CVPR VIRTUAL

RANSAC in 2020: A CVPR Tutorial

Mathematical Programming Approaches

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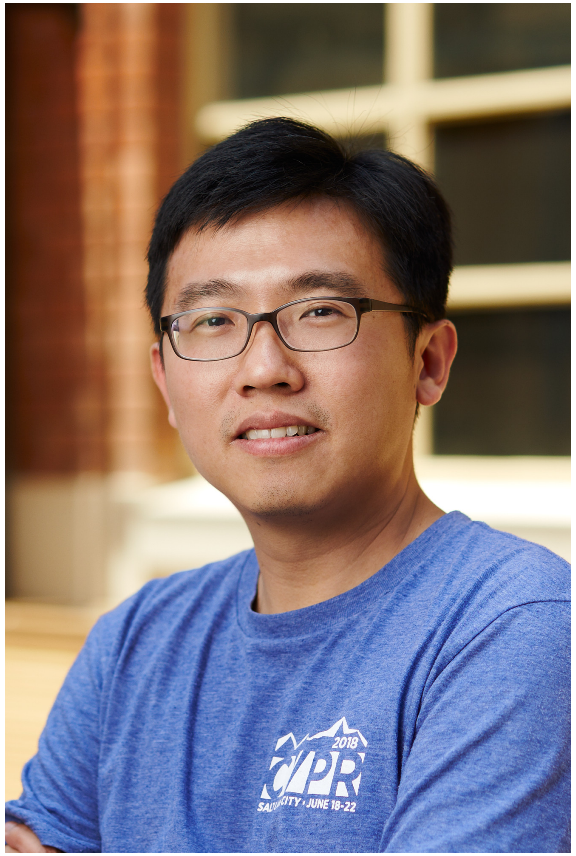
THE UNIVERSITY of ADELAIDE

In this talk



Tat-Jun ("TJ") Chin

Zhipeng Cai
"Wingman" (soon to be "Commander")

Photo from: [Wild Sulphur Crested Cockatoos]

Photo from: [Cockatoos Botanic Garden - Sydney]

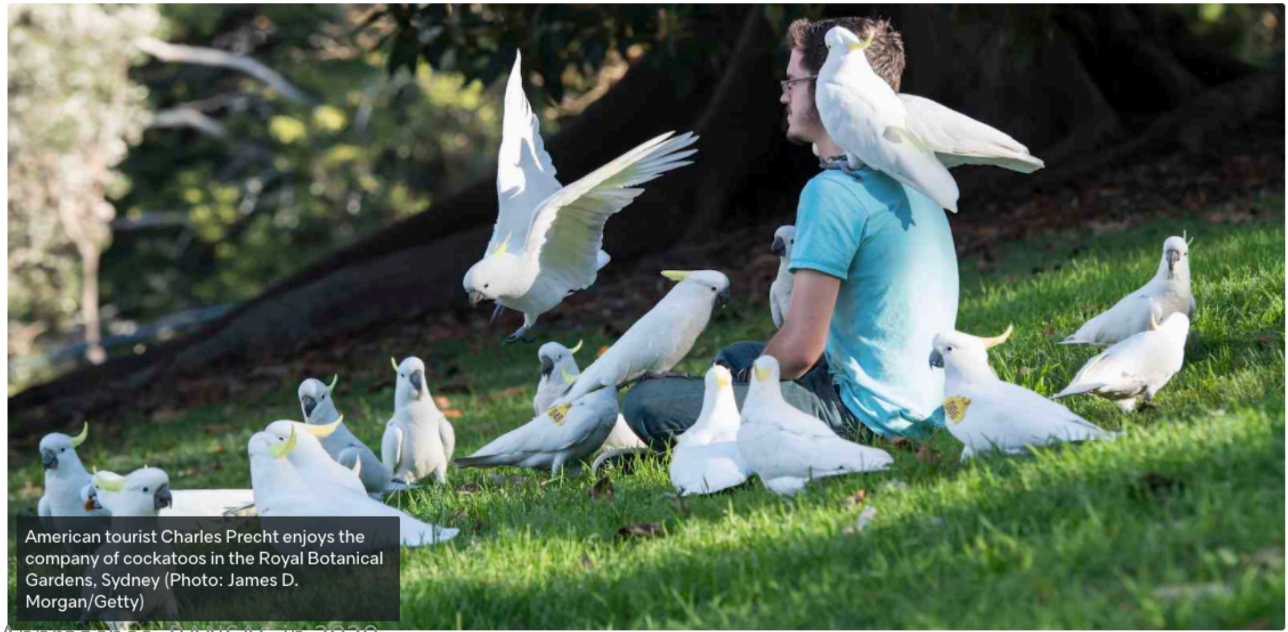
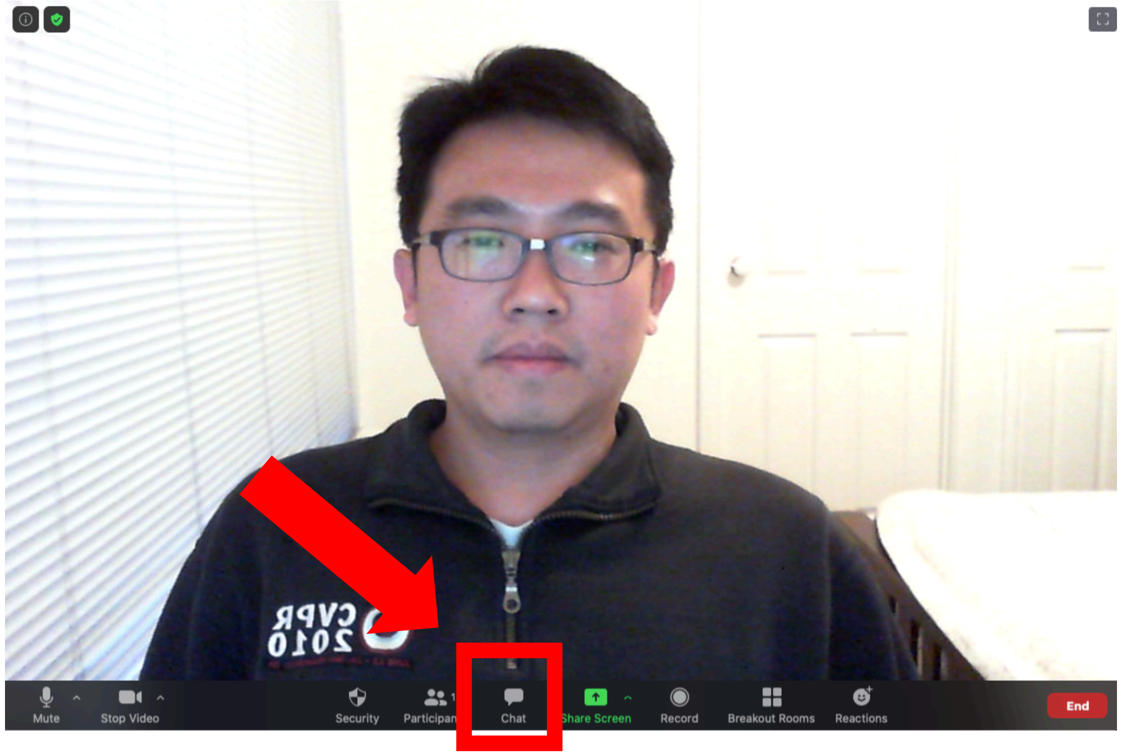


Photo from: [Soft power and reviewing Australia's global appeal]

In this talk

- Feel free to
  - Ask questions directly through voice chat (if you can unmute yourself).
  - Type your questions in the chat window.
- This talk is allocated 1.5 hours---we **may** have a short break (5-10 minutes) halfway through.



Outline

- What is and isn't fundamentally achievable
- Global algorithms
- Deterministic outlier removal
- Deterministic refinement
- Evaluation
- RANSAC in 2040---Quantum RANSAC?

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Consensus maximisation

Problem 1 (MAXCON). Given input data $\mathcal{D} = \{(\mathbf{a}_i, b_i)\}_{i=1}^N$, where $\mathbf{a}_i \in \mathbb{R}^d$ and $b_i \in \mathbb{R}$, and an inlier threshold $\epsilon \in \mathbb{R}_+$, find the $\mathbf{x} \in \mathbb{R}^d$ that maximises

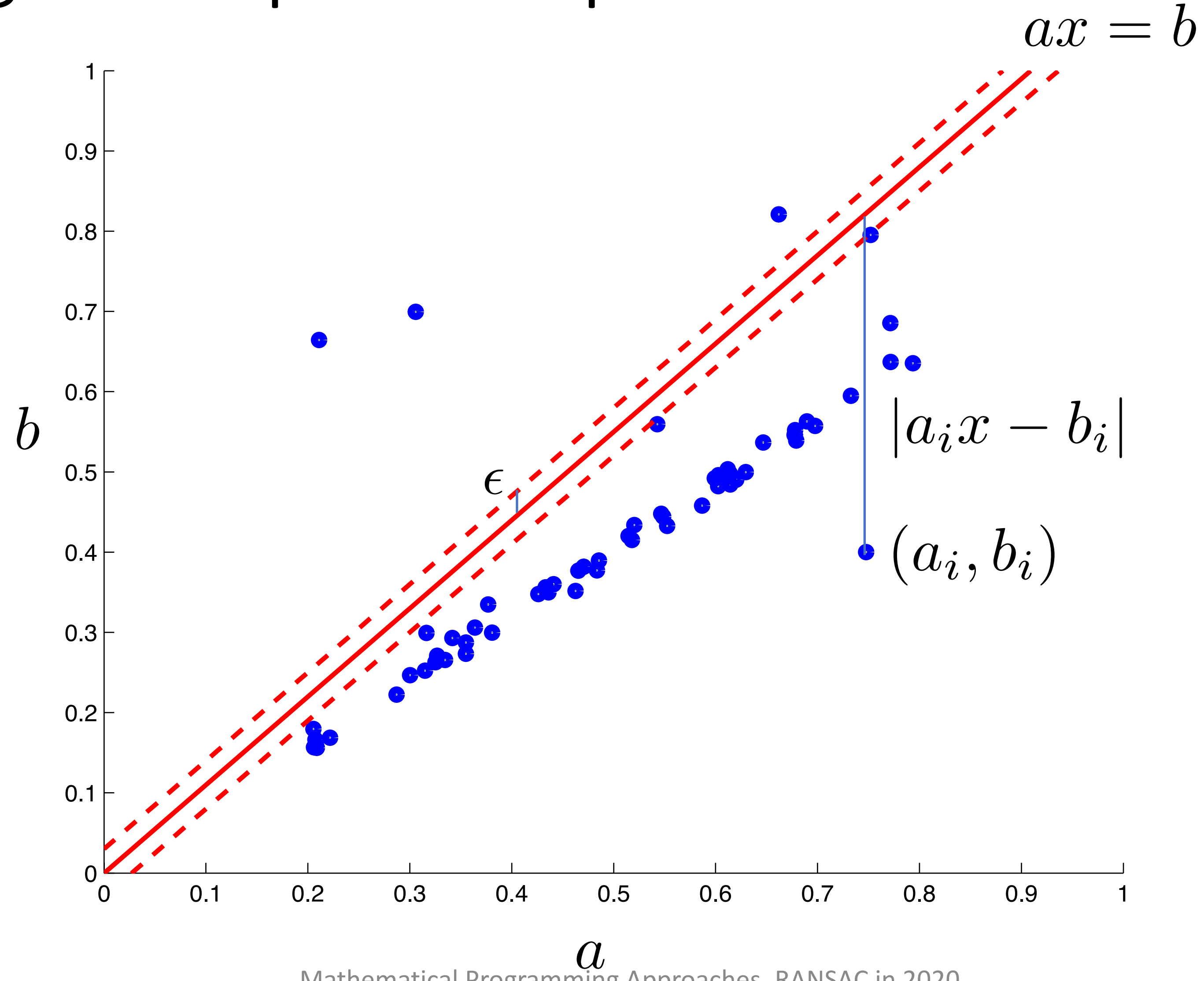
$$\Psi_\epsilon(\mathbf{x} \mid \mathcal{D}) = \sum_{i=1}^{N} \mathbb{I}\left(|\mathbf{a}_i^T \mathbf{x} - b_i| \leq \epsilon\right),$$

where $\mathbb{I}$ returns 1 if its input predicate is true, and 0 otherwise.

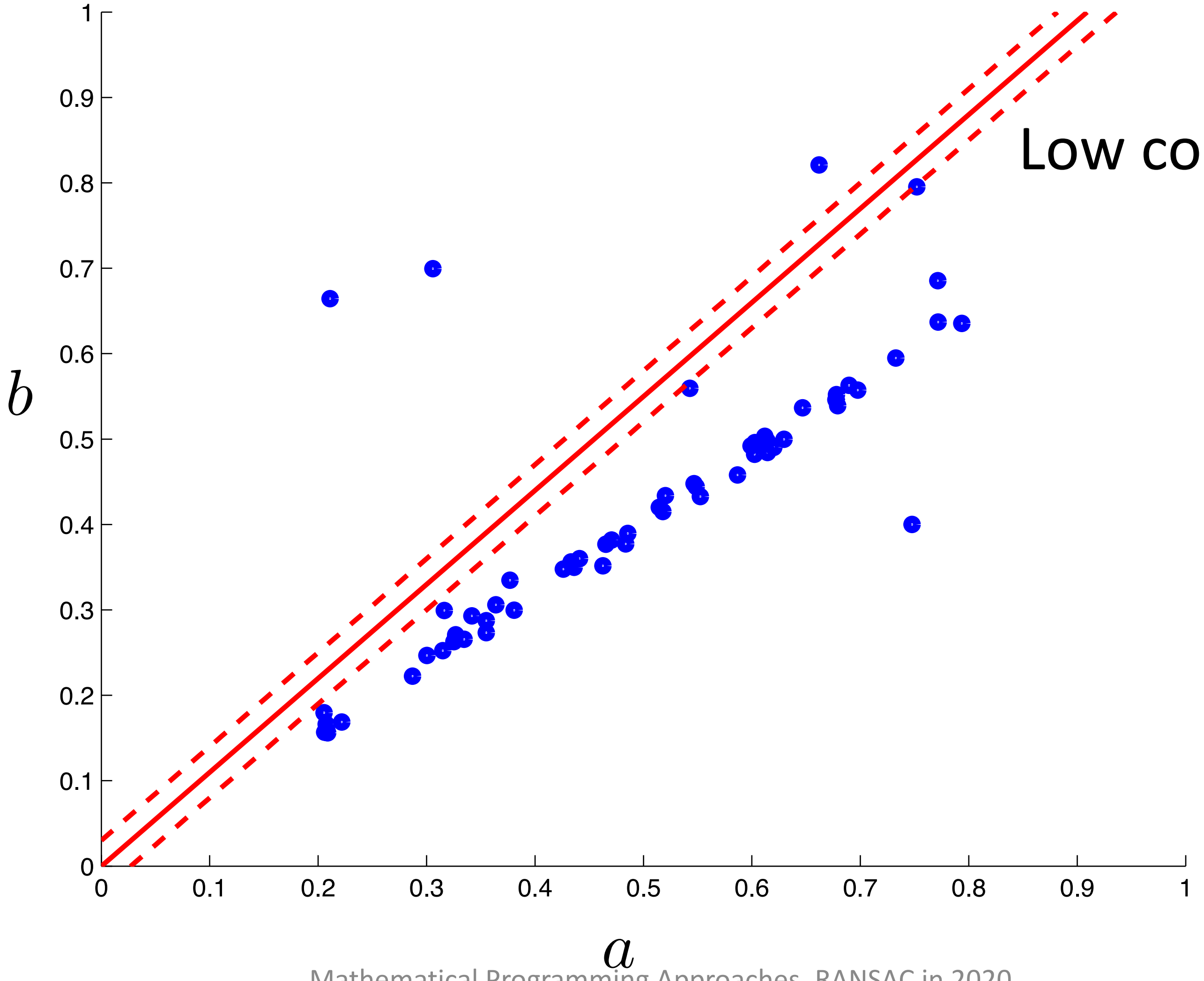
- Simple case to facilitate analysis (most vision problems do not have a "dependent" variable like b_i above).

T.-J. Chin and D. Suter. The maximum consensus problem: recent algorithmic advances. Synthesis Lectures on Computer Vision (Eds. Gerard Medioni and Sven Dickinson). Morgan & Claypool Publishers, San Rafael, CA, U.S.A., Feb 2017.

Running example: 1D problem

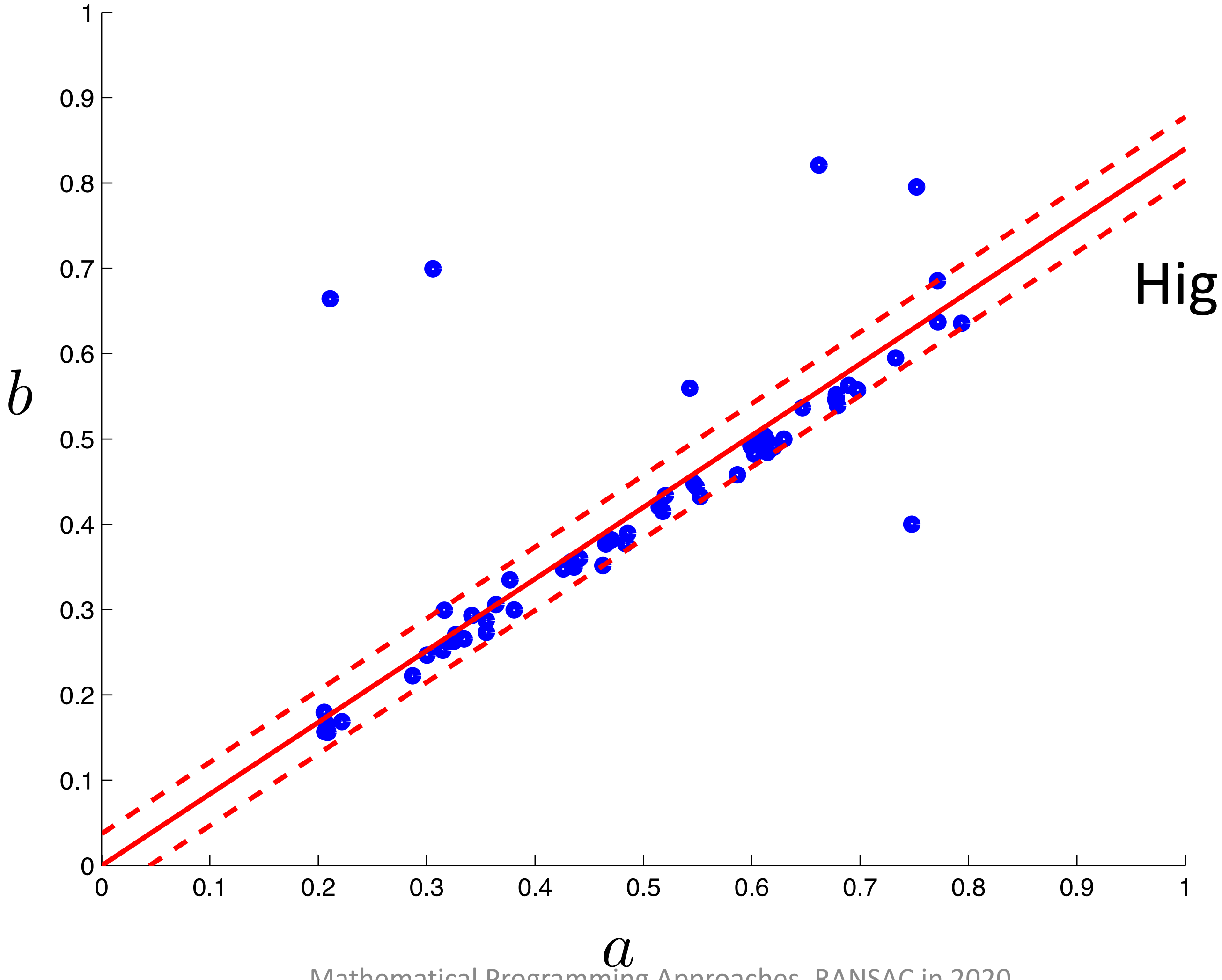


Running example: 1D problem



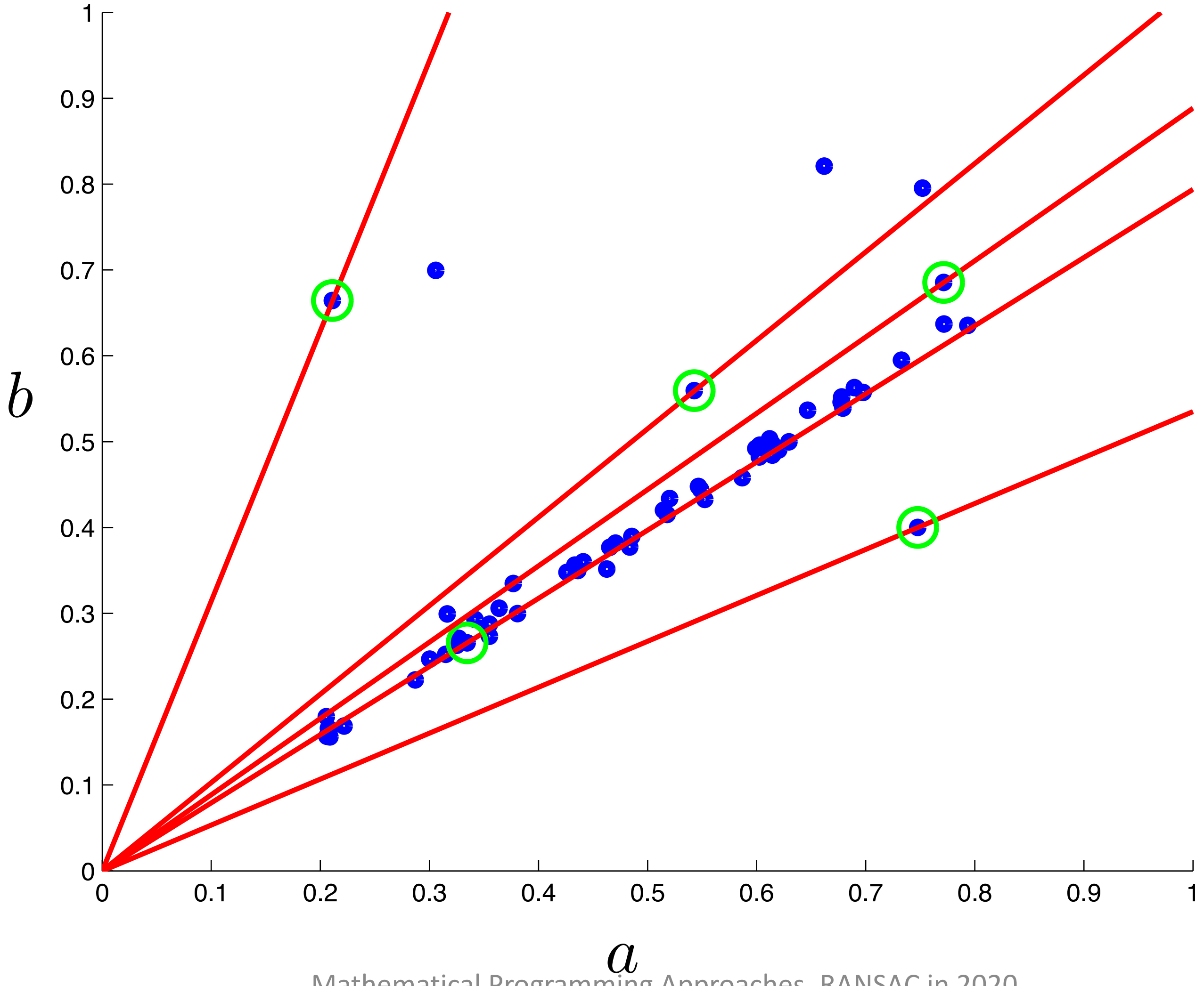
Low consensus $\Psi_{\epsilon}(x \mid \mathcal{D})$

