



## Light-fields: Beyond the Lambertian

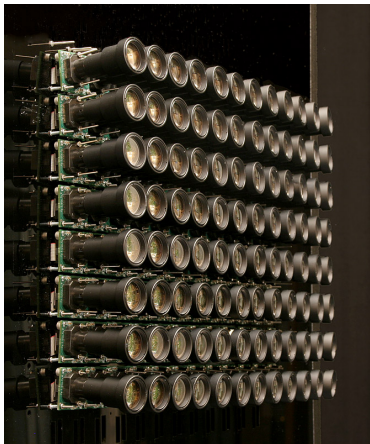
Antonin Sulc

April 3, 2016





# Recording Light Fields



Stanford Camera Array



Lytro Illum



## Changing focus - focus in front





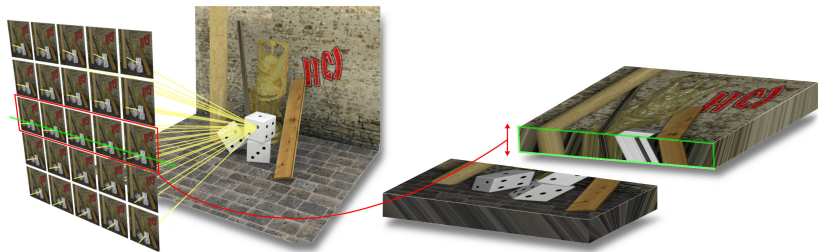
## Changing focus - focus in back







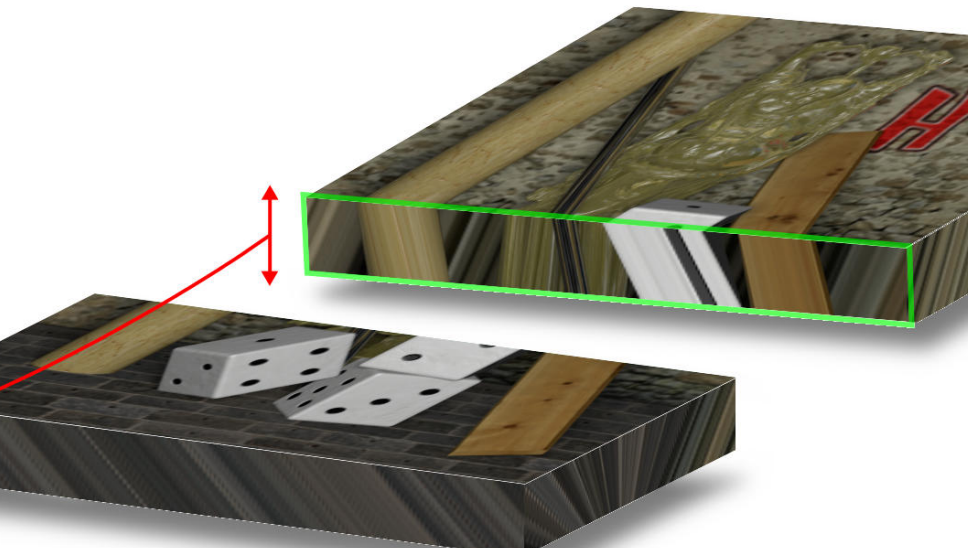
# Light Fields



[Figure: Wanner and Goldluecke 2012]

