Enumerate!
Don't Estimate

Shai Avidan

Tel-Aviv University

Based on PhD work of Simon Korman

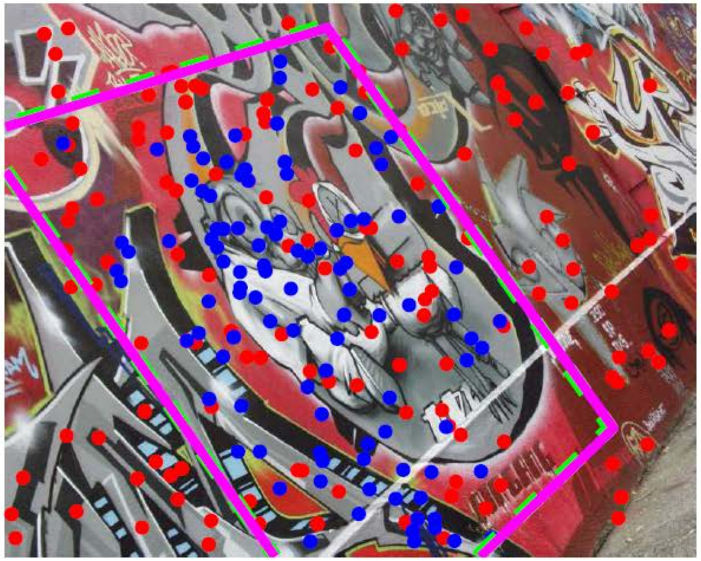
Prague, 12/11/2015

Image Matching

Image Matching (Optic Flow)

- Initial guess
- Local minima

Image Matching (Interest Points)



Outliers → RANSAC

- **Random**
- **No global guarantees**
- **Inlier error threshold**

Goal of this Talk

Deterministic Algorithms with
global approximation guarantees

How?

Enumerate all possible transformations
and pick the best one!

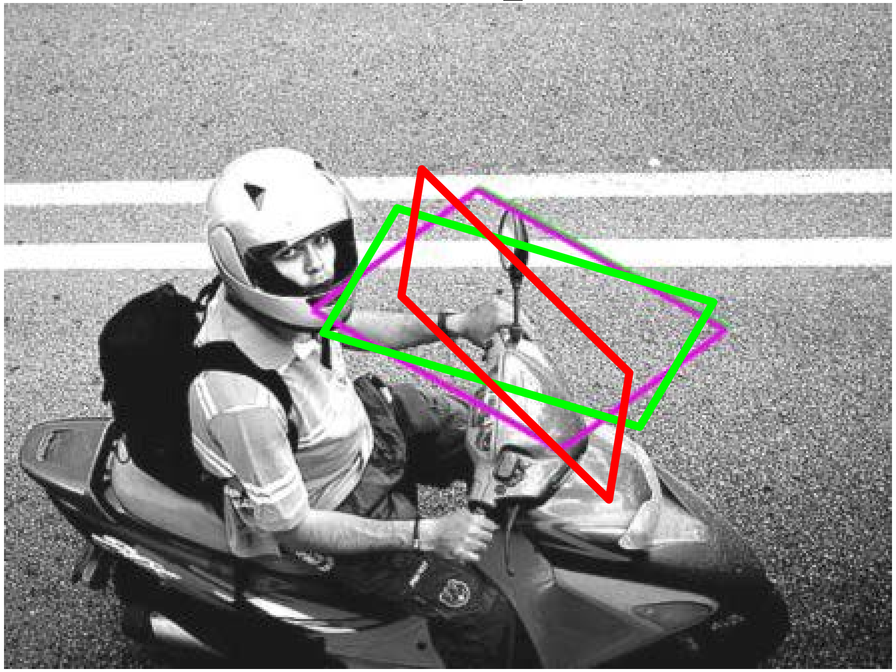
2D Affine Template Matching

FAST-Match. Korman, Reichman, Tsur, Avidan. 2013

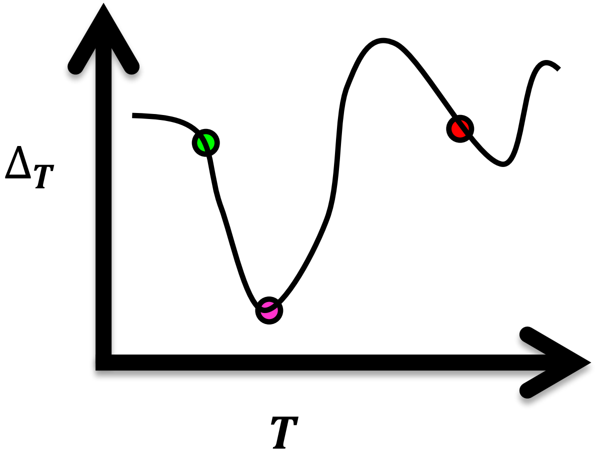
The main idea

Template (I_1)

Image (I_2)

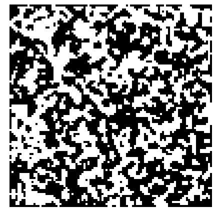
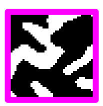
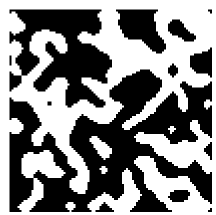
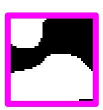


Trans. space



$$\Delta_T = \sum_{p \in I_1} |I_1(p) - I_2(T(p))|$$

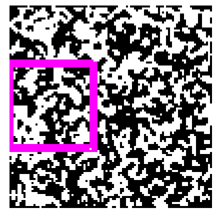
Simulation

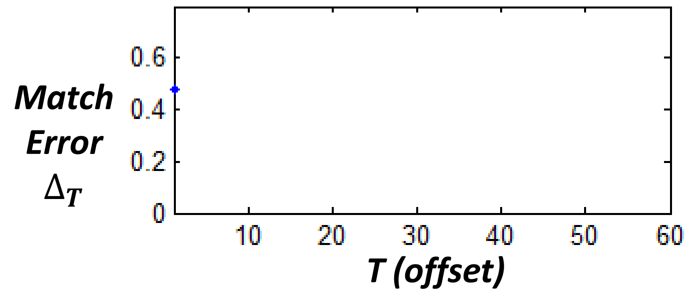
Total Variation	Template	Image
High		
Med		
Low		

