

# 3D with Rolling Shutter

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**CIIRC**

CZECH INSTITUTE OF INFORMATICS  
ROBOTICS AND CYBERNETICS

AAG – Applied Algebra & Geometry



CENTER FOR MACHINE  
PERCEPTION

GVR – Geometry of Vision & Robotics

# What is the Rolling Shutter Effect?

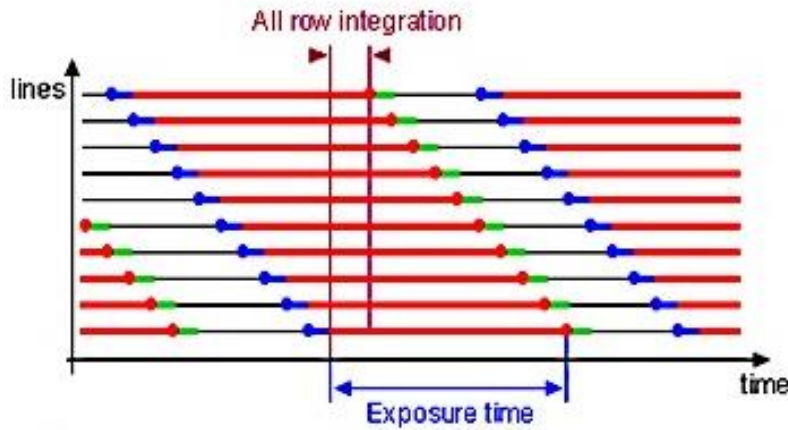
**GS - Global shutter**

**RS - Rolling shutter  
(most cameras)**



# How does the Rolling Shutter work?

- Images scanned line by line
- The effect



## The good

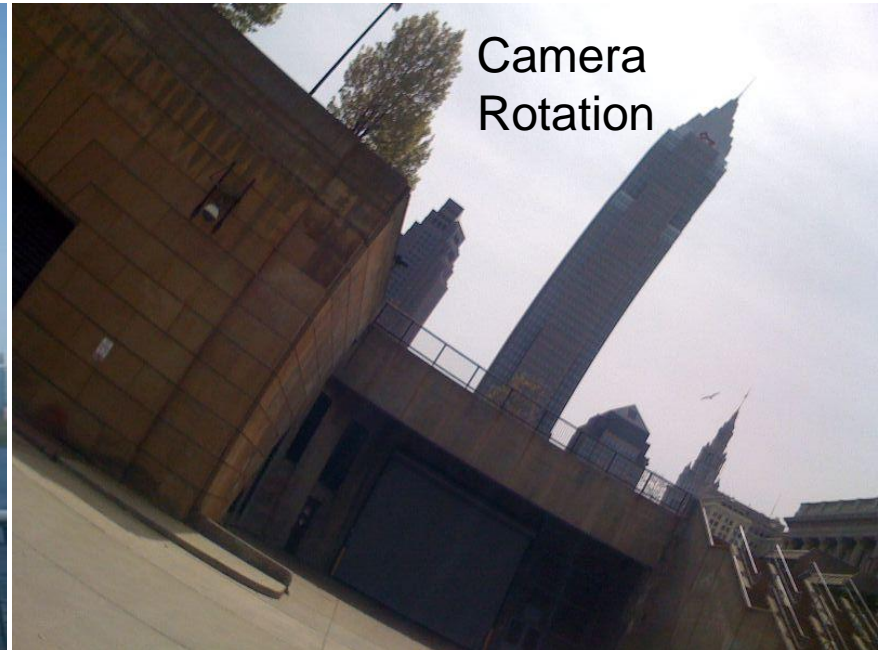
- Higher frame rate
- Longer exposure time
- Cheaper and easier to manufacture

## The bad

- Image distortions
- Non-perspective projections

# How does the Rolling Shutter look?

## And the ugly...

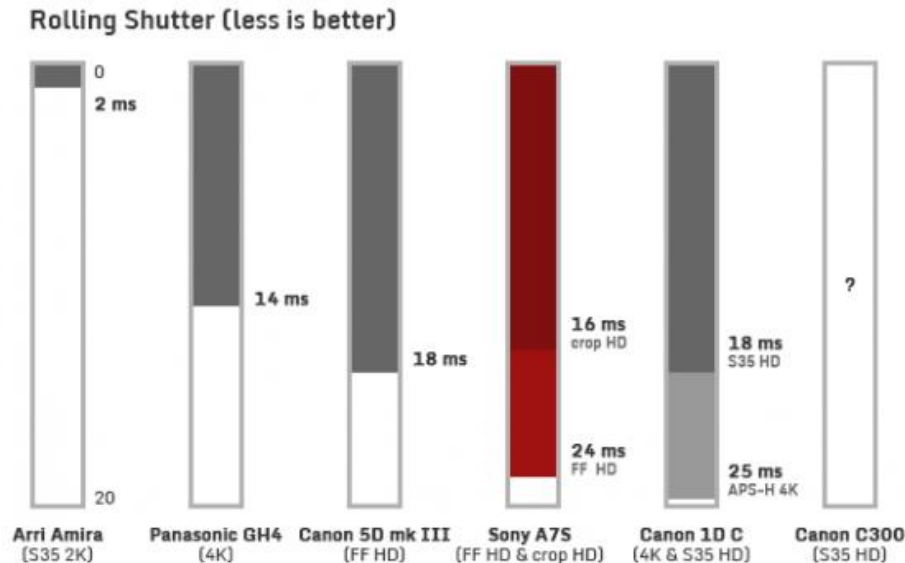


# Rolling shutter is ubiquitous

- It is in majority of cameras today ranging from cellphones, industrial cameras to professional DSLR



- Affects both videos AND single images
  - Difference between top and bottom can be  $\sim 1/30$ s

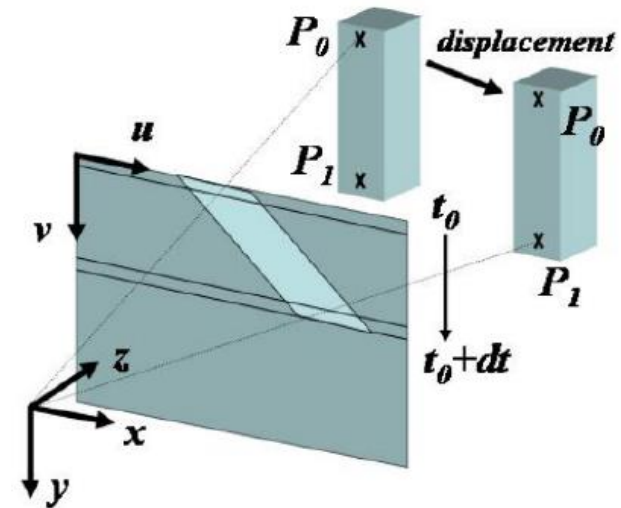


Tested with a rotary chart developed by cinema5D. Approximate values in milliseconds.

# Can we take advantage of Rolling shutter?

Object pose and velocity estimation

- O. Ait-Aider et al., ECCV'06
- Shape of the object known
- Image distortion -> object motion



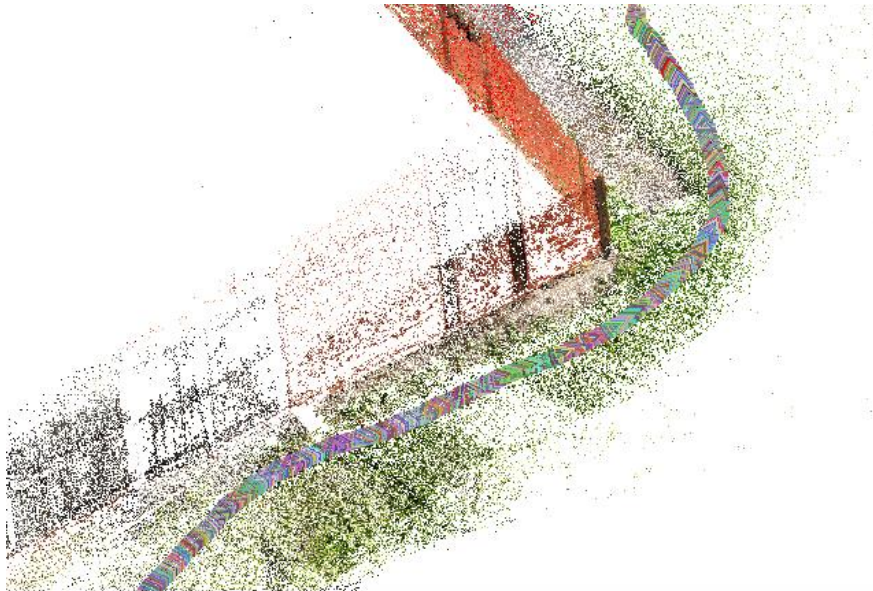
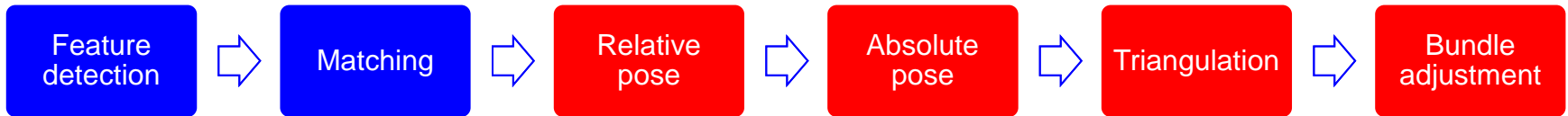
Multi-camera synchronization  
from flashes

- Smid et al., VISAPP'17
- Frames captured at different times -> different lines illuminated

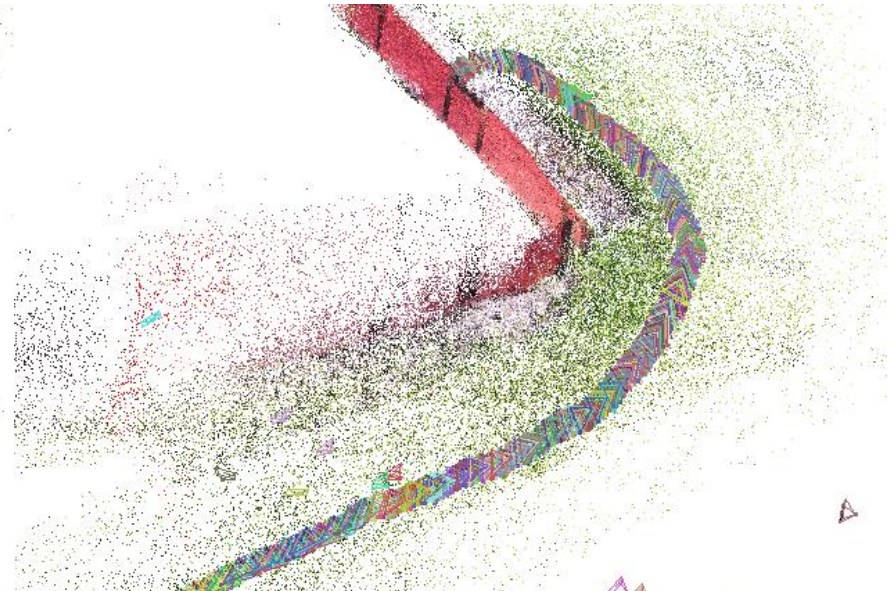
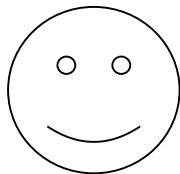


# 3D Reconstruction with Rolling Shutter

3D reconstruction from RS images ... degraded if ignored



Global shutter (Canon)