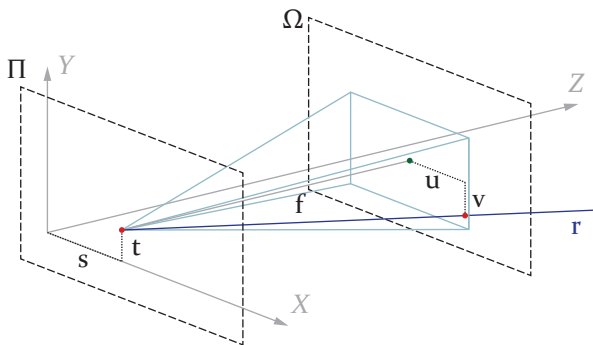


# Light Fields





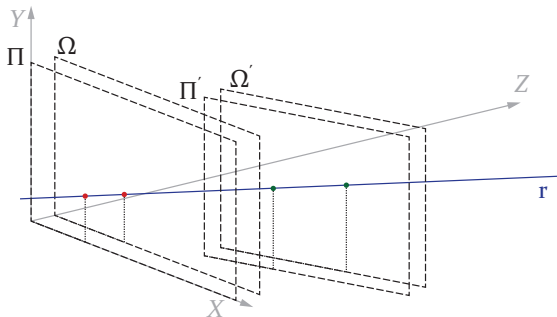
# Light Field Parametrization



- Angular coordinates on *focal plane*  $(s, t, 0) \in \Pi$
- Spatial coordinates on *image plane*  $(u, v, f) \in \Omega$
- Light field coordinates  $\mathbf{l} = [u, v, s, t]^T$

# Structure from Motion in Light fields

- How to find extrinsic parameters between two cameras?
- How find features in light fields?
- How to make refocusable light-field panoramas?



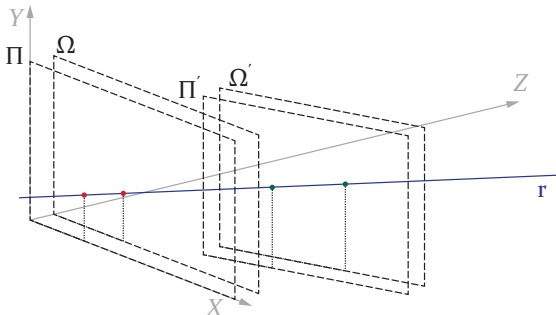
[Johannsen, Sulc and Goldluecke, ICCV 2015]



# Rigid transform in Plücker coordinates

- No easy way to transform light field coordinates  $\mathbf{l}$ .
- We can represent rays  $\mathbf{l}$  as homogeneous Plücker rays  $(\mathbf{q}, \mathbf{m})$

$$\begin{bmatrix} \mathbf{q}' \\ \mathbf{m}' \end{bmatrix} = \begin{bmatrix} R\mathbf{q} \\ R\mathbf{m} + E\mathbf{q} \end{bmatrix} \quad (1)$$

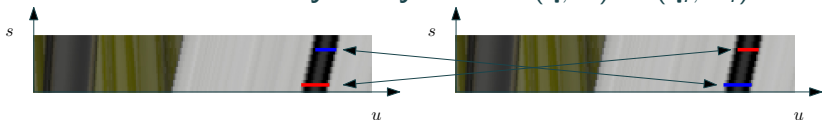


[Pless 2003]



# Generalized Epipolar Constraint

Consider  $n$ -to- $m$  ray-to-ray matches  $(\mathbf{q}, \mathbf{m}) \leftrightarrow (\mathbf{q}_i, \mathbf{m}_i)$



- $(\mathbf{q}, \mathbf{m}) \leftrightarrow (\mathbf{q}_i, \mathbf{m}_i)$  in the same coordinate frame intersect iff

$$\mathbf{q}^T \mathbf{m}_i + \mathbf{m}^T \mathbf{q}_i = 0 \quad (2)$$

- $(\mathbf{q}_i, \mathbf{m}_i)$  is in a coordinate frame  $R, t$ 
  - Transform the  $(\mathbf{q}', \mathbf{m}') = (R\mathbf{q}, R\mathbf{m} + E\mathbf{q})$
  - By plugging  $(\mathbf{q}', \mathbf{m}')$  into Eq. 2 we get:

$$\mathbf{q}_i^T E\mathbf{q} + \mathbf{q}_i^T R\mathbf{m} + \mathbf{m}_i^T R\mathbf{q} = 0 \quad (3)$$

- The Eq. 3 is called *Generalized Epipolar Constraint (GEC)*
- Huge system of ray-to-ray matches  $\mathcal{O}(nm)$

[Pless 2003]