Simulation

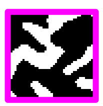
Total Variation | Template | Image

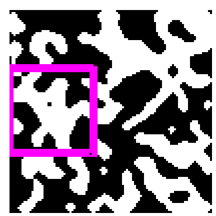
High

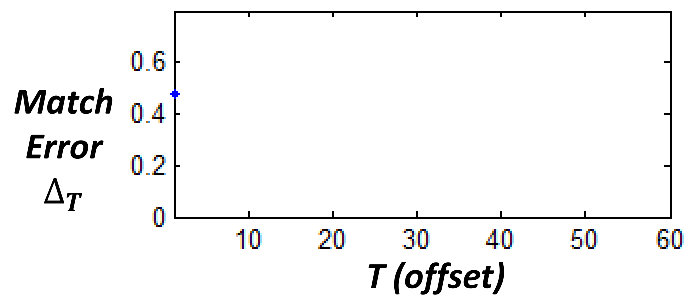




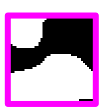
Med

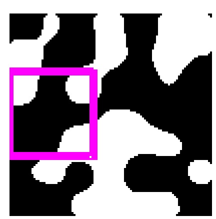


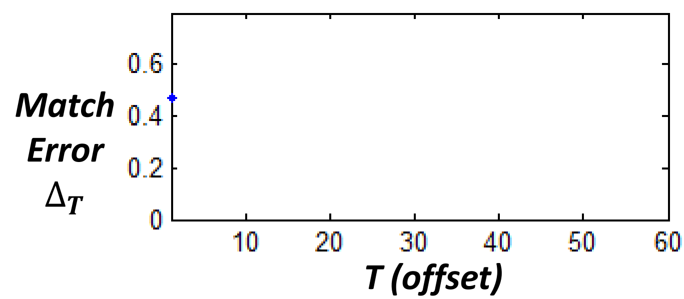




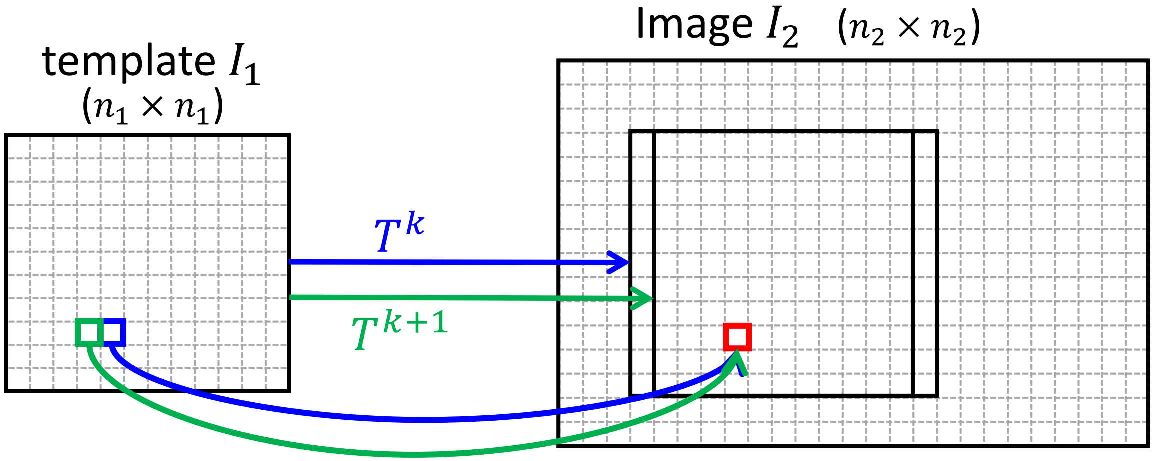
Low







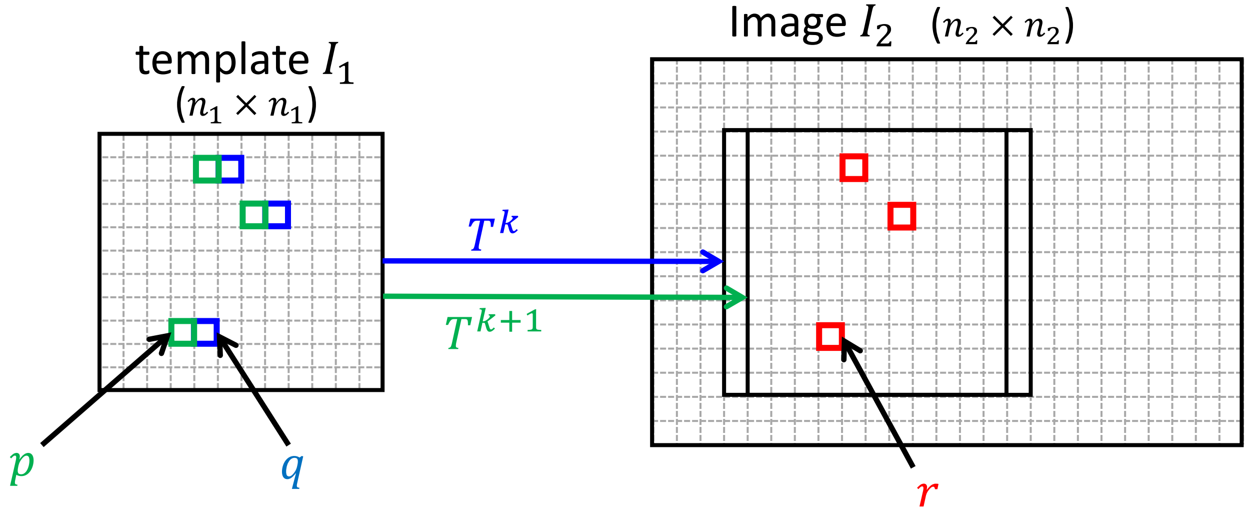
Intuition



- T^k: Shift by *k* pixels
- T^{k+1}: Shift by *k+1* pixels

$$dist\,(T^{k+1}, T^k) = 1$$

Main Insight - Intuition

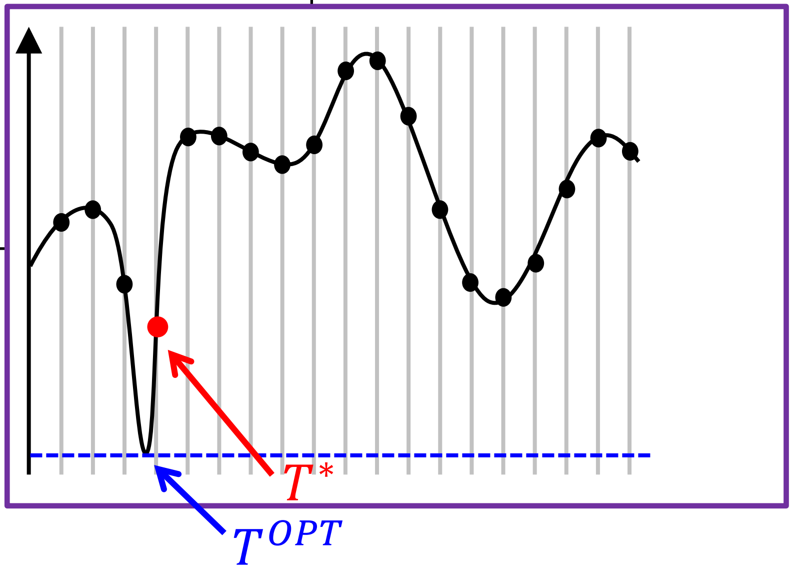


$$dist\,(T^{k+1}, T^k) < \delta n_1 \;\Rightarrow\; \left|\Delta_{T^{k+1}}(I_1, I_2) - \Delta_{T^k}(I_1, I_2)\right| < O\left(\delta \frac{TV}{n_1}\right)$$

$$\left|\Delta_{T^{k+1}} - \Delta_{T^k}\right| \leq \frac{1}{{n_1}^2} * TV(I_1)$$

The Algorithm

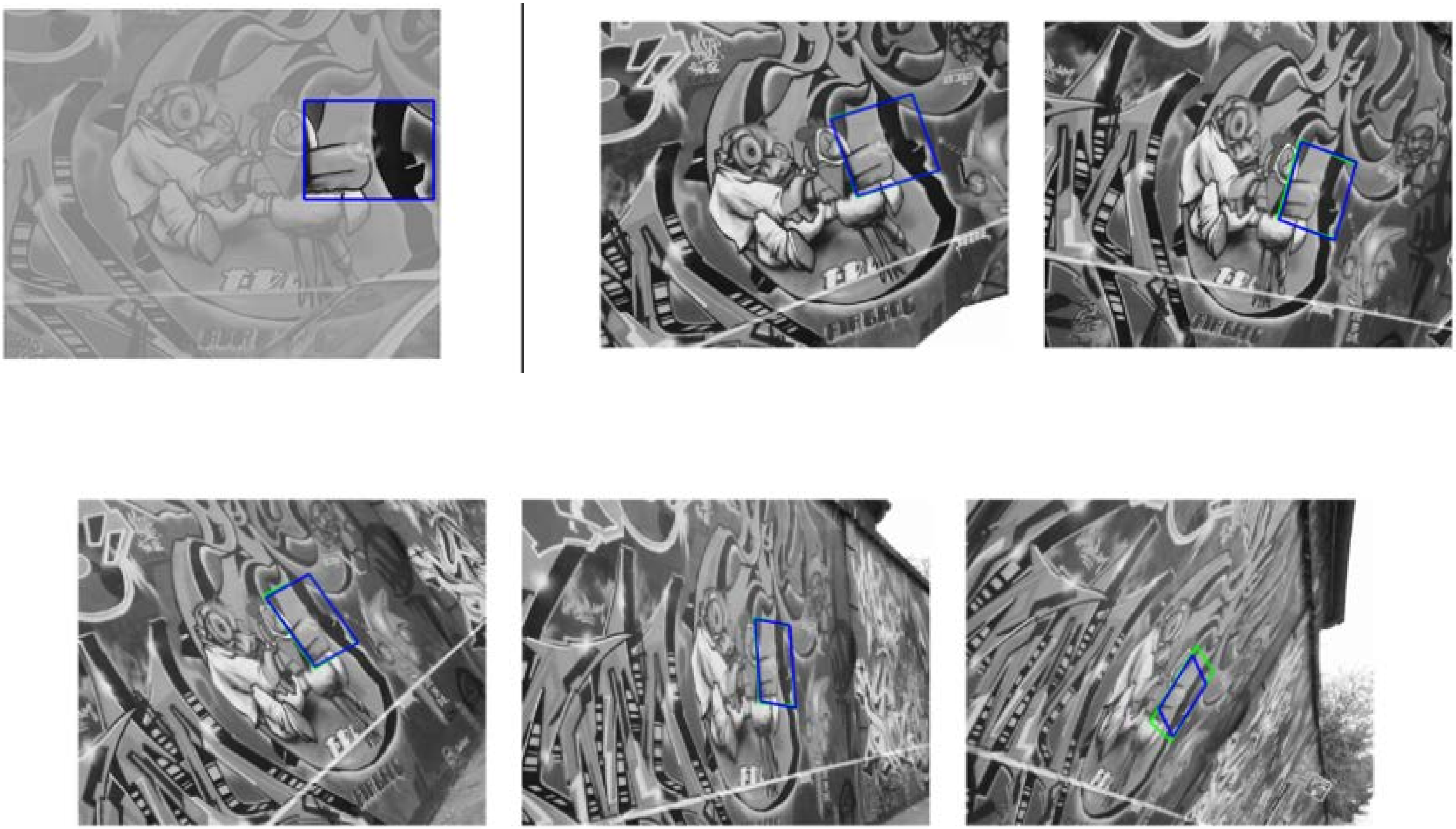
- For each affine transformation T in a Net A_δ
  - Compute the error $\Delta_T(I_1, I_2)$
- Return T^* with smallest error



- Sample transformation space
  - build a δn_1-Net A_δ of transformations
  - The size of the net is $\left(\frac{1}{\delta}\right)^6 \cdot \left(\frac{n_2}{n_1}\right)^2$
- **Guarantee**
  - 'δ – away' from best possible error

$$\left|\Delta_{T^{OPT}} - \Delta_{T^*}\right| = O(\delta)$$

Mikolajczyk– graffiti (viewpoint)