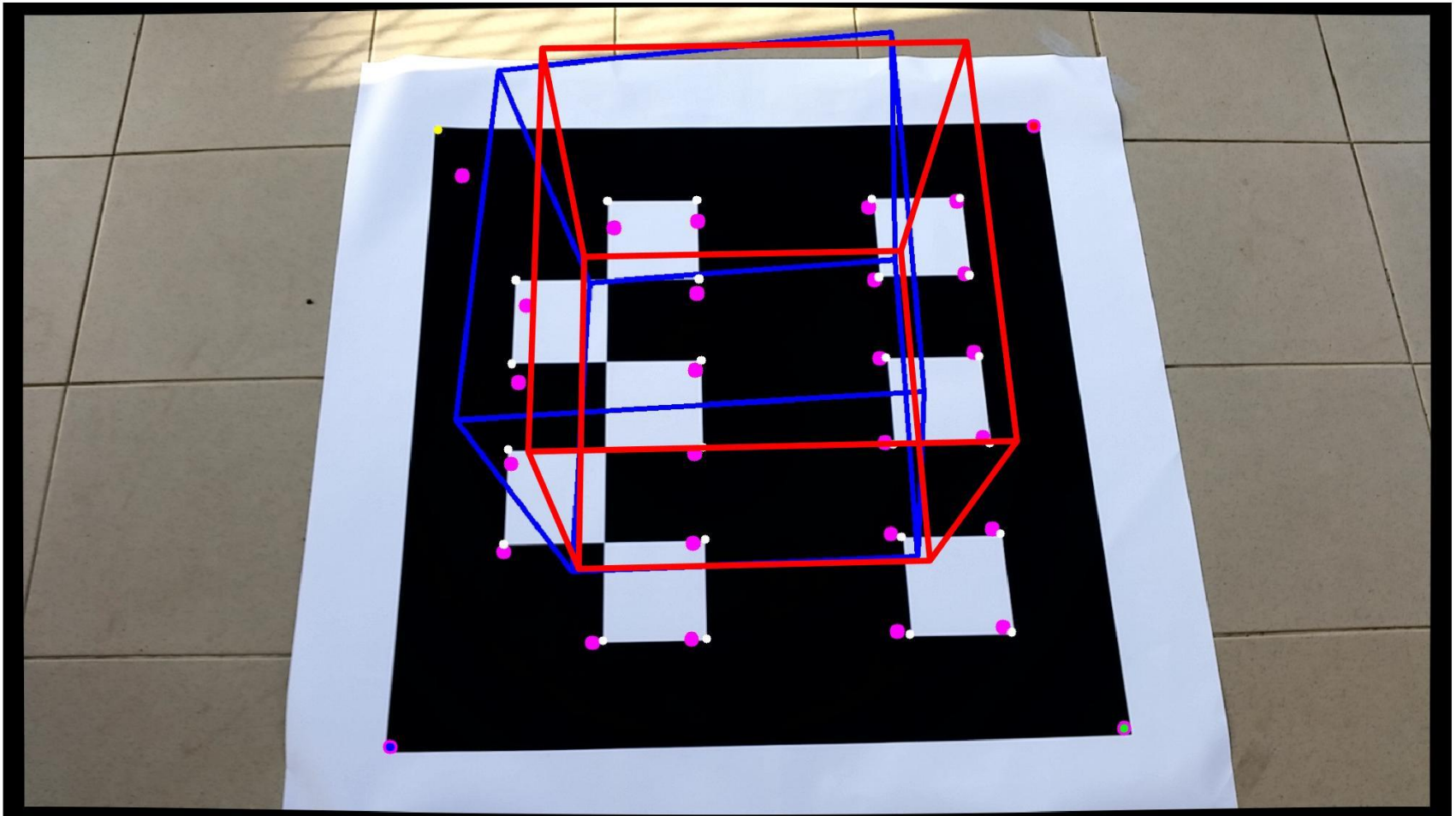
Rolling shutter (iPhone 4)



# Augmented reality, localization with RS

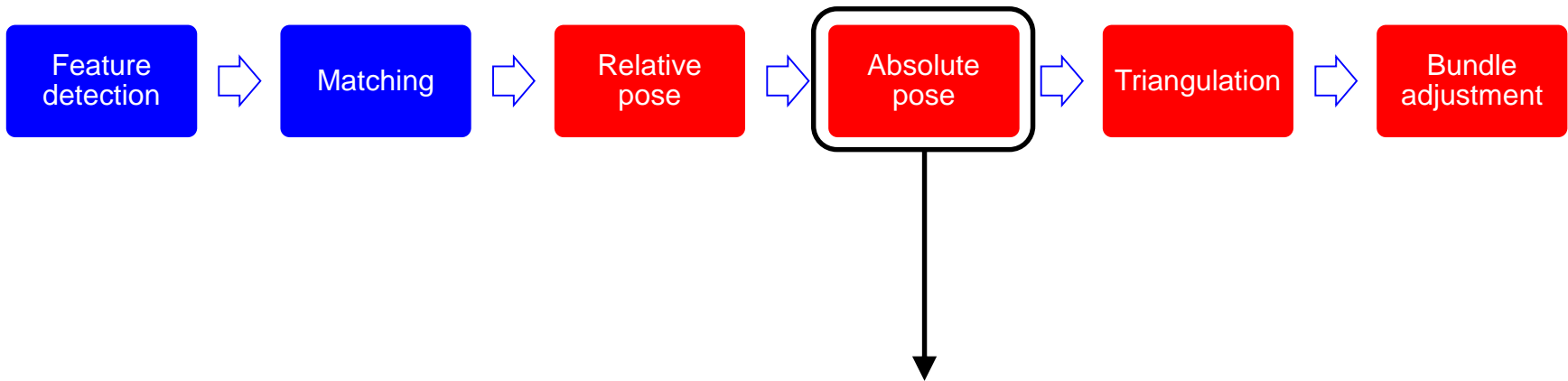
- Determining the camera pose
- Placing objects in the image

Perspective camera model  
Rolling shutter camera model





# Absolute Camera Pose with Rolling Shutter



## Absolute camera pose with RS

1. ***R6P - Rolling Shutter Absolute Camera Pose.***  
*C. Albl, Z. Kukelova, T Pajdla. CVPR 2015.*
2. ***Rolling Shutter Absolute Camera Pose Problem with known Vertical Direction.***  
*C. Albl, Z. Kukelova, T Pajdla. ICCV 2015.*

# Previous Work

- Klein et al. ISMAR'09, Hedborg et al. CVPR'12



Video sequences only

- Ait-aider et al. ECCV'06

- Non-linear optimization
- Initial guess – 8,5 points + planar scene

- Magerand et al. ECCV'12

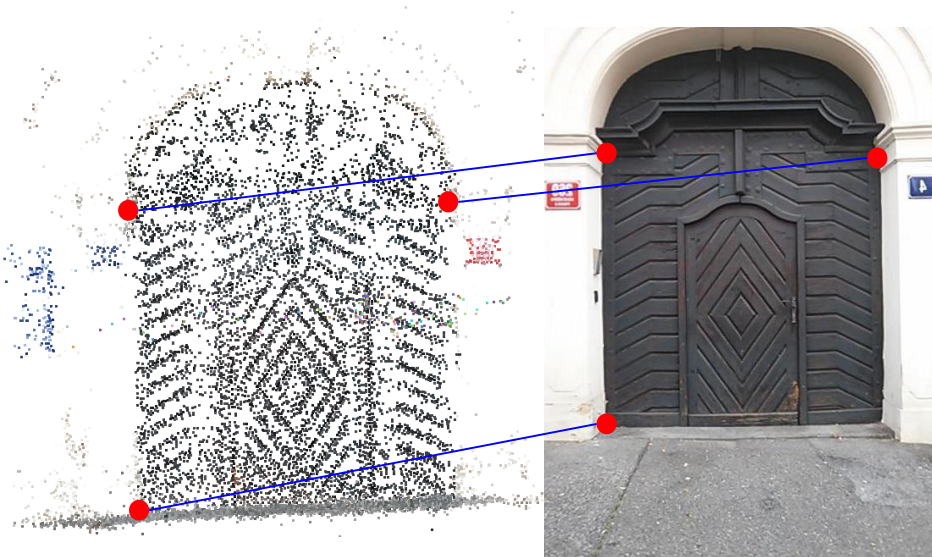
- Globally optimal
- 7 points
- Sensitive to outliers
- Slow for RANSAC

# Absolute Camera Pose with Rolling Shutter

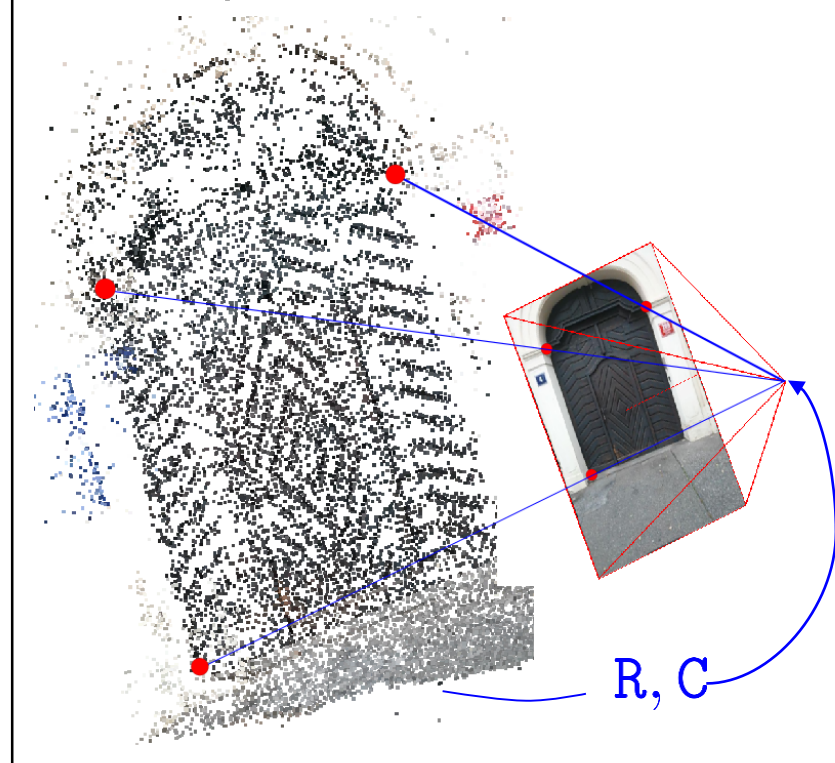
3D points



2D points



Perspective camera – **P3P**



3 correspondences

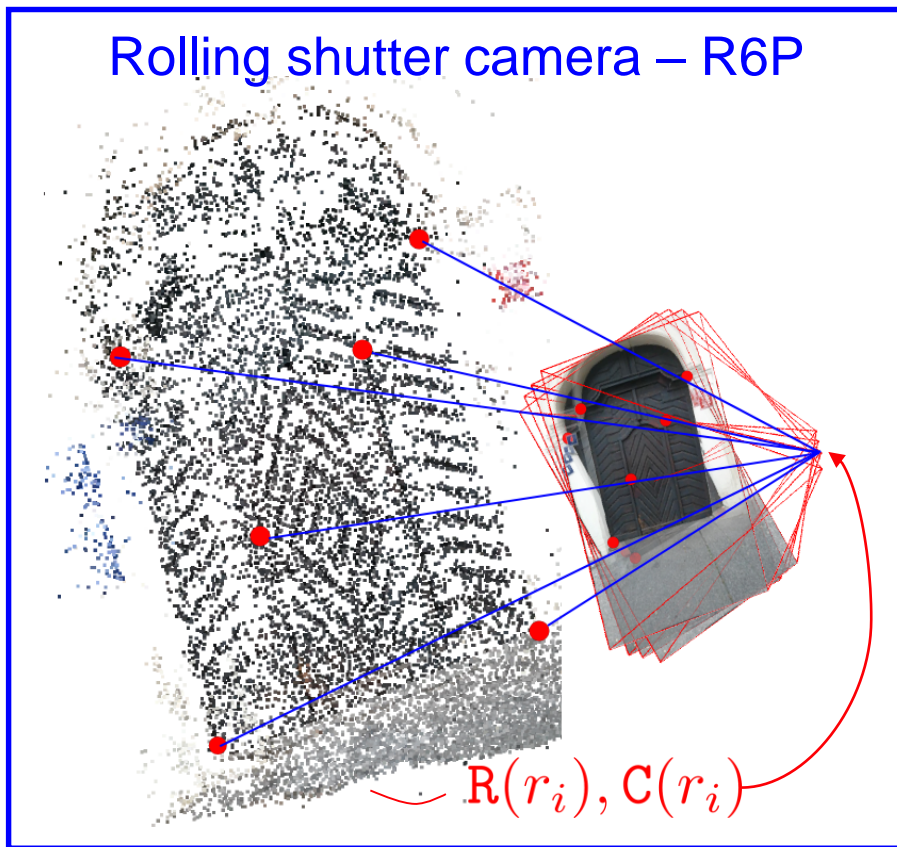
[Haralick CVPR 1991][Quan PAMI 1999]

[Triggs IJCV 1999][Wut JMIV 2006]

