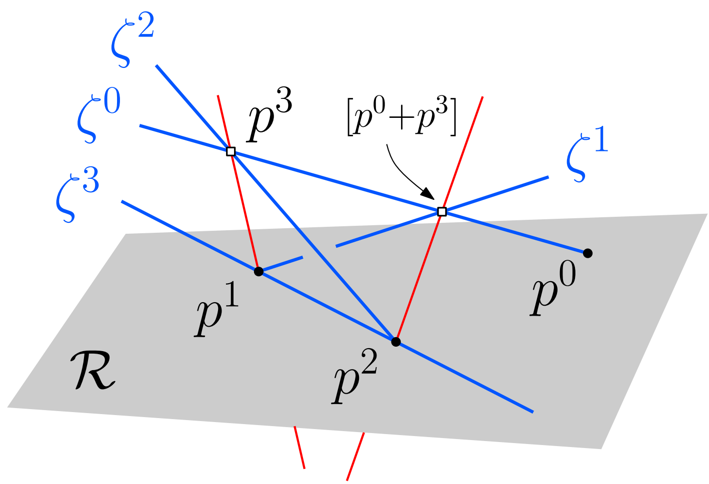
$$\begin{cases} \zeta^0 = p^0 \vee p^3 \\ \zeta^1 = p^1 \vee (p^0 + p^3) \\ \zeta^2 = p^2 \vee p^3 \\ \zeta^3 = p^1 \vee p^2 \end{cases}$$

basis of the bag of lines

Normalized coordinates

Idea: the bag of lines is spanned by at most 4 lines

⟶ express π_i in that "base" of lines



X-slit camera

$(p^0, p^1, p^2, p^3) =$ basis of the camera

$(p^0, p^1, p^2) =$ basis of the retina

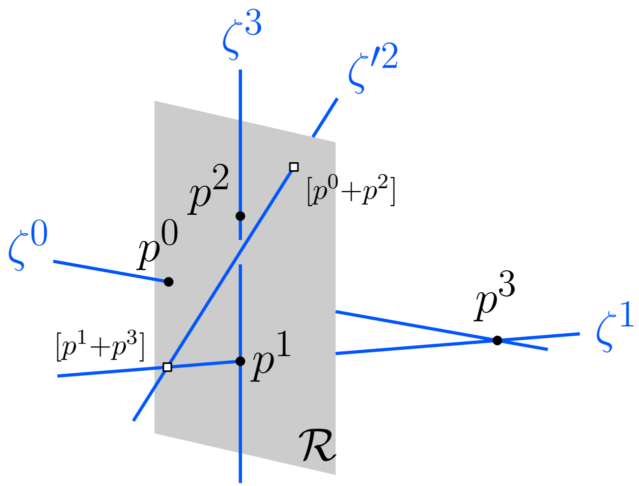
$$\begin{cases} \zeta^0 = p^0 \vee p^3 \\ \zeta^1 = p^1 \vee (p^0 + p^3) \\ \zeta^2 = p^2 \vee p^3 \\ \zeta^3 = p^1 \vee p^2 \end{cases}$$

basis of the bag of lines

$$A_\mathrm{s} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & -1 \end{pmatrix}$$

$$\widetilde{P}_\mathrm{s} \times \mu_\mathrm{s}(u) = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} {u_0}^2 \\ u_0 u_1 \\ u_0 u_2 \\ u_1 u_2 \end{pmatrix}$$

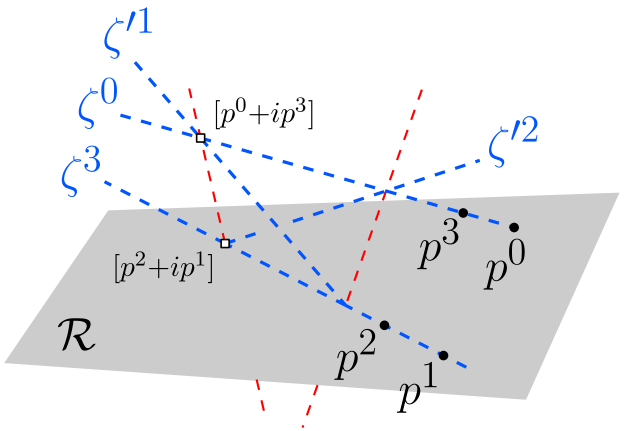
Normalized coordinates



Pencil camera

$$\widetilde{P}_{\mathrm{s}} \times \mu_{\mathrm{s}}(u) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \times \begin{pmatrix} {u_0}^2 \\ u_0 u_1 \\ u_0 u_2 \\ {u_2}^2 \end{pmatrix}$$

$\widetilde{\mathcal{P}}_{\mathrm{s}}$ and μ_{s} depend only on the type of the camera.