## Features in Light fields

A 3D point is projected into multiple sub-aperture views  $(s, t)$



$$\{\mathbf{l}_i\}_{i=1\dots n} = \{[u_i, v_i, s_i, t_i]^T\}_{i=1\dots n}$$



## Light field subspace constraint

- Pinhole projection equations impose an affine relationship between  $(u, v)$  and  $(s, t)$
- Rays  $\{\mathbf{l}_i\}_{i=1\dots n}$  which are intersecting the same 3D point  $\mathbf{X}$  form a linear 2D subspace  $M\mathbf{l}_i = 0$

$$\underbrace{\begin{bmatrix} 1 & 0 & \frac{f}{Z} & 0 & -\frac{fX}{Z} \\ 0 & 1 & 0 & \frac{f}{Z} & -\frac{fY}{Z} \end{bmatrix}}_{M(\mathbf{X}, f)} \begin{bmatrix} u \\ v \\ s \\ t \\ 1 \end{bmatrix} = 0 \quad (4)$$



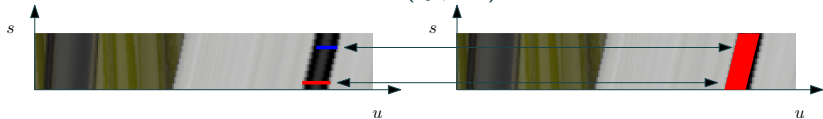




## Our method

Consider  $n$ -to- $m$  **ray-to-subspace** matches

$$M \leftrightarrow (\mathbf{q}_i, \mathbf{m}_i)$$



- $M$  and  $I$  represent the same 3D point iff

$$MI = 0$$

- No straightforward way to transform light field coordinates in different coordinate frames **but**
- A  $I$  as Plücker  $(\mathbf{q}, \mathbf{m})$  ray can be transformed

$$M'P(f) \begin{bmatrix} R\mathbf{q} \\ R\mathbf{m} + E\mathbf{q} \end{bmatrix} = 0$$

- ray-to-subspace matches  $\mathcal{O}(m+n)$

[Johannsen, Sulc and Goldluecke, ICCV 2015]



## Our method

- Direct utilization of light field geometry.
- Linear complexity  $\mathcal{O}(m + n)$  with number  $n, m$  rays (matches)
- Robust (a lot of outliers can be removed in  $M$  estimation)

Method	Rotation error	Translation error	Time	Complexity
3DPC	1.31	9.49	<b>0.00</b>	
R2R	1.55	2.37	0.07	$\mathcal{O}(mn)$
Ours	<b>0.58</b>	<b>1.22</b>	0.04	$\mathcal{O}(m + n)$









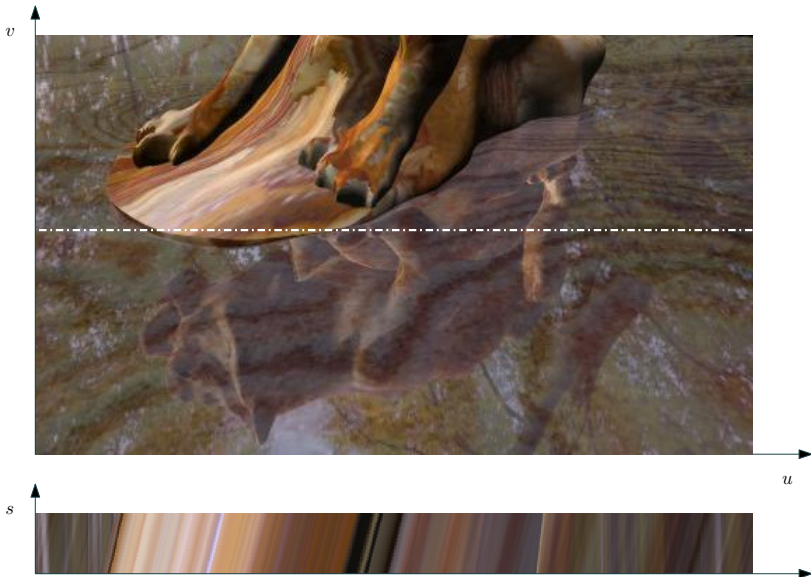
# Non-Lambertian surfaces

- How to calculate depth of a reflective/transparent surface.
- How to find a reflection mask.
- How to separate foreground from reflection.