Running example: 1D problem



High consensus $\Psi_\epsilon(x \mid \mathcal{D})$

RANSAC

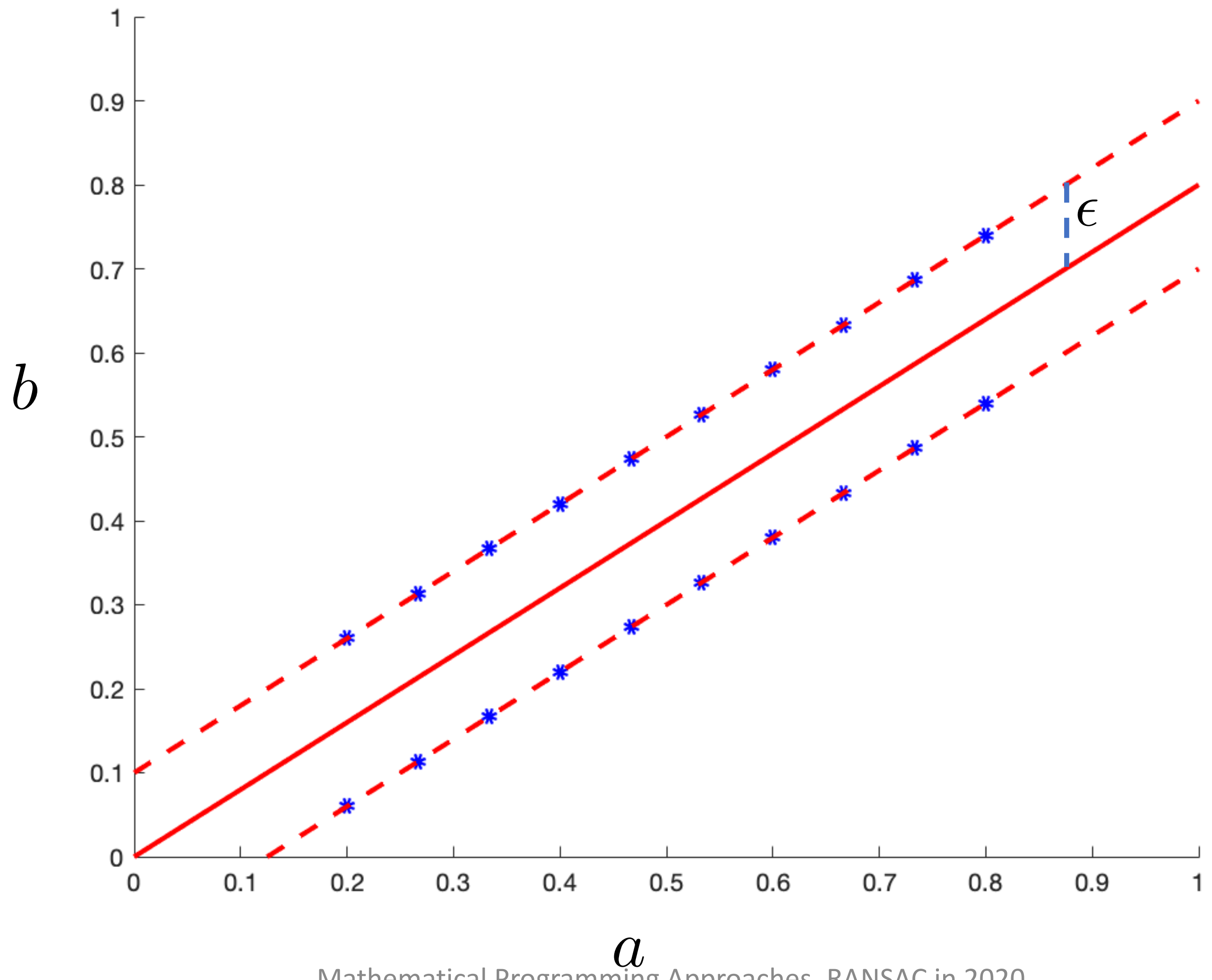


Assuming an inlier rate of η and dimensionality of d, if we draw

$$T = \left\lceil \frac{\log(1-\gamma)}{\log(1-\eta^d)} \right\rceil$$

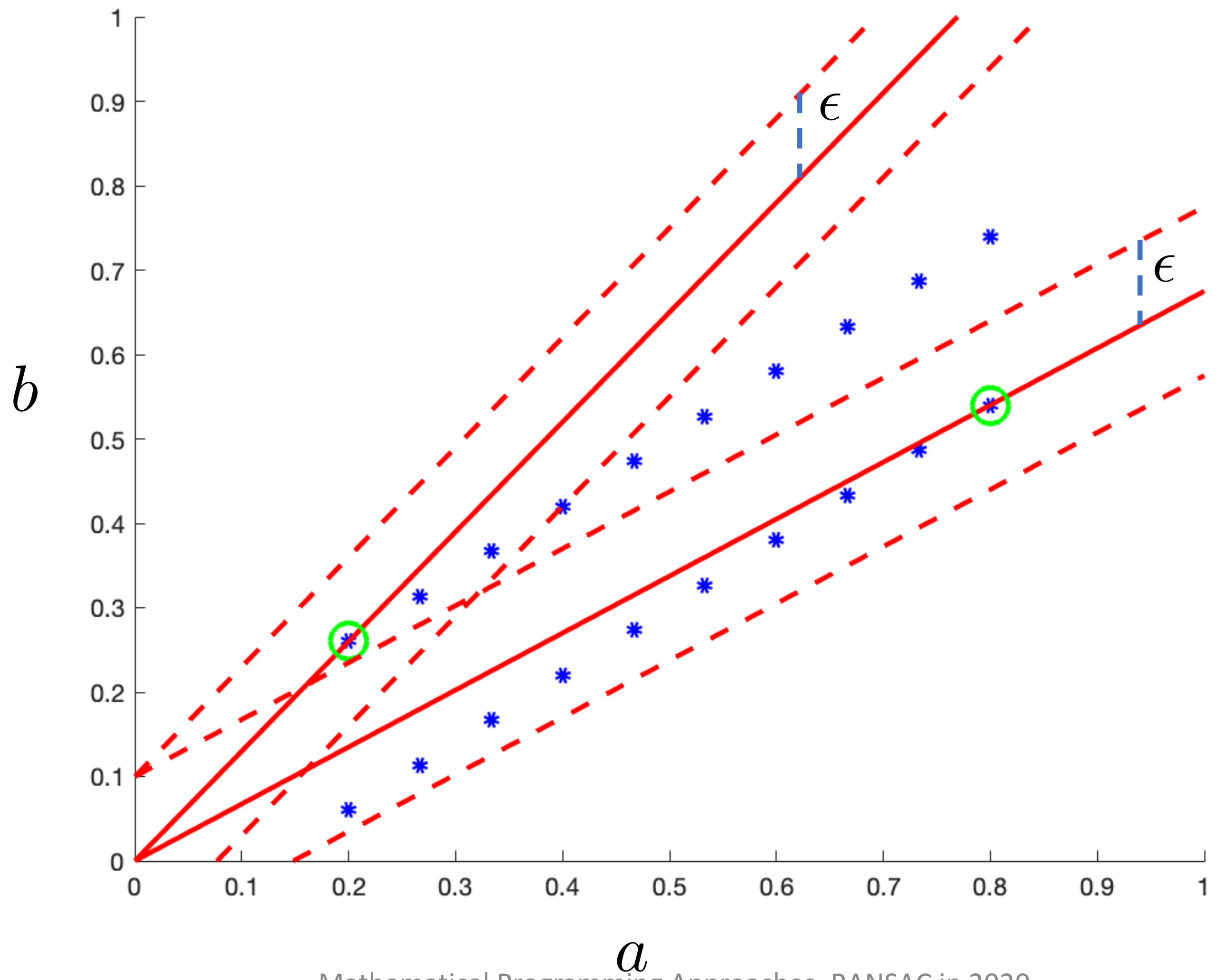
samples, with probability γ we will get **at least one** minimal subset that contains only inliers.

Adversarial case for RANSAC



Desired solution (max consensus)

Adversarial case for RANSAC



Fundamental limitations

$\boxed{\text{Maxcon is NP-hard}}$

$\Rightarrow$ Cannot be solved in polynomial time.

$\Rightarrow$ In experiments, should also examine how often worst-case results happen.

$\boxed{\text{Maxcon is W[1]-hard in dimension}}$

$\Rightarrow$ Cannot remove d from the exponent, i.e., $N^{f(d)}$.

$\Rightarrow$ Exact solvers are practical for (very) low-dimensions only.

$\boxed{\text{Maxcon is APX-hard}}$

$\Rightarrow$ There are no polynomial time approximation algorithms.

(RANSAC is not an approximation algorithm---no error bounds)

T.-J. Chin, Z. Cai and F. Neumann. Robust fitting in computer vision: easy or hard? European Conference on Computer Vision (ECCV) 2018.

Minimally trimmed squares (MTS)

$$|\mathcal{O}^*| = \min_{\mathcal{M},\mathcal{O}\subset\mathcal{D}} |\mathcal{O}|$$

$$\text{s.t.} \quad \sum_{\mathbf{d}\notin\mathcal{O}} r(\mathbf{d} \mid \mathcal{M})^2 \leq \epsilon$$

There are no quasi-polynomial-time algorithm that can find $\mathcal{O}$ with

$$|\mathcal{O}| \leq \lambda |\mathcal{O}^*|$$

for any factor $\lambda > 1$.

Vasileios Touzmas, Pasquale Antonante, and Luca Carlone. Outlier-robust spatial perception: hardness, general-purpose algorithms, and guarantees. In IROS 2019.

M-estimators

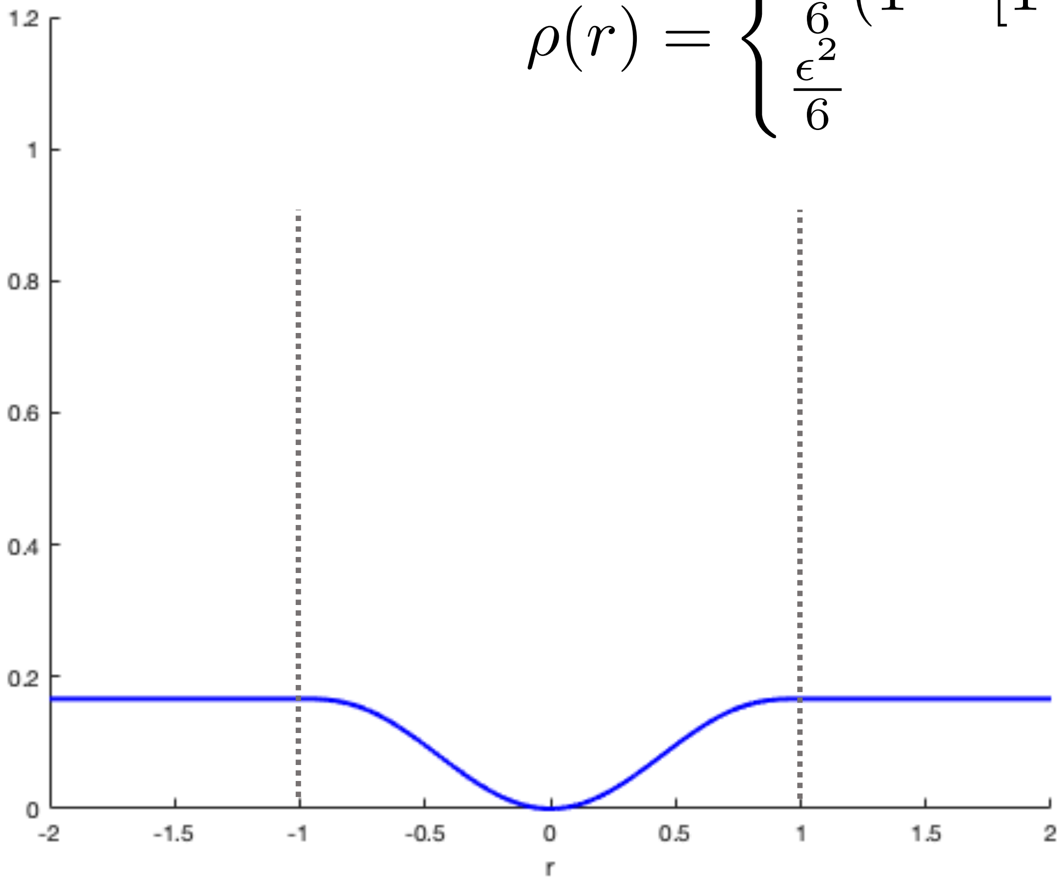
- Given data $\mathcal{D} = \{(\mathbf{a}_i, b_i)\}_{i=1}^{N}$, estimate vector $\mathbf{x} \in \mathbb{R}^d$:

$$\min_{\mathbf{x} \in \mathbb{R}^d} \sum_{i=1}^{N} \rho(|\mathbf{a}_i^T \mathbf{x} - b_i|)$$

Robust loss function

Tukey's biweight:

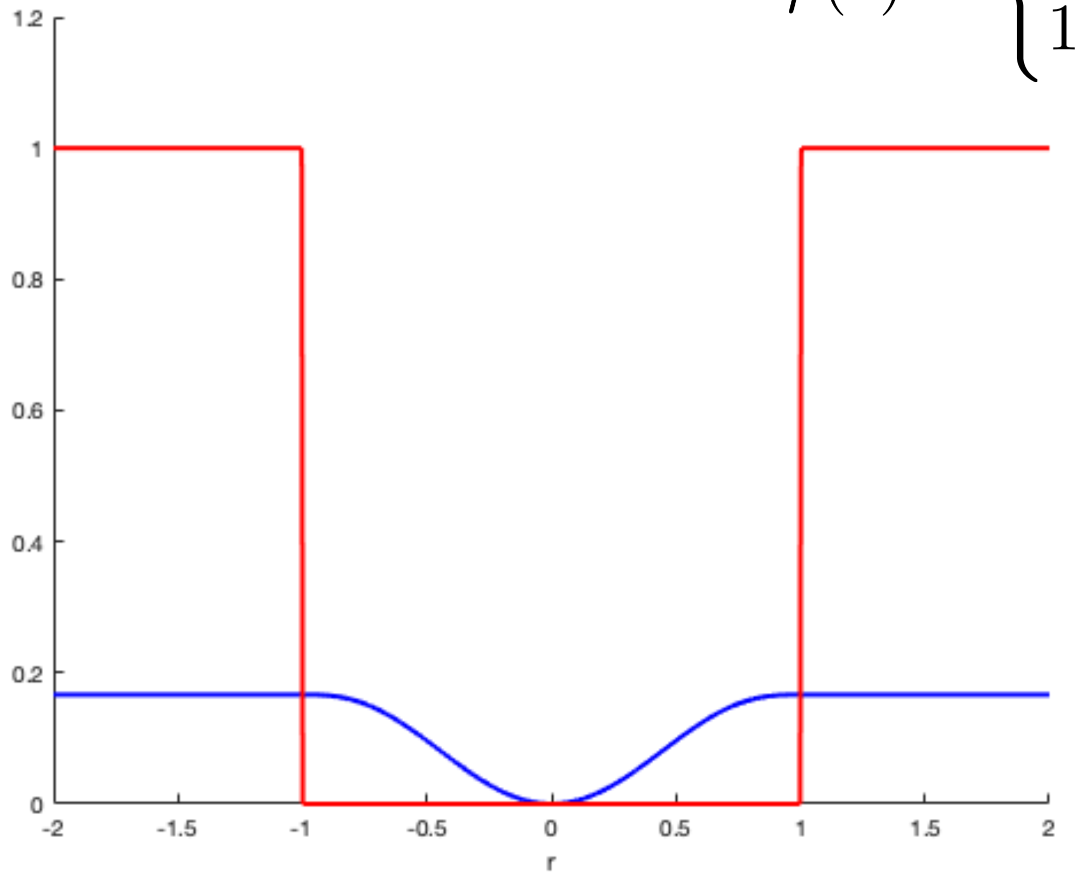
$$\rho(r) = \begin{cases} \frac{\epsilon^2}{6}(1 - [1 - (\frac{r}{\epsilon})^2]^3) & \text{if } |r| \leq \epsilon \\ \frac{\epsilon^2}{6} & \text{otherwise,} \end{cases}$$



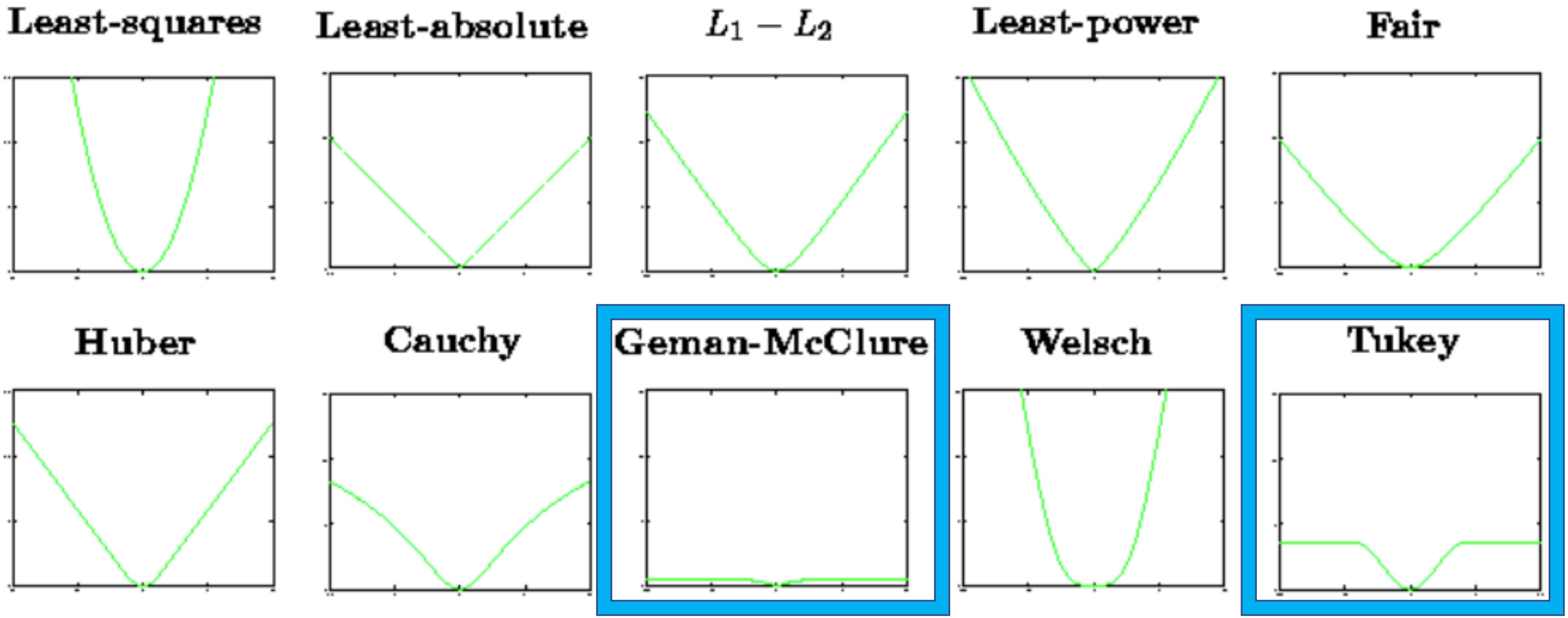
Robust loss function

Outlier counting:

$$\rho(r) = \begin{cases} 0 & \text{if } |r| \leq \epsilon \\ 1 & \text{otherwise;} \end{cases}$$



Robust loss function



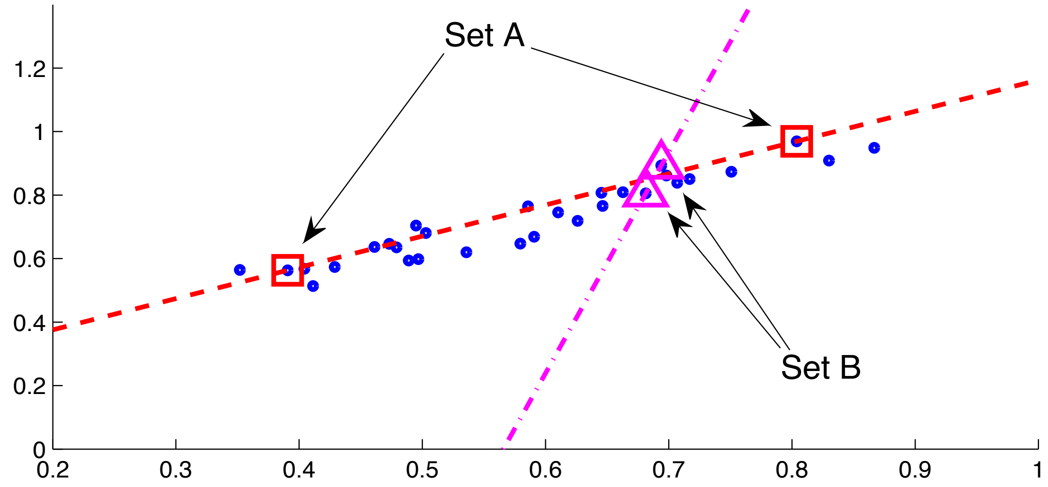
http://www-sop.inria.fr/odyssee/software/old_robotvis/Tutorial-Estim/node24.html

"Hard redescenders" (non-convex)

“B…but, RANSAC works well”

- RANSAC and variants work well (usually); it’s when they don’t work that’s the issue.
- RANSAC aims to find an all-inlier minimal subset, with the assumption that the ALL all-inlier minimal subsets will give high consensus.
  - Degeneracy (see Ondra’s talk) and adversarial cases will cause problems.
  - Even if there is no degeneracy, the assumption does not hold.
- When RANSAC does not return a model with high consensus, you don’t know if it is because there are no high-consensus models to be found, or if you are unlucky.
- By all means, use RANSAC, but that doesn’t mean that the science should stop.