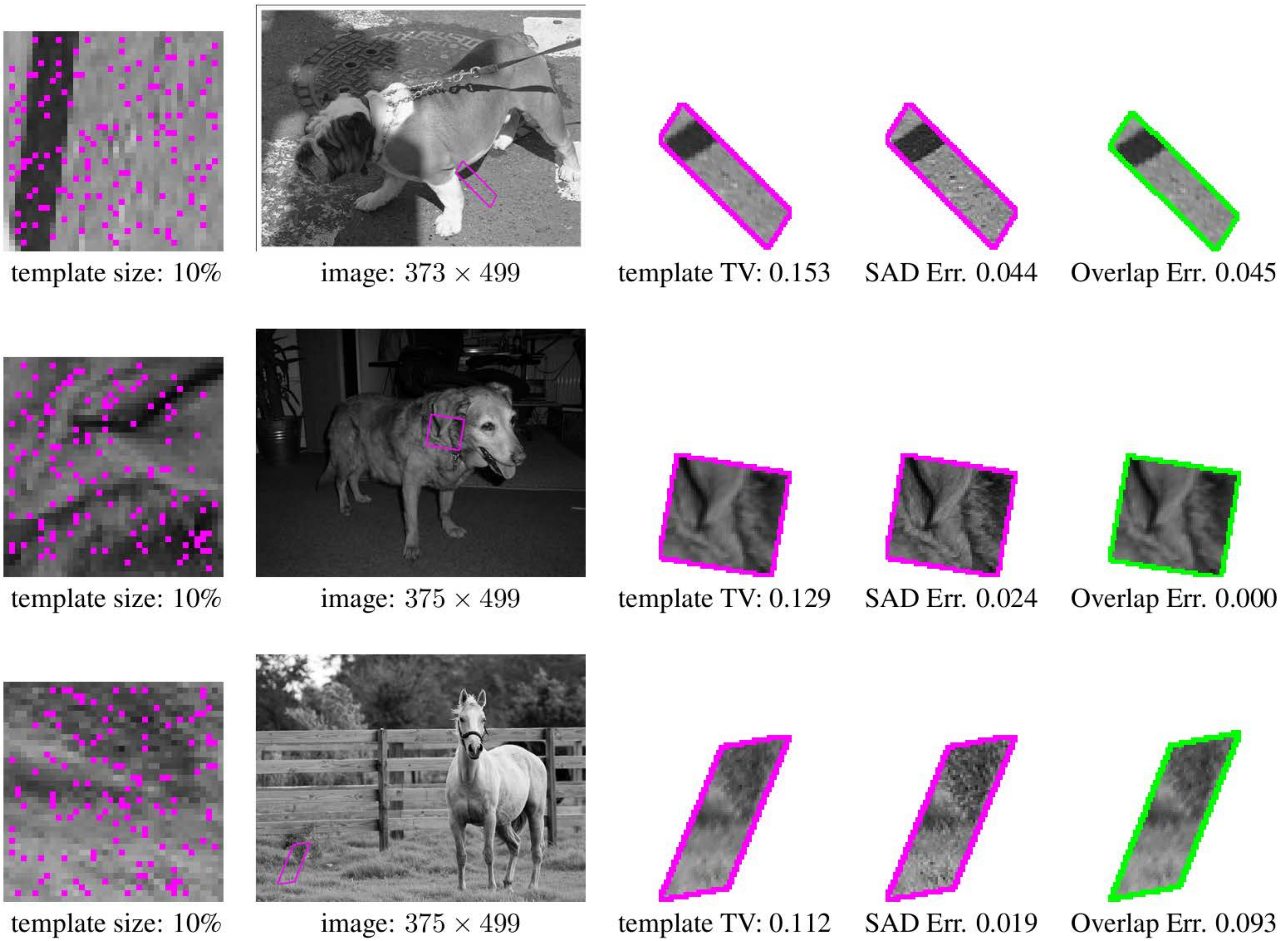
Template Dim: 45%

template size: 45% | image: 375 × 499 | template TV: 0.045 | SAD Err. 0.013 | Overlap Err. 0.015

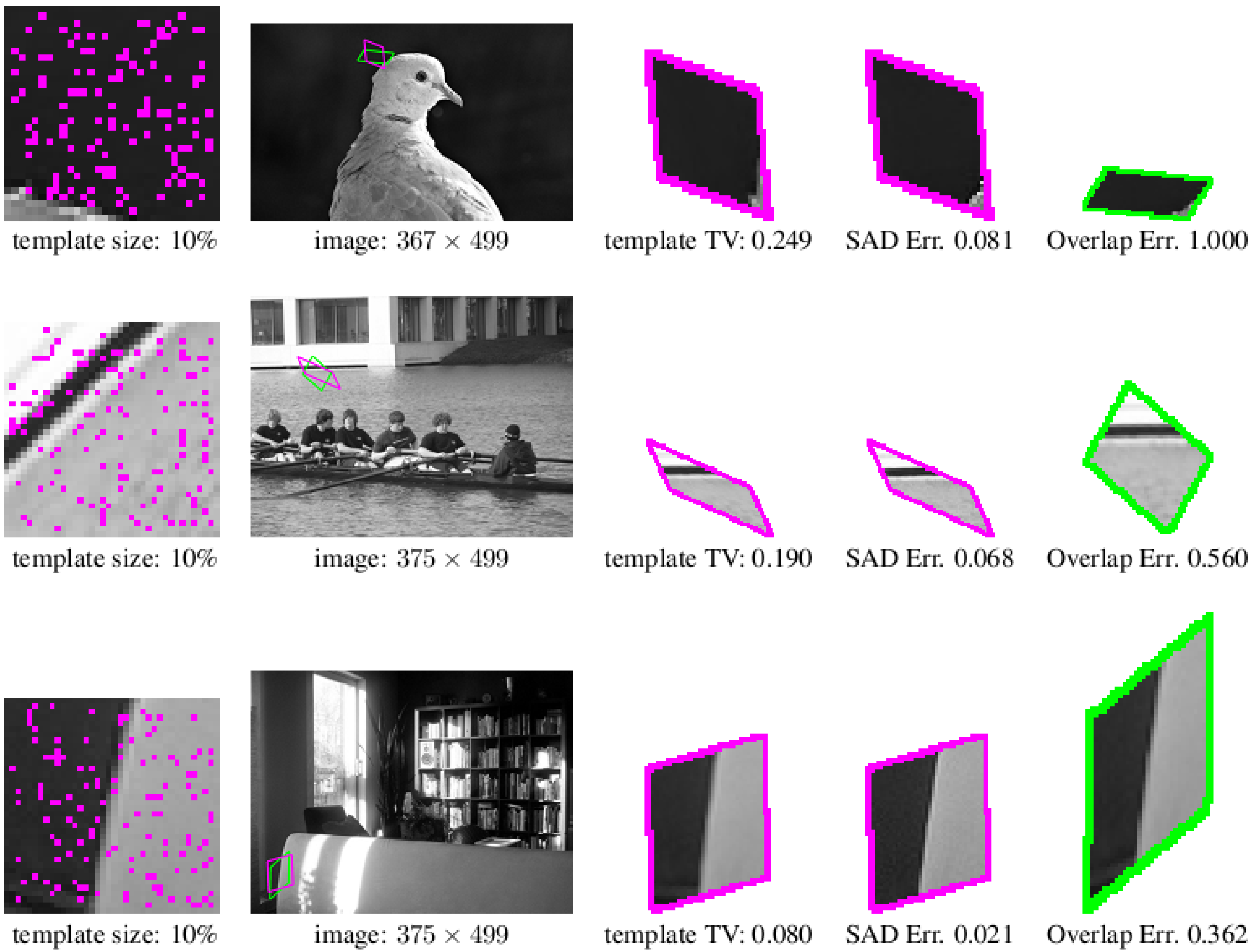
template size: 45% | image: 375 × 499 | template TV: 0.146 | SAD Err. 0.095 | Overlap Err. 0.114

template size: 35% | image: 375 × 499 | template TV: 0.071 | SAD Err. 0.020 | Overlap Err. 0.017