# Non-Lambertian surfaces in light-fields







## Depth estimation - Structure tensor

First order structure tensor:

$$f(x) = f(x + \alpha \mathbf{v}) \quad (5)$$



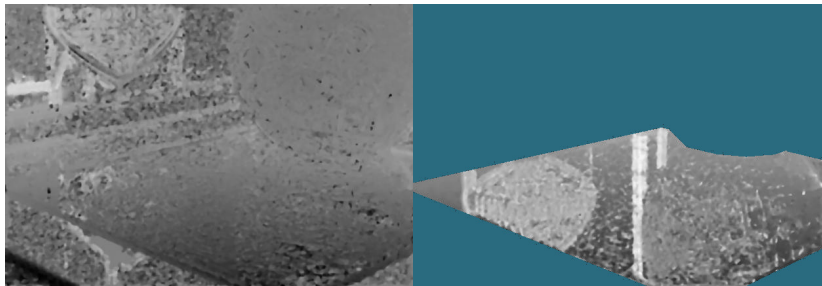
Second order structure tensor:

What if  $f = f_u + f_v$ ?

$$\mathbf{u} \nabla f_u = 0 \quad \text{and} \quad \mathbf{v} \nabla f_v = 0 \quad (6)$$

Eigensystem analysis of the second order structure tensor

[Aach et.al. 2006; Wanner and Goldluecke 2013]



[Johannsen, Sulc and Goldluecke, VMV 2015]



# Robust depth estimation - Sparse codes

- Structure tensor is very sensitive to noise and texture-less regions



- **Idea** Represent EPI patches as a linear combination of atoms with fixed disparity



[Johannsen, Sulc and Goldluecke, CVPR 2016]



## Robust depth estimation - Sparse codes

Each patch in EPI can be represented as a linear combination of atoms with fixed disparity.



- **Dictionary** consists of sheared center view patches  $a$  with fixed (known) disparity.



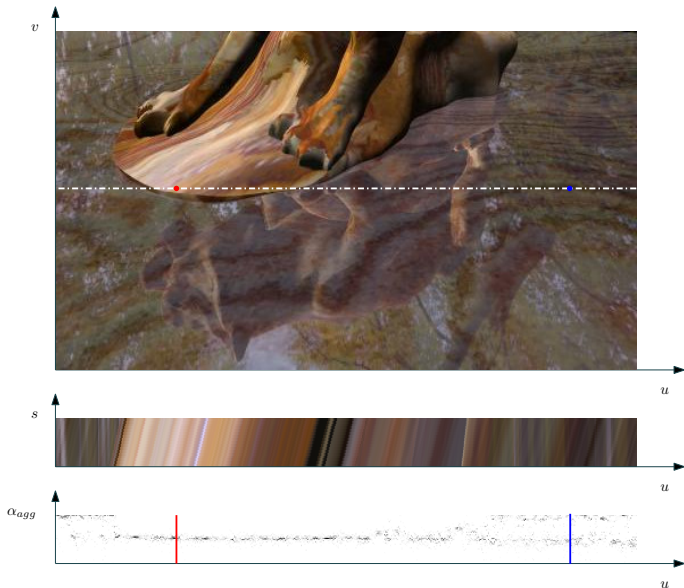
- **Disparity** is weighted average of patch disparities from *sparse codes*  $\alpha$  (lasso,  $L_1$ )

$$\arg \min_{\alpha} \|\mathbf{x} - D\alpha\|_2^2 + \|\alpha\|_1 \quad (7)$$

[Johannsen, Sulc and Goldluecke, CVPR 2016]