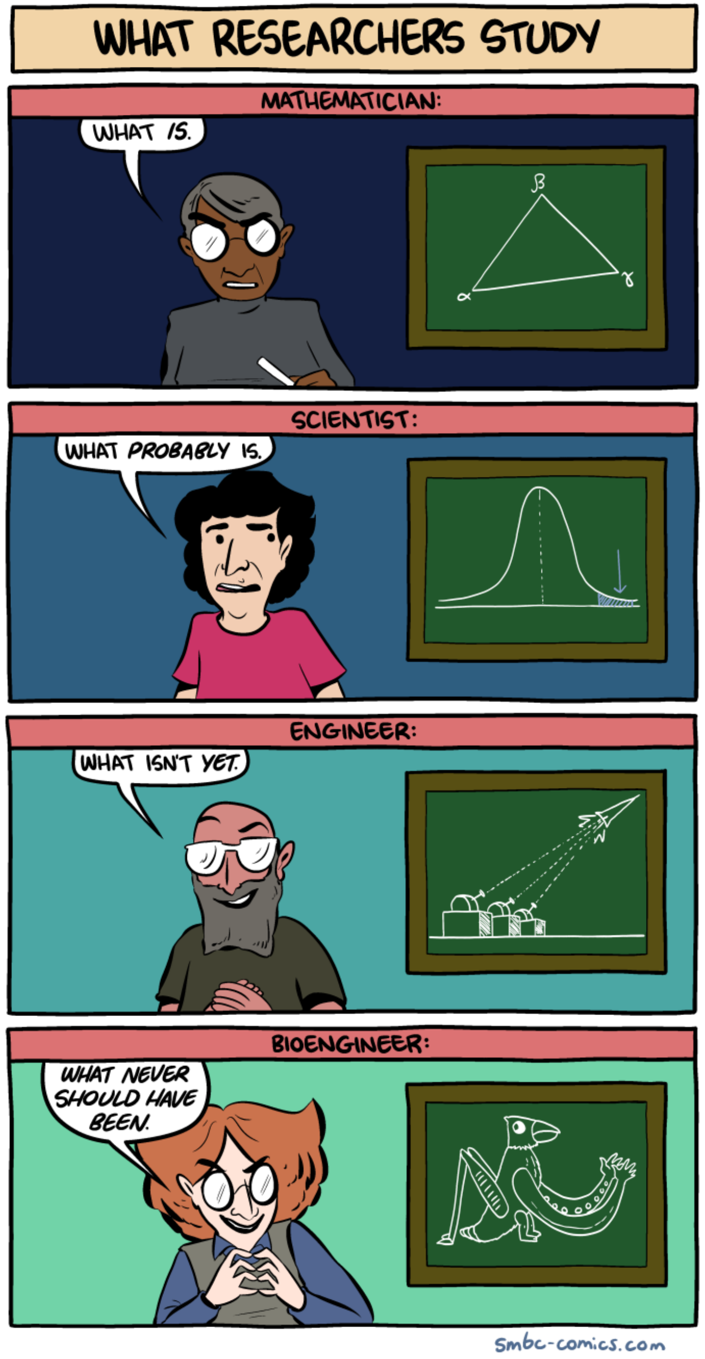
Not all “all-inlier” minimal subsets give a good fit



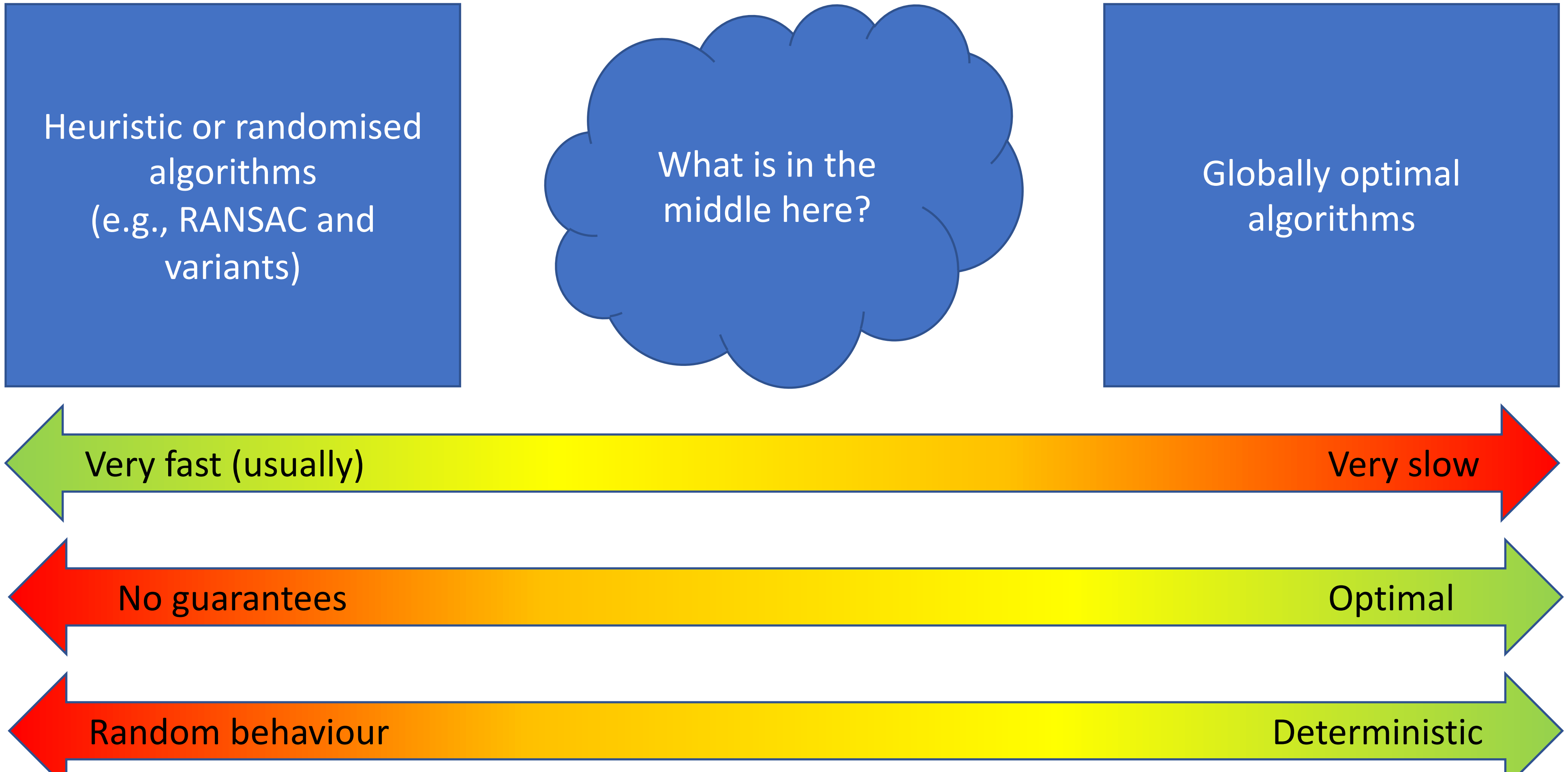
Quoc-Huy Tran, Tat-Jun Chin, Wojciech Chojnacki, David Suter: Sampling Minimal Subsets with Large Spans for Robust Estimation. Int. J. Comput. Vis. 106(1): 93-112 (2014)

Why should we care about theoretical hardness?

- We want to
  - Classify algorithms.
  - Predict performance in *general settings*
    - *e.g., Dr XXX claims that his/her algorithm can do this; is that probable according to the theory?*
    - What aspects of the algorithm should our experiments test? (outlier rate, inlier noise, number of outliers, dimensionality, distribution of inliers, distribution of outliers, etc. etc.)
- For future work
  - What kind of new algorithms to develop?
  - Where should we devote our efforts?
    - Should we just use RANSAC variants but carefully preprocess or postprocess?
    - Is deep learning the way to go for robust fitting? There is increasing evidence that they don't generalise well. <u>Should we care about the tables with bold numbers?</u>



Back to fundamental limitations: what does the theory allow us we do?



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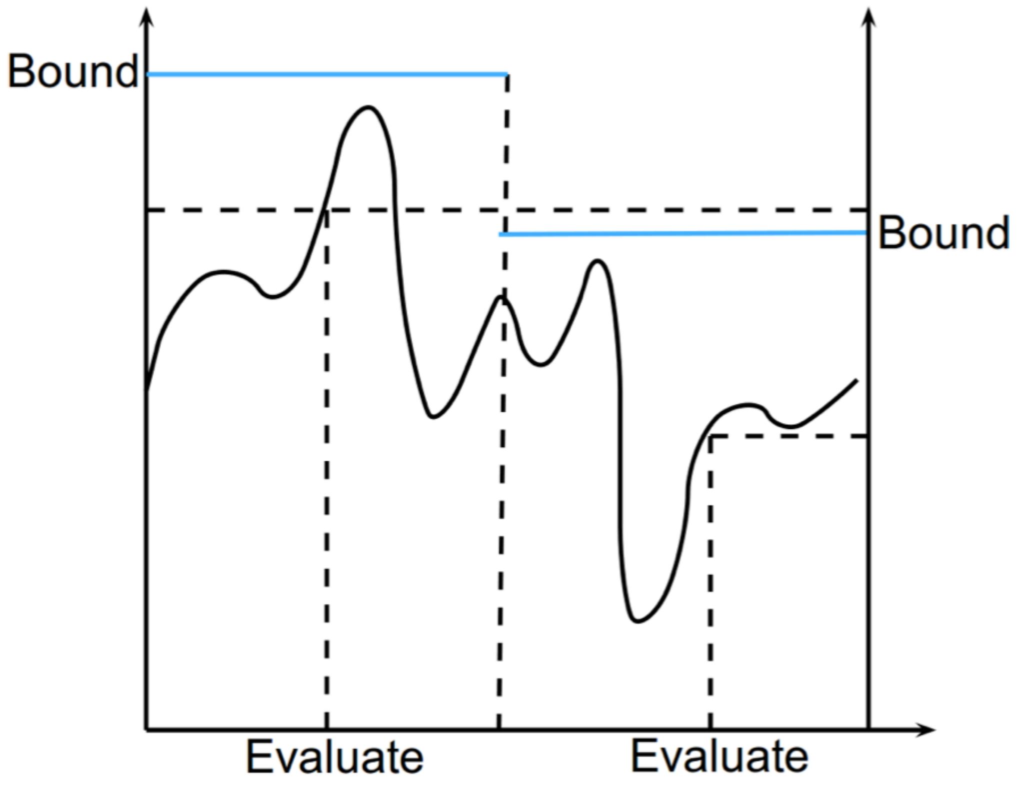
Branch and bound (BnB)

- Recursively test and prune the search space.
- Given a region in the search space, bound the objective value and compare with the best solution so far – discard whole region or subdivide and test further.
- Two key issues:
  - How to parametrise and subdivide the search domain;
  - How to bound the objective value.
- Note: does not have sub-exponential bound in general.



Applications of globally optimal methods

- 3DOF and 6DOF point cloud registration.
  - Correspondence-based
  - Correspondence-free
- Epipolar geometry estimation.
- Perspective-N-point (PnP) problem.
- LiDAR-camera registration.
- Conformal registration.
- Contrast maximisation for event-based motion estimation.
- Many others.

Not necessarily consensus maximisation.

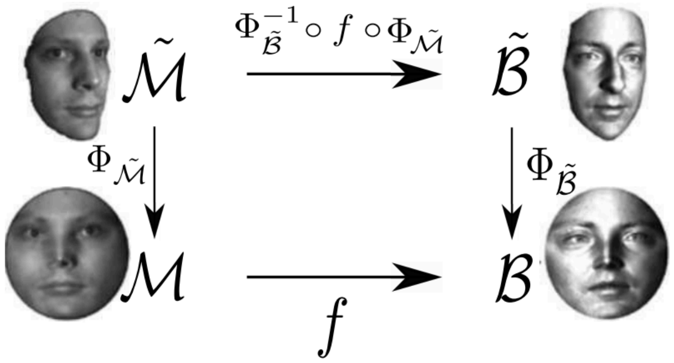
Tends to be more ”bespoke” for the particular problem (i.e., special cases).

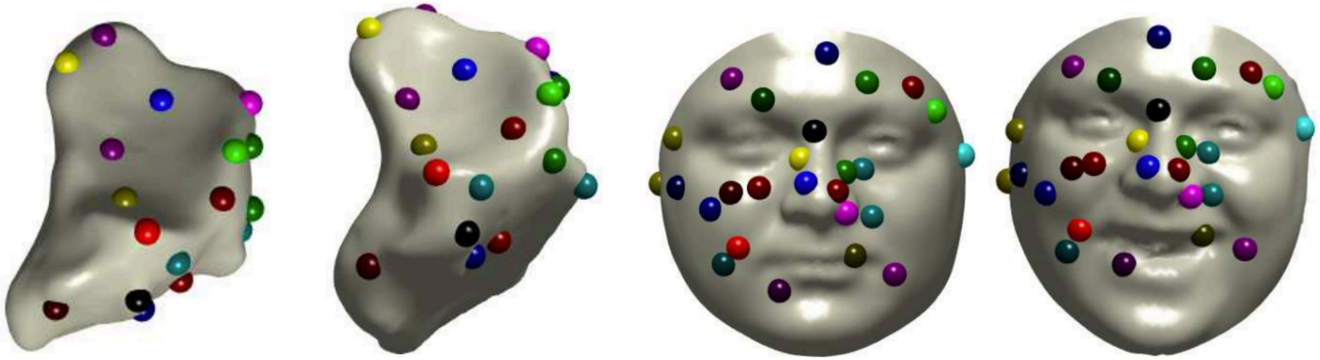
Typically slower than approximate methods. Speeding up requires creativity ---can’t just dump data on GPU!

Still a fruitful research direction (in my opinion).

Some "exotic" applications of global methods

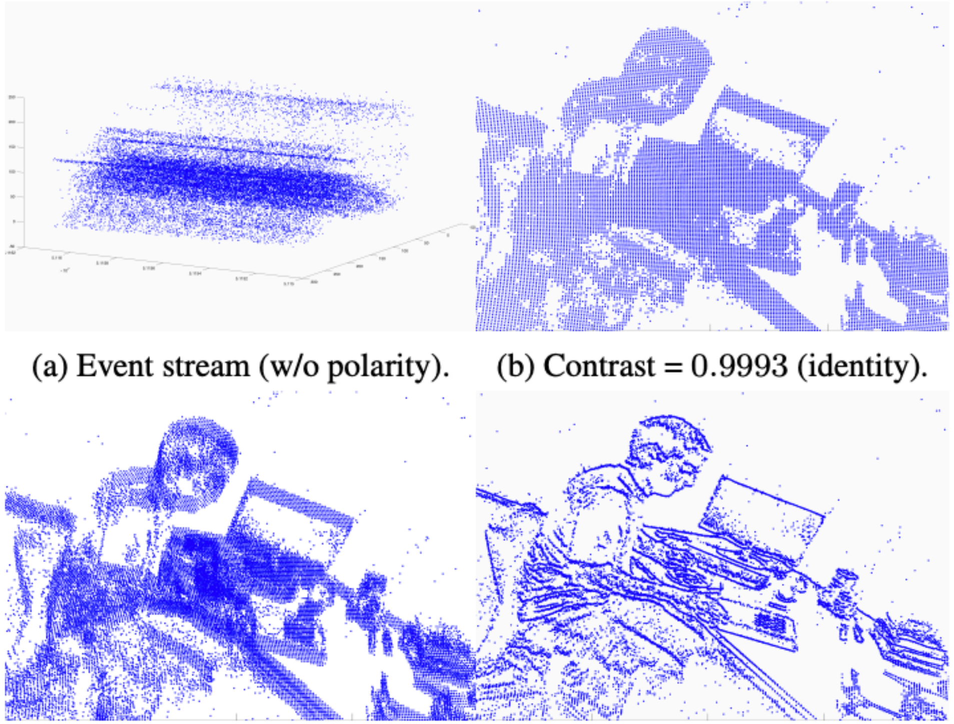
Conformal surface alignment





H. M. Le, T.-J. Chin and D. Suter. Conformal surface alignment with optimal Mobius search. CVPR 2016.

Contrast maximisation (event-based motion estimation)



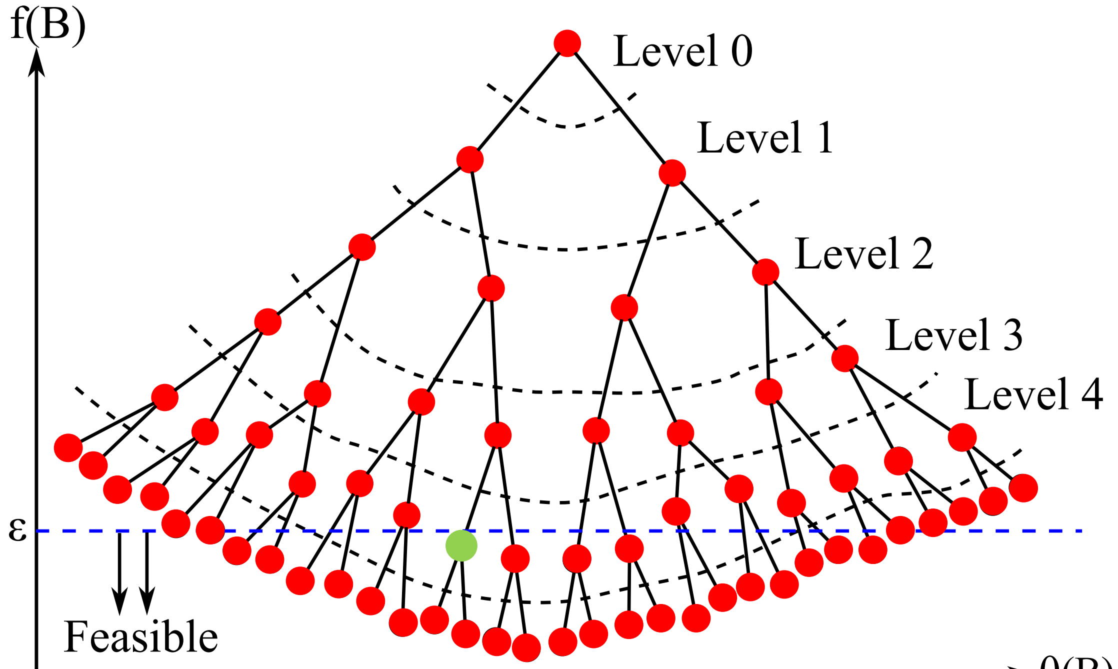
(a) Event stream (w/o polarity). (b) Contrast = 0.9993 (identity).

(c) Contrast = 1.0103 (local). (d) Contrast = 1.9748 (global).

D. Liu, A. Parra and T.-J. Chin. Globally Optimal Contrast Maximisation for Event-based Motion Estimation. CVPR 2020.

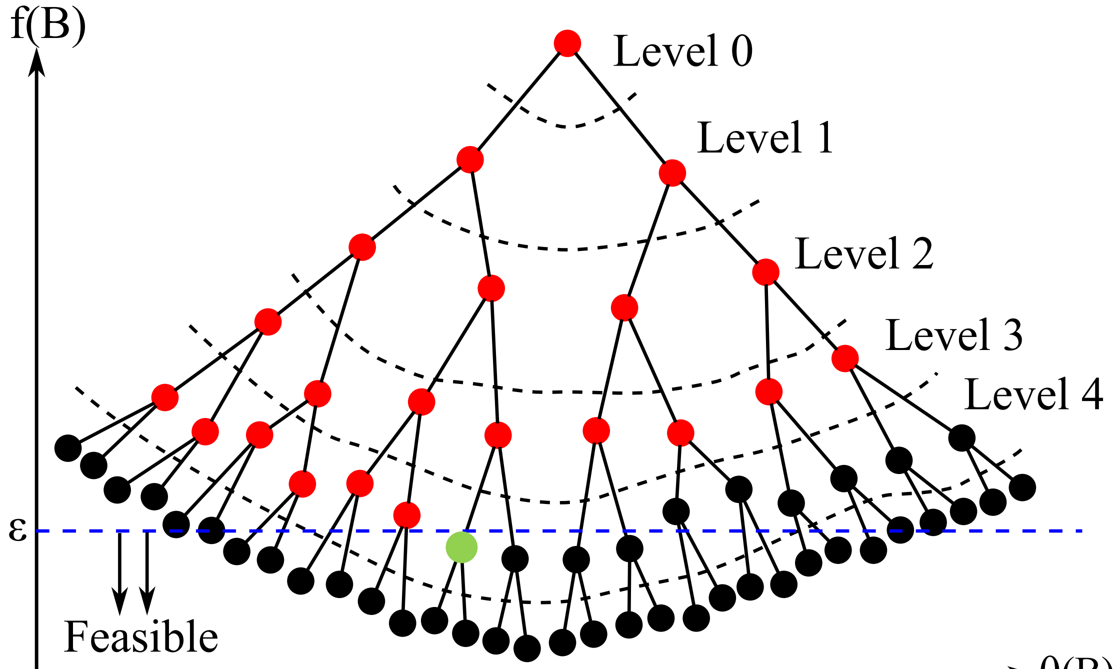
Subset (basis) search methods

- Enumeration



Olof Enqvist, Erik Ask, Fredrik Kahl, Kalle Åström: Tractable Algorithms for Robust Model Estimation. Int. J. Comput. Vis. 112(1): 115-129 (2015).

- Tree search



Tat-Jun Chin, Pulak Purkait, Anders P. Eriksson, David Suter: Efficient globally optimal consensus maximisation with tree search. CVPR 2015: 2413-2421.

Positive result

- The tree search method has runtime

$$\mathcal{O}((o+1)^{d+1}poly(N,d))$$

Maxcon is FPT in the number of outliers and dimension.