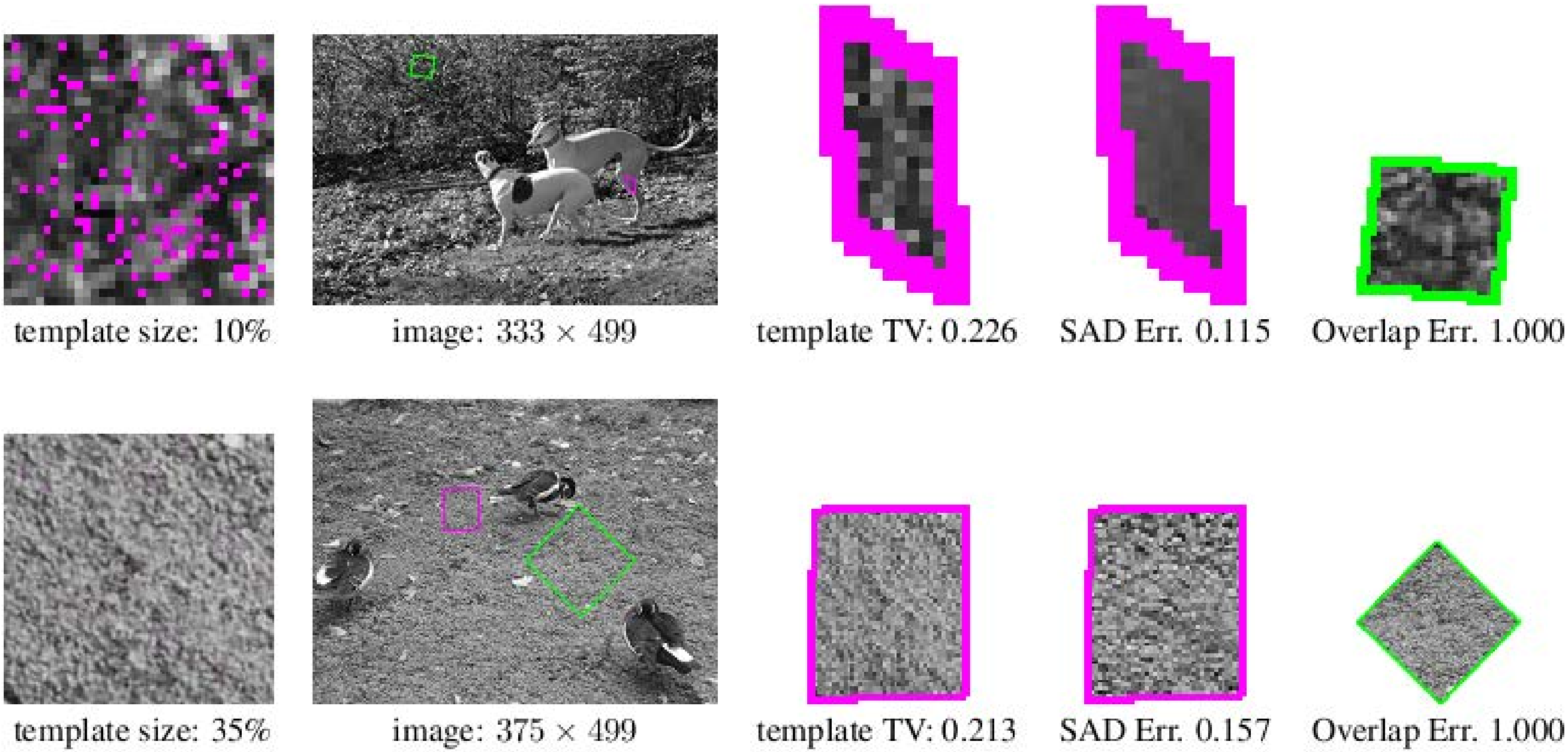
Template Dim: 10%



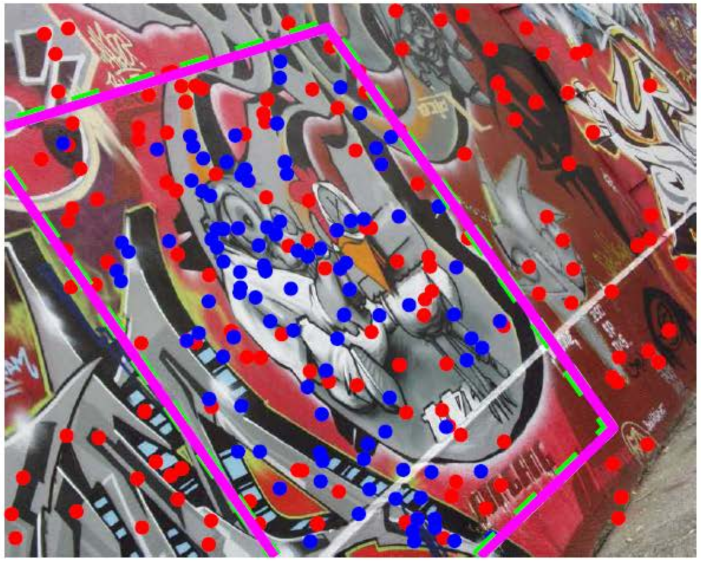
Bad overlap due to ambiguity



High SAD due to high TV and ambiguity

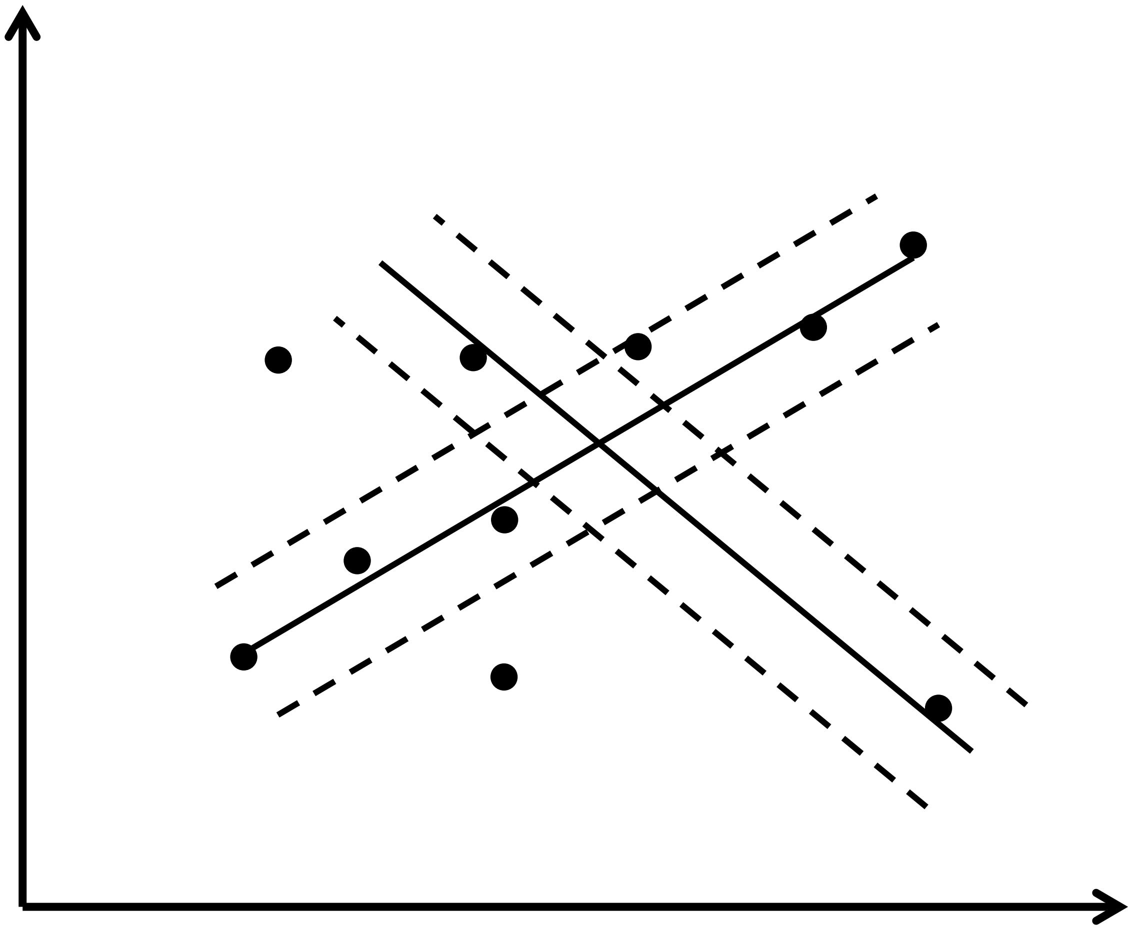


Replacing RANSAC



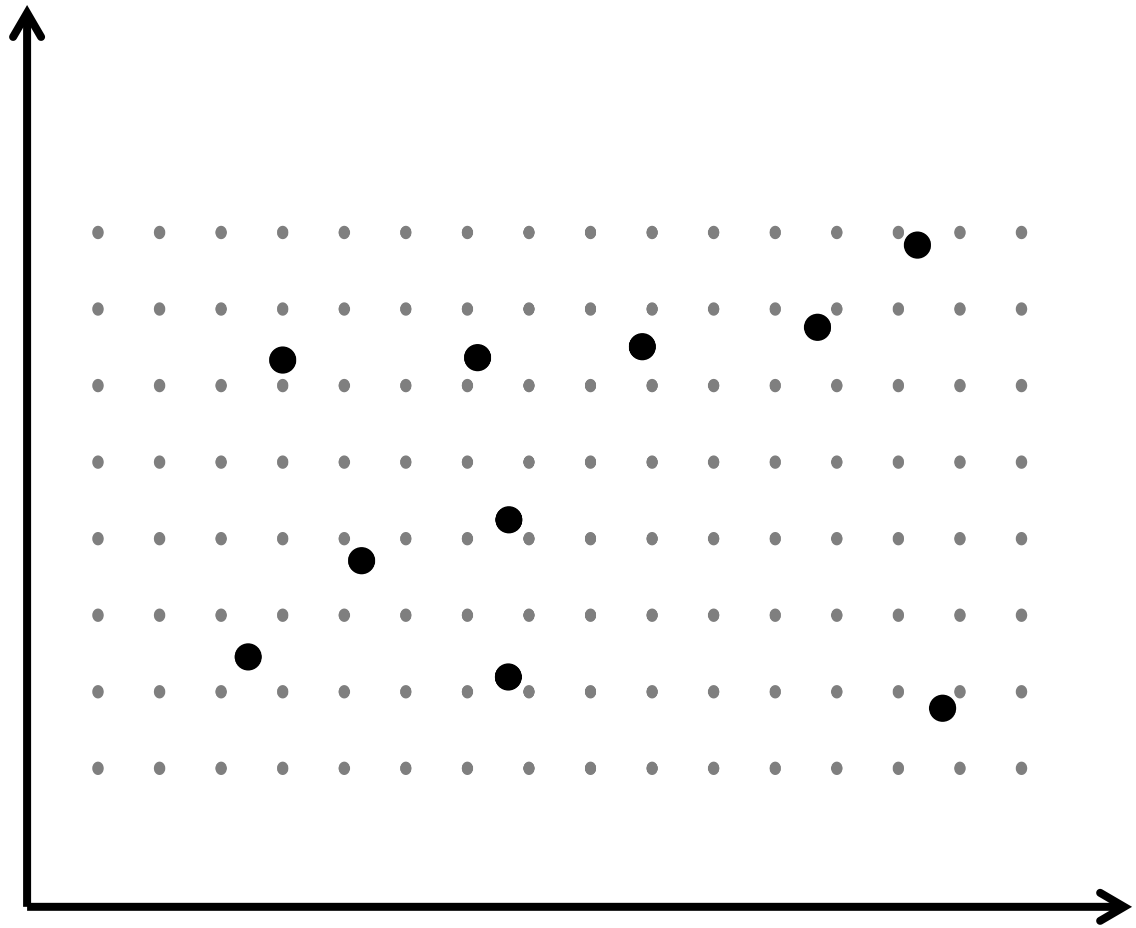
Litman, Korman, Bronstein, Avidan. CVPR 2015

Canonical Example (RANSAC)