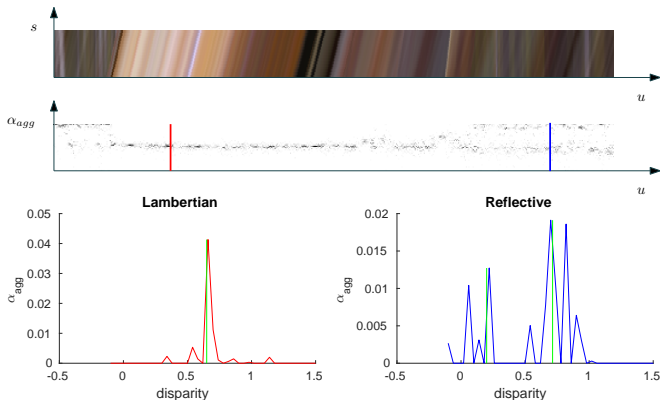
# Robust depth estimation - Sparse codes

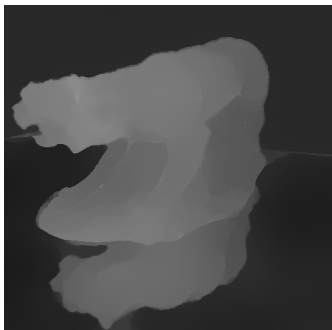
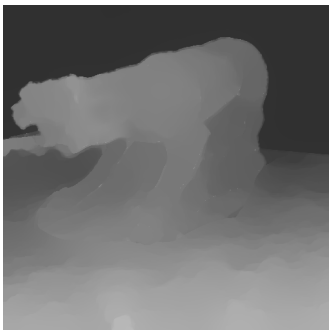


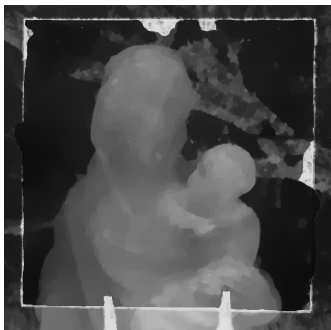
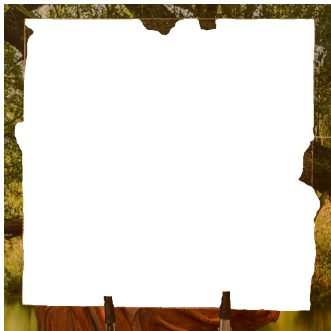


# Robust depth estimation - Sparse codes

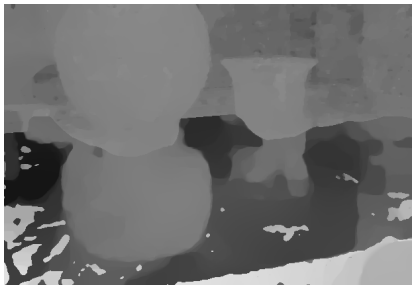
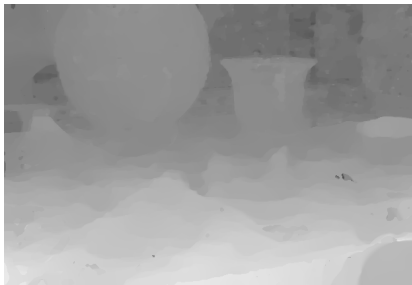
- Classes of aggregated sparse codes:
  - One peak - Lambertian surface
  - Two peaks - Reflective/Transparent surface
  - Uniform - Textureless region



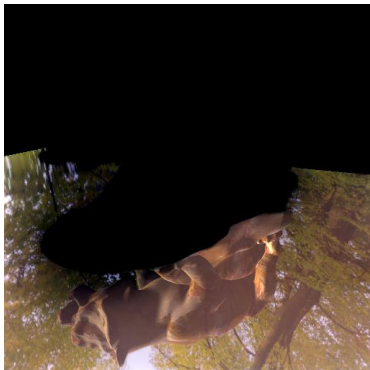
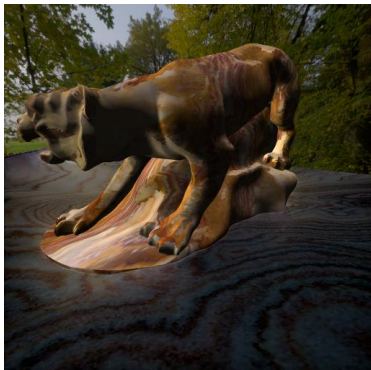