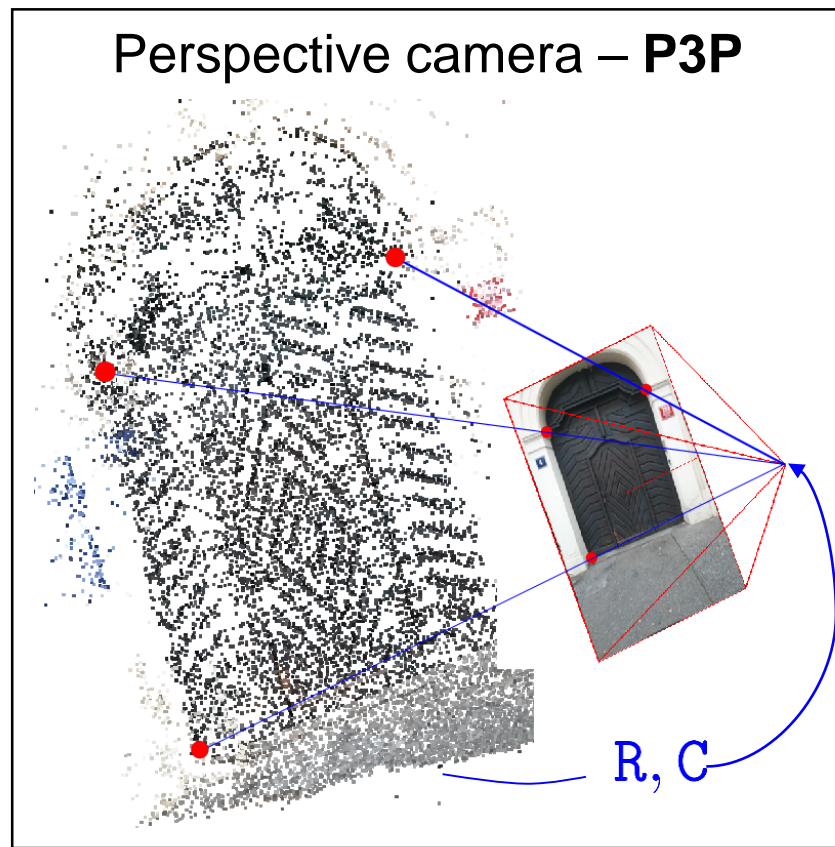
[Zhi MMRC 2002][Lepetit IJCV 2009]

# Absolute Camera Pose with Rolling Shutter

**This work = R6P**



**6 correspondences**



**3 correspondences**

[Haralick CVPR 1991][Quan PAMI 1999]  
[Triggs IJCV 1999][Wut JMIV 2006]  
[Zhi MMRC 2002][Lepetit IJCV 2009]

# Rolling Shutter Camera Projection

Standard (calibrated) perspective projection

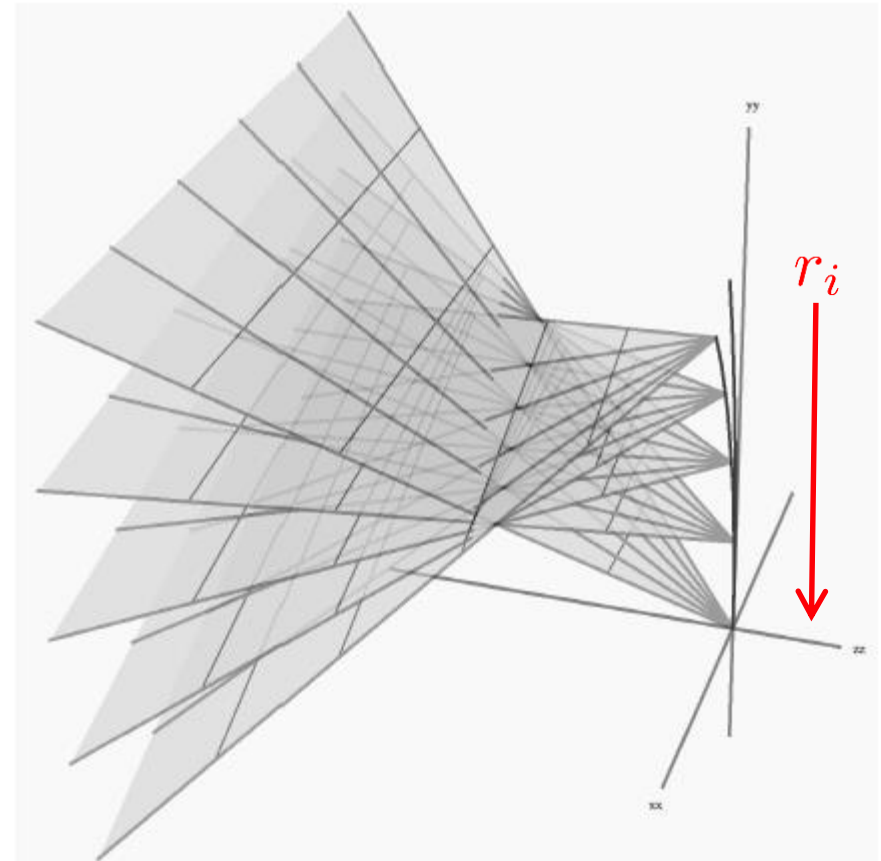
$$\lambda_i \mathbf{x}_i = \mathbf{R}\mathbf{X}_i + \mathbf{C}$$

RS camera undergoing motion

$$\lambda_i \mathbf{x}_i = \begin{bmatrix} r_i \\ c_i \\ 1 \end{bmatrix} = \mathbf{R}(r_i)\mathbf{X}_i + \mathbf{C}(r_i)$$

Camera pose changes for every row

How to model  $\mathbf{R}(r_i)$  and  $\mathbf{C}(r_i)$ ?



Picture from Meingast et al.

# Rolling Shutter Camera Projection

$$\lambda_i \mathbf{x}_i = \begin{bmatrix} r_i \\ c_i \\ 1 \end{bmatrix} = \mathbf{R}(r_i) \mathbf{X}_i + \mathbf{C}(r_i)$$
$$\lambda_i \mathbf{x}_i = \begin{bmatrix} r_i \\ c_i \\ 1 \end{bmatrix} = \mathbf{R}_m(r_i) \mathbf{R}_0 \mathbf{X}_i + \mathbf{C} + \mathbf{C}_m(r_i)$$

Diagram annotations:

- Blue arrows point from "Camera initial pose" to  $\mathbf{R}_0$  and  $\mathbf{C}$ .
- Red arrows point from "Motion during capture" to  $\mathbf{R}_m(r_i)$  and  $\mathbf{C}_m(r_i)$ .

Solving in general leads to **complicated** polynomials

We analyzed several models

- SLERP
- Cayley parameterization
- Linearized
- ...
- **Double linear model**

$$\mathbf{C}_m(r_i) = (r_i - r_0) \mathbf{t}$$

[Hedborg CVPR-2012]

# Rolling Shutter Double-Linearized Projection

Full projection model

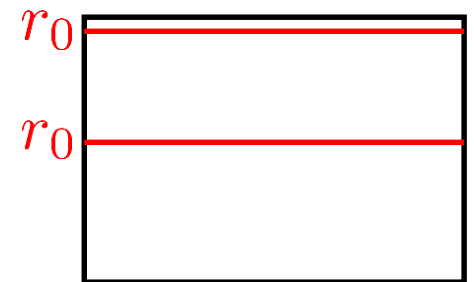
$$\lambda_i \mathbf{x}_i = \begin{bmatrix} r_i \\ c_i \\ 1 \end{bmatrix} = \mathbf{R}_m(r_i) \mathbf{R}_0 \mathbf{X}_i + \mathbf{C} + \mathbf{C}_m(r_i)$$

Double-linearized projection model

$$\lambda_i \begin{bmatrix} r_i \\ c_i \\ 1 \end{bmatrix} = (\mathbf{I} + (r_i - r_0) [\mathbf{w}]_x) (\mathbf{I} + [\mathbf{v}]_x) \mathbf{X}_i + \mathbf{C} + (r_i - r_0) \mathbf{t}$$

Diagram annotations:

- Blue arrows point from "Camera initial pose" to  $\mathbf{I} + [\mathbf{v}]_x$  and  $\mathbf{C}$ .
- Red arrows point from "Motion during capture" to  $(\mathbf{I} + (r_i - r_0) [\mathbf{w}]_x)$  and  $(r_i - r_0) \mathbf{t}$ .
- A black arrow points from "known" to the left-hand side of the equation.



# R6P Minimal Solver

$$\lambda_i \begin{bmatrix} r_i \\ c_i \\ 1 \end{bmatrix} = (\mathbf{I} + (r_i - r_0)[\mathbf{w}]_x)(\mathbf{I} + [\mathbf{v}]_x) \mathbf{X}_i + \mathbf{C} + (r_i - r_0)\mathbf{t}$$

Diagram annotations:

- Blue arrows point from "Camera initial pose" to  $\mathbf{I} + [\mathbf{v}]_x$  and  $\mathbf{C}$ .
- Red arrows point from "Motion during capture" to  $(r_i - r_0)[\mathbf{w}]_x$  and  $(r_i - r_0)\mathbf{t}$ .
- A black arrow points from "known" to the left-hand side of the equation.

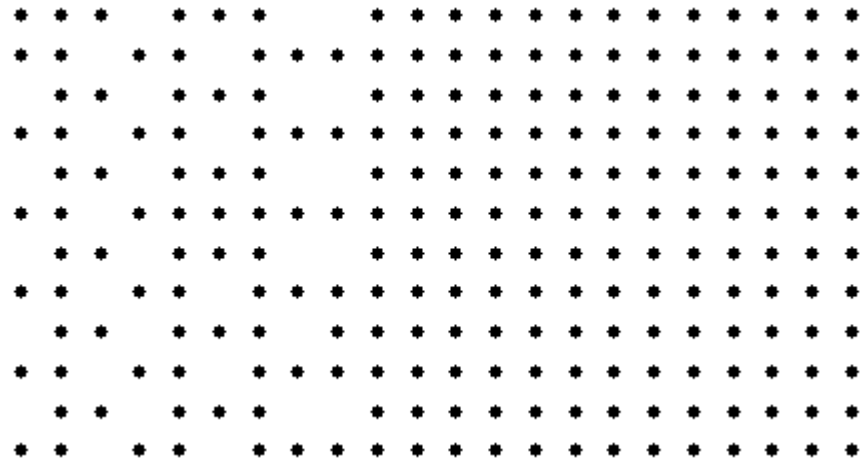
- 12 unknowns  $\rightarrow$  6 3D-2D correspondences
- A system of 12 equations in 12 unknowns and 22 monomials
- Automatic generator of Gröbner basis solvers [Kukelova ECCV 2008]
- Can we do better?