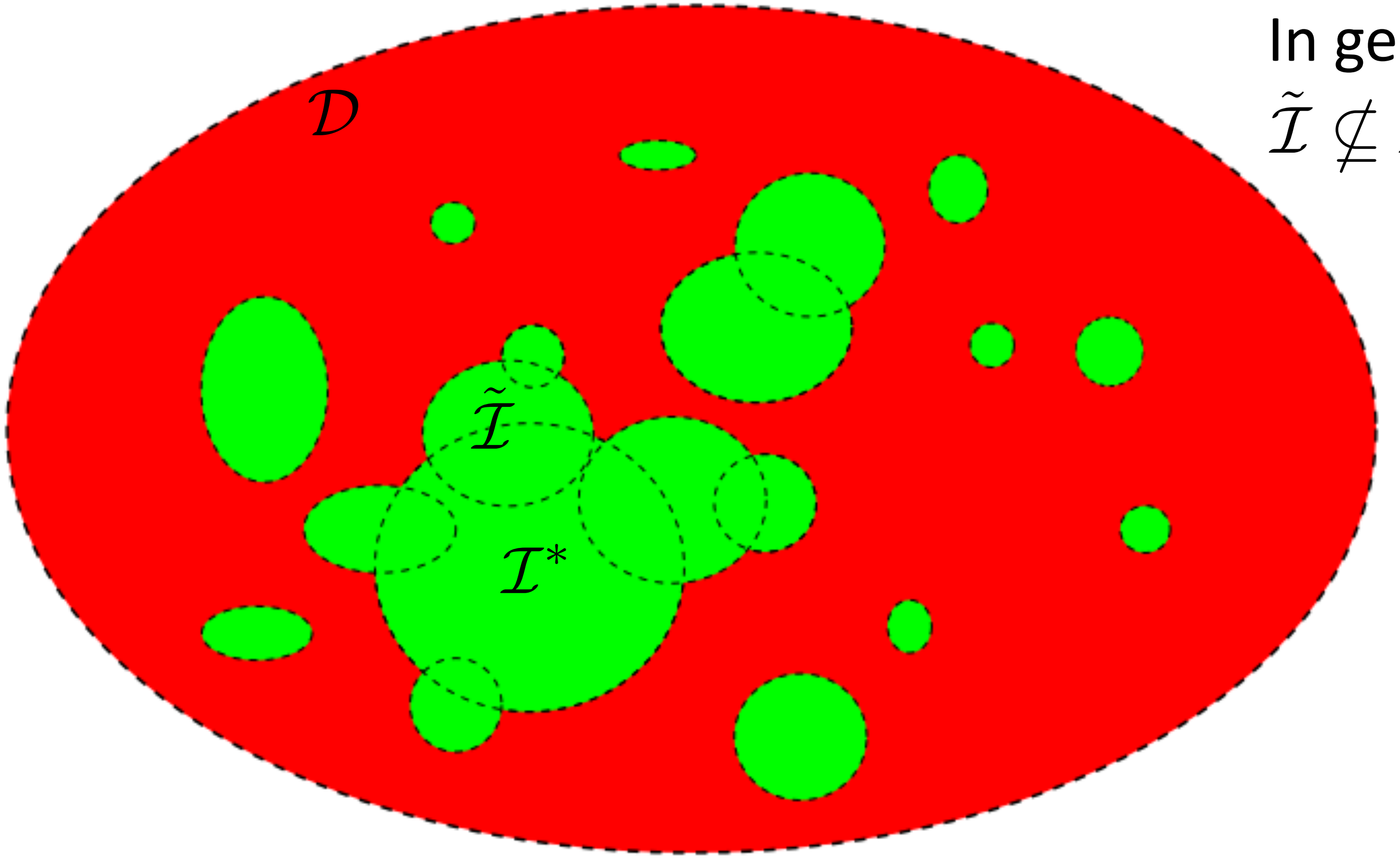
- Useful to exploit problem structure/additional knowledge to speed up algorithm or derive sub-exponential bound in runtime.

Suboptimal algorithm

$|\tilde{\mathcal{I}}| \leq |\mathcal{I}^*|$

In general

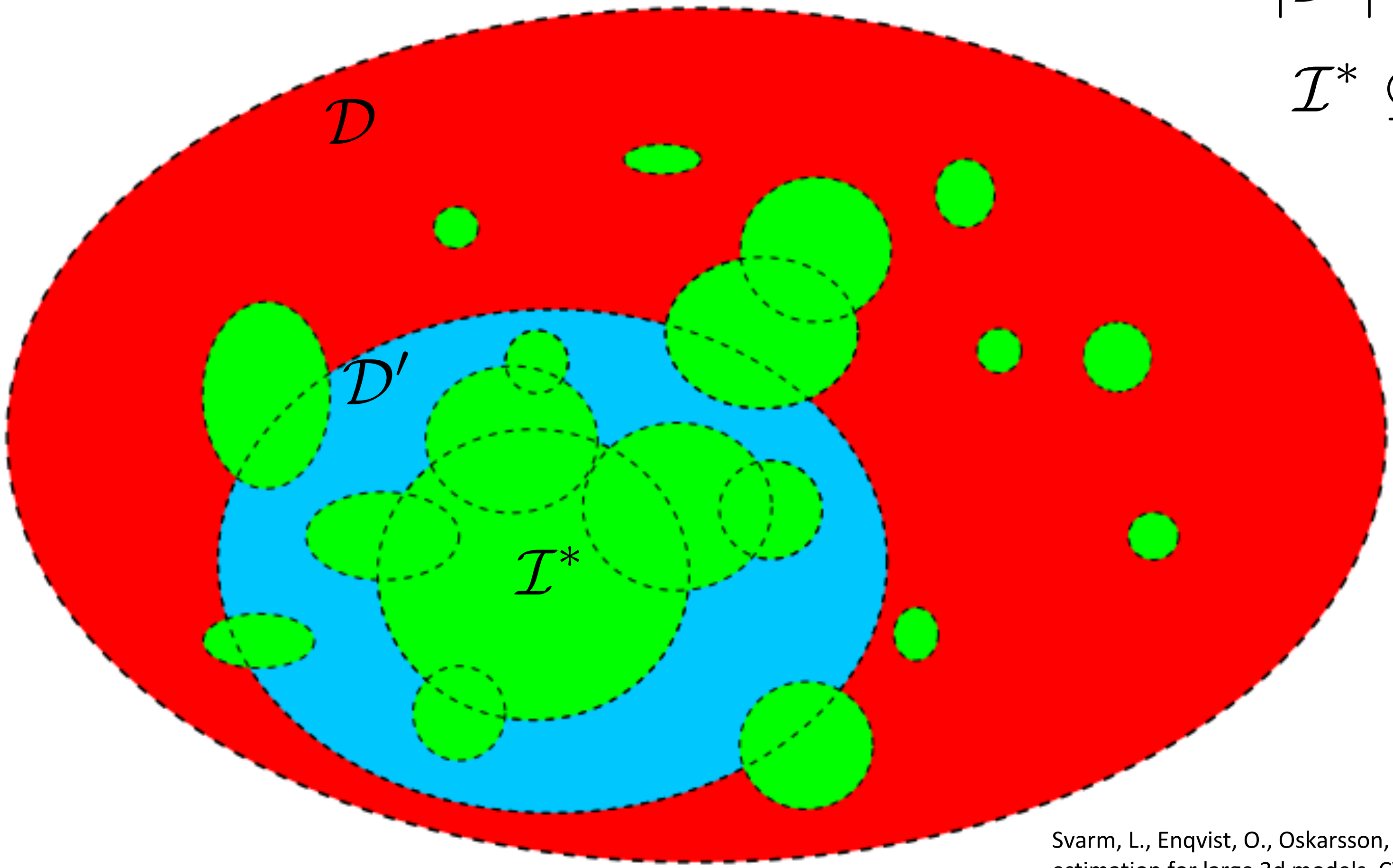
$\tilde{\mathcal{I}} \not\subseteq \mathcal{I}^*$



Preprocessing algorithm

$$|\mathcal{D}'| \leq \mathcal{D}$$

$$\mathcal{I}^* \subseteq \mathcal{D}'$$

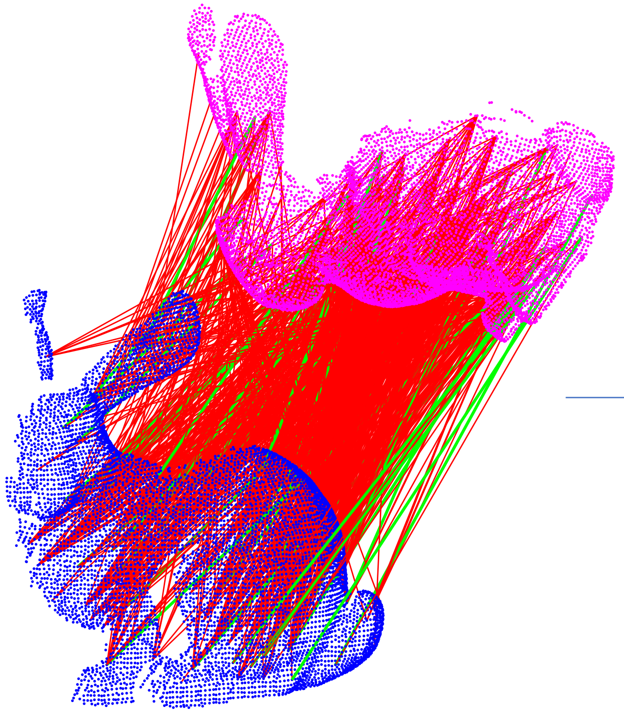


Svarm, L., Enqvist, O., Oskarsson, M., Kahl, F.: Accurate localization and pose estimation for large 3d models. CVPR 2014.

Álvaro Parra Bustos, Tat-Jun Chin: Guaranteed Outlier Removal for Point Cloud Registration with Correspondences. TPAMI 40(12): 2868-2882 (2018)

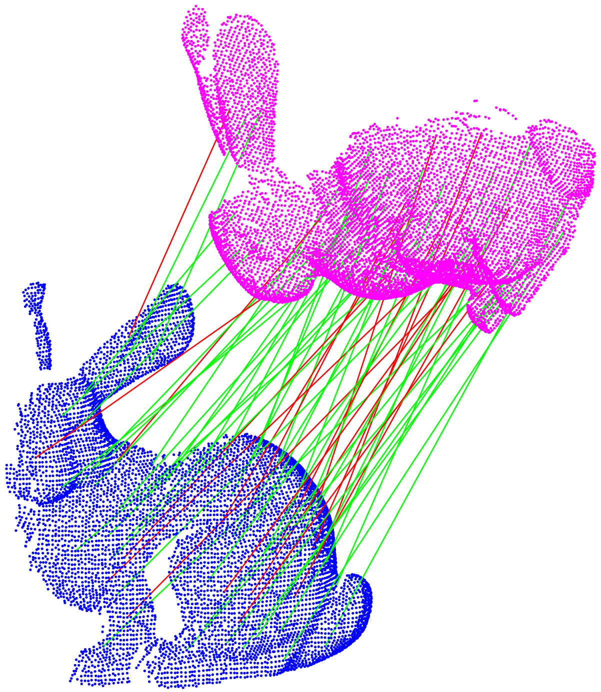
Tat-Jun Chin, Yang Heng Kee, Anders P. Eriksson, Frank Neumann: Guaranteed Outlier Removal with Mixed Integer Linear Programs. CVPR 2016: 5858-5866

Preprocessing method (GORE) for point cloud registration



2000 correspondences, 98% outliers

4.62 seconds



60 correspondences, 28% outliers

Álvaro Parra Bustos, Tat-Jun Chin: Guaranteed Outlier Removal for Point Cloud Registration with Correspondences. TPAMI 40(12): 2868-2882 (2018)

Certifiably optimal relaxations

- Relax the robust objective function and solve using convex solvers (e.g, SDP).
- Check for strong duality to certify optimality of the result.

TEASER: Fast and Certifiable Point Cloud Registration

Heng Yang, Jingnan Shi, Luca Carlone

Abstract—**We propose the first fast and certifiable algorithm for the registration of two sets of 3D points in the presence of large amounts of outlier correspondences. A *certifiable algorithm* is one that attempts to solve an intractable optimization problem (*e.g.*, robust estimation with outliers) and provides readily checkable conditions to verify if the returned solution is optimal (*e.g.*, if the algorithm produced the most accurate estimate in the face of outliers) or bound its sub-optimality or accuracy.**

Towards this goal, we first reformulate the registration problem using a *Truncated Least Squares* (TLS) cost that makes the estimation insensitive to a large fraction of spurious correspondences. Then, we provide a general graph-theoretic framework to decouple scale, rotation, and translation estimation, which allows solving in cascade for the three transformations. Despite the fact that each subproblem (scale, rotation, and translation estimation) is still non-convex and combinatorial in nature, we show that (i) TLS scale and (component-wise) translation estimation can be solved in polynomial time via an *adaptive voting* scheme, (ii) TLS rotation estimation can be relaxed to a semidefinite program (SDP) and the relaxation is tight, even in the presence of extreme outlier rates. We name the resulting algorithm TEASER (*Truncated least squares Estimation And SEmidefinite Relaxation*). While solving large SDP relaxations is typically slow, we develop a second certifiable algorithm, named TEASER++, that circumvents the need to solve an SDP and runs in *milliseconds*.

For both algorithms, we provide theoretical bounds on the estimation errors, which are the first of their kind for robust registration problems. Moreover, we test their performance on standard benchmarks, object detection datasets, and the 3DMatch scan matching dataset, and show that (i) both algorithms dominate the state of the art (*e.g.*, RANSAC, branch-&-bound, heuristics) and are robust to more than 99% outliers, (ii) TEASER++ can run in milliseconds and it is currently the fastest robust registration algorithm, (iii) TEASER++ is so robust it can also solve problems without correspondences (*e.g.*, hypothesizing all-to-all correspondences) where it largely outperforms ICP. We release a fast open-source C++ implementation of TEASER++.

Index Terms—**3D registration, scan matching, point cloud alignment, robust estimation, certifiable algorithms, outliers-robust estimation, object pose estimation, 3D robot vision.**

Supplementary Material

- Video: `https://youtu.be/xib1RSUoeeQ`
- Code: `https://github.com/MIT-SPARK/TEASER-plusplus`

I. Introduction

Point cloud registration (also known as *scan matching* or *point cloud alignment*) is a fundamental problem in robotics and computer vision and consists in finding the best transformation (rotation, translation, and potentially scale) that aligns two point clouds. It finds applications in motion estimation and 3D reconstruction [3], [4], [5], [6], object recognition and localization [7], [8], [9], [10], panorama stitching [11], and medical imaging [12], [13], to name a few.

H. Yang, J. Shi, and L. Carlone are with the Laboratory for Information & Decision Systems (LIDS), Massachusetts Institute of Technology, Cambridge, MA 02139, USA, Email: {hankyang,jnshi,lcarlone}@mit.edu

This work was partially funded by ARL DCIST CRA W911NF-17-2-0181, ONR RAIDER N00014-18-1-2828, Lincoln Laboratory "Resilient Perception in Degraded Environments", and the Google Daydream Research Program.

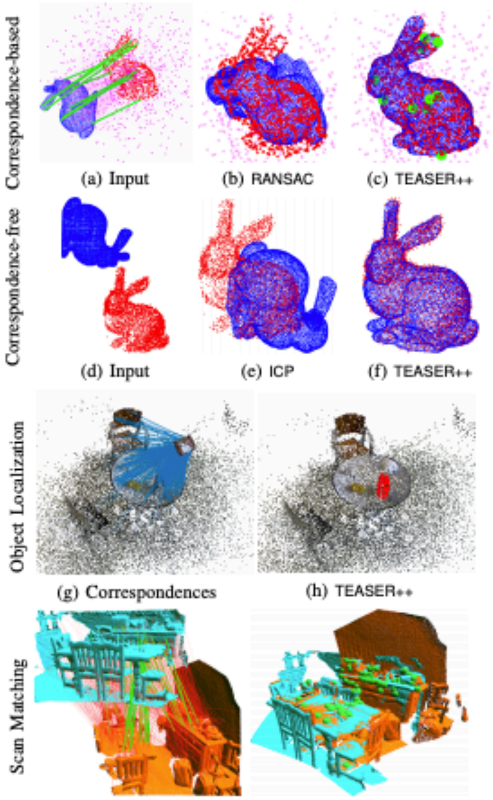
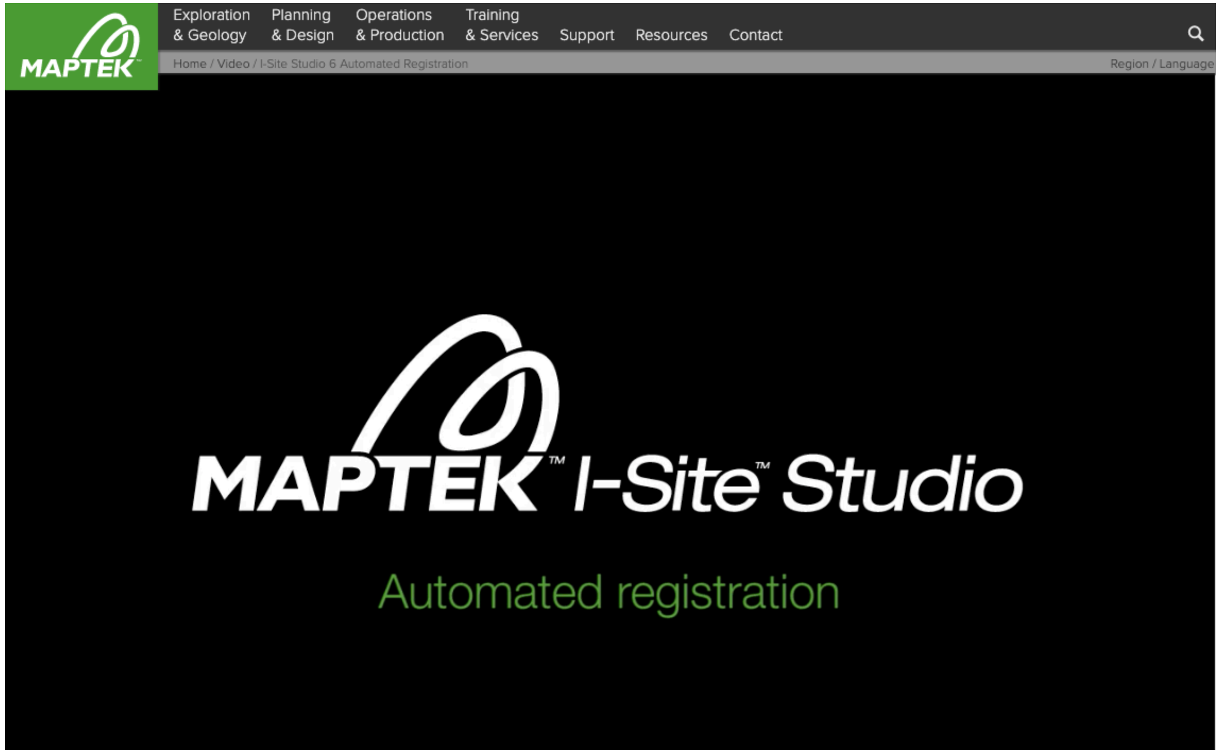
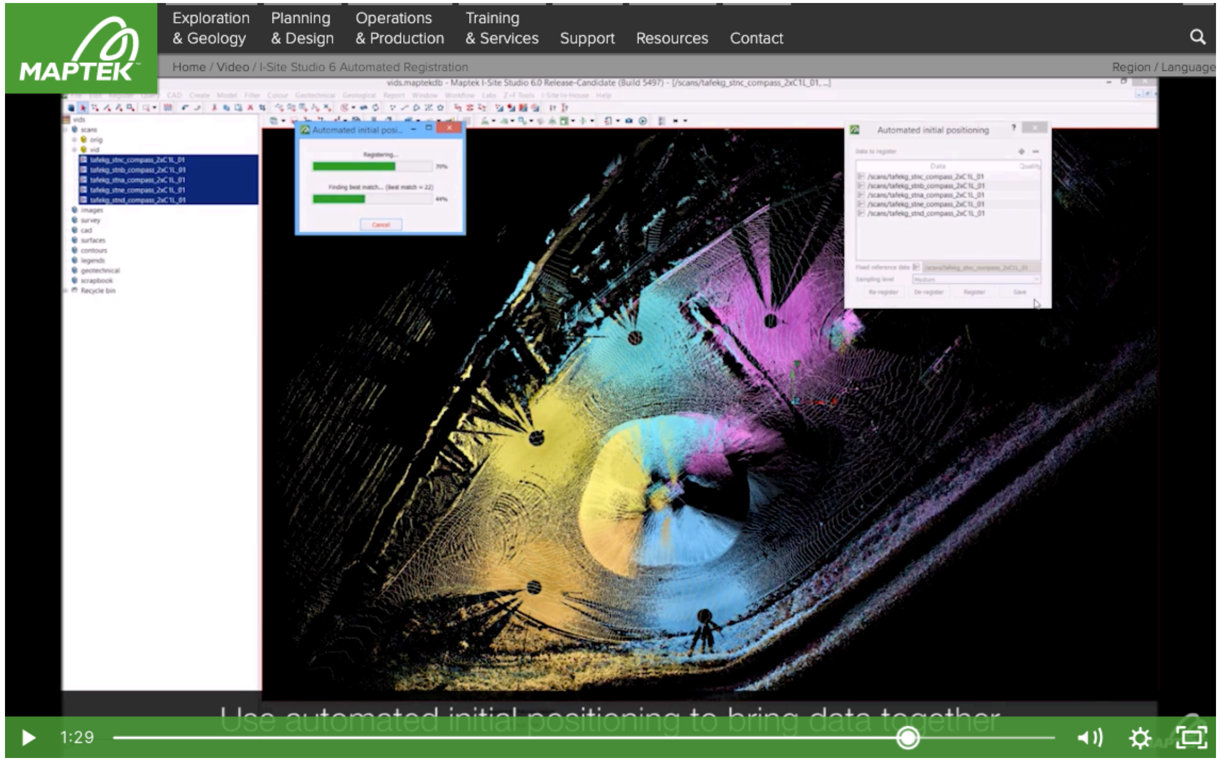


Fig. 1. We address 3D registration in the realistic case where many point-to-point correspondences are outliers due to incorrect matching. (a) Bunny dataset (source cloud in blue and target cloud in red) with 95% outliers (shown as magenta points) and 5% inliers (shown as green lines). Existing algorithms, such as RANSAC (b), can produce incorrect estimates without notice even after running for 10,000 iterations. Our certifiable algorithm, TEASER, largely outperforms the state of the art in terms of robustness and accuracy, and a fast implementation, named TEASER++ (c), computes accurate estimates in milliseconds even with extreme outlier rates and finds the small set of inliers (shown as green dots). The unprecedented robustness of TEASER++ enables the solution of correspondence-free registration problems (d), where ICP (e) fails without a good initial guess, while TEASER++ (f) succeeds without requiring an initial guess. We test our approach in challenging object localization (g-h) and scan matching (i-j) RGB-D datasets, using both traditional features (*i.e.*, FPFH [1]) and deep-learned features (*i.e.*, 3DSmoothNet [2]).