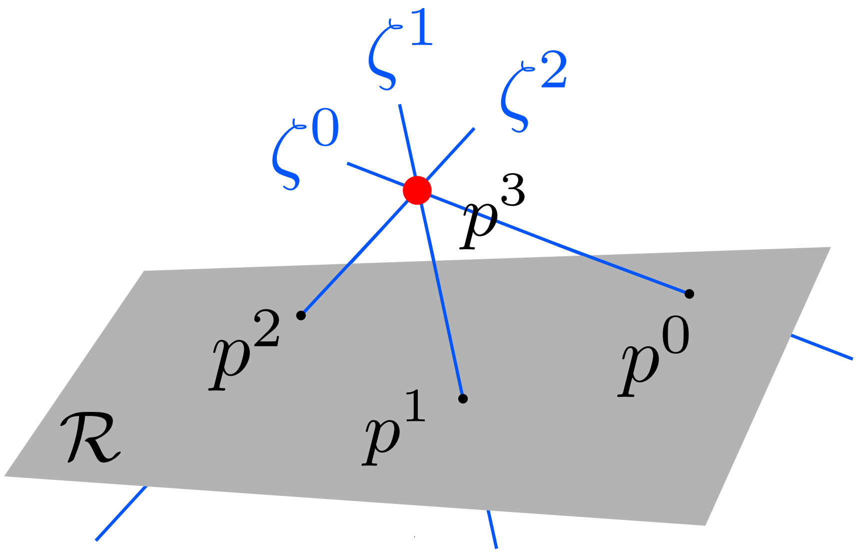


$$\widetilde{P}_{\mathrm{s}} \times \mu_{\mathrm{s}}(u) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \times \begin{pmatrix} {u_0}^2 \\ u_0 u_1 \\ u_0 u_2 \\ {u_1}^2 + {u_2}^2 \end{pmatrix}$$

Linear oblique camera

Normalized coordinates

Also for pinhole cameras!



$$\widetilde{P}_{\mathrm{s}} \times \mu_{\mathrm{s}}(u) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{pmatrix} u_0 \\ u_1 \\ u_2 \end{pmatrix}$$

Calibration

⋆ position of the camera in the world $\longrightarrow W$

camera coordinates in (p^0, p^1, p^2, p^3) $\xrightarrow{\text{invertible } 4\times 4 \text{ matrix}}$ world coordinates in (e^0, e^1, e^2, e^3)

⋆ position of the captor lattice in the retina $\longrightarrow K$

retina coordinates in (p^0, p^1, p^2) $\xrightarrow{\text{invertible } 3\times 3 \text{ matrix}}$ physical / pixel coordinates

Calibration

⋆ position of the camera in the world $\longrightarrow W$

camera coordinates in (p^0, p^1, p^2, p^3)	invertible 4×4 matrix $\longmapsto$	world coordinates in (e^0, e^1, e^2, e^3)

⋆ position of the captor lattice in the retina $\longrightarrow K$

retina coordinates in (p^0, p^1, p^2)	invertible 3×3 matrix $\longmapsto$	physical / pixel coordinates

$$\pi_i(u) = (\Lambda^2 W)\, \widetilde{\mathcal{P}}_{\mathrm{s}} \times \mu_{\mathrm{s}}(K^{-1} u)$$

$\Lambda^2 W$ is a 6×6 matrix encoding the action of W on lines.