# Constructing R6P Solver

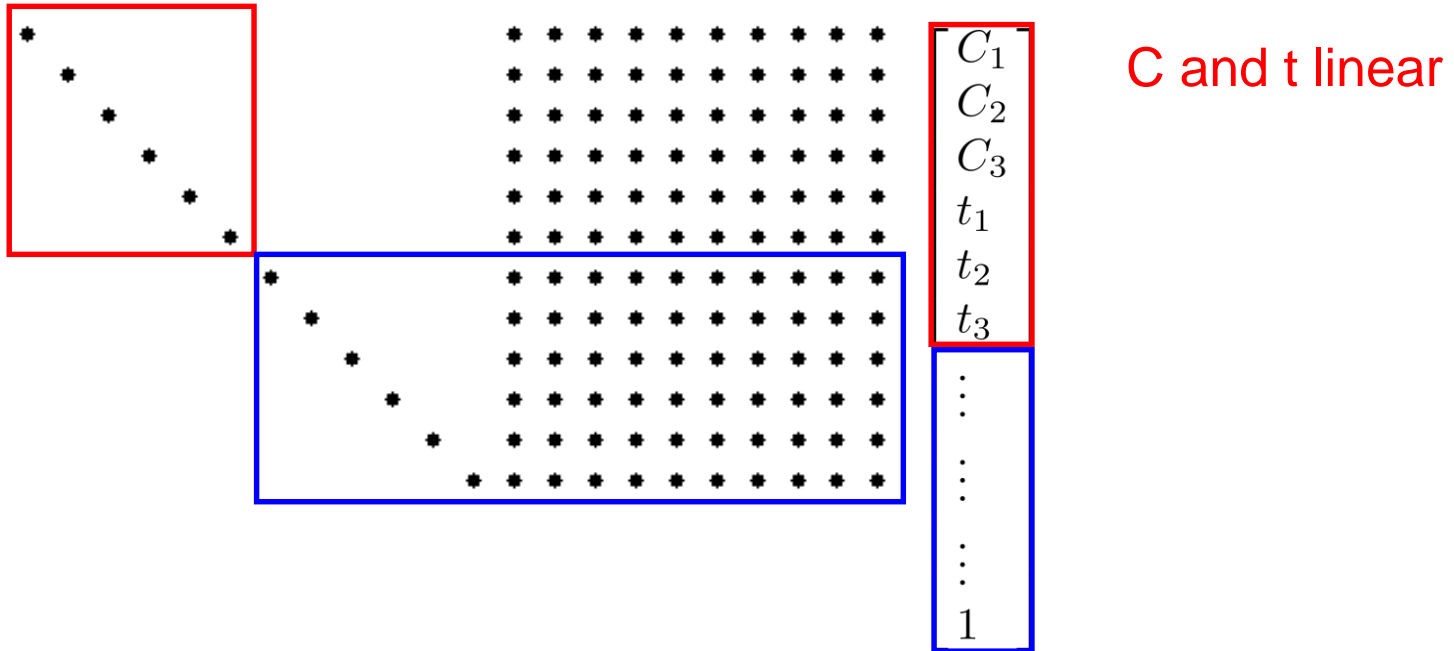
12 linearly independent equations (12x22 matrix ... 22 monomials)

Matrix form


$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ t_1 \\ t_2 \\ t_3 \\ \vdots \\ \vdots \\ \vdots \\ 1 \end{bmatrix} = 0$$

# Constructing R6P Solver

Simplify by Gauss-Jordan elimination



6 equations, 6 unknowns  $v$  &  $w$  (16 monomials)

Solve for  $v$  &  $w$   $\rightarrow$  back-substitution  $\rightarrow$   $C$  &  $t$

Can we do even better?

# Constructing R6P Solver

The remaining 16 monomials are bilinear in  $v$  and  $w$

$v_1, v_2, v_3, w_1, w_2, w_3, v_1w_1, v_1w_2, v_2w_1, v_1w_3, v_2w_2, v_3w_1, v_2w_3, v_3w_2, v_3w_3$

We can write  $M(v) \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ 1 \end{bmatrix} = 0$ , where  $M(v)$  is a 6x4 matrix

4x4 subdeterminants of  $M(v)$  must be zero



15 equations in 3 variables and 35 monomials

Use automatic generator of Gröbner basis solvers [Kukelova ECCV 2008] to solve for  $v$

0.3ms in C++ (Eigen)

# Double linearization ... Initialization need

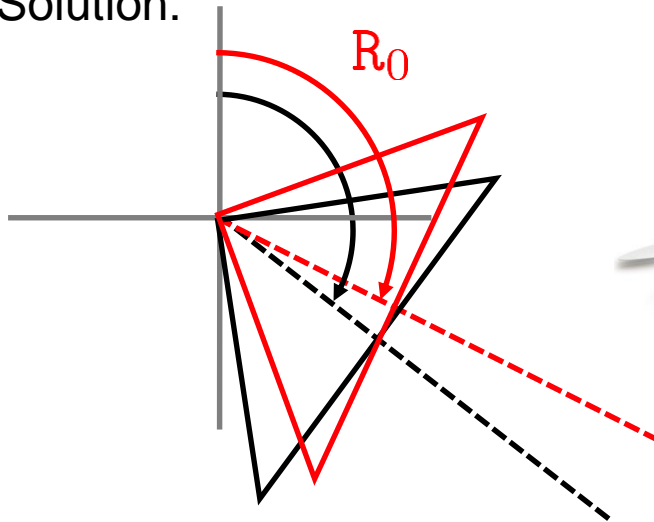
Linearization of rotation

$$\lambda_i \begin{bmatrix} r_i \\ c_i \\ 1 \end{bmatrix} = (\mathbf{I} + (r_i - r_0)[\mathbf{w}]_x)(\mathbf{I} + [\mathbf{v}]_x)\mathbf{X}_i + \mathbf{C} + (r_i - r_0)\mathbf{t}$$

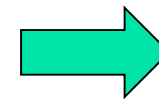
OK – small rotation during the capture

NOT OK – rotation can be arbitrary

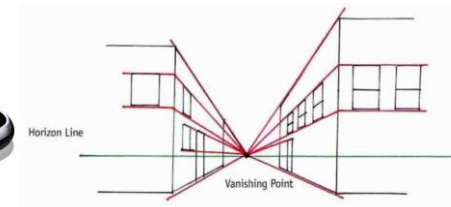
Solution:



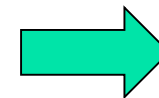
P3P



R6P



IMU



R6P

# Synthetic Experiments

## Synthetic data

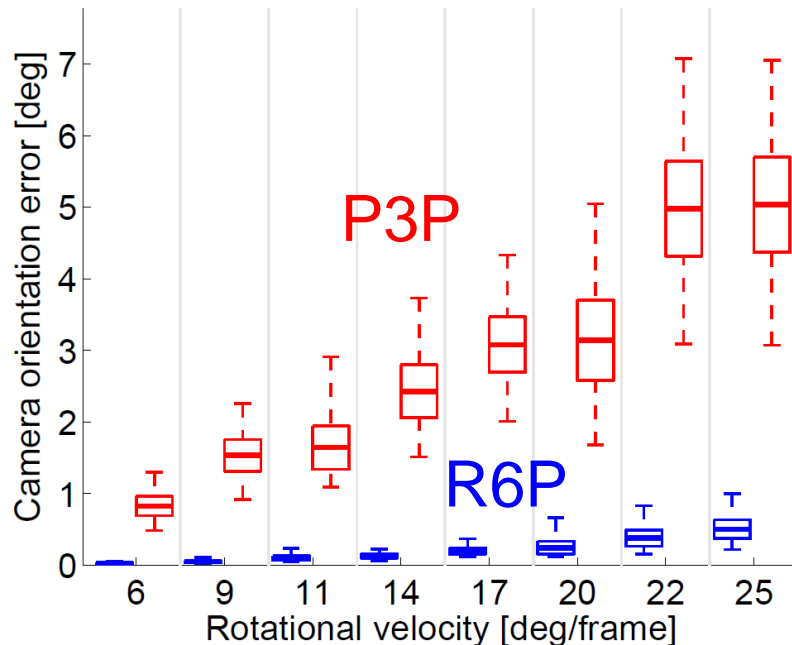
- Compared  $R(v)$ ,  $R(w)$ ,  $C$ ,  $t$  to GT
- Camera pose accuracy

## Camera pose accuracy

- Orientation  $< 0.5^\circ$
- Position  $< 2\%$

Significant improvement over P3P

Increasing RS effect



# Synthetic Experiments

## Synthetic data

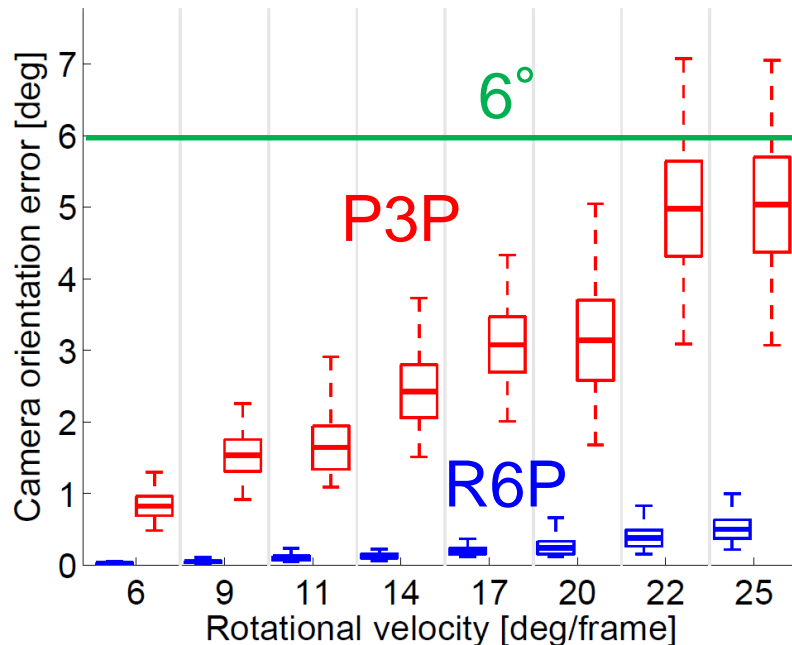
- Compared  $R(v)$ ,  $R(w)$ ,  $C$ ,  $t$  to GT
- Camera pose accuracy

## Camera pose accuracy

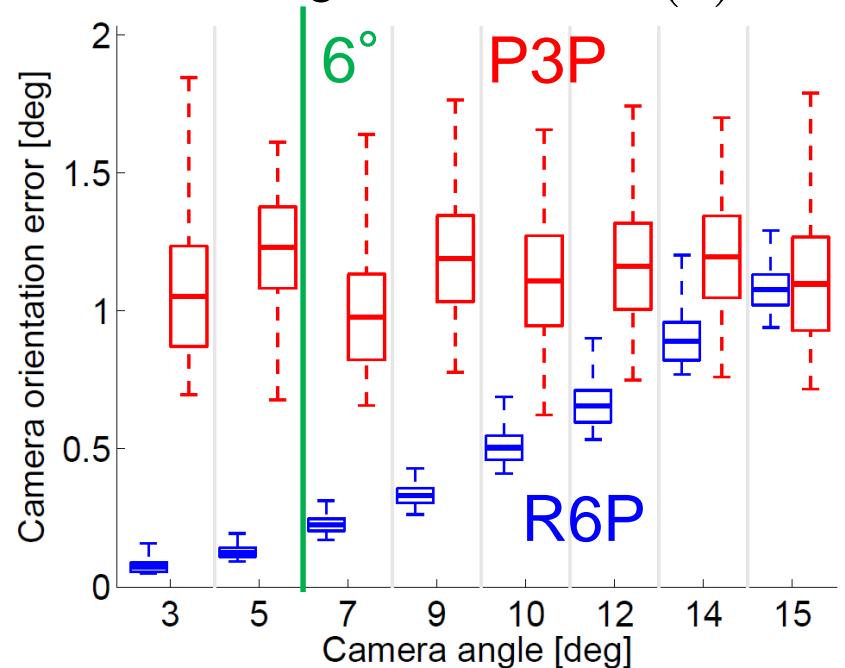
- Orientation  $< 0.5^\circ$
- Position  $< 2\%$