Commercial product based on globally optimal robust fitting





https://www.maptek.com/video/i-site-studio-6-automated-registration/

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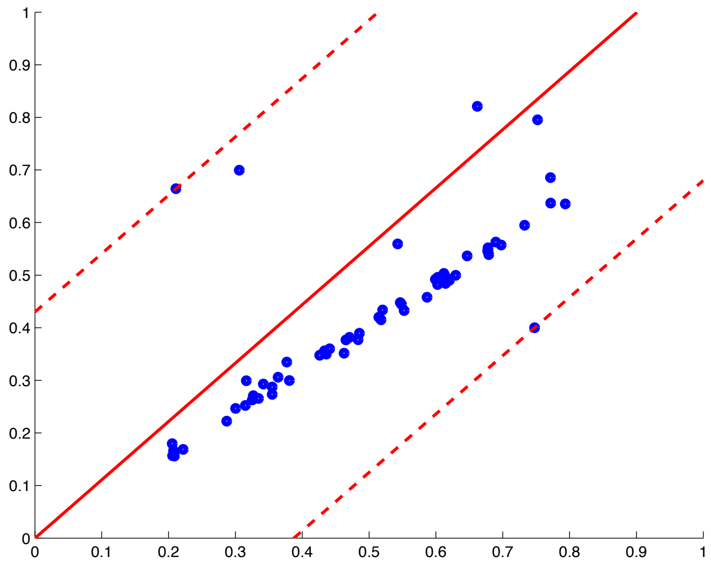
Chebyshev problem

- Given data $\mathcal{D} = \{(\mathbf{a}_i, b_i)\}_{i=1}^N$, find

$$\hat{\mathbf{x}} = \underset{\mathbf{x}\in\mathbb{R}^d}{\operatorname{argmin}} \; \max_i \left|\mathbf{a}_i^T\mathbf{x} - b_i\right| \quad \text{Linear program}$$

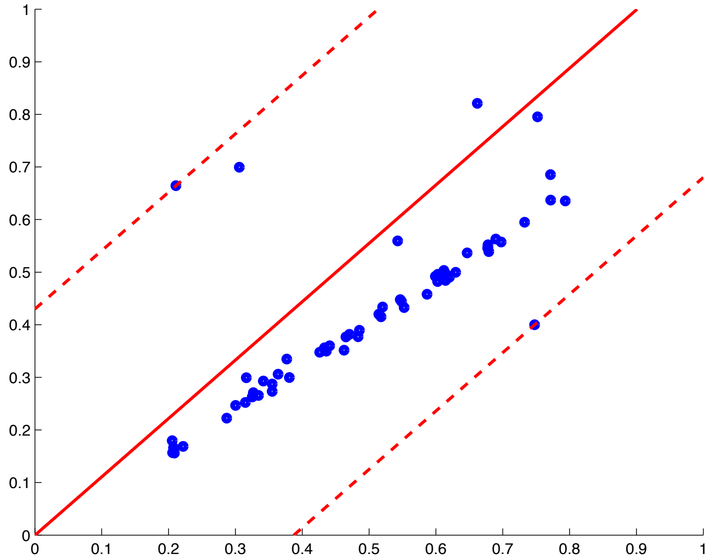


Find the thinnest slab and encloses all points.

Chebyshev problem

- Given data $\mathcal{D} = \{(\mathbf{a}_i, b_i)\}_{i=1}^N$, find

$$\hat{\mathbf{x}} = \underset{\mathbf{x}\in\mathbb{R}^d}{\operatorname{argmin}} \underbrace{\left\| \begin{bmatrix} |\mathbf{a}_1^T\mathbf{x} - b_1| \\ \vdots \\ |\mathbf{a}_N^T\mathbf{x} - b_N| \end{bmatrix} \right\|_\infty}_{\text{Linear program}}$$



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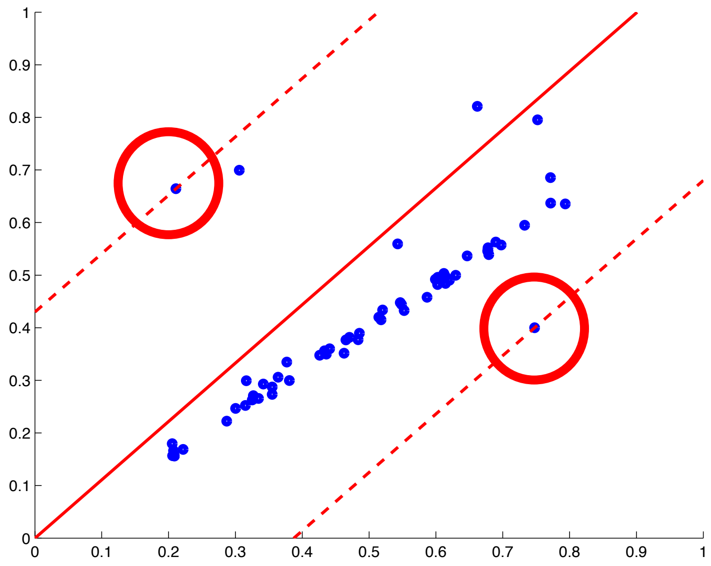
$$\hat{\mathbf{x}} = \underset{\mathbf{x}\in\mathbb{R}^d}{\operatorname{argmin}} \left\| \begin{bmatrix} |\mathbf{a}_1^T\mathbf{x} - b_1| \\ \vdots \\ |\mathbf{a}_N^T\mathbf{x} - b_N| \end{bmatrix} \right\|_\infty$$

Linear program



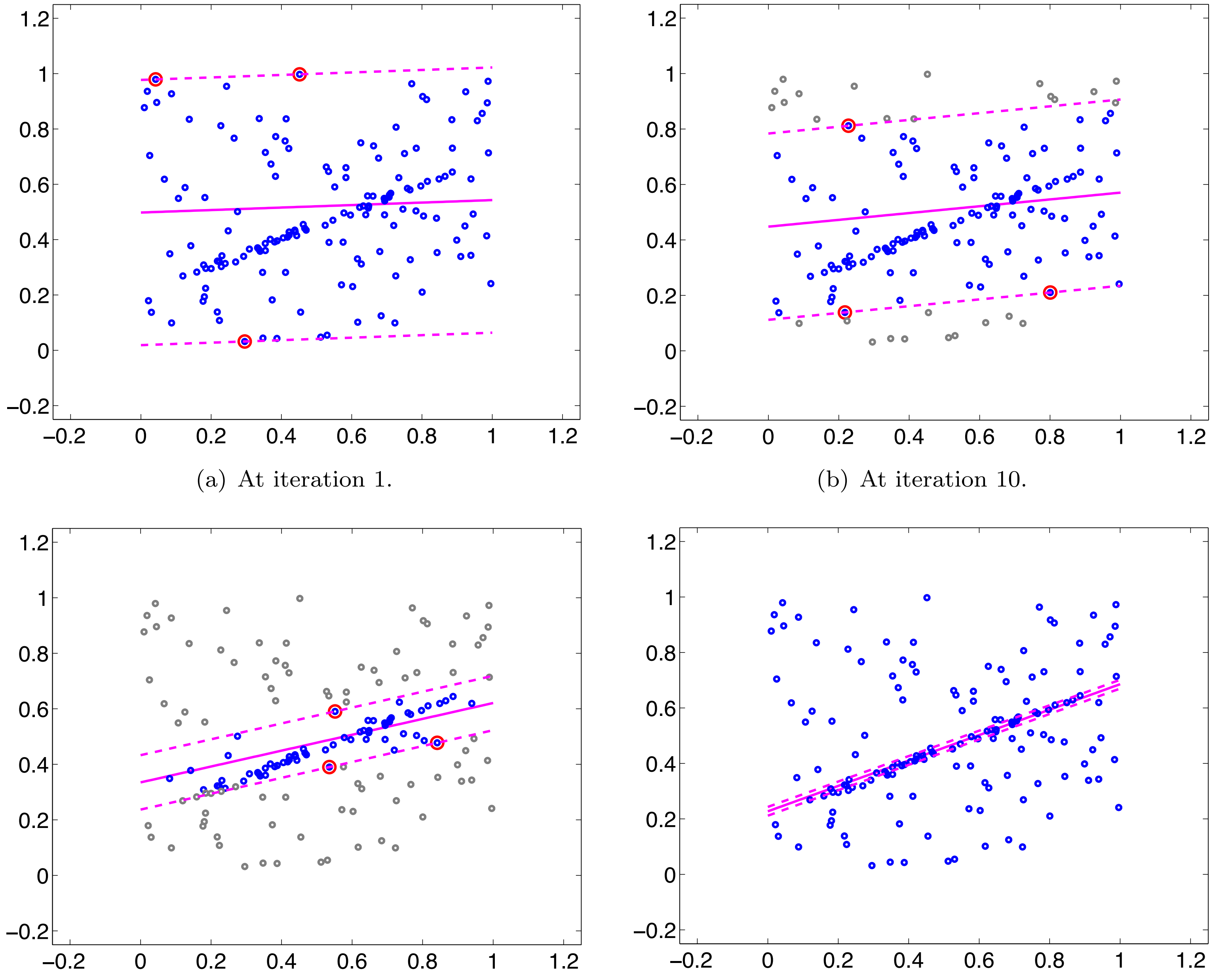
Active points tend to be outliers.

So

- Solve Chebyshev problem;
- Remove the active points;
- Repeat until the error is below ϵ.

Kristy Sim, Richard I. Hartley: Removing Outliers Using The L\infty Norm. CVPR (1) 2006: 485-494

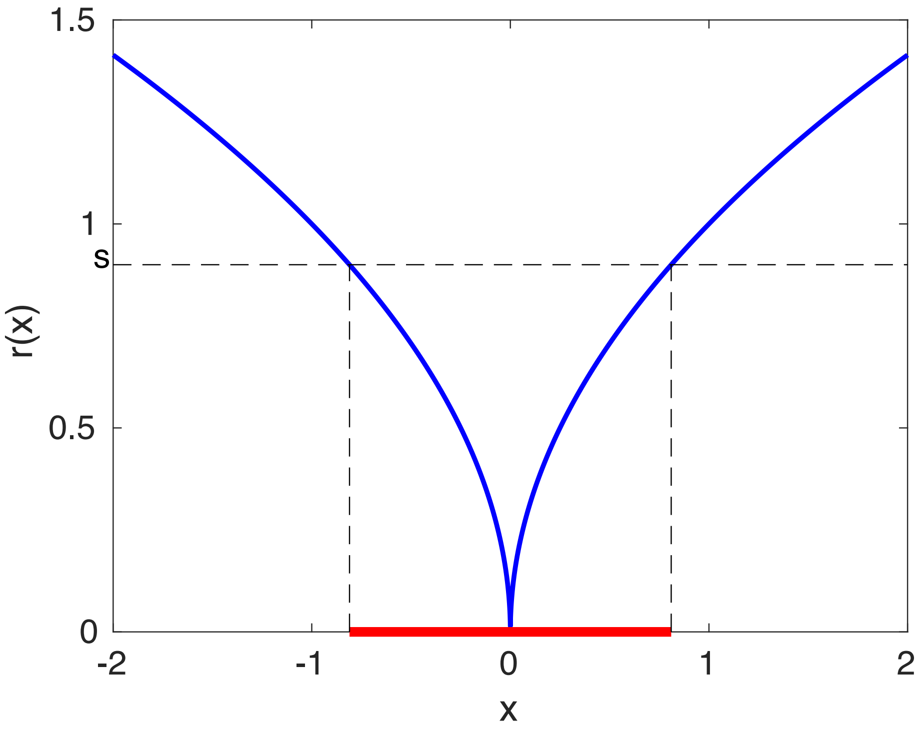
Recursive outlier removal – line fitting



(a) At iteration 1. (b) At iteration 10. (c) At iteration 30. (d) Final result.

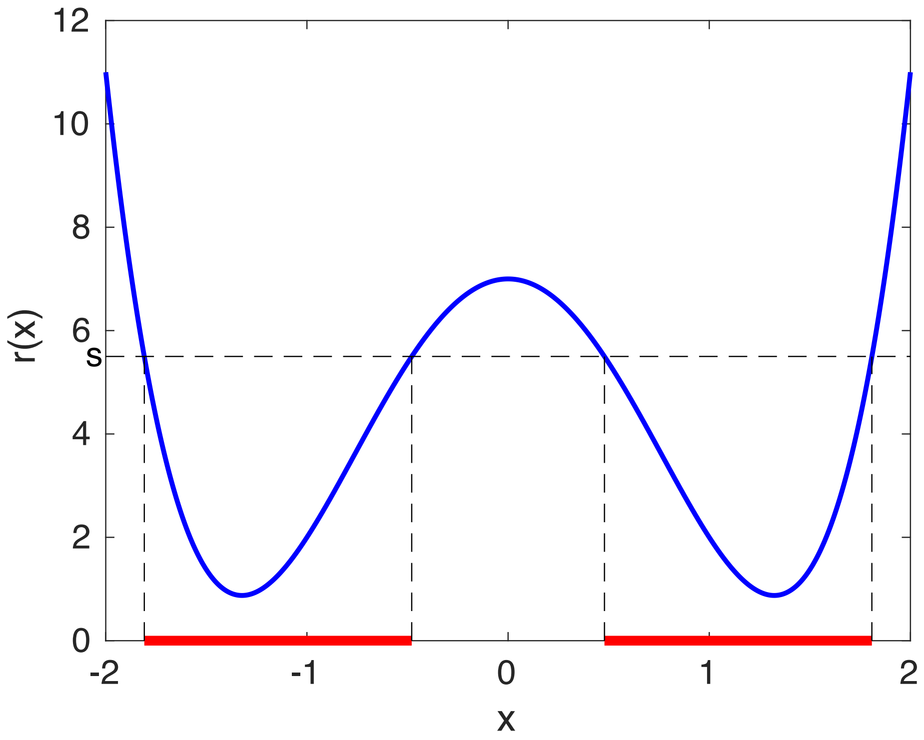
T.-J. Chin and D. Suter. The maximum consensus problem: recent algorithmic advances. Morgan & Claypool Publishers, San Rafael, CA, U.S.A., Feb 2017.

Quasiconvex function



(a)

Quasiconvex



(b)

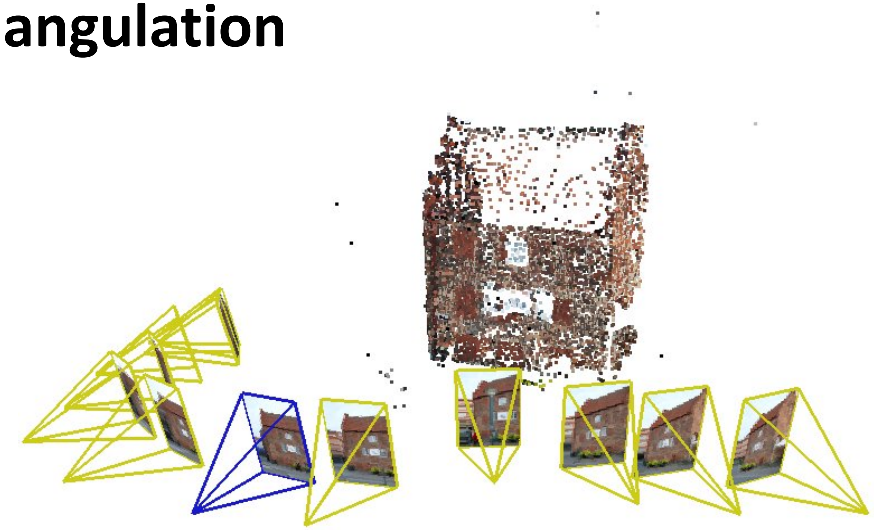
Not quasiconvex

https://www.youtube.com/watch?v=XANb1KUs5NU

T.-J. Chin and D. Suter. The maximum consensus problem: recent algorithmic advances. Morgan & Claypool Publishers, San Rafael, CA, U.S.A., Feb 2017.

Problems with quasiconvex residuals

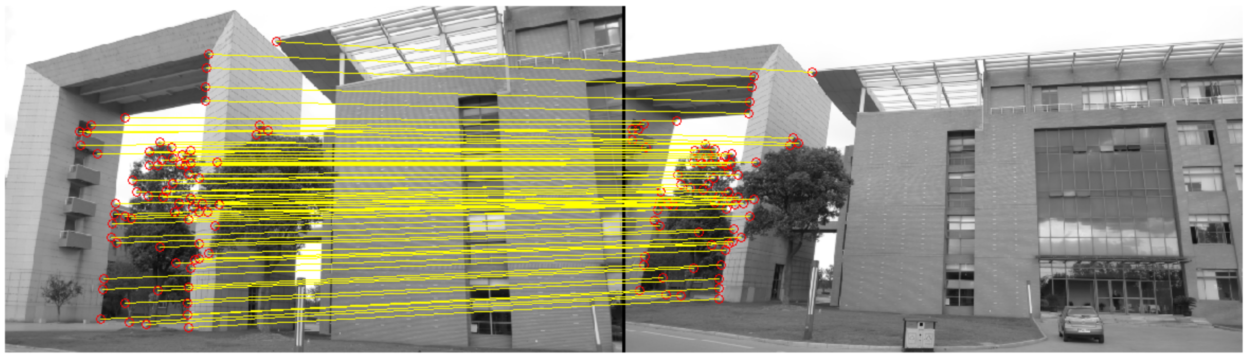
Triangulation



Reprojection error:

$$r_i(\mathbf{x}) = \left\| \mathbf{p}_i - \frac{\mathbf{P}_i^{1:2}\tilde{\mathbf{x}}}{\mathbf{P}_i^3\tilde{\mathbf{x}}} \right\|_2$$

Homography fitting



Transfer error:

$$r_i(\mathbf{x}) = \frac{\|(\mathbf{H}^{1:2} - \mathbf{v}_i\mathbf{H}_3)\tilde{\mathbf{u}}_i\|_2}{\mathbf{H}^3\tilde{\mathbf{u}}_i}$$

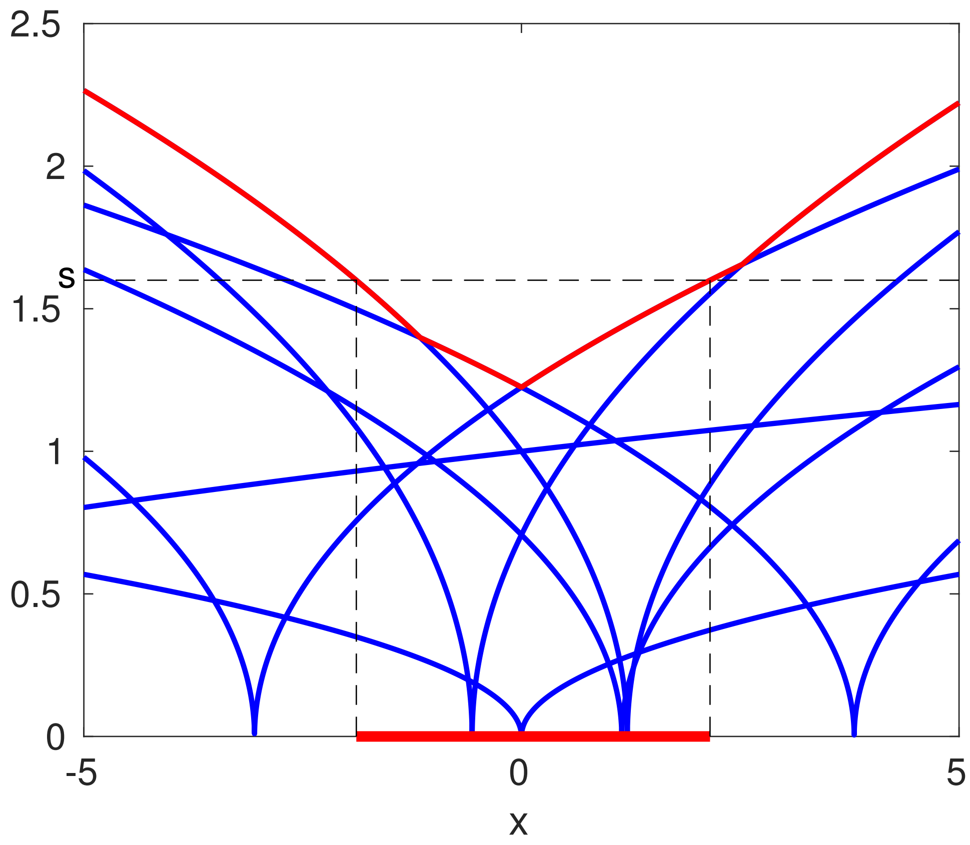
Chebyshev with quasiconvex residuals

- The maximum of a set of quasiconvex residuals is still quasiconvex.
- Hence, the corresponding Chebyshev problem

$$\min_{\mathbf{x} \in \mathbb{R}^d} \; \max_i \; r_i(\mathbf{x})$$

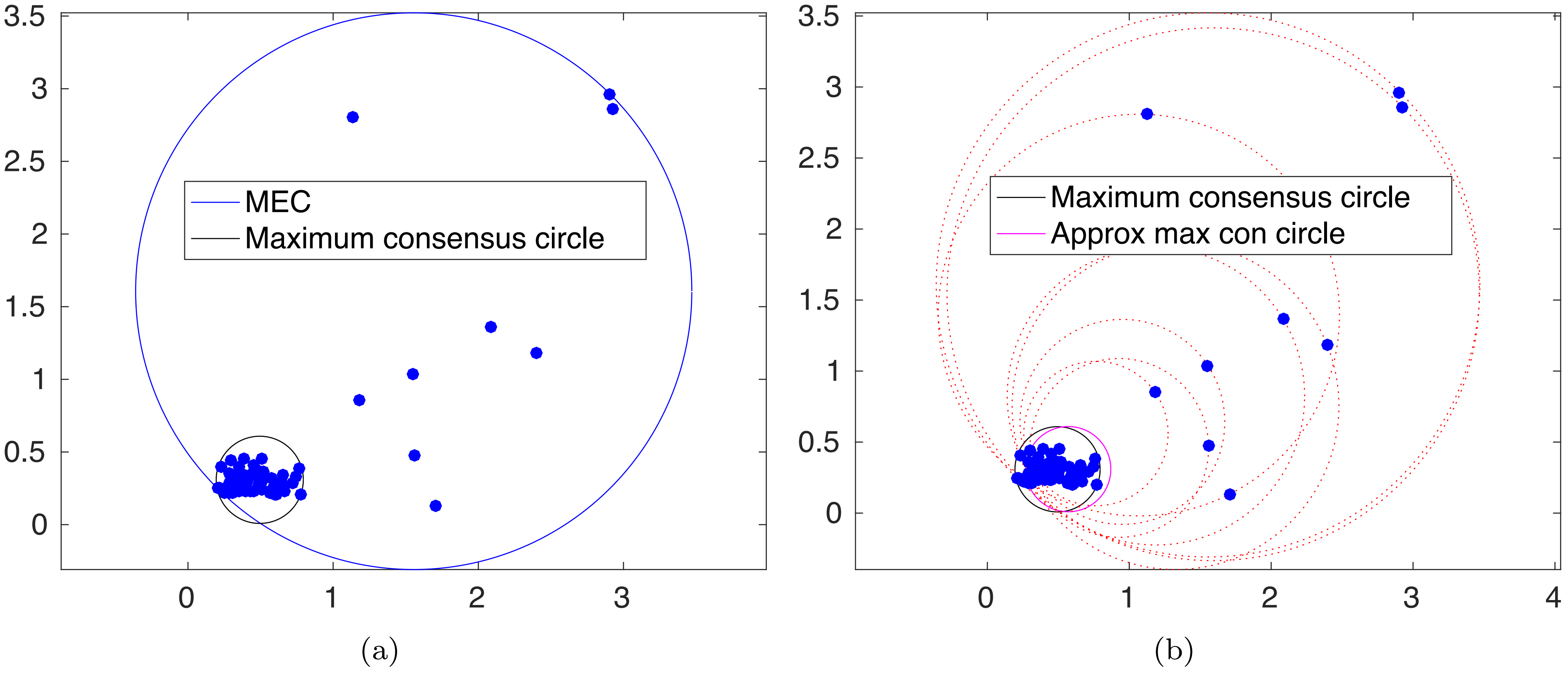
can be solved efficiently, e.g.,

  - Bisection with convex feasibility tests.
  - Sequential quadratic programming.
  - Generalised simplex method.