Essential matrix

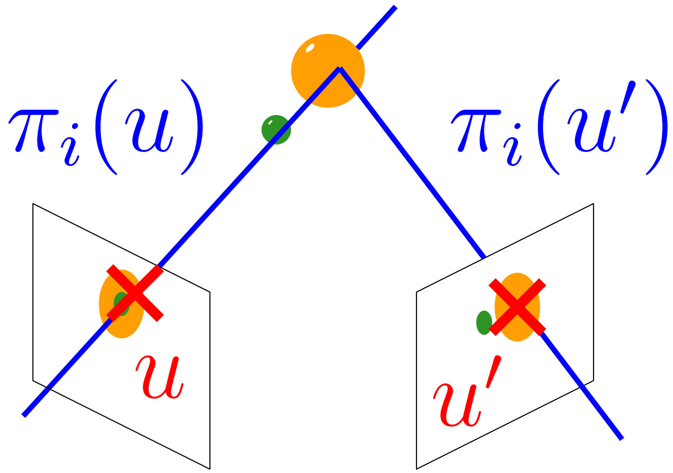


Image coordinates u and u' are in stereo correspondance

$$\iff$$

$$\pi_i(u) \odot \pi_i(u') = 0$$

$$\mu_{\mathrm{s}}(K^{-1}u)^T \; \widetilde{\mathcal{P}}_{\mathrm{s}}^T \; \Lambda^2 W^T \; (\Lambda^2 W')^* \; (\widetilde{\mathcal{P}}'_{\mathrm{s}})^* \; \mu_{\mathrm{s}}(K'^{-1}u') = 0$$

4×4 essential matrix

15 correspondances needed

Euclidean framework

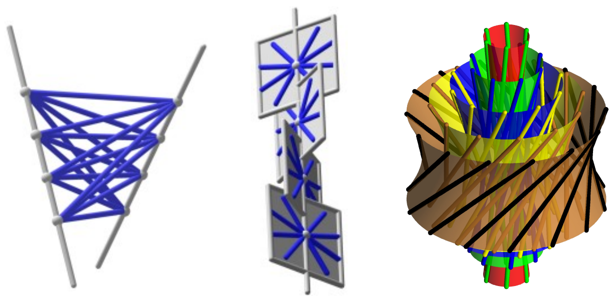
Exactly the same machinery

Only 6 correspondances needed to build $\mathcal{E}$

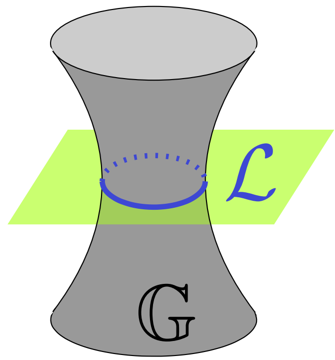
Need *orthonormal* normalized basis (p^0, p^1, p^2, p^3)

Induce more involved intrinsic parameters

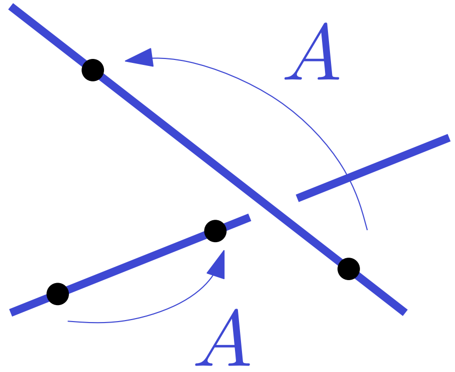
Linear Congruences



Grassmannian Sections



Admissible Maps



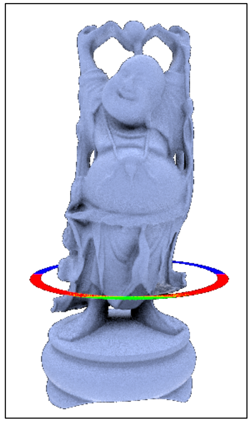
$$\text{Camera} = \begin{cases} \text{bag of lines} \\ + \\ \text{retina} \end{cases}$$

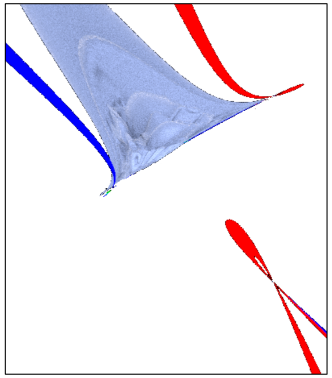
6×6 *fundamental matrix* (uncalibrated case)

4×4 *essential matrix* (calibrated case)