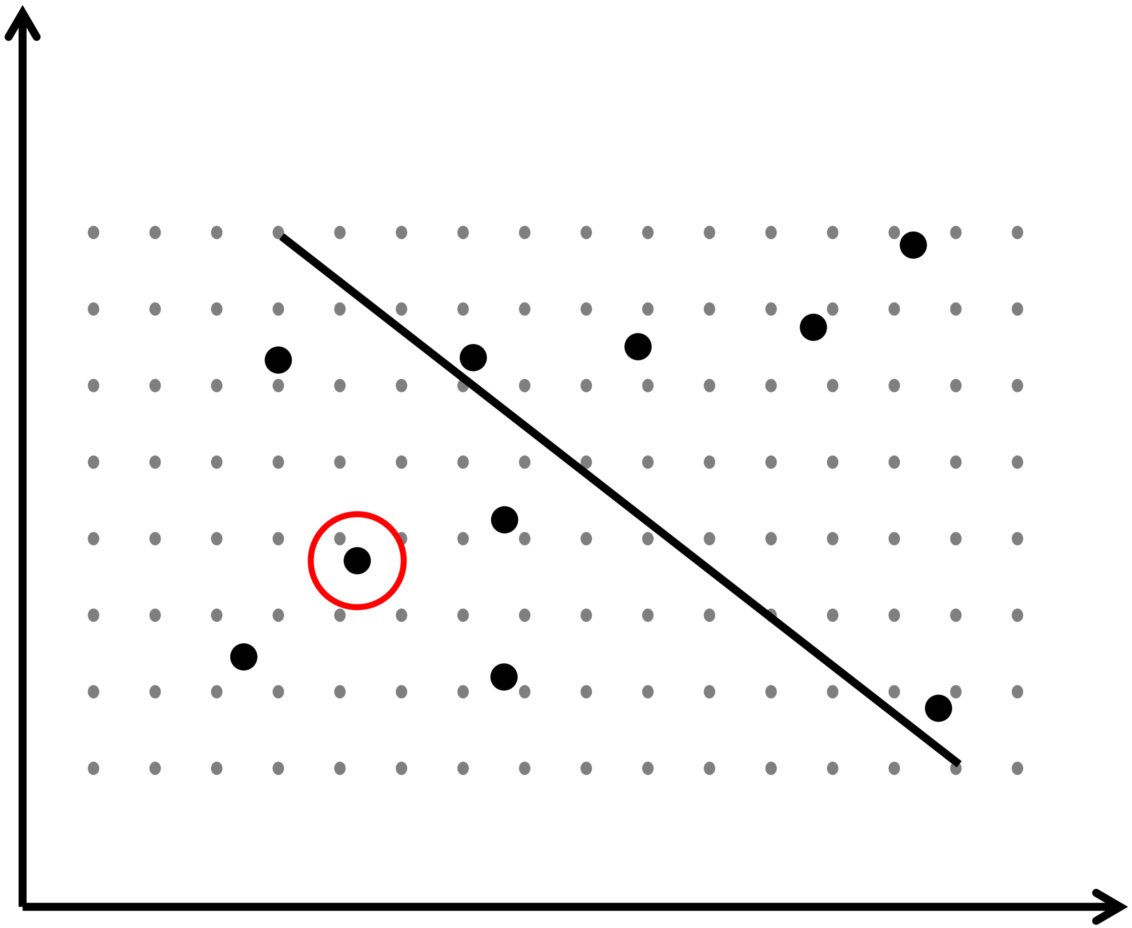


Accept inlier error r* as input

Canonical Example
Global Model Detection (GMD)