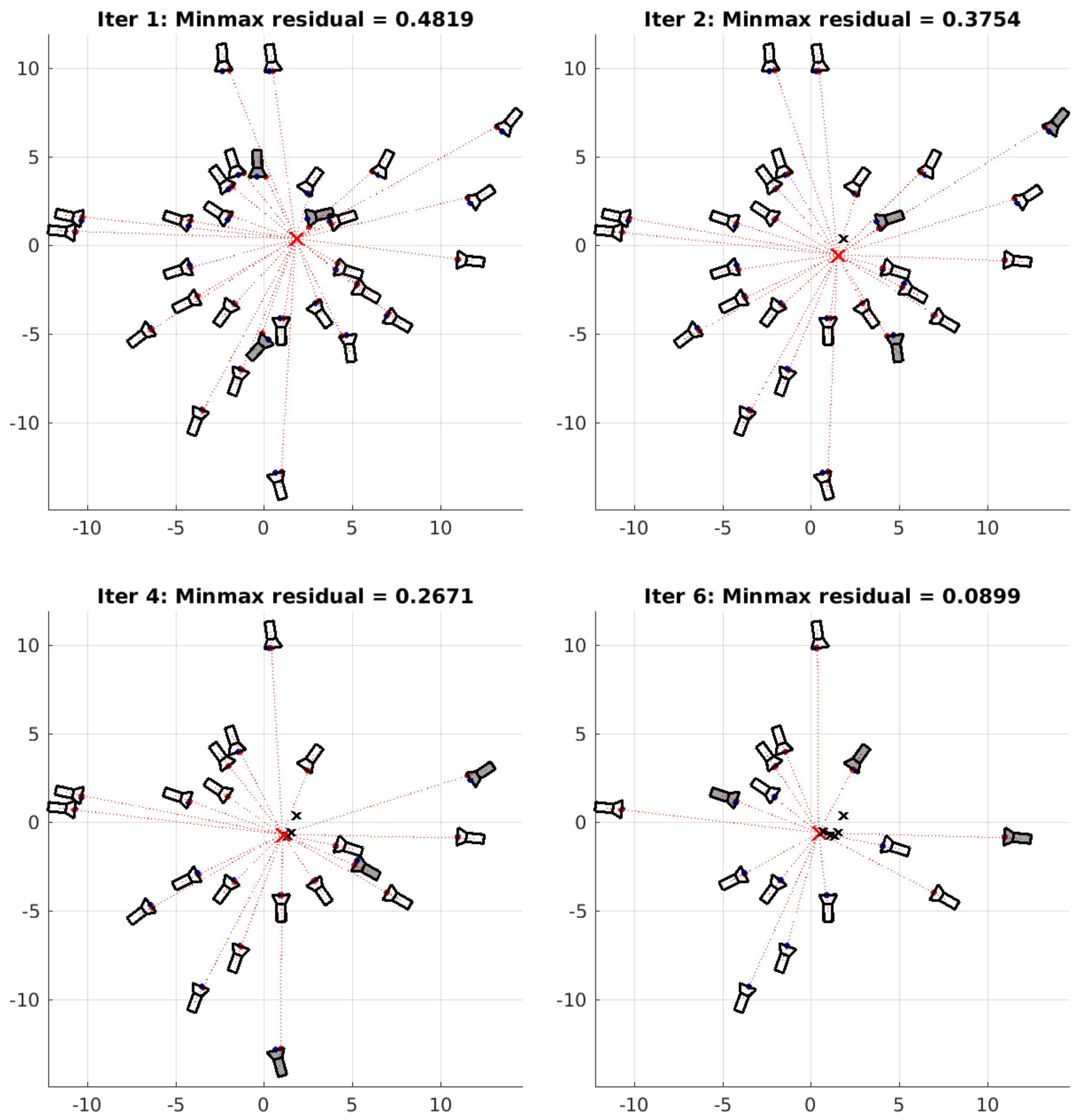
T.-J. Chin and D. Suter. The maximum consensus problem: recent algorithmic advances. Morgan & Claypool Publishers, San Rafael, CA, U.S.A., Feb 2017.

Recursive outlier removal – circle fitting



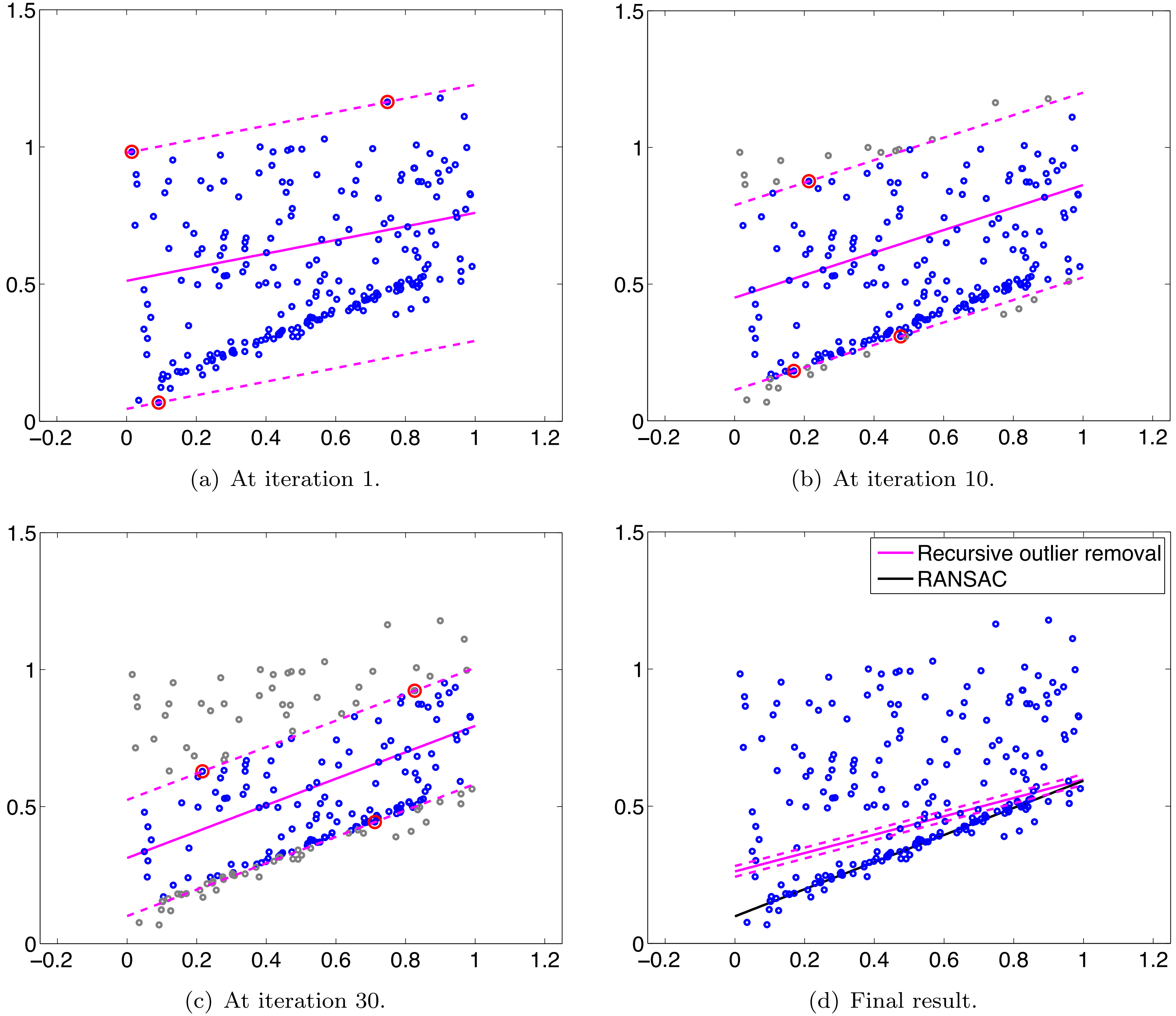
T.-J. Chin and D. Suter. The maximum consensus problem: recent algorithmic advances. Morgan & Claypool Publishers, San Rafael, CA, U.S.A., Feb 2017.

Recursive outlier removal - triangulation



T.-J. Chin and D. Suter. The maximum consensus problem: recent algorithmic advances. Morgan & Claypool Publishers, San Rafael, CA, U.S.A., Feb 2017.

Weakness of recursive outlier removal – unbalanced data



(a) At iteration 1.

(b) At iteration 10.

(c) At iteration 30.

(d) Final result.

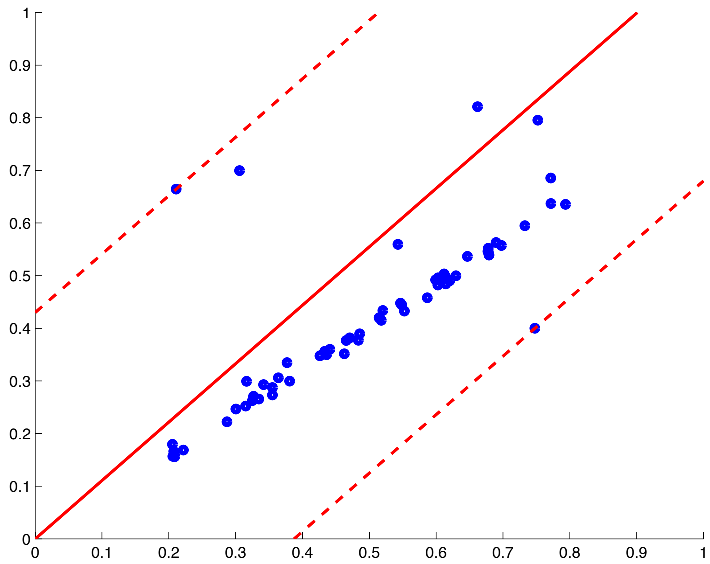
Outline

- What is and isn't fundamentally achievable
- Global algorithms
- Deterministic outlier removal
  - L-infinity method
  - L1 method
  - K-slack method
- Deterministic refinement
- Evaluation
- RANSAC in 2040---Quantum RANSAC?

Chebyshev problem

- Given data $\mathcal{D} = \{(\mathbf{a}_i, b_i)\}_{i=1}^{N}$, find

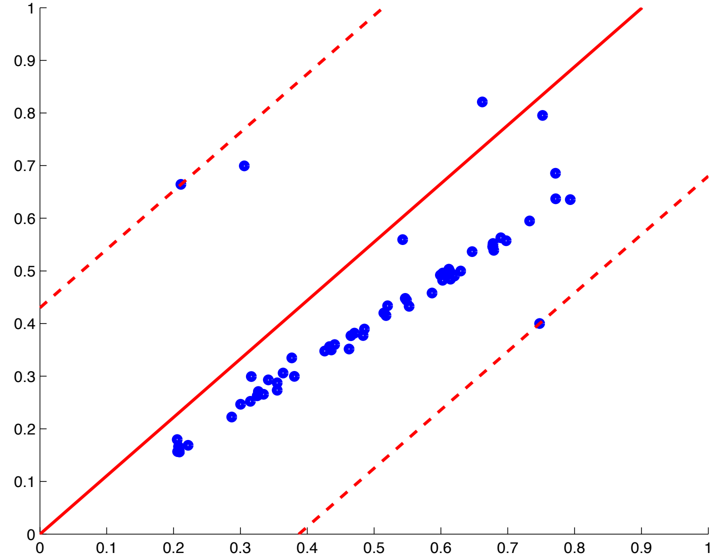
$$\left.\hat{\mathbf{x}} = \underset{\mathbf{x}\in\mathbb{R}^d}{\operatorname{argmin}} \max_i \left|\mathbf{a}_i^T\mathbf{x} - b_i\right| \right\} \text{Linear program}$$



Chebyshev problem

- Given data $\mathcal{D} = \{(\mathbf{a}_i, b_i)\}_{i=1}^{N}$, find

$$\left.\begin{aligned} \hat{\mathbf{x}} = \operatorname*{argmin}_{\mathbf{x}\in\mathbb{R}^d} \quad & s \\ \text{subject to} \quad & |\mathbf{a}_i^T\mathbf{x} - b_i| \leq s, \;\; \forall i \end{aligned}\right\} \text{Linear program}$$



Introduce multiple slacks and sum them up

- Given data $\mathcal{D} = \{(\mathbf{a}_i, b_i)\}_{i=1}^{N}$, find

$$\left.\begin{aligned} \hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathbb{R}^d} \quad & \sum_i s_i \\ \text{subject to} \quad & |\mathbf{a}_i^T\mathbf{x} - b_i| \leq s_i, \ \forall i \end{aligned}\right\} \text{Linear program}$$

Introduce multiple slacks and sum them up

- Given data $\mathcal{D} = \{(\mathbf{a}_i, b_i)\}_{i=1}^N$, find

$$\left.\begin{aligned}\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathbb{R}^d} \quad & \left\| \begin{bmatrix} s_1 \\ \vdots \\ s_N \end{bmatrix} \right\|_1 \\ \text{subject to} \quad & |\mathbf{a}_i^T\mathbf{x} - b_i| \leq s_i, \; \forall i \end{aligned}\right\} \text{Linear program}$$

Add a threshold

- Given data $\mathcal{D} = \{(\mathbf{a}_i, b_i)\}_{i=1}^N$, find

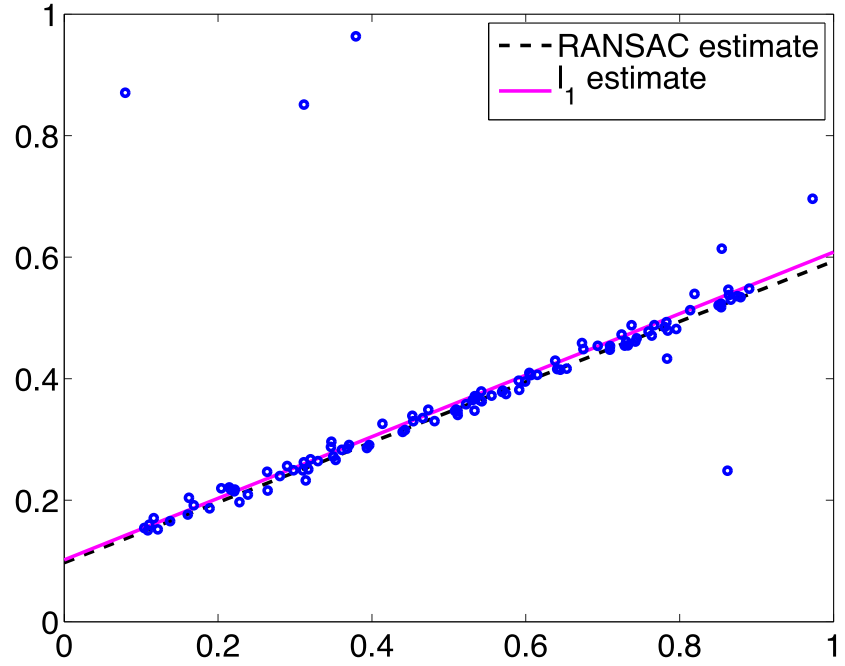
$$\left.\begin{aligned} \hat{\mathbf{x}} &= \arg\min_{\mathbf{x}\in\mathbb{R}^d} \sum_i s_i \\ &\text{subject to } |\mathbf{a}_i^T\mathbf{x} - b_i| \le \epsilon + s_i, \quad \forall i \end{aligned}\right\} \text{Linear program}$$

Solve the problem and remove the points with positive slacks.

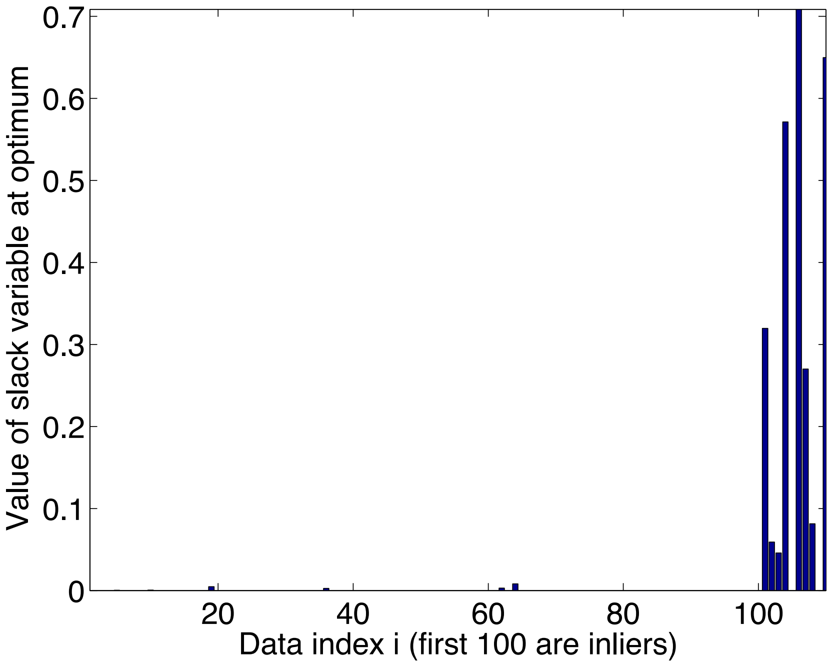
Carl Olsson, Anders P. Eriksson, Richard I. Hartley: Outlier removal using duality. CVPR 2010.

Y. Seo, H. Lee, and S. W. Lee. Outlier removal by convex optimization for l-infinity approaches. In PSIVT ’09: Pacific Rim Symposium on Advances in Image and Video Technology, 2009.

L1 method



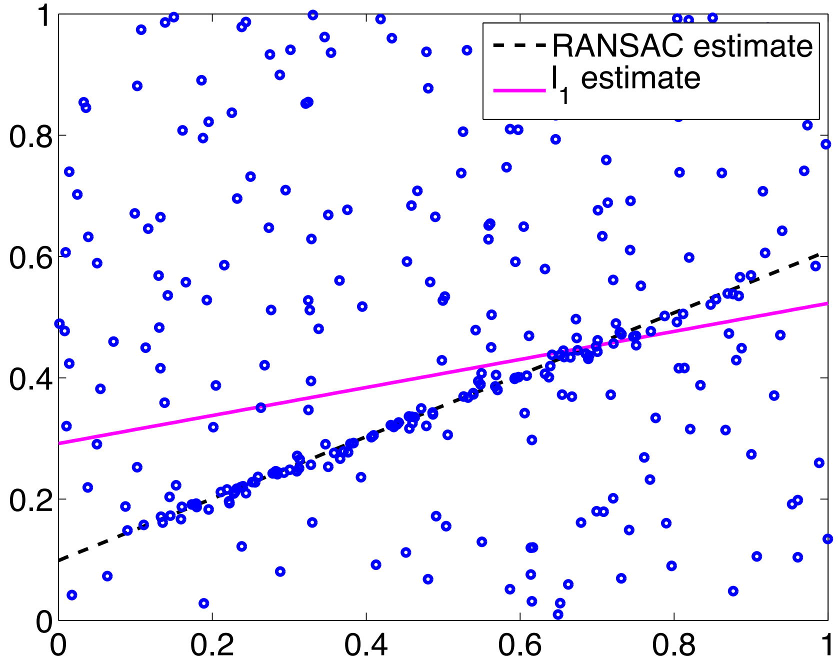
(a)



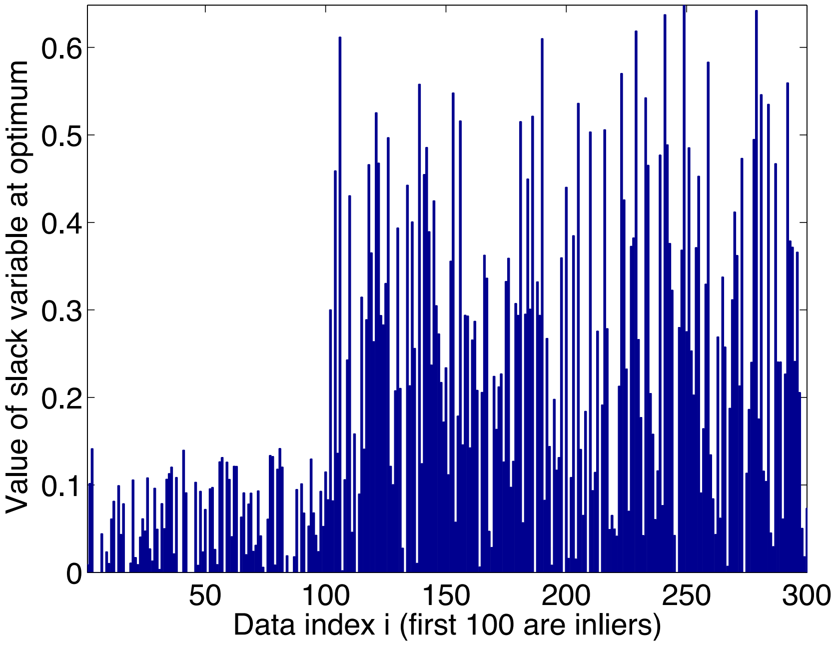
(b)

T.-J. Chin and D. Suter. The maximum consensus problem: recent algorithmic advances. Morgan & Claypool Publishers, San Rafael, CA, U.S.A., Feb 2017.

L1 method



(a)



(b)

T.-J. Chin and D. Suter. The maximum consensus problem: recent algorithmic advances. Morgan & Claypool Publishers, San Rafael, CA, U.S.A., Feb 2017.