between *ANY* pair of linear cameras

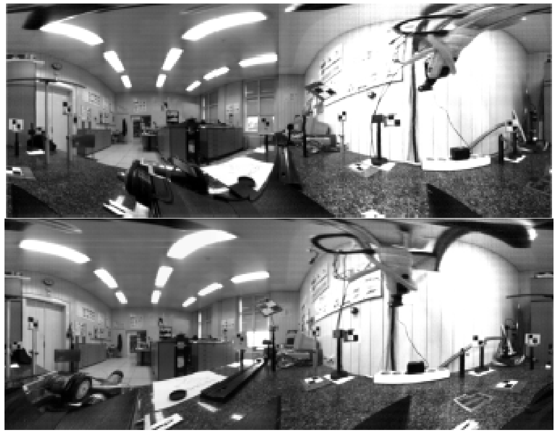
FURTHER QUESTIONS

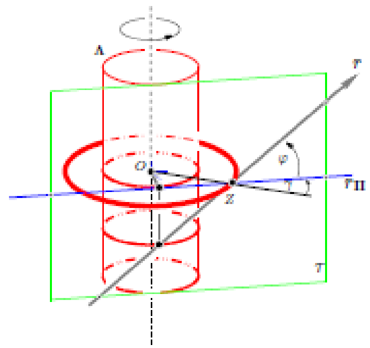




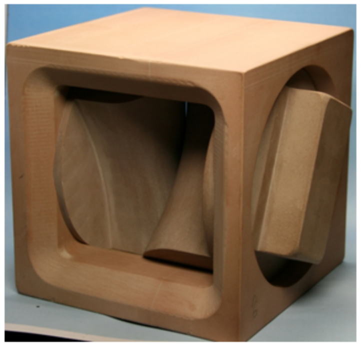
What are the positions of the retina that minimize the distorsions of the image?

Do algebraic line congruences suggest interesting imaging devices? Can they admit analogues of admissible maps?





[Pajdla et al.]



Can we get clear pictures from our pencil camera?

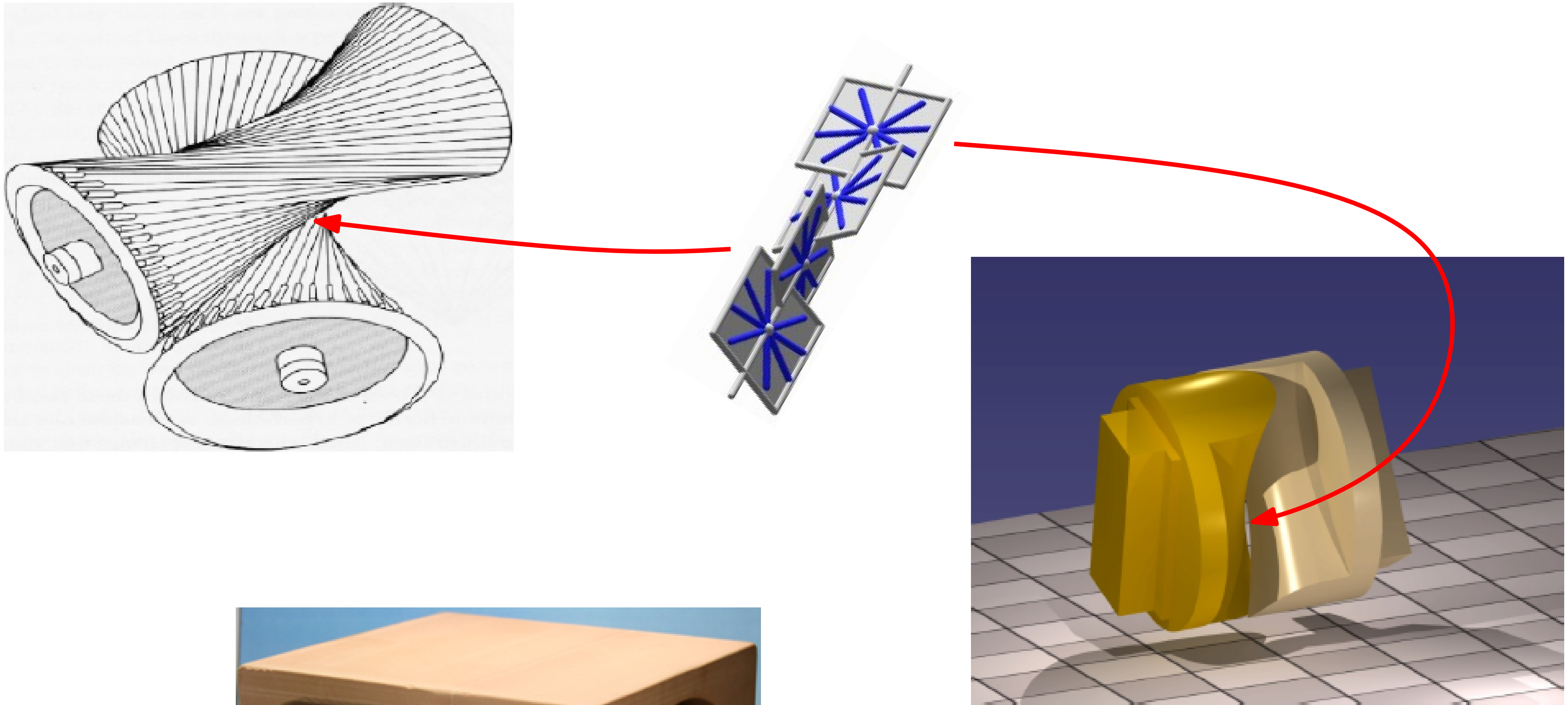
Thank you!