Significant improvement over P3P

### Increasing RS effect

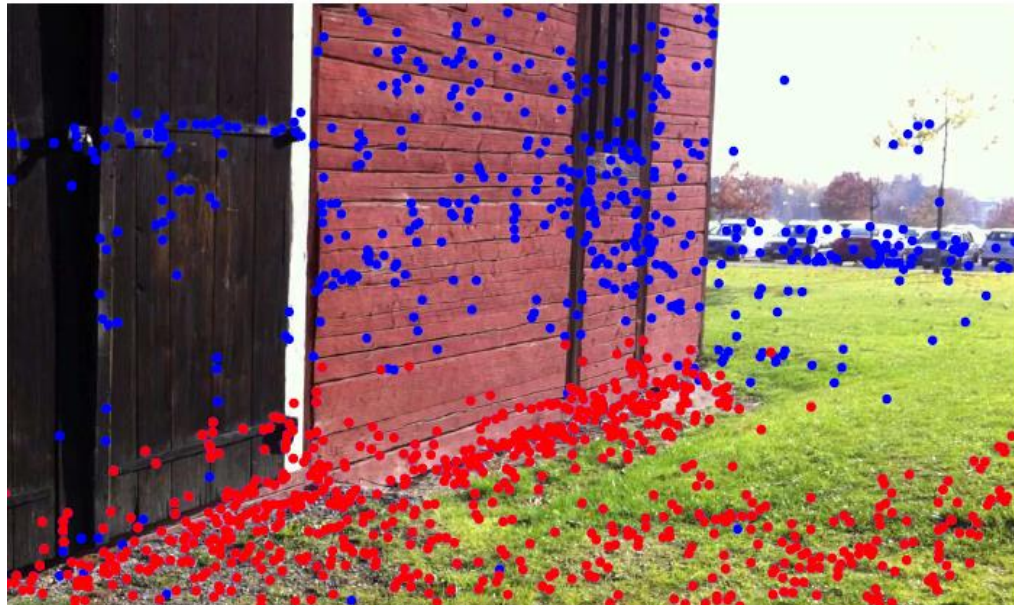


### Increasing distance of $R(v)$ from I



# Real Experiments

P3P inliers  
788



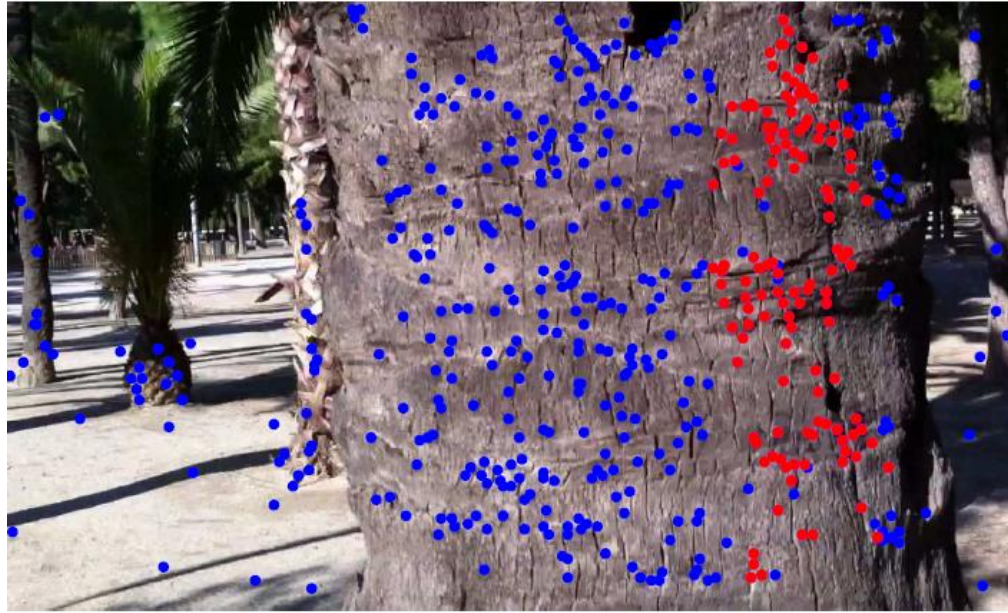
R6P inliers  
1152



Data from  
Hedborg et.al,  
CVPR12

# Real Experiments

P3P inliers  
139



R6P inliers  
465

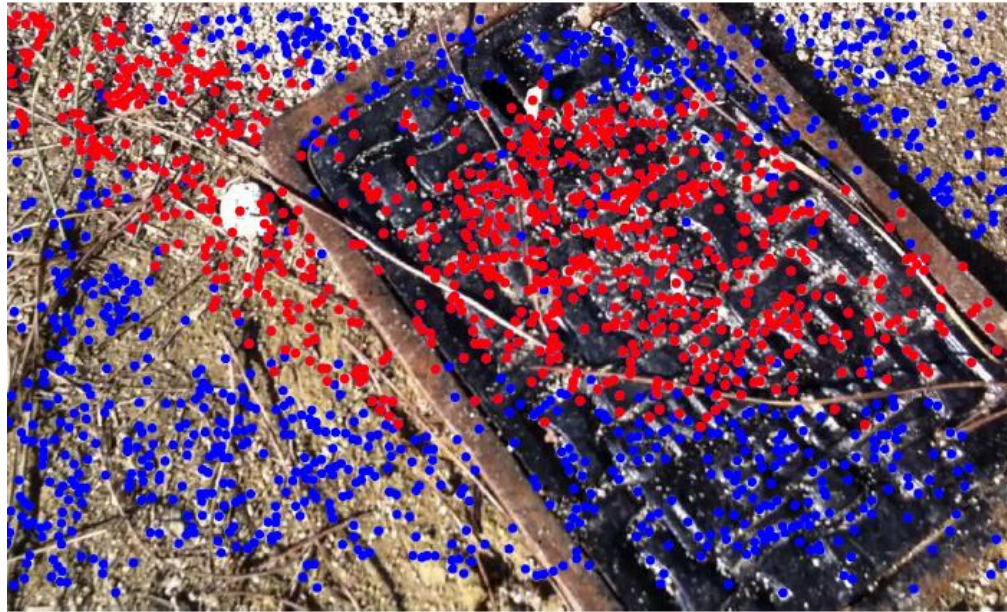


Data from  
Hedborg et.al,  
CVPR12

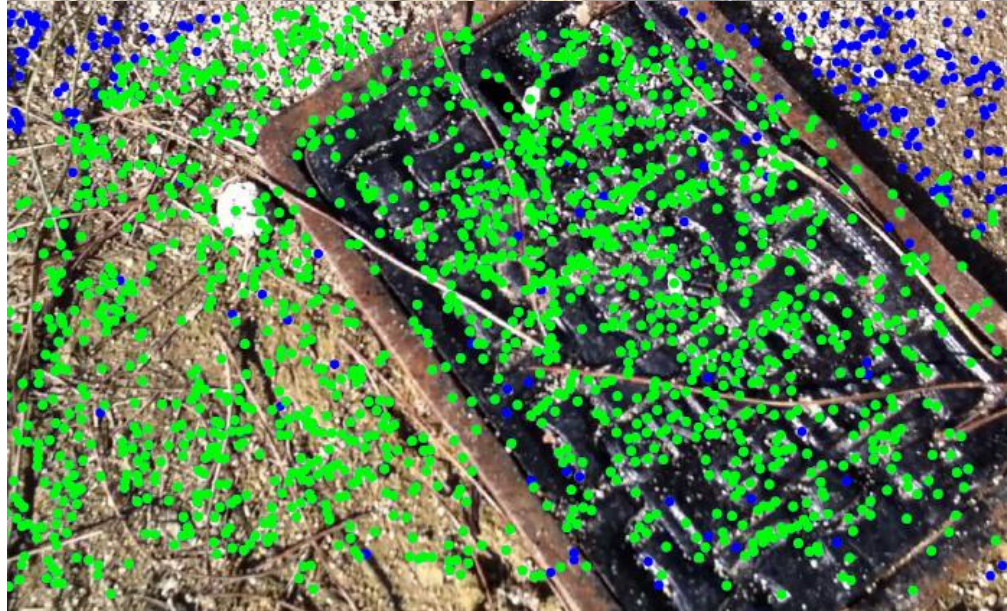


# Real Experiments

P3P inliers  
937



R6P inliers  
1742



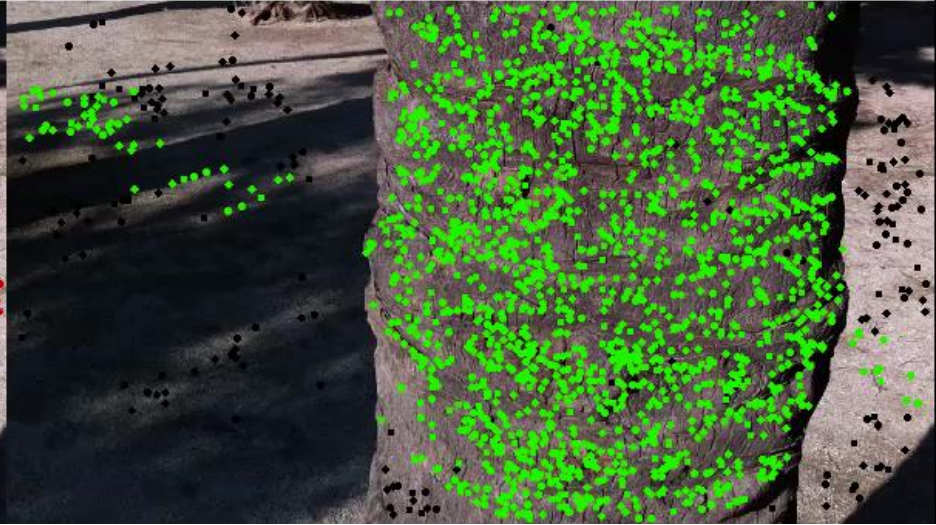
Data from  
Hedborg et.al,  
CVPR12

# Real experiments

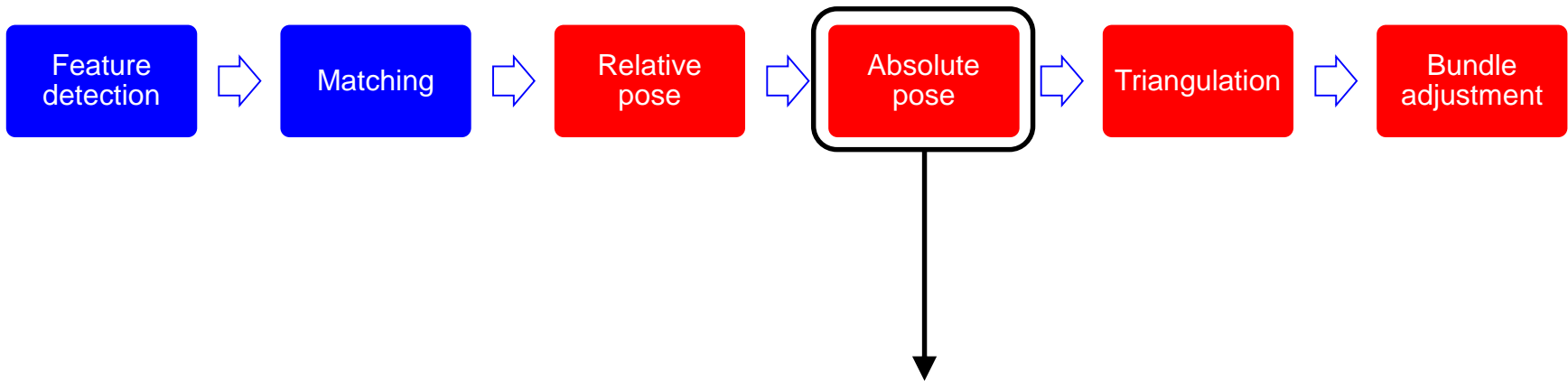
P3P (inliers in red)



R6P (inliers in green)



# Absolute Camera Pose with Rolling Shutter



## Absolute camera pose with RS

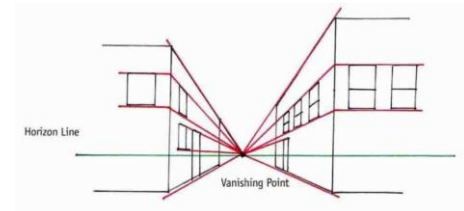
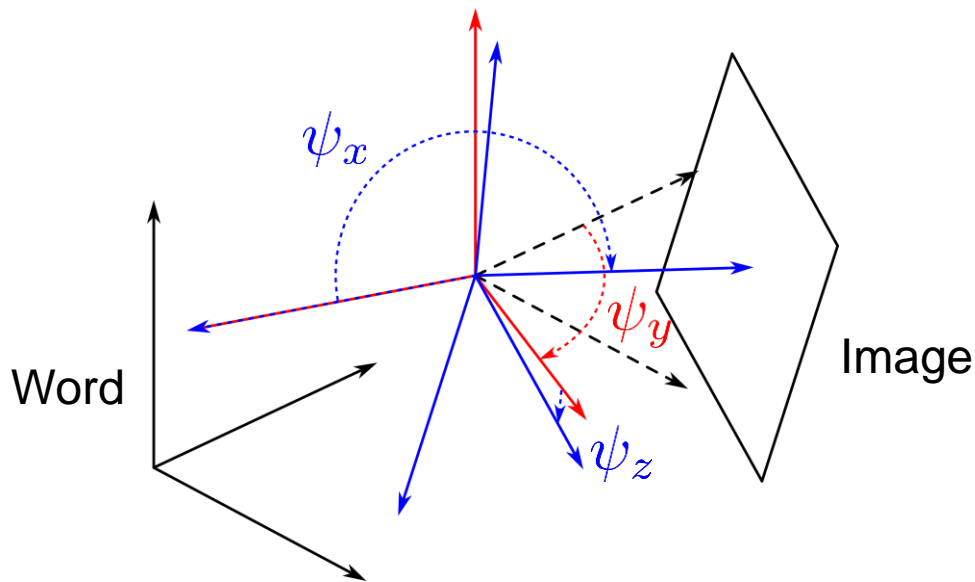
1. ***R6P - Rolling Shutter Absolute Camera Pose.***  
*C. Albl, Z. Kukelova, T Pajdla. CVPR 2015.*