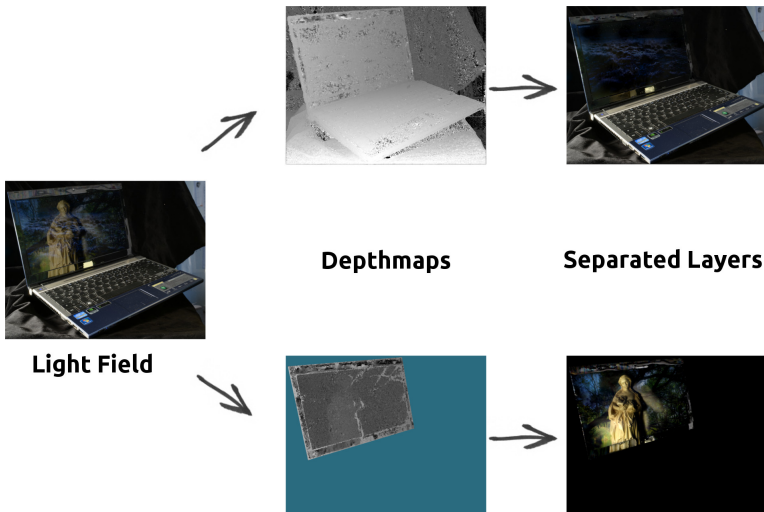


# How to separate light-field images with reflection or transparent surface





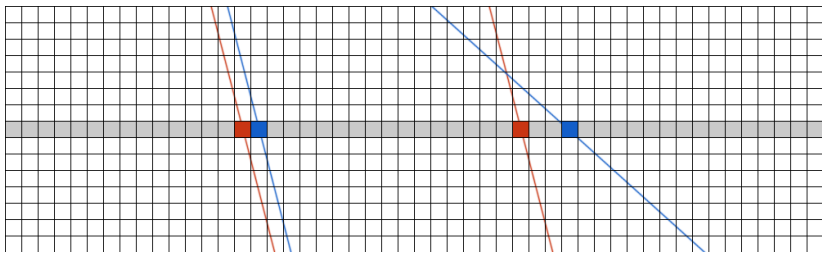
# Layer separation



[Johannsen, Sulc and Goldluecke, VMV 2015]



# Generative Model for EPIs: Lambertian Surface



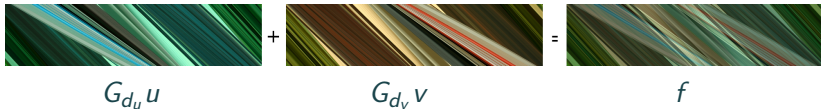
Depth dependent linear relation between center view and full EPI



## Variational Inverse Problem: Dataterm

- $f$  input EPI
- $u, v$  center view layers restricted to this EPI
- $d_u, d_v$  layer disparities restricted to this EPI
- $G_{d_u}, G_{d_v}$  matrices to compute full EPI from layers

$$D_{EPI}(u, v) = \|G_{d_u}u + G_{d_v}v - f\|_p^p$$



[Johannsen, Sulc and Goldluecke, VMV 2015]



# Variational Inverse Problem

- **Dataterm:** Goes through horizontal (W) and vertical (H) EPIs

$$D(u, v) = \sum_{x=1}^W D_x(u_x, v_x) + \sum_{y=1}^H D_y(u_y, v_y) \quad (8)$$

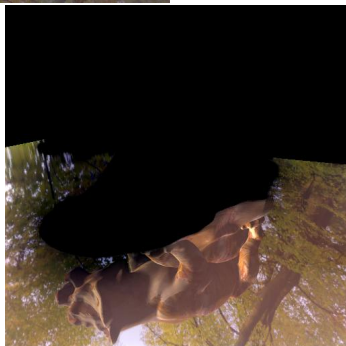
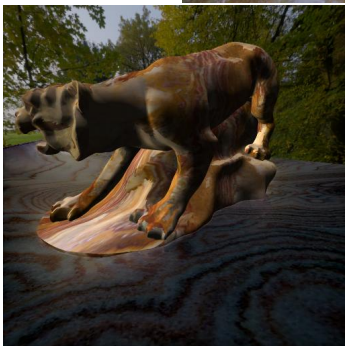
- **Regularization:** TGV favors linear solutions