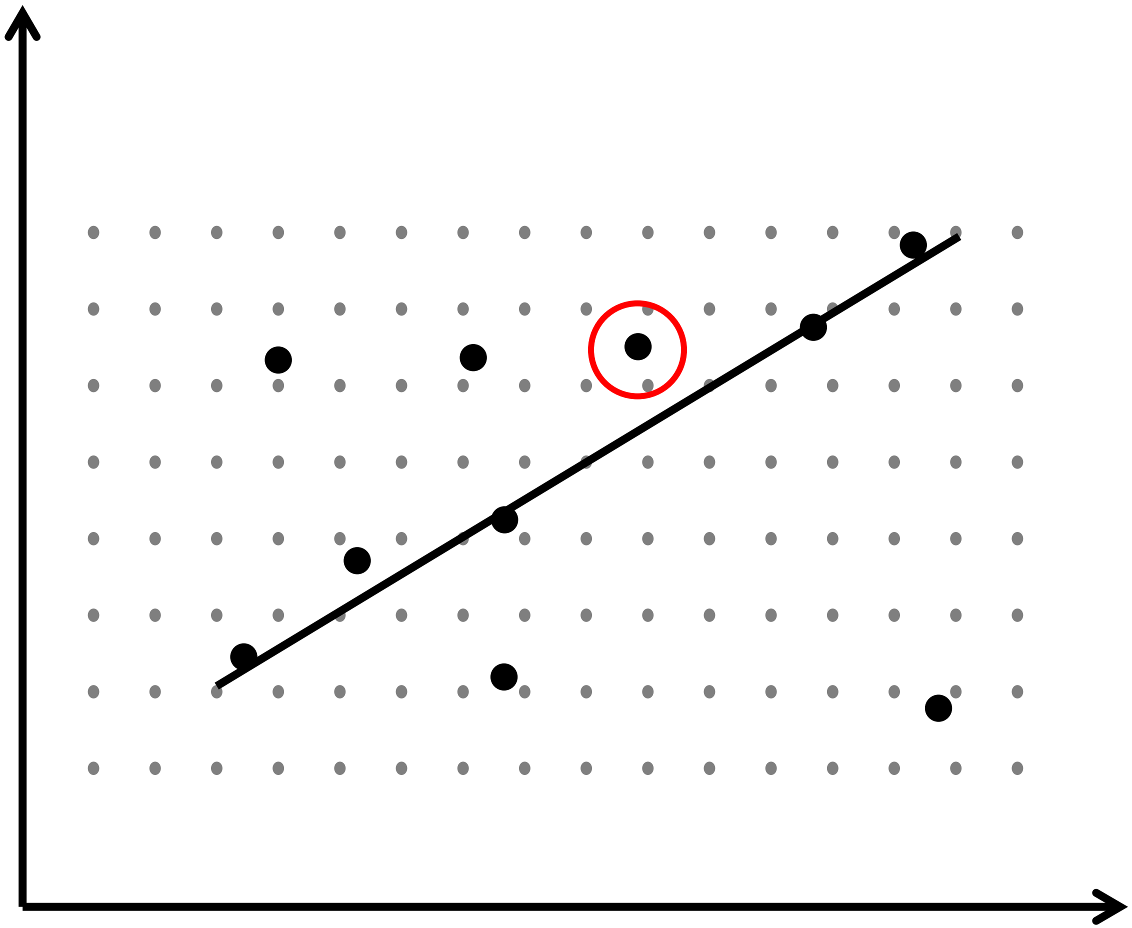


Suppose we know p*=6

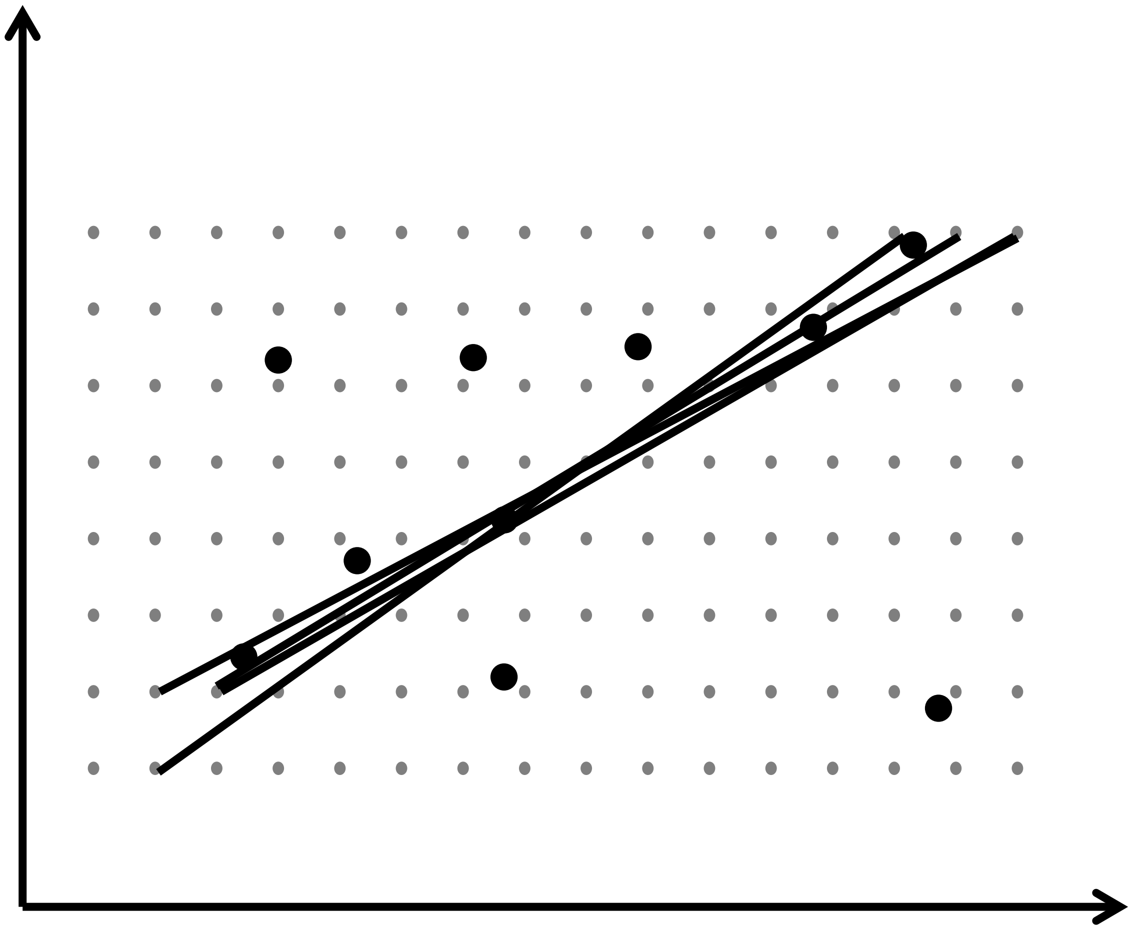


Instead of inlier error r*, assume p* is given

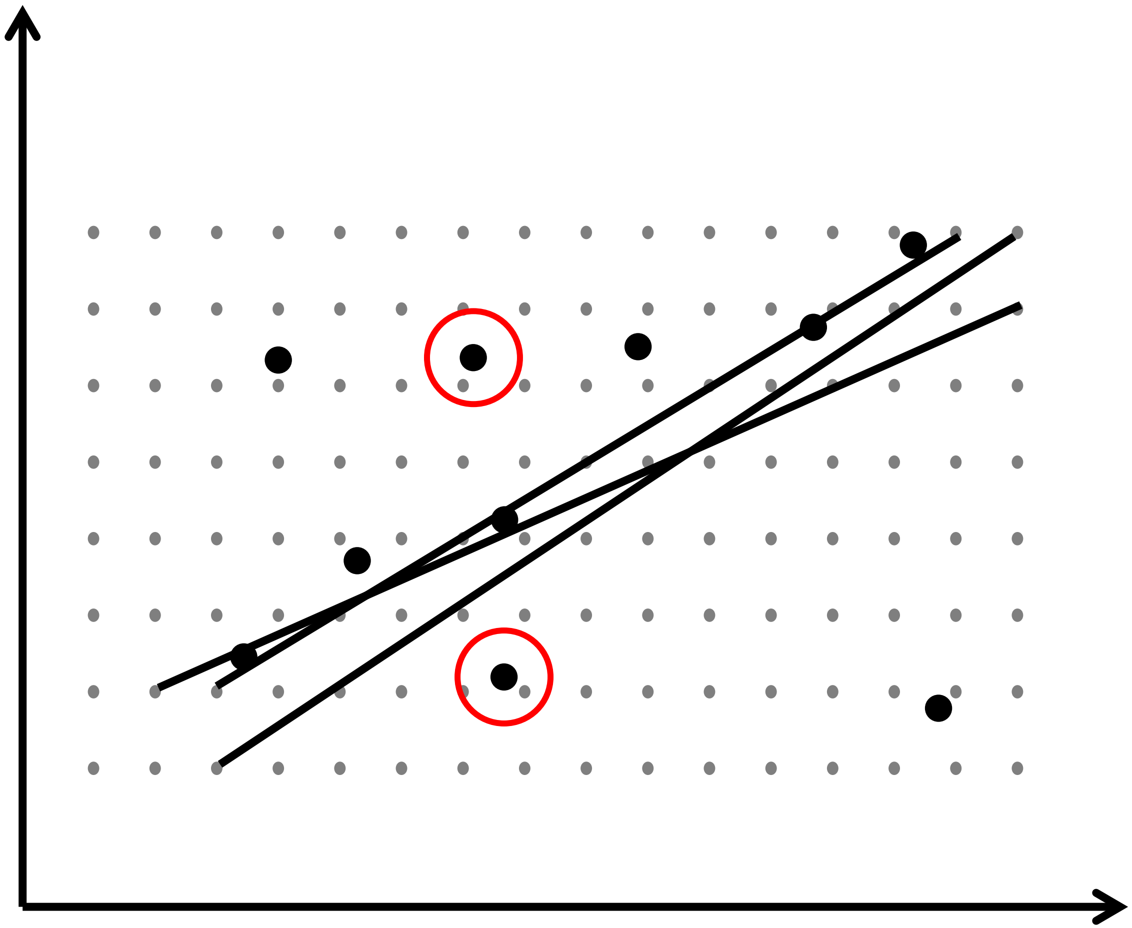
Suppose we know p*=6



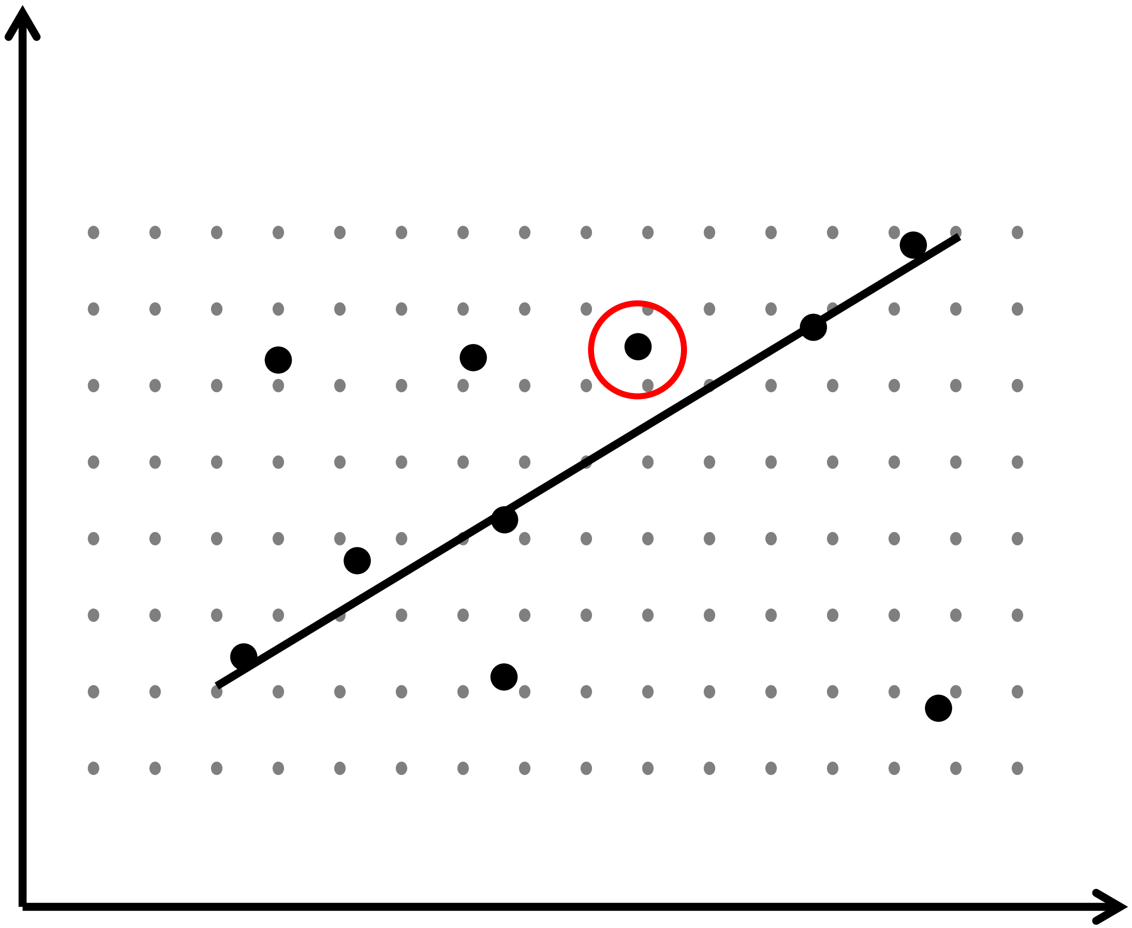
Suppose we guess p*=4



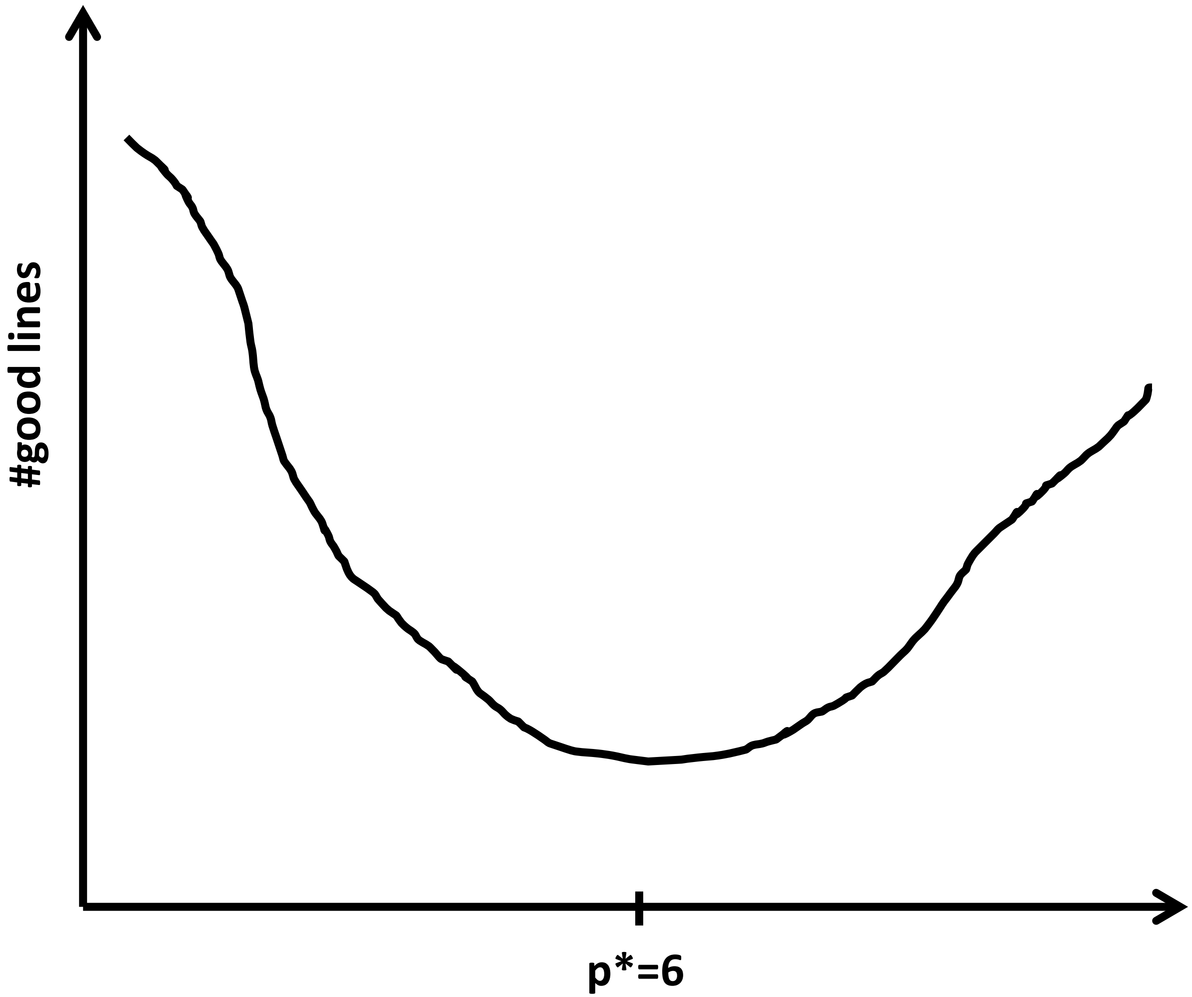
Suppose we guess p*=8



Suppose we guess p*=6

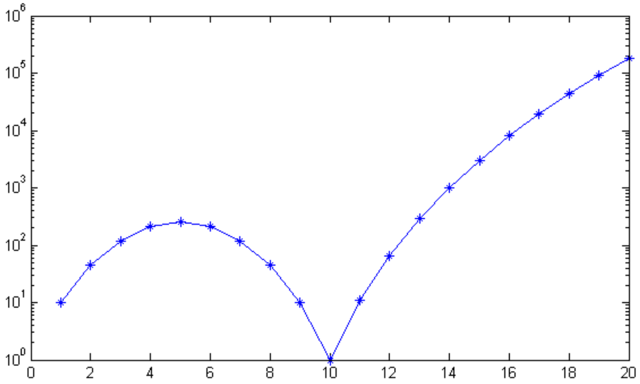


Correct p* → #good lines is minimal



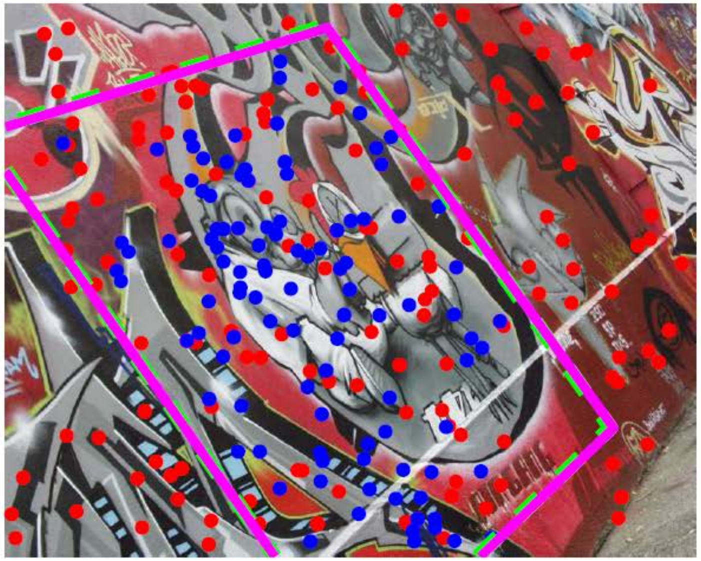
A combinatorial metaphor

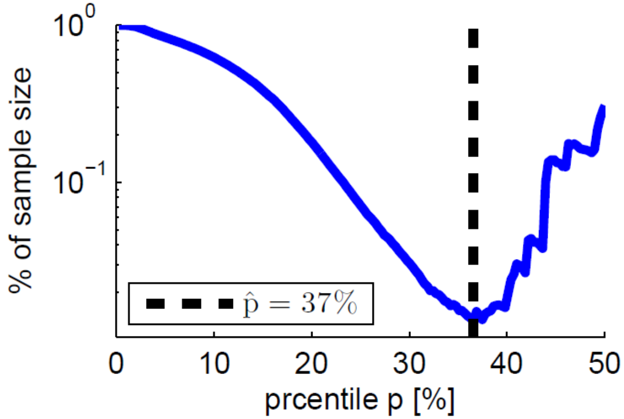
- **Bag with N balls:**
  - k **white (inliers) ,** $\mathrm{N}-k$ **black (outliers)**