2. ***Rolling Shutter Absolute Camera Pose Problem with known Vertical Direction.***  
*C. Albl, Z. Kukelova, T Pajdla. ICCV 2015.*

# R5P – Rolling Shutter Absolute Pose (with UP Vector)

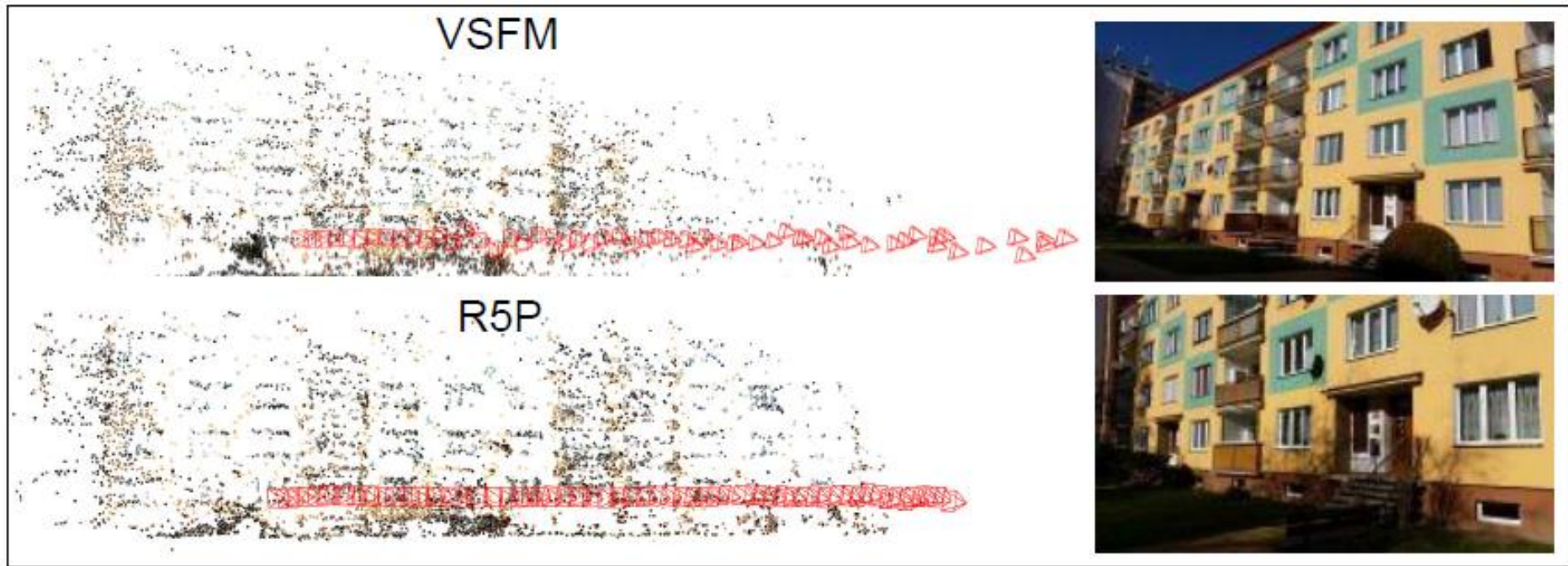
- UP-vector known (IMU, vanishing points, ...)
- Needs only 5 correspondences
- Solved by hidden variable resultant method
- Faster – 0.1 ms

Unkown      Known (IMU, vanishing points, ...)



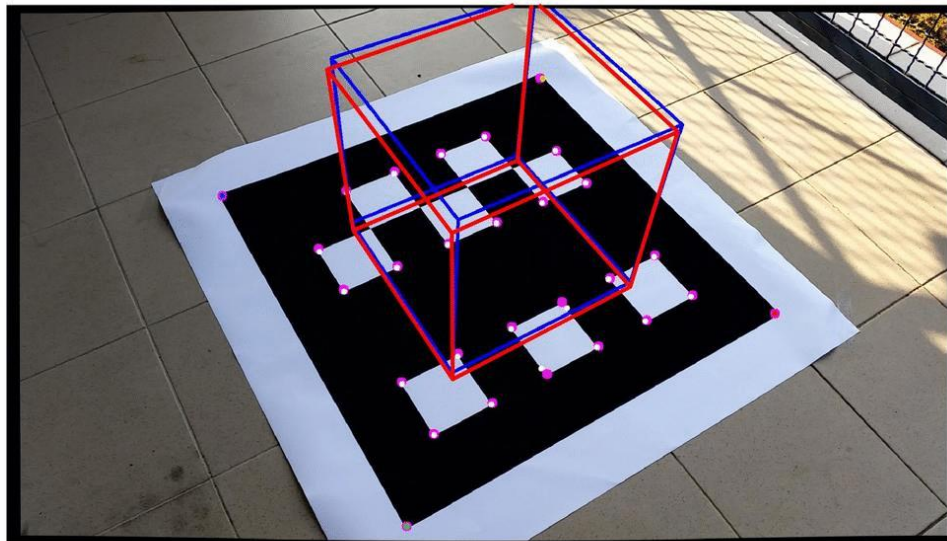
***Rolling Shutter Absolute Camera Pose Problem with known Vertical Direction.***  
C. Albl, Z. Kukelova, T Pajdla. ICCV 2015.

# R5P – RS Absolute Pose with UP Vector – SFM & VR

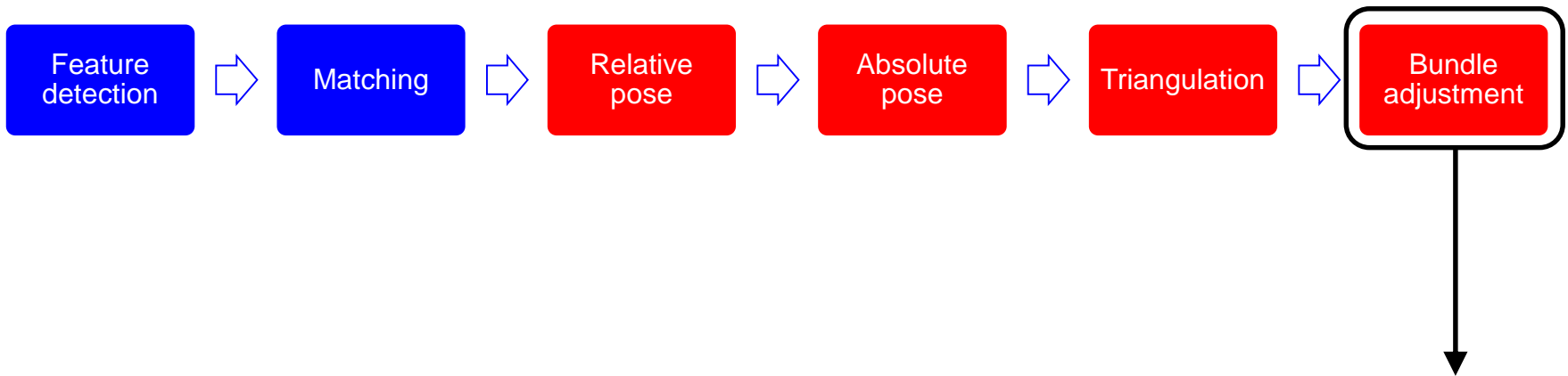


P3P

R5Pup



# Rolling Shutter Bundle Adjustment



## RS Bundle Adjustment

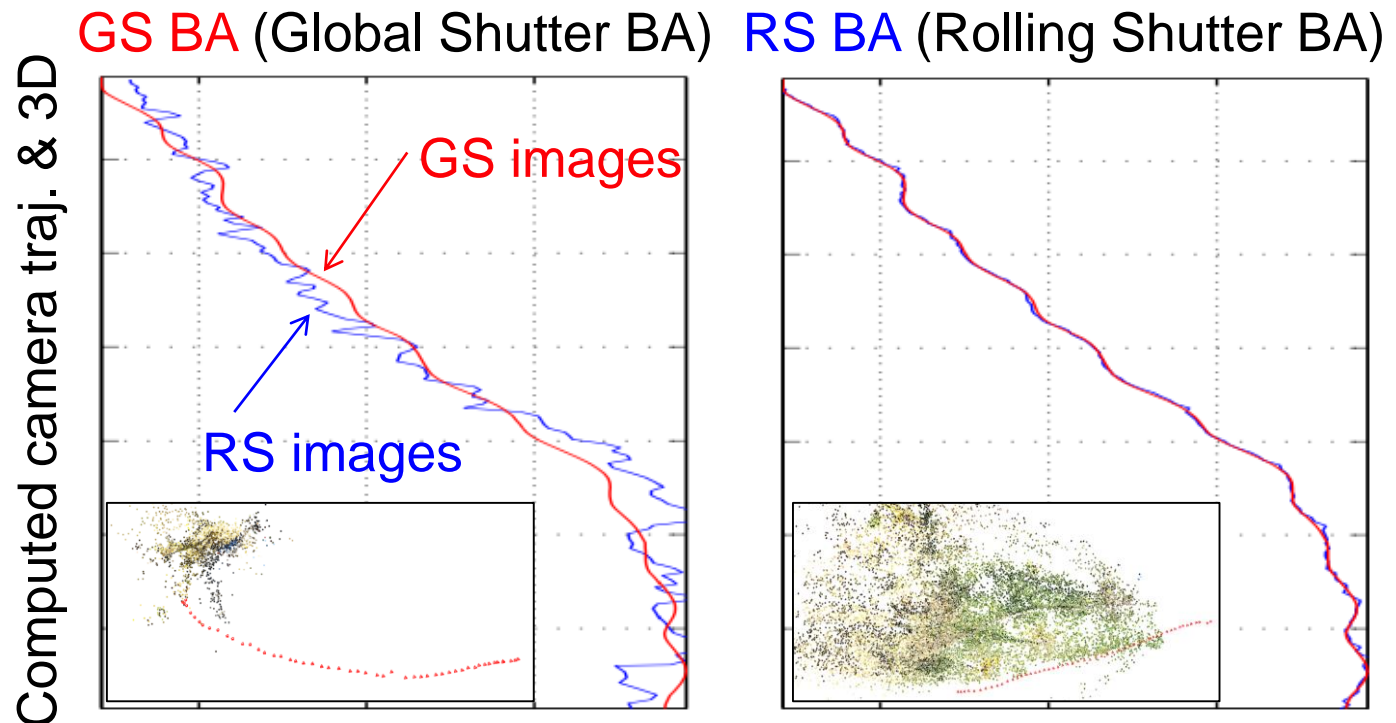
*Degeneracies in Rolling Shutter SfM*  
C. Albl, A. Sugimoto, T Pajdla. ECCV  
2016.