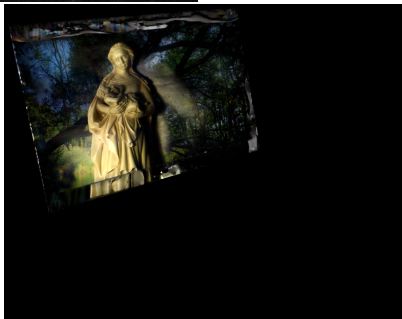
$$\lambda(J(u) + J(v)) \quad (9)$$

- **Energy:**

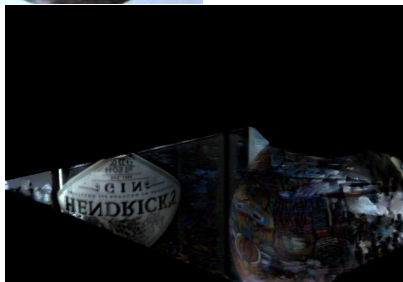
$$E(u, v) = D(u, v) + \lambda(J(u) + J(v))$$

Solved by primal-dual algorithm [Chambolle and Pock 2010]











## Conclusion

- LF Light field samples luminance of a subset of **rays which go through aperture.**
- SfM Rays from a 3D point form a **2D subspace.**
- SfM Linear algorithm for SfM with **ray-to-subspace** matches.
- ST Structure tensor can find a **disparity of two superimposed** light fields but is too sensitive.
- DL Distribution of grouped **sparse codes encode disparities.**
- LS Knowing **disparity and mask** of respective light field components, light field can be **separated in two components.**

# Thank you for your attention

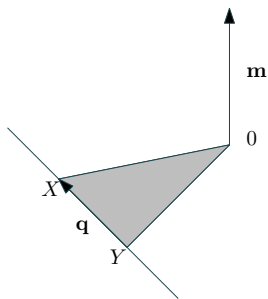
Joint work of Ole Johannsen and Prof. Bastian Goldluecke

Funded by ERC grant *Light Field Imaging and Analysis*



## Notes : Plücker coordinates

- Direction  $\mathbf{q} \in \mathbb{R}^3 - \{0\}$
- Moment  $\mathbf{m} \in \mathbb{R}^3$
- A point  $X \in \mathbb{R}^3$  lies on the  $(\mathbf{q}, \mathbf{m})$  ray iff  $\mathbf{m} = X \times \mathbf{q}$



Two coordinates  $(\mathbf{q}_1, \mathbf{m}_1)$  and  $(\mathbf{q}_2, \mathbf{m}_2)$  represent the same ray iff  
 $\exists w \neq 0 : \mathbf{q}_1 = w\mathbf{q}_2$  and  $\mathbf{m}_1 = w\mathbf{m}_2$



## Note : Light Field Subspace constraint

$$\underbrace{\begin{bmatrix} 1 & 0 & \frac{f}{Z} & 0 & -\frac{fX}{Z} \\ 0 & 1 & 0 & \frac{f}{Z} & -\frac{fX}{Z} \end{bmatrix}}_{M(\mathbf{X},f)} \begin{bmatrix} u \\ v \\ s \\ t \\ 1 \end{bmatrix} = 0 \quad (10)$$

$$\begin{bmatrix} u + s\frac{f}{Z} - \frac{fX}{Z} \\ v + t\frac{f}{Z} - \frac{fX}{Z} \end{bmatrix} = 0 \quad (11)$$

$$\begin{bmatrix} (u - sd) - x \\ (v - td) - y \end{bmatrix} = 0 \quad (12)$$