- **Pick** $\widehat{k}$ **balls (an estimate of** k**) with a max number of whites**
- **How many options are there?**
  - **If** $\widehat{k} < k$ **:** $\binom{k}{\widehat{k}}$ **options**
  - **If** $\widehat{k} > k$ **:** $\binom{N-k}{\widehat{k}-k}$ **options**



- **Metaphor explained**
  - **white/black balls → inliers / outliers**
  - **`selection of balls' → transformation**
  - **# of possible selections → our measure**

Image Matching (interest points)

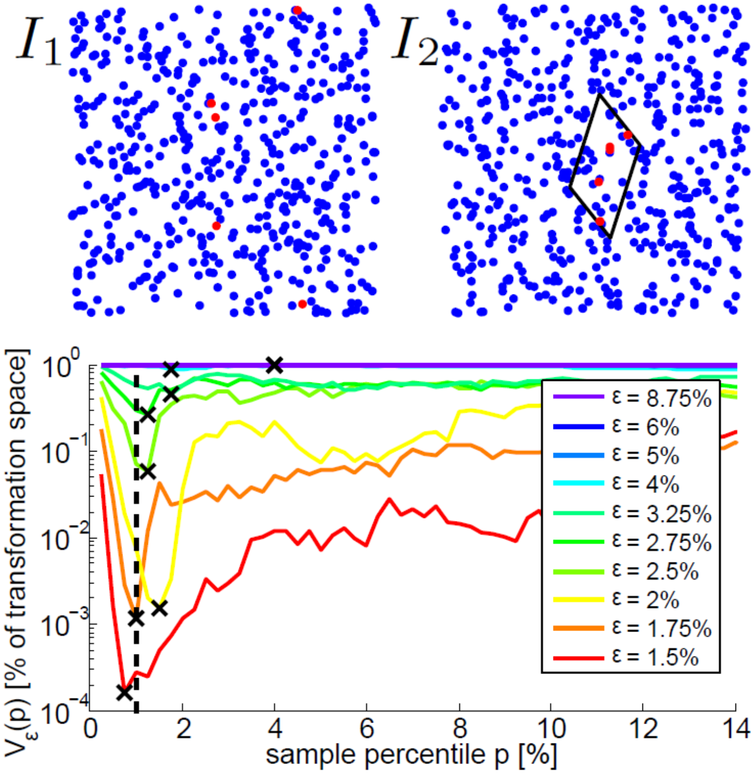




Method

1. Sample homography space
2. Estimate inlier rate p^*
3. Find best transformation for p^*

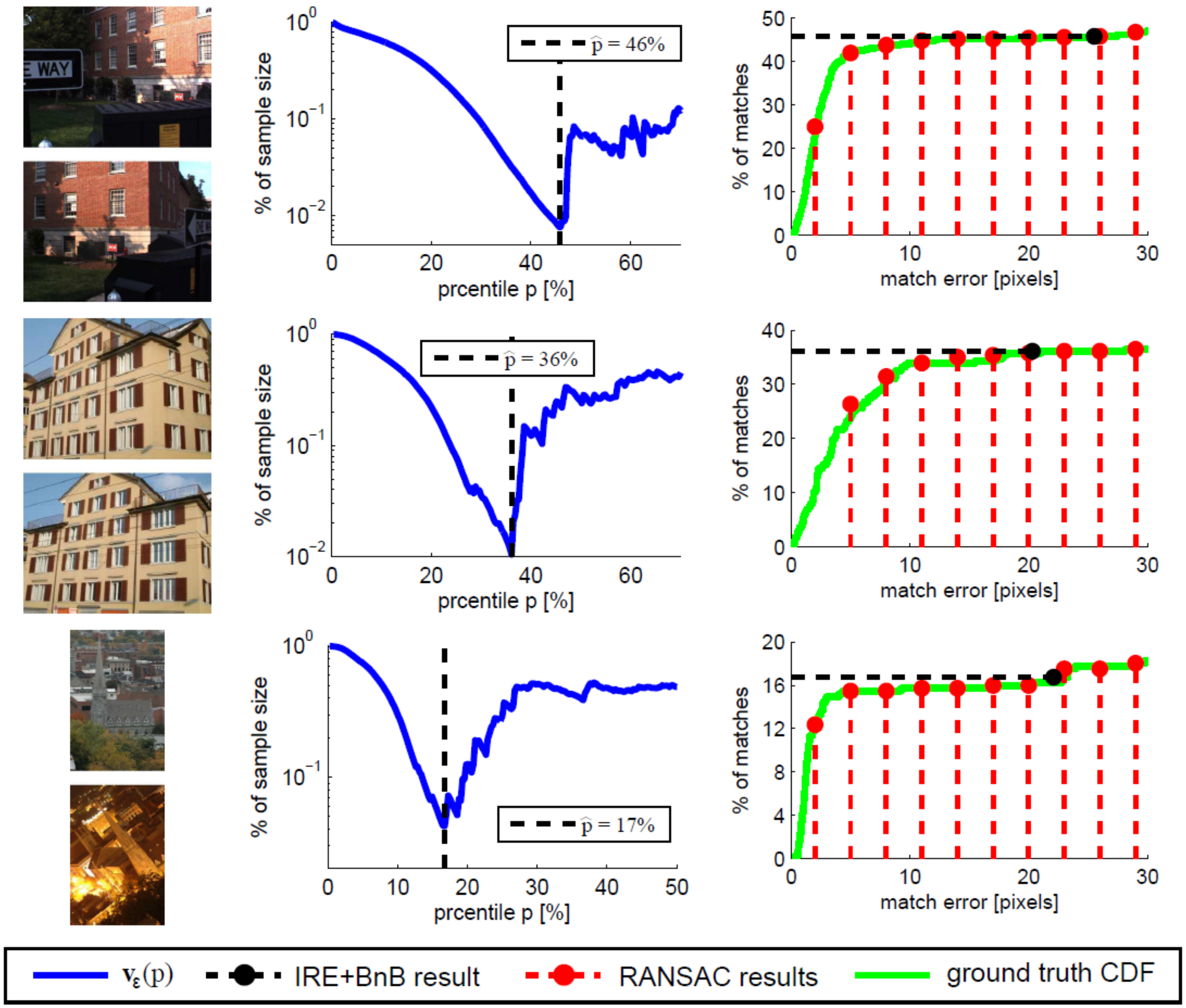
2D-Affine: synthetic data



(low inlier rate $p^\star$)