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Avoid summing all the slacks

- Sum only the K-largest slacks:

$$\begin{aligned} \underset{\mathbf{x}\in\mathbb{R}^d, \mathbf{s}\in\mathbb{R}^N}{\text{minimise}} \quad & \max_{\boldsymbol{\pi}\in\{0,1\}^N} \boldsymbol{\pi}^T\mathbf{s} \\ \text{subject to} \quad & \left|\mathbf{a}_i^T\mathbf{x} - b_i\right| \leq \epsilon + s_i, \\ & s_i \geq 0, \\ & i = 1, \ldots, N, \\ & \boldsymbol{\pi}^T\mathbf{1} = K. \end{aligned}$$

Has an equivalent linear programing formulation

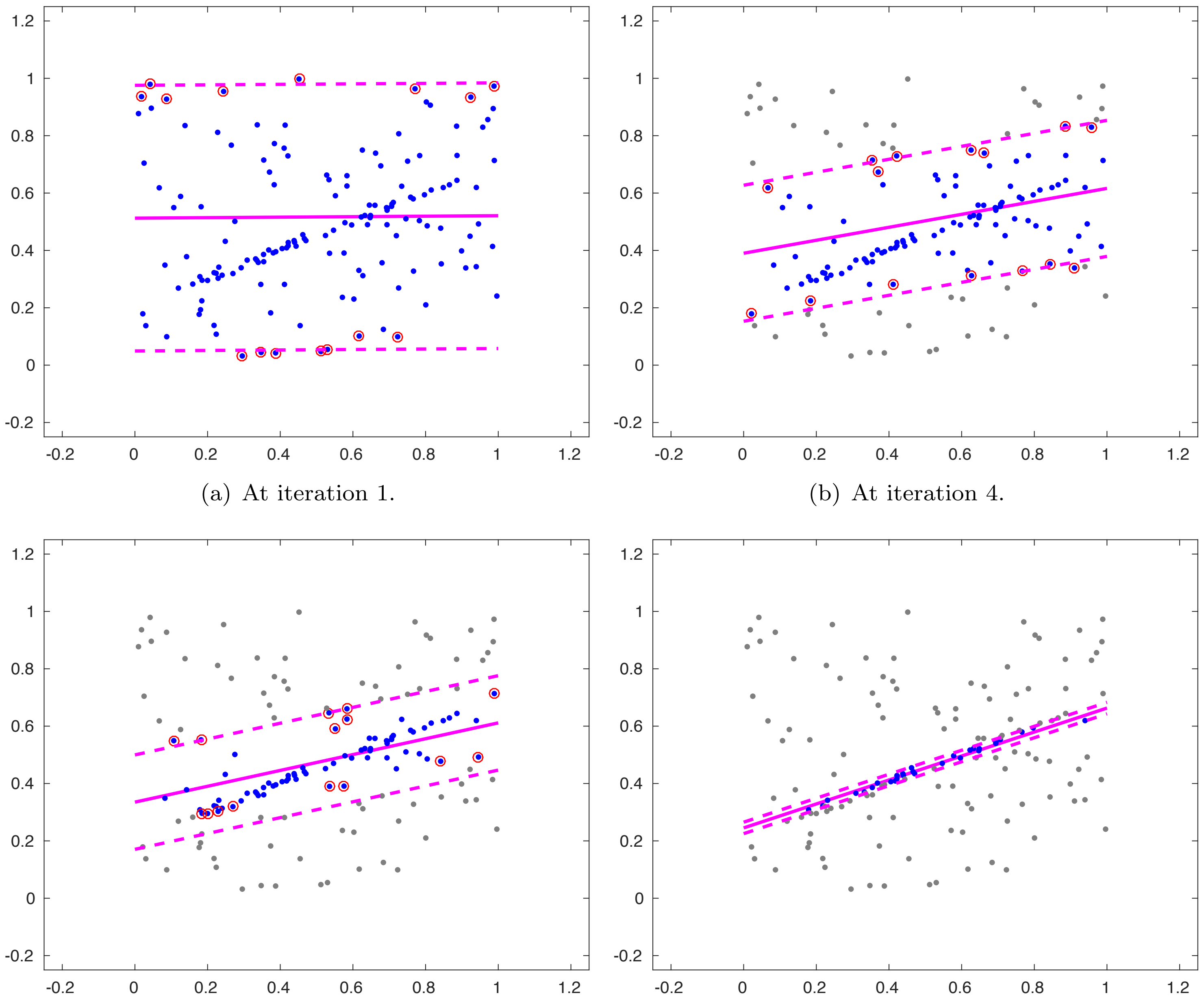
- K is a hyperparameter;
  - K = 1 ➔ Chebyshev problem
  - K = N ➔ L1 formulation

J. Yu, A. Eriksson, T.-J. Chin, and D. Suter. An Adversarial Optimization Approach to Efficient Outlier Removal. ICCV 2011.

K-slack method

- Solve the ”sum of K-largest slack” formulation.
- Remove the points with the K-largest slacks.
- Repeat until all slacks are zero.

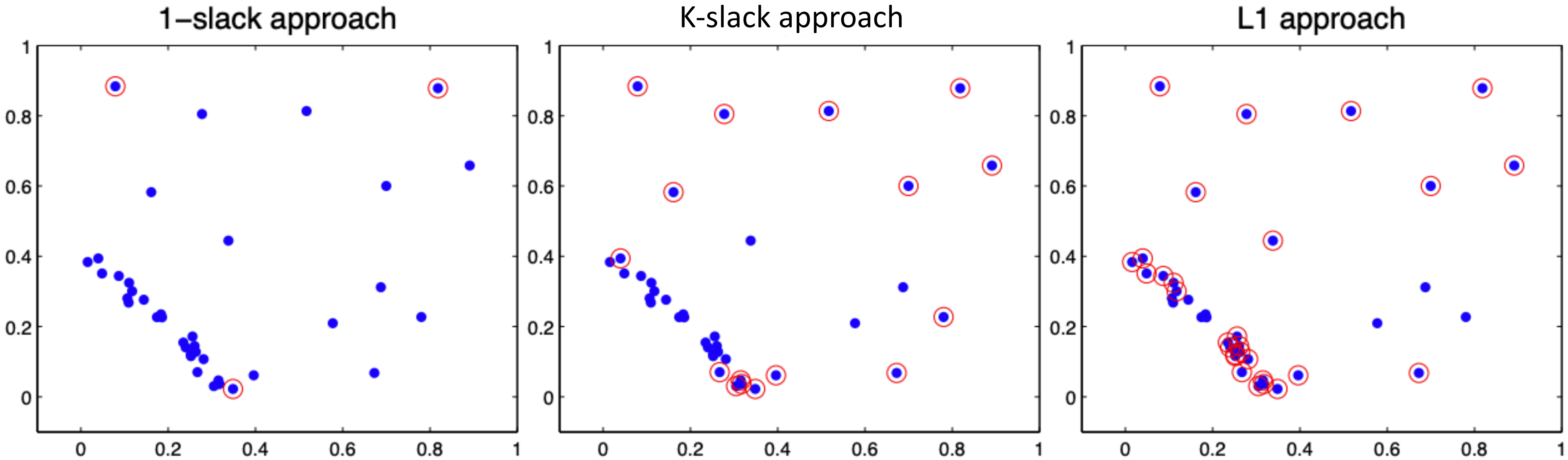
J. Yu, A. Eriksson, T.-J. Chin, and D. Suter. An Adversarial Optimization Approach to Efficient Outlier Removal. ICCV 2011.



(a) At iteration 1. (b) At iteration 4. (c) At iteration 6. (d) Final result (after 8 iterations).

T.-J. Chin and D. Suter. The maximum consensus problem: recent algorithmic advances. Morgan & Claypool Publishers, San Rafael, CA, U.S.A., Feb 2017.

Effects of varying K



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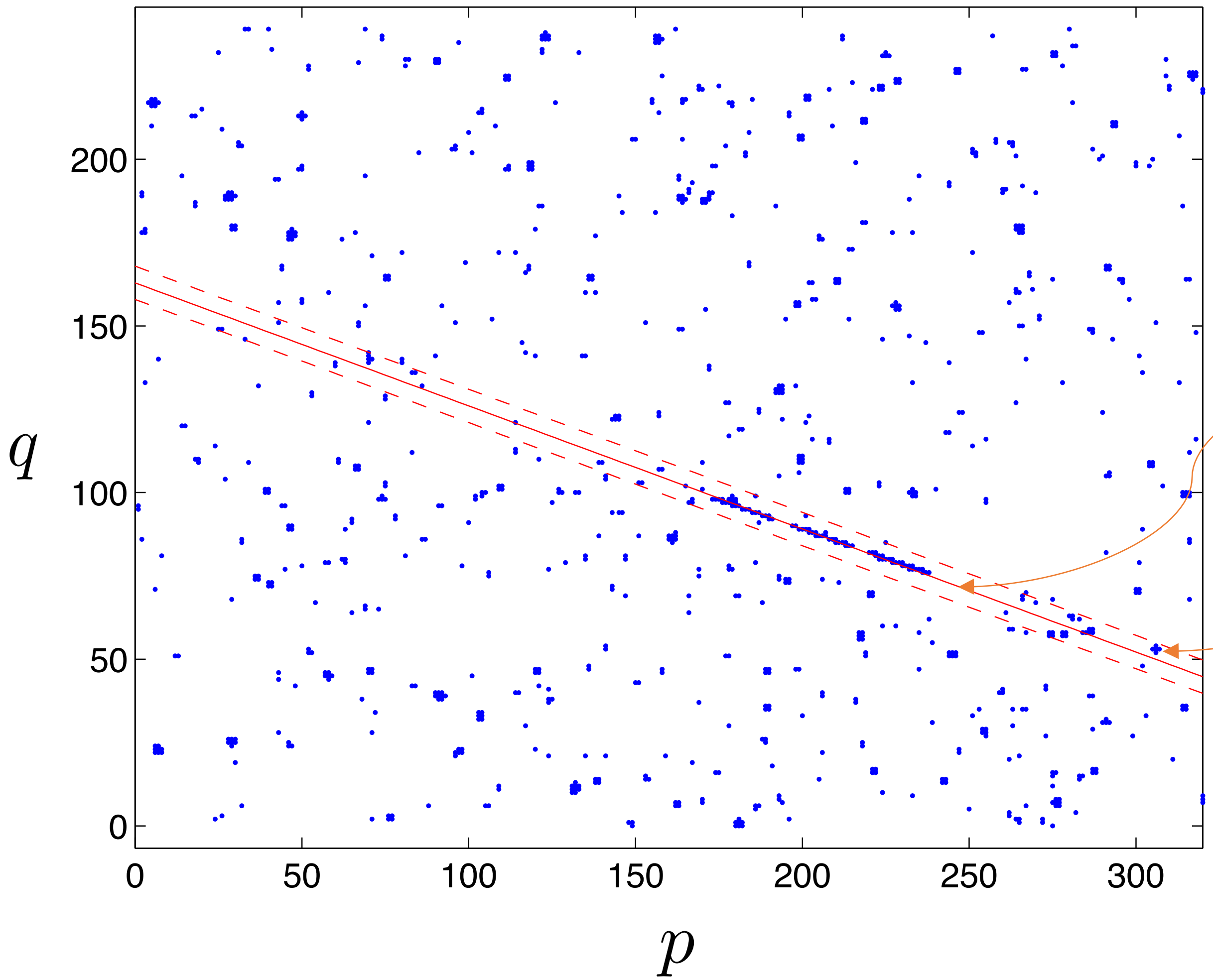
Consensus maximisation



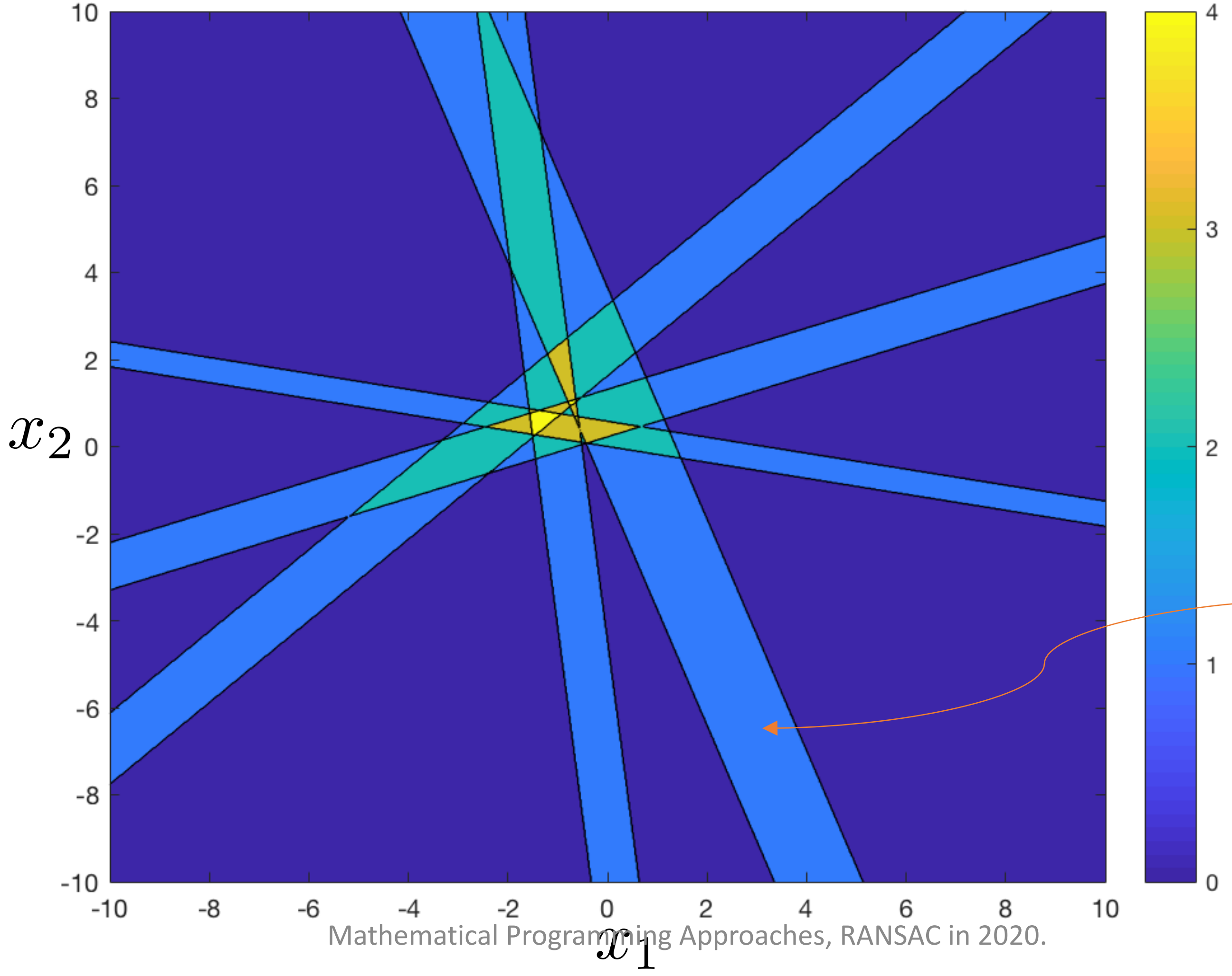
$$\mathbf{a} = \begin{bmatrix} p \\ 1 \end{bmatrix}, \quad b = q$$

$$\mathbf{a}^T\mathbf{x} - b = 0$$

$$|\mathbf{a}_i^T\mathbf{x} - b_i| \leq \epsilon$$

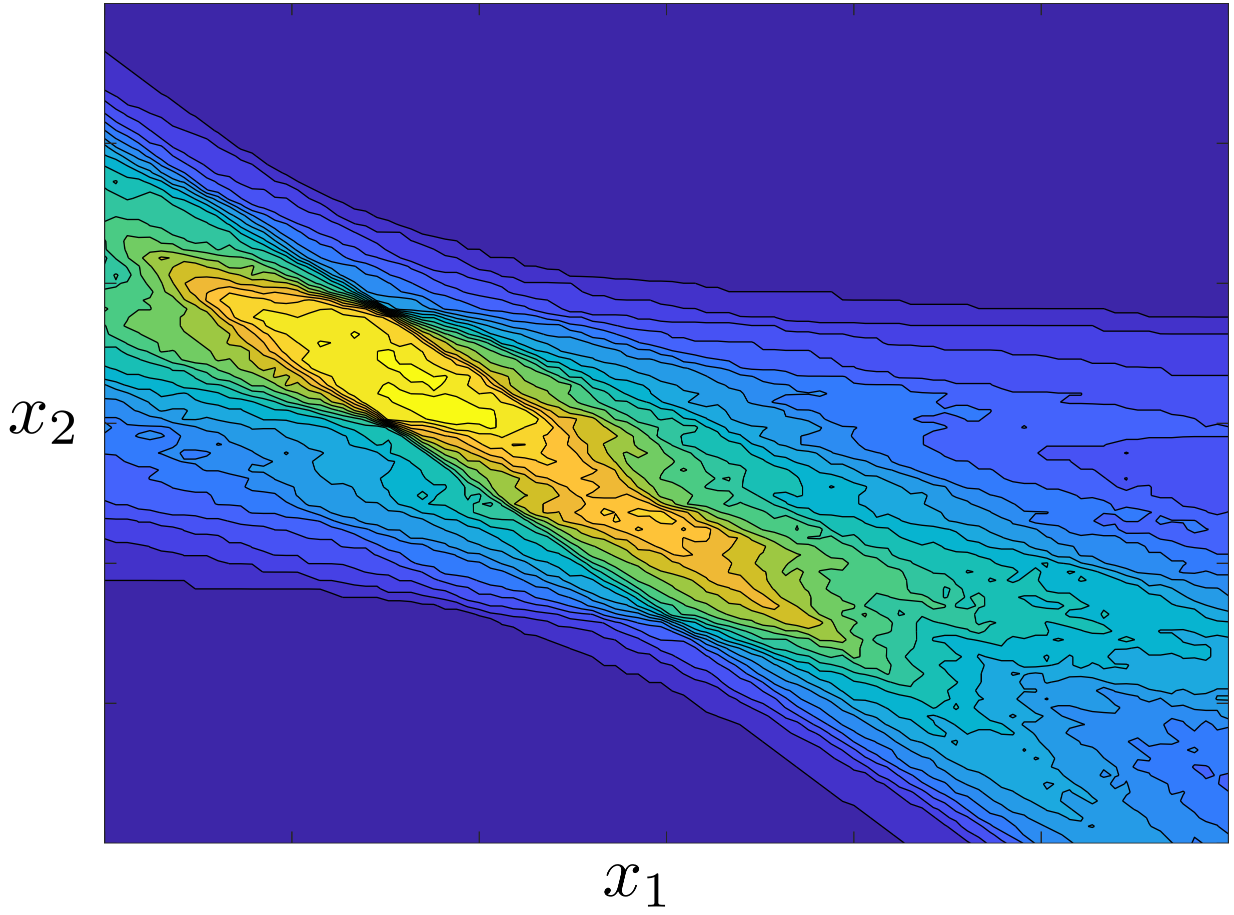
Find the slab of thickness ϵ that encloses the most number of points.

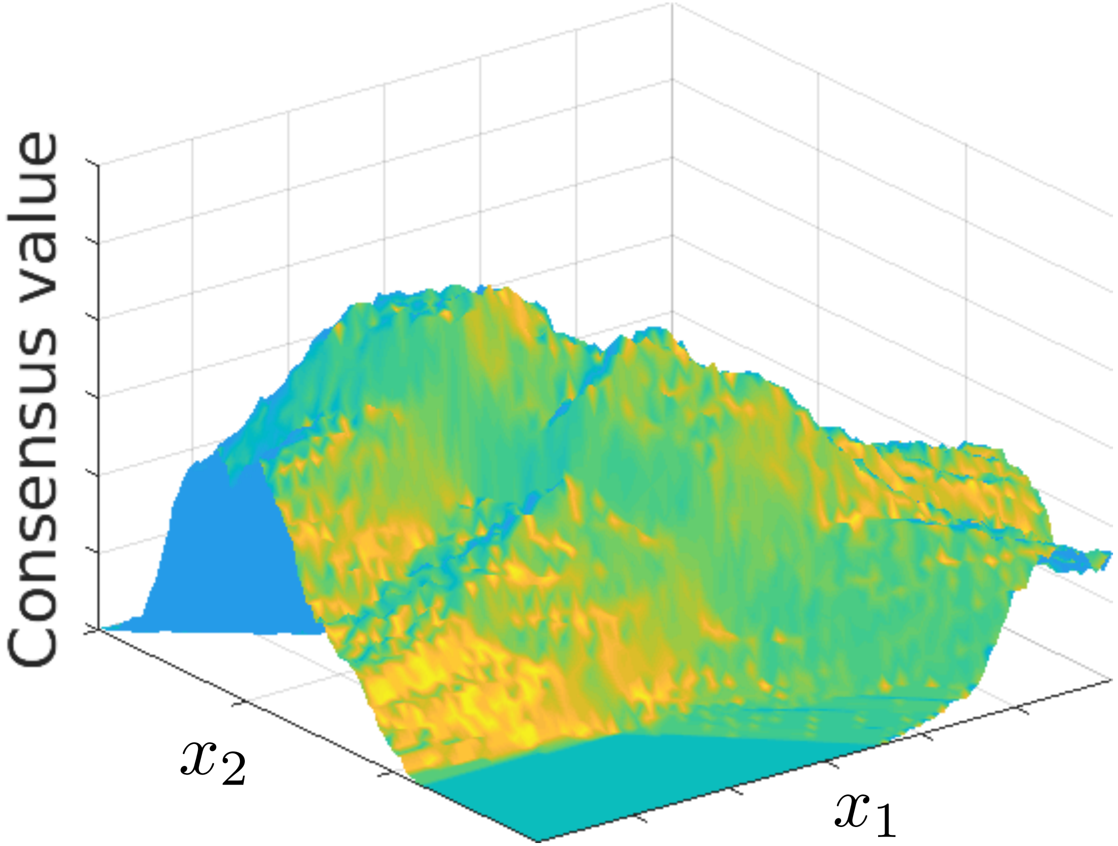
Objective function



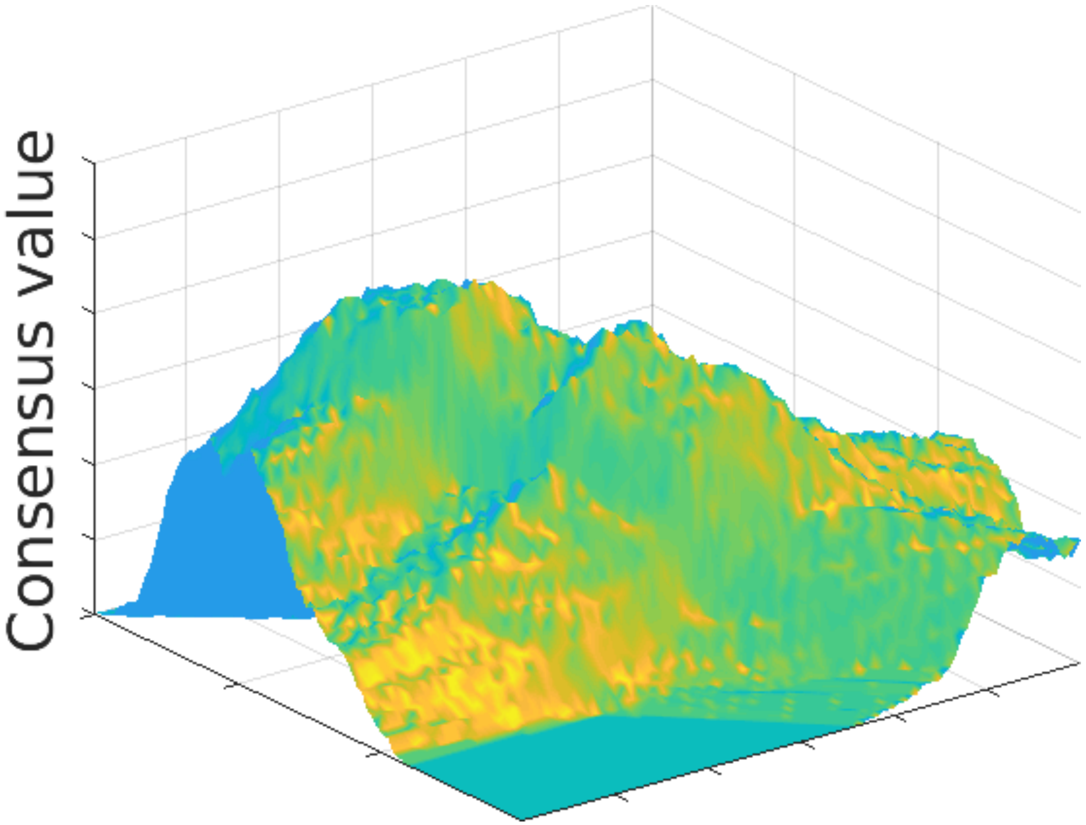
$|\mathbf{a}_i^T \mathbf{x} - b_i| \leq \epsilon$