# RS BA on RS Images helps for videos



Global shutter camera  
moves together with  
Rolling shutter camera

RS BA improves over GS BA  
on videos  
when  
*motion during capture constrained  
motion between captures*



# Rolling Shutter BA - Motivation

- Can we reconstruct general unordered sets of images?
  - motions during and between image capture are independent
- Why would we do that?
  - Rolling shutter is present even in still images
  - Computing entire video is expensive
- We need Rolling Shutter Bundle Adjustment for unordered image sets



# Bundle Adjustment with RS Model

Projection model (full, linearized, ...)

$$\alpha_i \mathbf{u}_i = \alpha_i [c_i, r_i, 1]^\top = \mathbf{R}_r(r_i) \mathbf{R}_0 \mathbf{X}_i + \mathbf{C}_0 + r_i \mathbf{t}$$

Motion between captures

Motion during captures

Image reprojection error

$$\mathbf{e}_i^j = \tilde{\mathbf{u}}_i^j - \mu(\pi(\mathbf{P}^j(\tilde{\mathbf{r}}_i), \mathbf{X}_i))$$

$$\mu([x, y, z]^\top) = [x/z, y/z]^\top$$

Image measurement

Projection model

Perspective projection

Bundle adjustment (minimizes the SOS of reprojection errors)

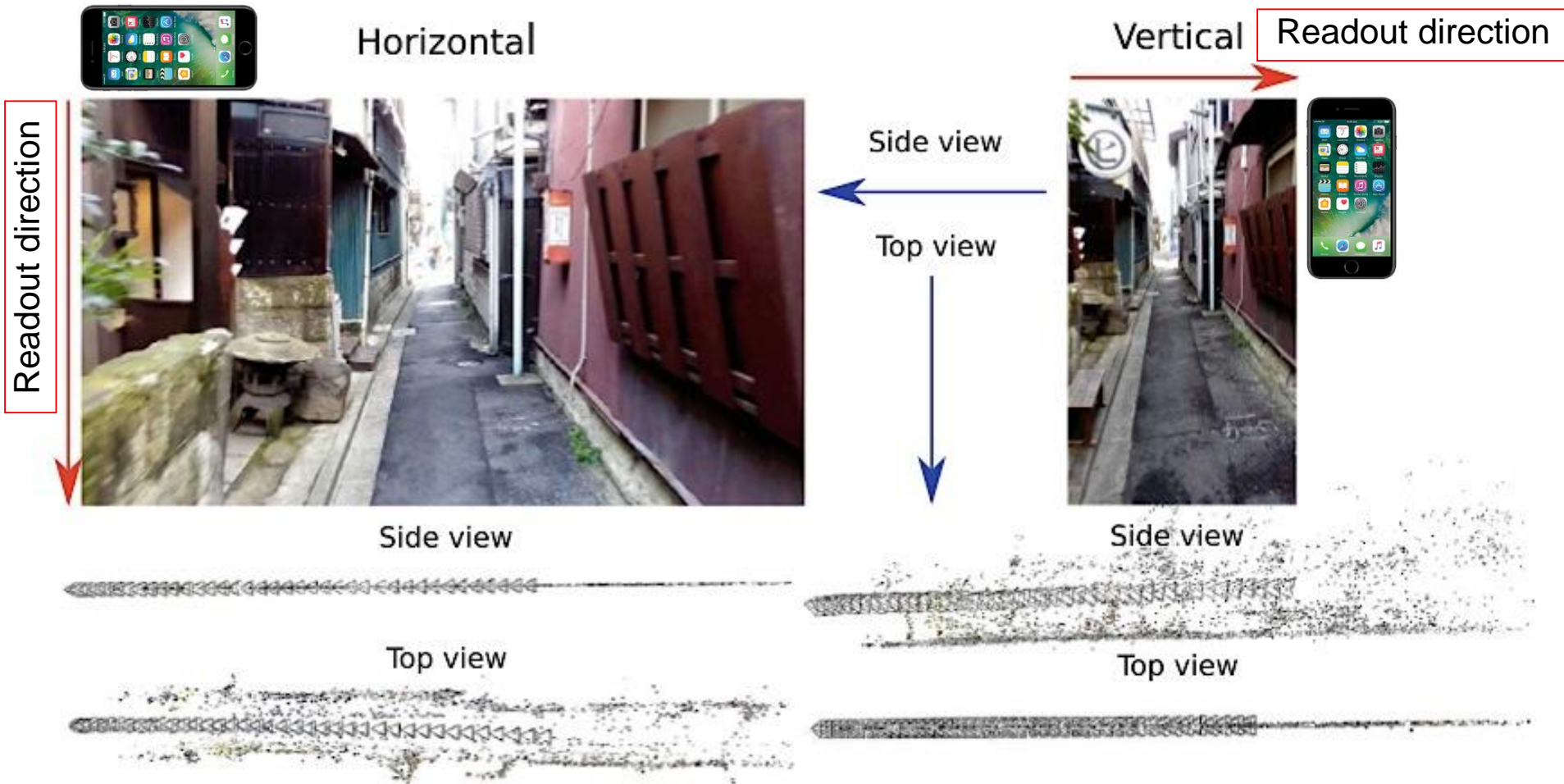
$$(\mathbf{P}^{j*}, \mathbf{X}_i^*) = \arg \min \sum_{(i,j)} \|\mathbf{e}_i^j\|^2$$

$$\mathbf{P}(r) = [\mathbf{R}_r(r), \mathbf{R}_0, \mathbf{C}_0, \mathbf{t}]$$

Many parameters per camera

# RS BA Fails for Unstructured Images

- Rolling Shutter BA flattens 3D in the readout direction

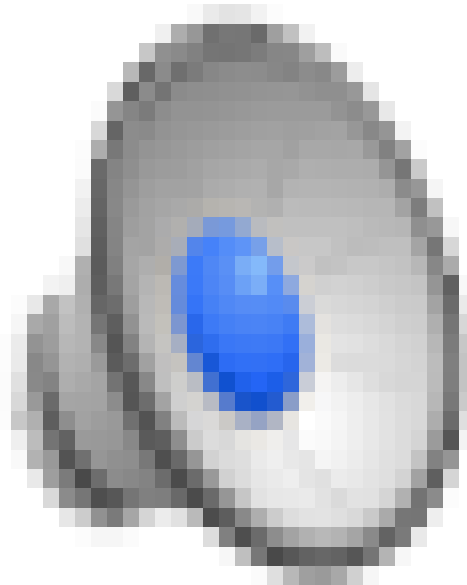


- for RS & GS cameras ... **problem with the RS model**
- RS model has more freedom ... **more degenerate situations**

# RS BA Fails for Unstructured Images



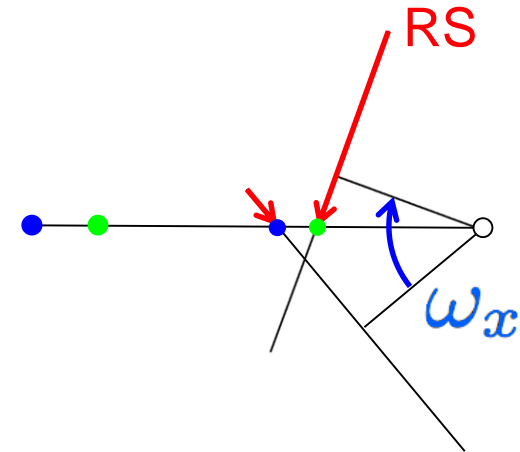
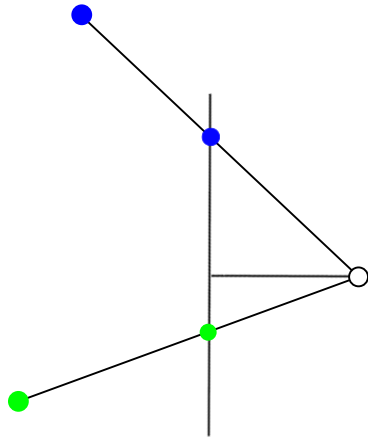
- Rolling Shutter BA flattens 3D in the readout direction



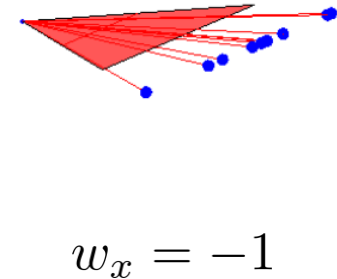
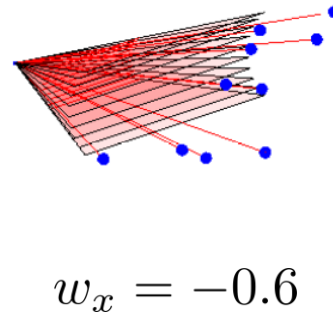
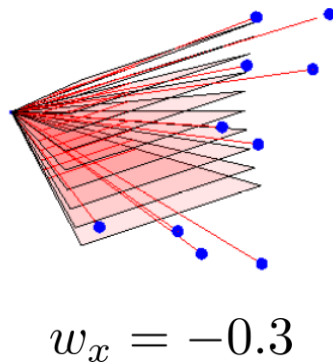
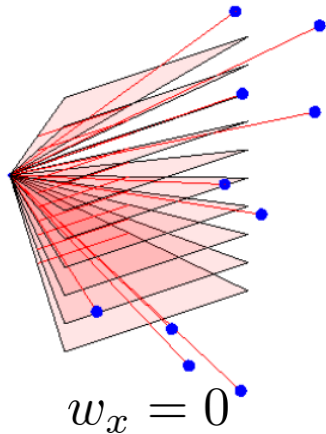
- for RS & GS cameras ... **problem with the RS model**
- RS model has more freedom ... **more degenerate situations**

# 1 image – Degenerate – 3D explained by 2D

Camera **rotation** compensates RS **scanning** to explain 3D by 2D



Global Shutter & 3D *explained by* Rolling Shutter & 2D  
*when camera motion speed matches RS scanning speed*