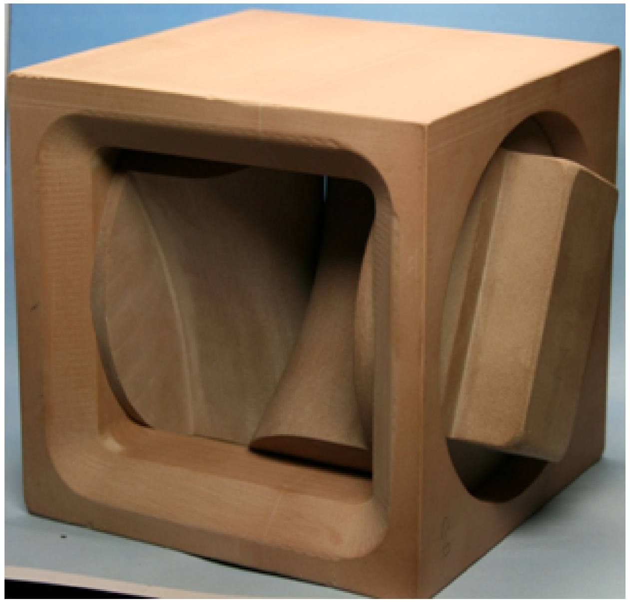
[1] J. Ponce, What is a Camera?, CVPR'09

[2] G. B., X. Goaoc, J. Ponce, Admissible Linear Map Model for Linear Cameras, CVPR'10

[3] The "linear camera" model (*in preparation*)

Pencil camera





our pencil camera

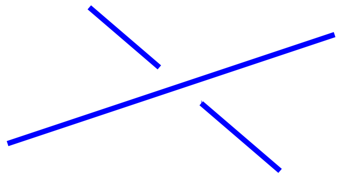
Synthesis

	projective linear camera	euclidean linear camera	euclidean pinhole camera
extrinsic parameters	15	6	6
intrinsic parameters	6	12	5
fundamental matrix	27		7

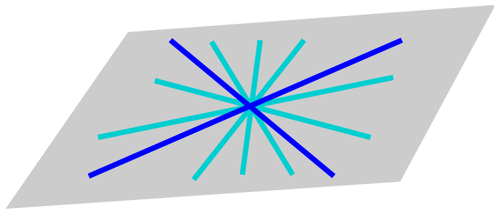
Geometric axiomatisation for linear dependance

[Veblen&Young,1910]

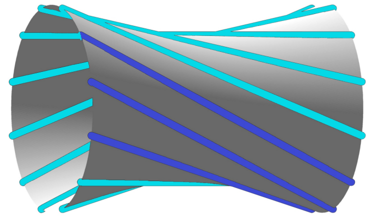
Initialisation



two lines



line pencil



regulus

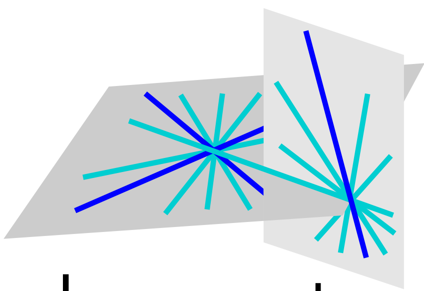
Heredity

ℓ depends linearly on $L = \{\ell_1, ..., \ell_k\}$ iff

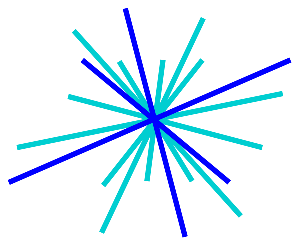
$$\exists\ L \rightarrow \ell_{k+1} \rightarrow \cdots \rightarrow \ell_{k+n} = \ell$$

where $\rightarrow$ is a line pencil/regulus construction.

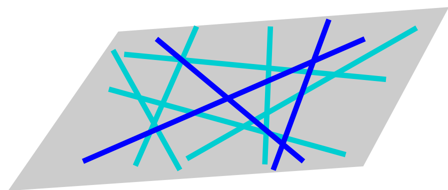
Exemples



degenerate regulus



bundle



field

(Need 2 line pencil steps)