$p^\star = 1\%$ and $r^\star = 1\%$

2D-Homography: real data



2D-Homography: real data

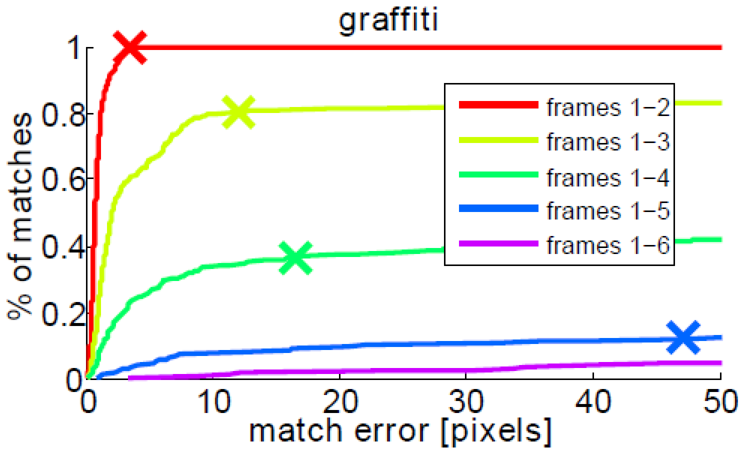
- **Mikolajczyk sequences: Sampson errors**

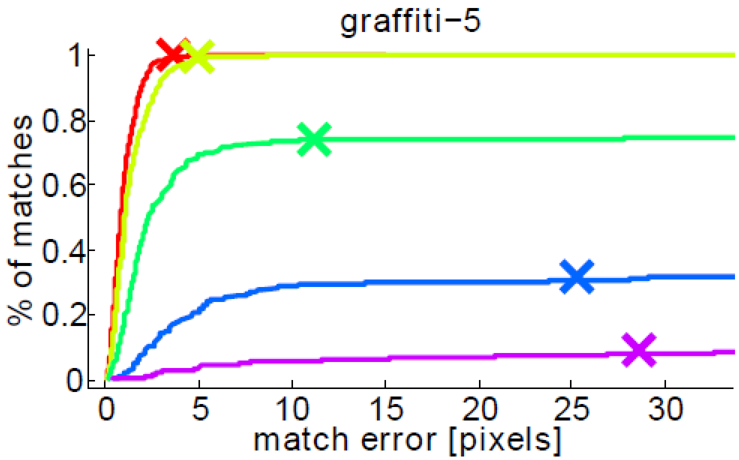
		Image pair				
sequence	method	1-2	1-3	1-4	1-5	1-6
bark	GMD	1.56	**3.45**	**2.53**	**1.14**	2.36
	USAC	**1.55**	3.49	**2.53**	1.15	**2.35**
graffiti	GMD	0.54	1.53	1.45	**6.55**	fail
	USAC	**0.53**	**0.85**	**0.98**	fail	fail
graffiti 4	GMD	**0.38**	**1.06**	**0.59**	**0.92**	**1.11**
	USAC	0.42	1.23	0.85	1.27	1.27
graffiti 5	GMD	0.75	**1.23**	**2.00**	**2.51**	**6.63**
	USAC	**0.62**	1.51	2.03	2.52	fail
wall	GMD	**1.24**	0.60	**1.29**	**1.56**	**2.12**
	USAC	**1.24**	**0.59**	1.33	1.66	2.81

2D-Homography: real data

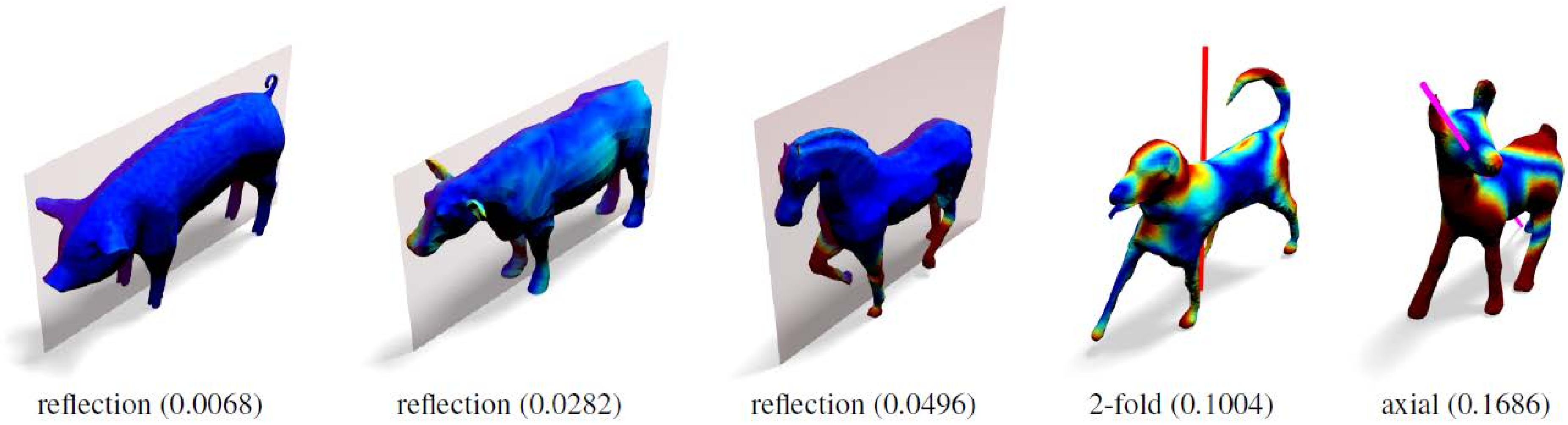
- **Mikolajczyk sequences: Inlier rates and respective errors**

		Image pair				
sequence	method	1-2	1-3	1-4	1-5	1-6
graffiti	GMD	99.8% 0.5±0.7	80.2% 1.8±1.8	36.6% 2.2±3.4	12.6% 8.5±14	fail
	USAC	87.8% 0.5±0.3	36.6% 0.7±0.3	19.5% 1.1±0.6	fail	fail
graffiti 5	GMD	99.8% 0.6±0.6	99.2% 0.9±0.8	74.2% 1.4±1.5	31.6% 3.0±4.9	9.3% 6.3±11
	USAC	89.2% 0.5±0.3	66.4% 0.6±0.3	33.6% 0.7±0.3	15.3% 1.5±0.7	fail



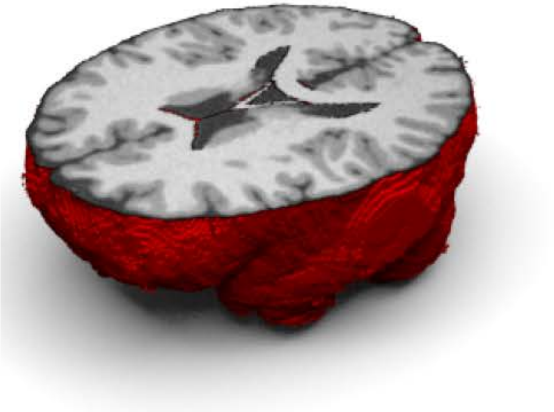


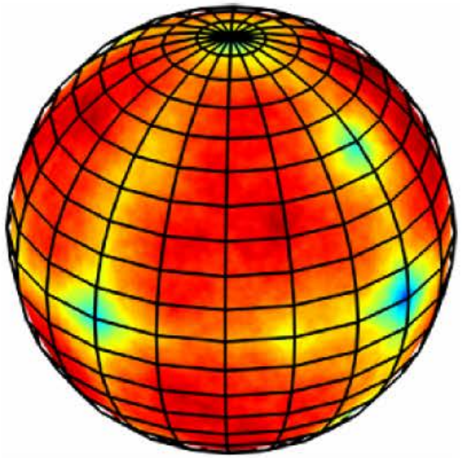
Probably Approximately Symmetric

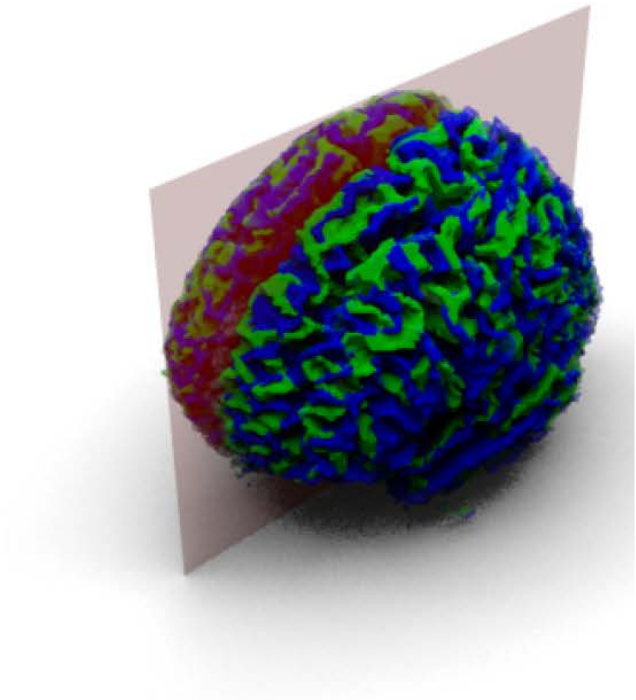


Korman , Litman, Avidan and Bronstein. 2014

Symmetry in MRI







Take home message

Enumerating is easier than estimating

Enumerating is guaranteed
to find globally approximate solution