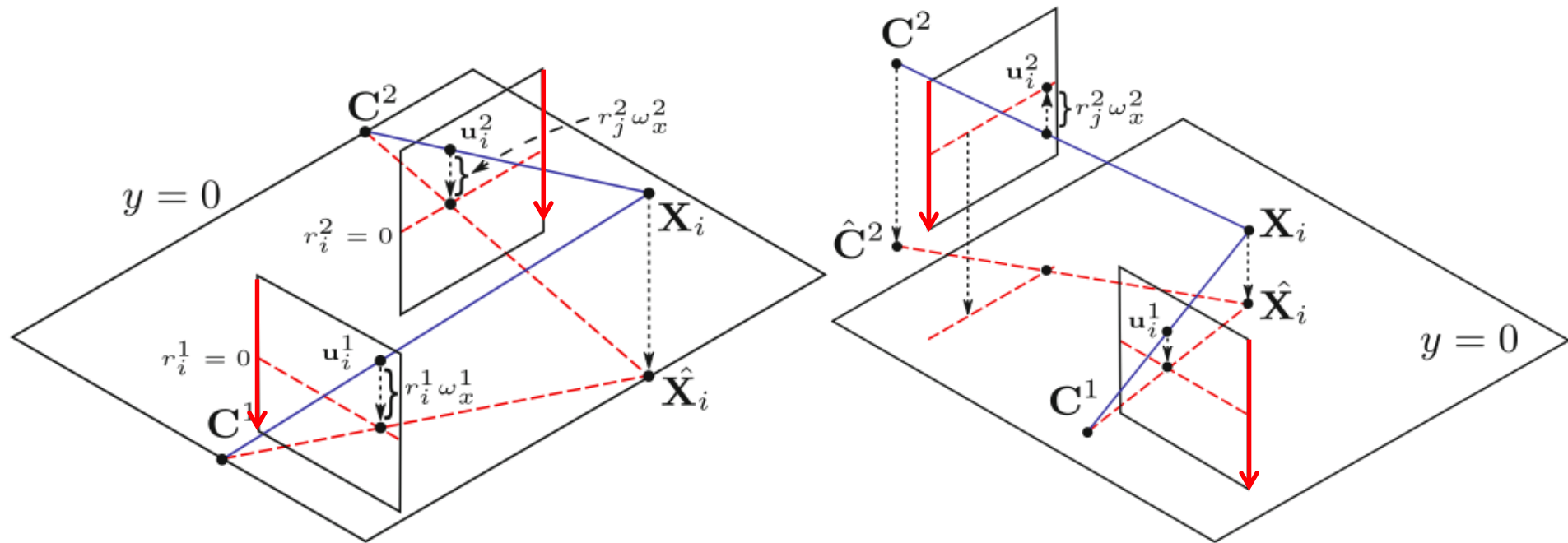


# N images – Degenerate – 3D explained by 2D

N images with aligned (parallel) **readout directions**

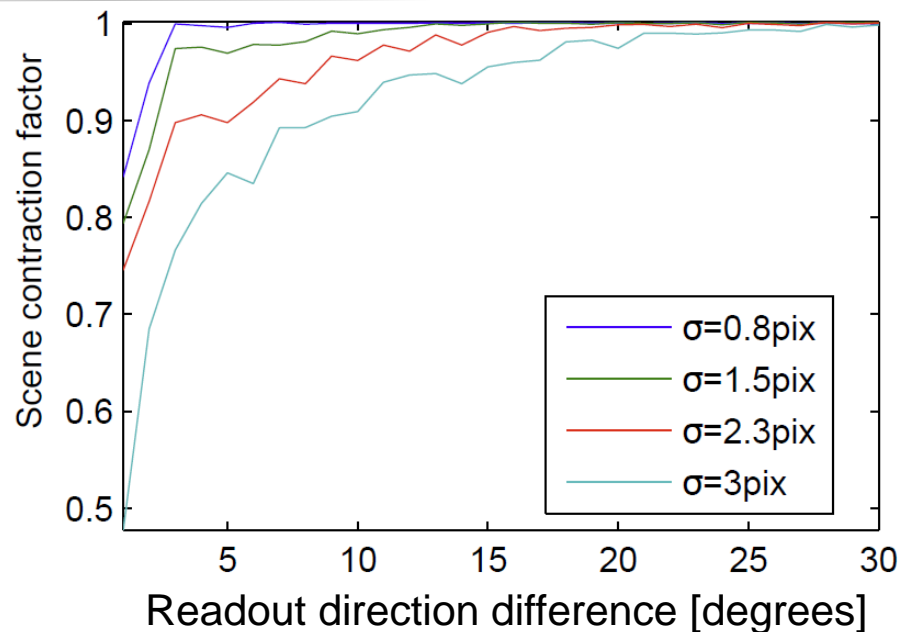
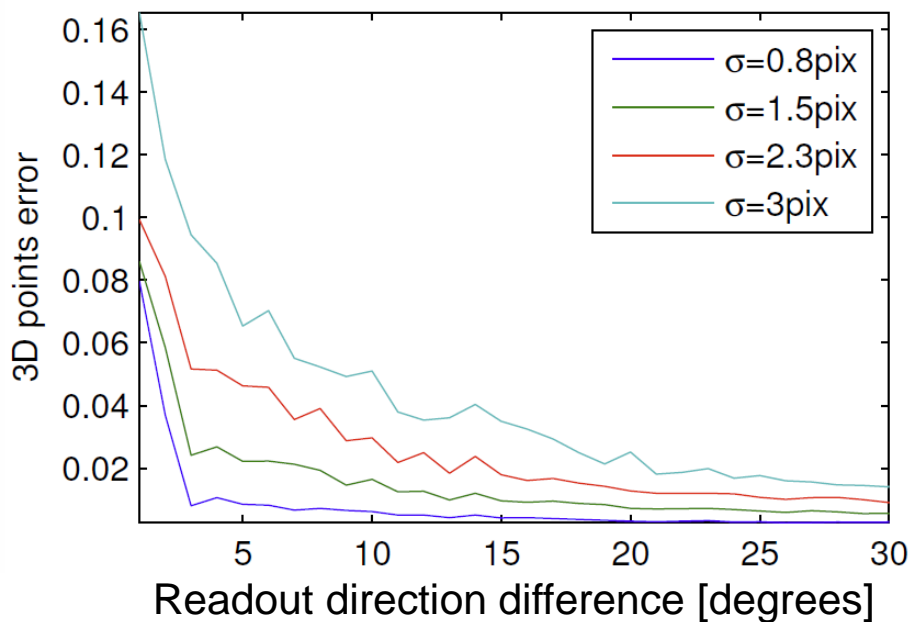


Images can be explained by a 2D scene

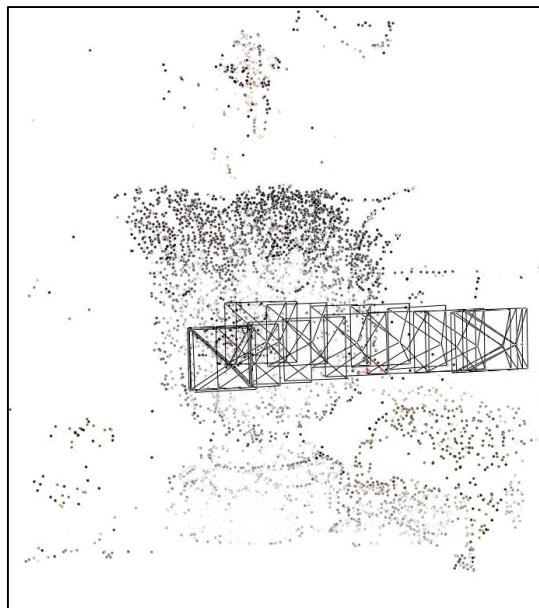


All images explained by a planar scene  
in the plane perpendicular to the scanning directions

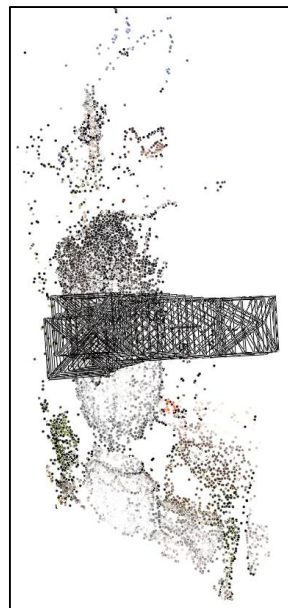
# Practice: Similar readout directions are BAD



“Ciorrect” reconstruction



“Distorted” reconstruction



# Why does BA do that?

Degenerate solution has usually lower re-projection error

Might increase

$$\mathbf{e}_i^j = \begin{bmatrix} e_{ix}^j \\ e_{iy}^j \end{bmatrix} = \tilde{\mathbf{u}}_i^j - \mu \left( \mathbf{R}_r^j(\tilde{r}_i^j) \mathbf{R}_0^j \mathbf{X}_i + \mathbf{C}^j + \tilde{r}_i^j \mathbf{t}^j \right) = \begin{bmatrix} \tilde{C}_i^j - \frac{C_x^j + x \cos(\phi^j) + z \sin(\phi^j)}{C_z^j + z \cos(\phi^j) - x \sin(\phi^j)} \\ 0 \end{bmatrix}$$

Gone

More similar readout directions – less increase in  $e_{ix}^j$

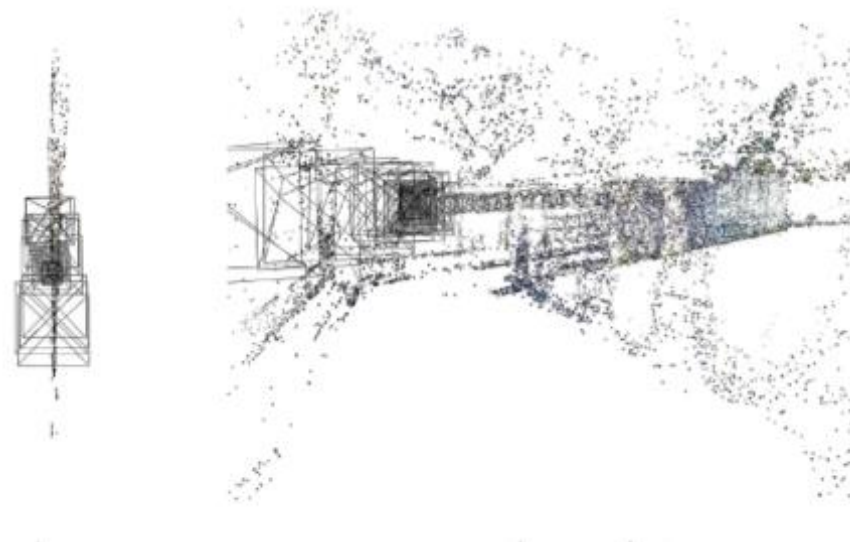
# A Practical Solution

1. Use images with many different readout directions
2. Use a camera pair with orthogonal readout directions.

Independent image sequences



2 coupled cameras with perpendicular readouts

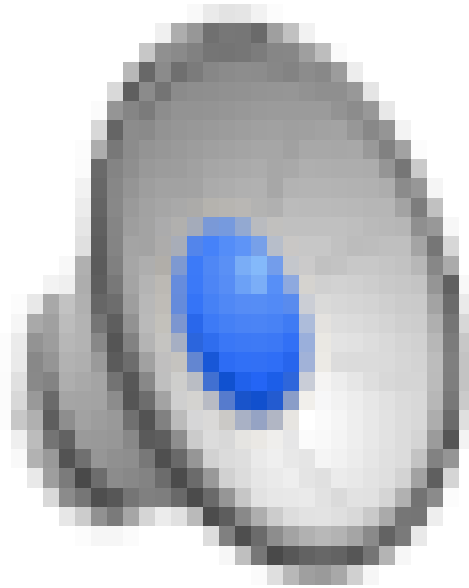




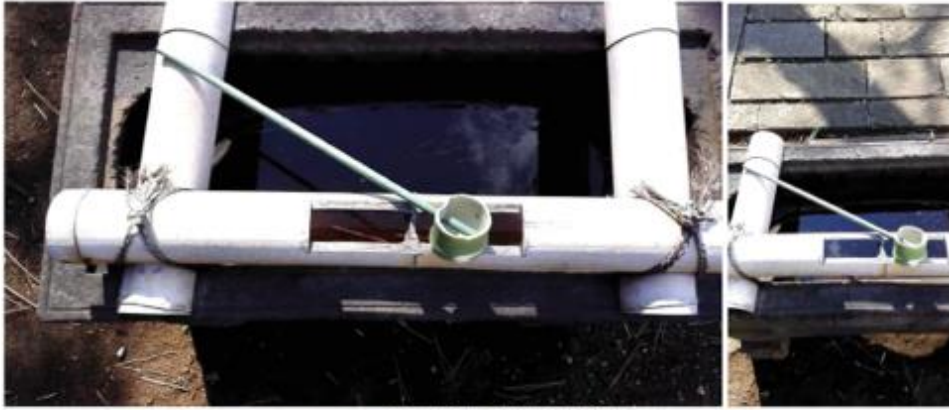
# A Practical Solution

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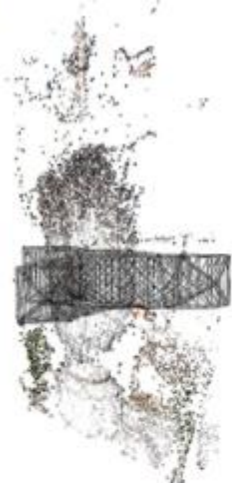
1. Use images with many different readout directions
2. Use a camera pair with orthogonal readout directions.



# A Practical Solution



# A Practical Solution



# 3D with Rolling Shutter

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ROBOTICS AND CYBERNETICS**

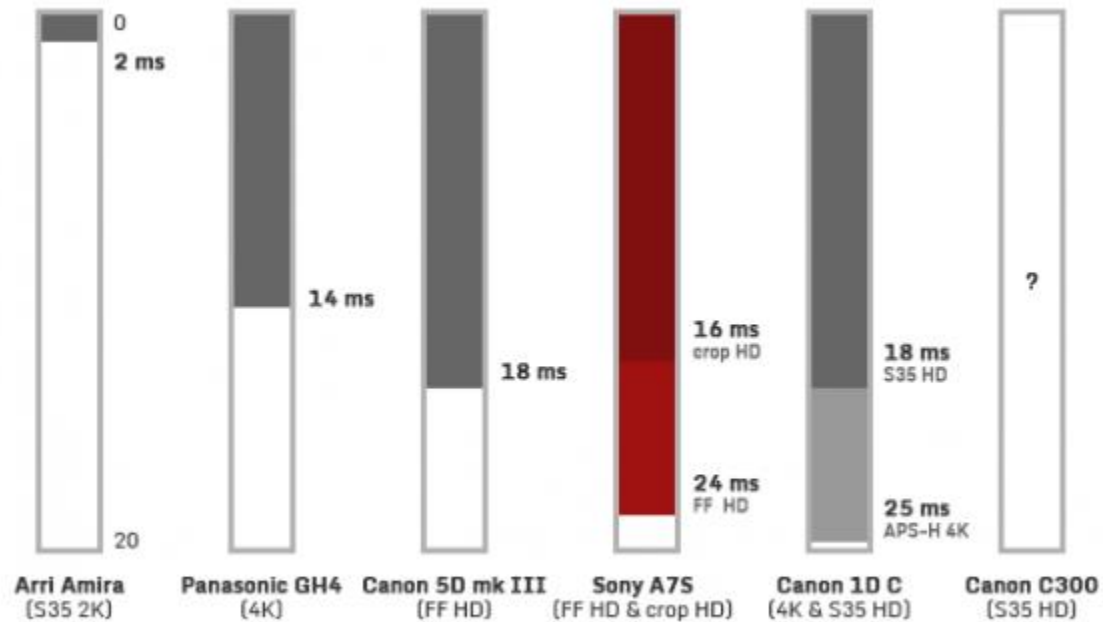
AAG – Applied Algebra & Geometry



**CENTER FOR MACHINE  
PERCEPTION**

GVR – Geometry of Vision & Robotics

## Rolling Shutter (less is better)



Tested with a rotary chart developed by cinema5D. Approximate values in milliseconds.