Objective function

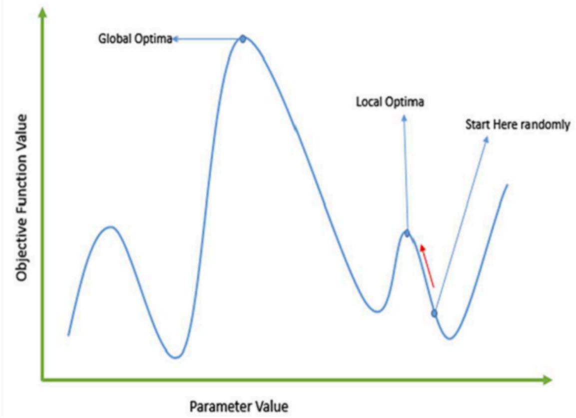


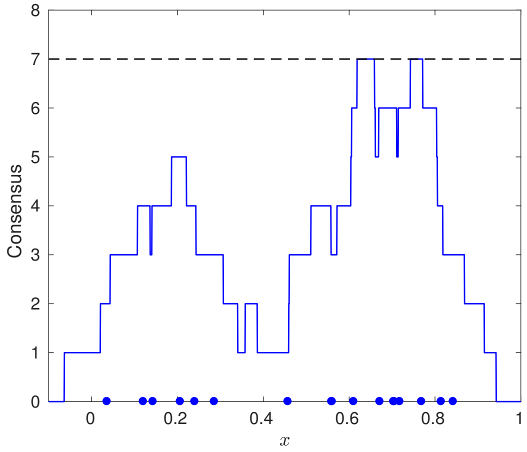


Objective function

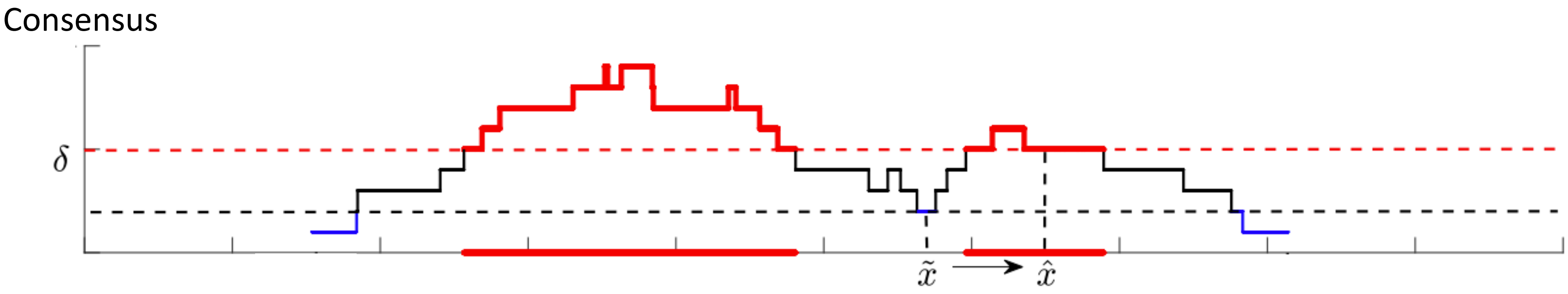


- Piece-wise constant function, no gradient.
- Local optimality is meaningless (every location within a constant patch looks the same).





The refinement problem



Can be from RANSAC
or least squares

- Given the current solution $\tilde{\mathbf{x}}$ find a better solution $\hat{\mathbf{x}}$.
- The domain for the better solution is non-convex.

Zhipeng Cai, Tat-Jun Chin, Huu Le, David Suter: Deterministic Consensus Maximization with Biconvex Programming. ECCV (12) 2018: 699-714

Reformulation using complementarity constraints

- The constraint $|\mathbf{a}_i^T\mathbf{x} - b_i| \leq \epsilon$ can be written as two linear constraints
$$\mathbf{a}_i^T\mathbf{x} - b_i \leq \epsilon, \quad -\mathbf{a}_i^T\mathbf{x} + b_i \leq \epsilon.$$

- Given input data $\mathcal{D} = \{(\mathbf{a}_i, b_i)\}_{i=1}^N$ the set of $|\mathbf{a}_i^T\mathbf{x} - b_i| \leq \epsilon$ can be rewritten as
$$\mathbf{\Lambda}^T\mathbf{x} - \boldsymbol{\beta} \leq \mathbf{0}$$

- The consensus maximisation problem can be rewritten as
$$\underset{\mathbf{x}\in\mathbb{R}^d}{\text{maximise}} \quad |\mathcal{J}(\mathbf{x})|$$

where
$$\mathcal{J}(\mathbf{x}) = \left\{ j \in \{1, \ldots, M\} \mid \boldsymbol{\alpha}_j^T\mathbf{x} - \beta_j \leq 0 \right\}$$

Reformulation using complementarity constraints

- Introducing indicator variables $\mathbf{u} \in \{0,1\}^M$ and slack variables $\mathbf{s} \in \mathbb{R}^M$, we can reformulate using complementarity constraints:

$$
\begin{aligned}
\min_{\mathbf{u},\mathbf{s}\in\mathbb{R}^M,\, \mathbf{x}\in\mathbb{R}^d} \quad & \sum_j u_j \\
\text{subject to} \quad & s_j - \boldsymbol{\alpha}_j^T\mathbf{x} + \beta_j \geq 0, \\
& u_j(s_j - \boldsymbol{\alpha}_j^T\mathbf{x} + \beta_j) = 0, \\
& s_j(1 - u_j) = 0, \\
& 1 - u_j \geq 0, \\
& s_j, u_j \geq 0.
\end{aligned}
$$

Reformulation using complementarity constraints

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& s_j(1 - u_j) = 0, \\
& 1 - u_j \geq 0, \\
& s_j, u_j \geq 0.
\end{aligned}
$$

Multiply with a scalar and move to the objective function

Penalty formulation

$$\begin{aligned} \min_{\mathbf{u},\mathbf{s}\in\mathbb{R}^M,\, \mathbf{x}\in\mathbb{R}^d} \quad & \sum_j u_j \; + \lambda\left(u_j(s_j - \boldsymbol{\alpha}_j^T\mathbf{x} + \beta_j)\right) \\ \text{subject to} \quad & s_j - \boldsymbol{\alpha}_j^T\mathbf{x} + \beta_j \geq 0, \\ & s_j(1 - u_j) = 0, \\ & 1 - u_j \geq 0, \\ & s_j, u_j \geq 0. \end{aligned}$$

Penalty formulation

$$
\begin{aligned}
\min_{\mathbf{u},\mathbf{s}\in\mathbb{R}^M,\ \mathbf{x}\in\mathbb{R}^d} \quad & \sum_j u_j \ +\lambda\left(u_j(s_j - \boldsymbol{\alpha}_j^T\mathbf{x} + \beta_j)\right) \\
\text{subject to} \quad & s_j - \boldsymbol{\alpha}_j^T\mathbf{x} + \beta_j \geq 0, \\
& s_j(1 - u_j) = 0, \\
& 1 - u_j \geq 0, \\
& s_j, u_j \geq 0.
\end{aligned}
$$

For fixed λ and $\mathbf{s}$, this is a linear program.

For fixed λ and $\mathbf{u}$, this is also a linear program.

Exact penalty method

$$\begin{aligned} \min_{\mathbf{u},\mathbf{s}\in\mathbb{R}^M,\ \mathbf{x}\in\mathbb{R}^d} \quad & \sum_j u_j \ + \lambda \left(u_j (s_j - \boldsymbol{\alpha}_j^T \mathbf{x} + \beta_j) \right) \\ \text{subject to} \quad & s_j - \boldsymbol{\alpha}_j^T \mathbf{x} + \beta_j \geq 0, \\ & s_j(1 - u_j) = 0, \\ & 1 - u_j \geq 0, \\ & s_j, u_j \geq 0. \end{aligned}$$

Huu Le, Tat-Jun Chin, David Suter: An Exact Penalty Method for Locally Convergent Maximum Consensus. CVPR 2017: 379-387.

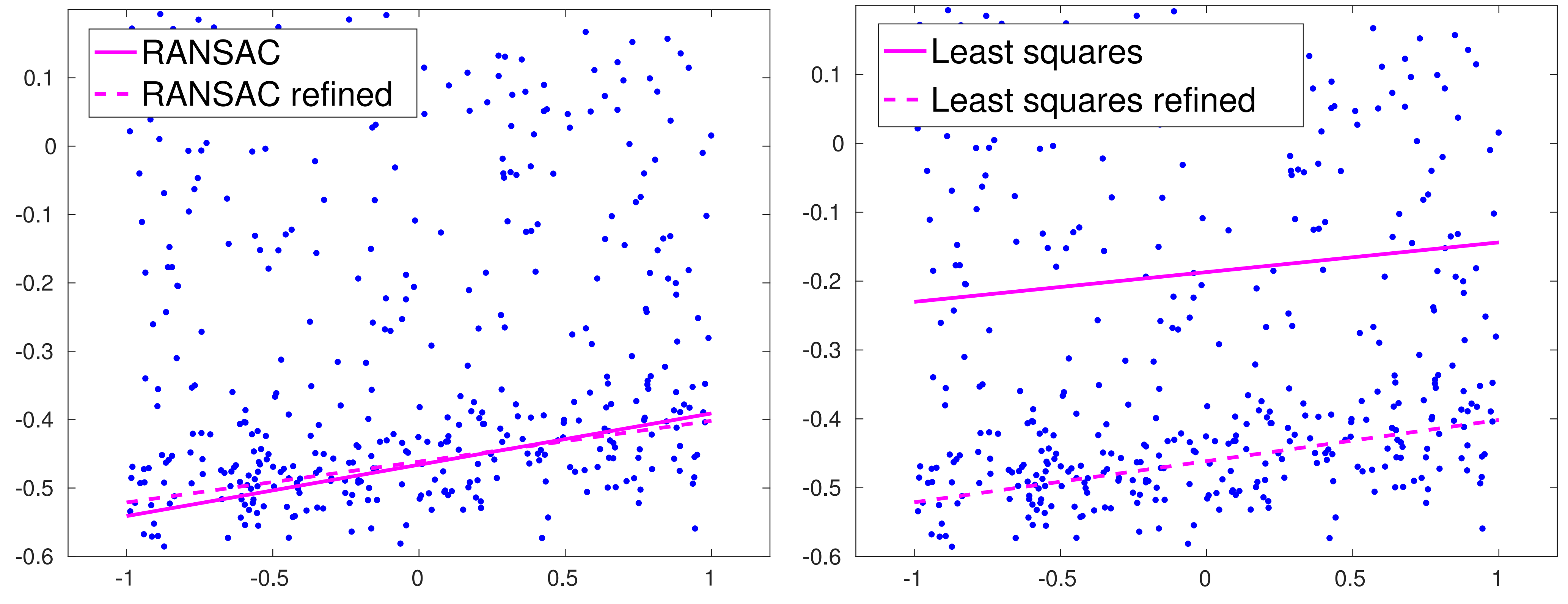
Zhipeng Cai, Tat-Jun Chin, Huu Le, David Suter: Deterministic Consensus Maximization with Biconvex Programming. ECCV (12) 2018: 699-714

For fixed λ and $\mathbf{s}$, this is a linear program.

For fixed λ and $\mathbf{u}$, this is also a linear program.

Alternate the two LPs,
Progressively increase λ.
When λ is large enough, but still finite, the algorithm will converge to a KKT point.

Exact penalty method



(a) (b)

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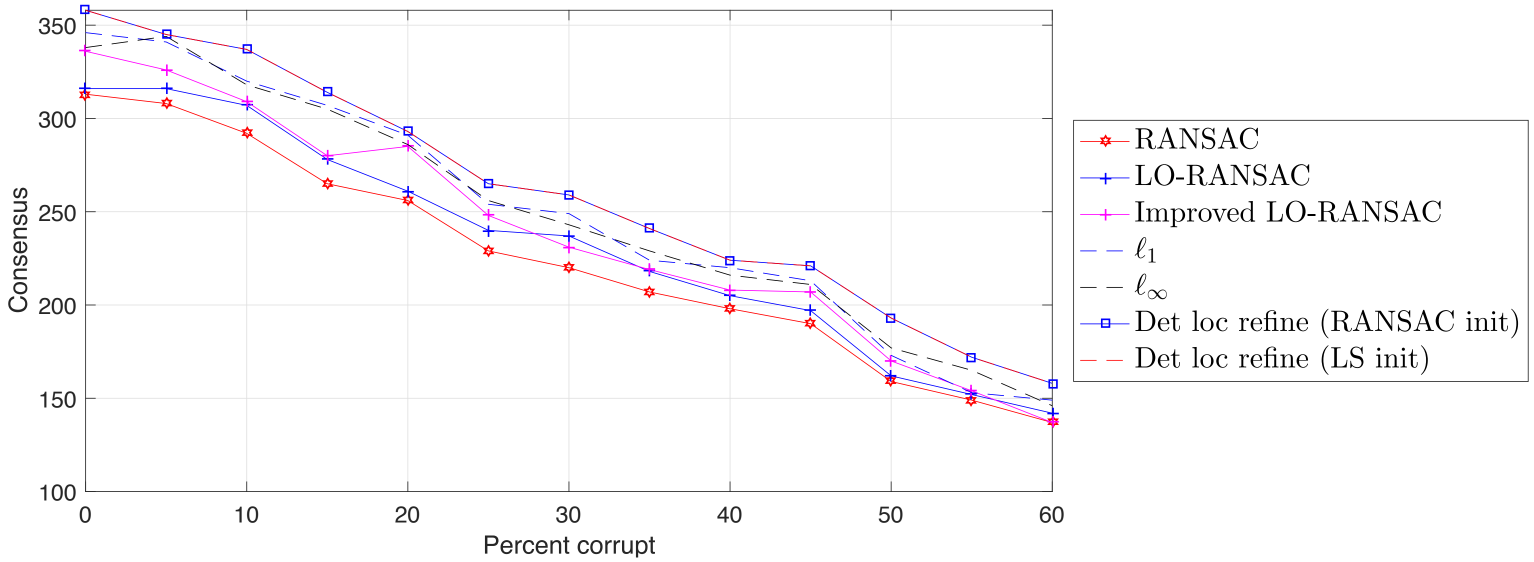
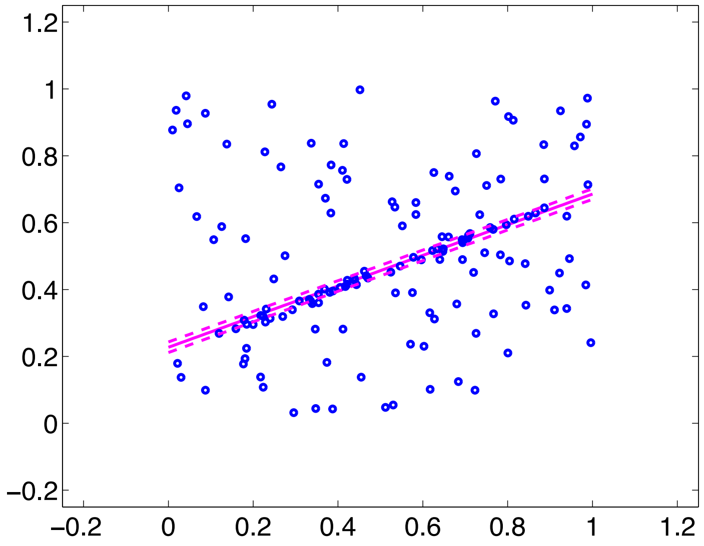
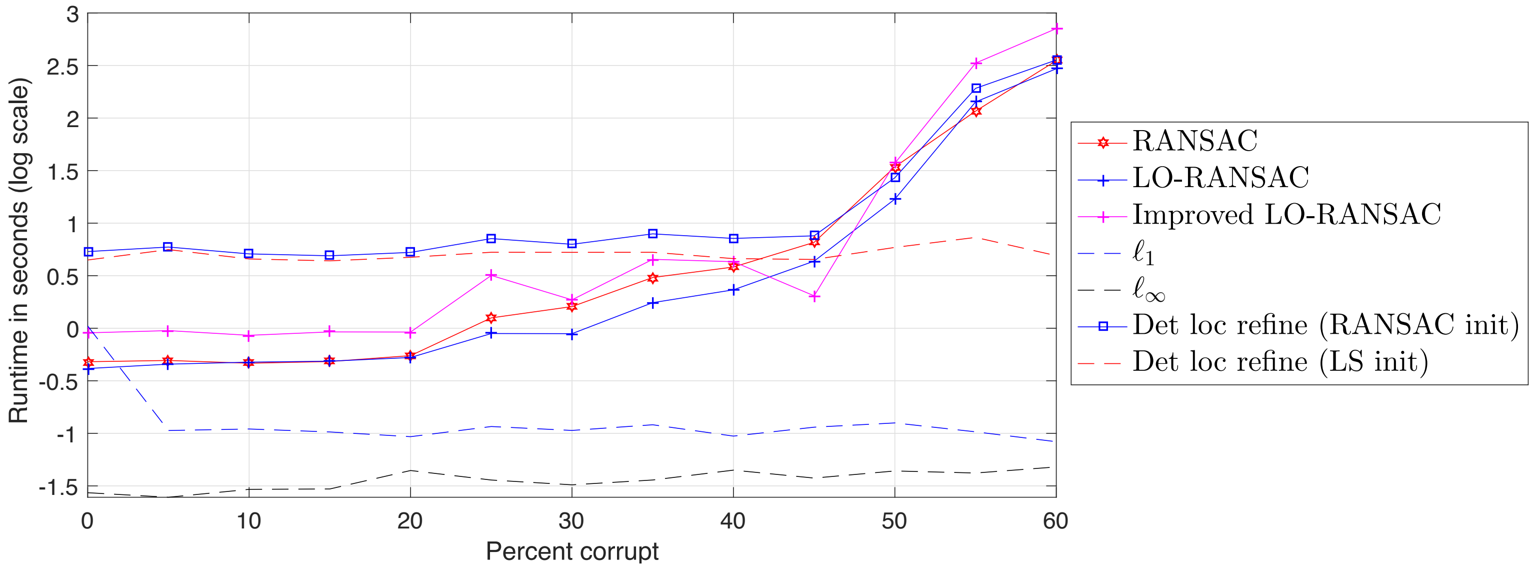
Balanced data

- 8 dimensions
- Vary outlier rate
- Record converged consensus and runtime



(a)

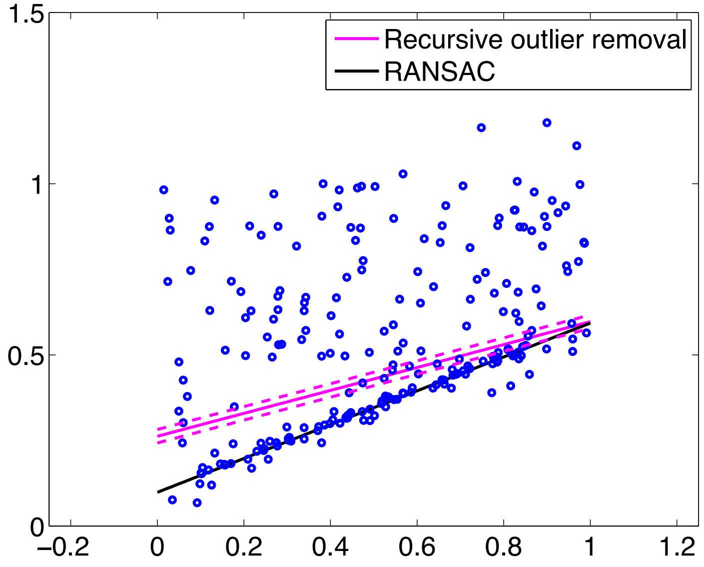


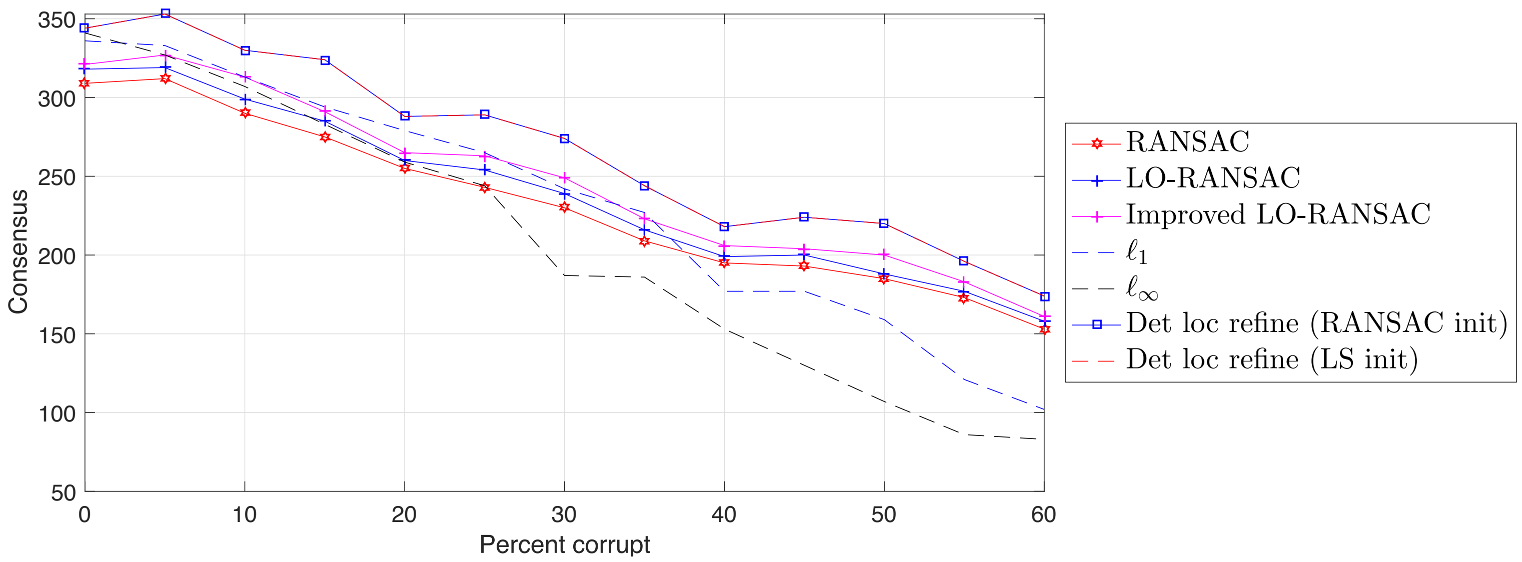


(b)

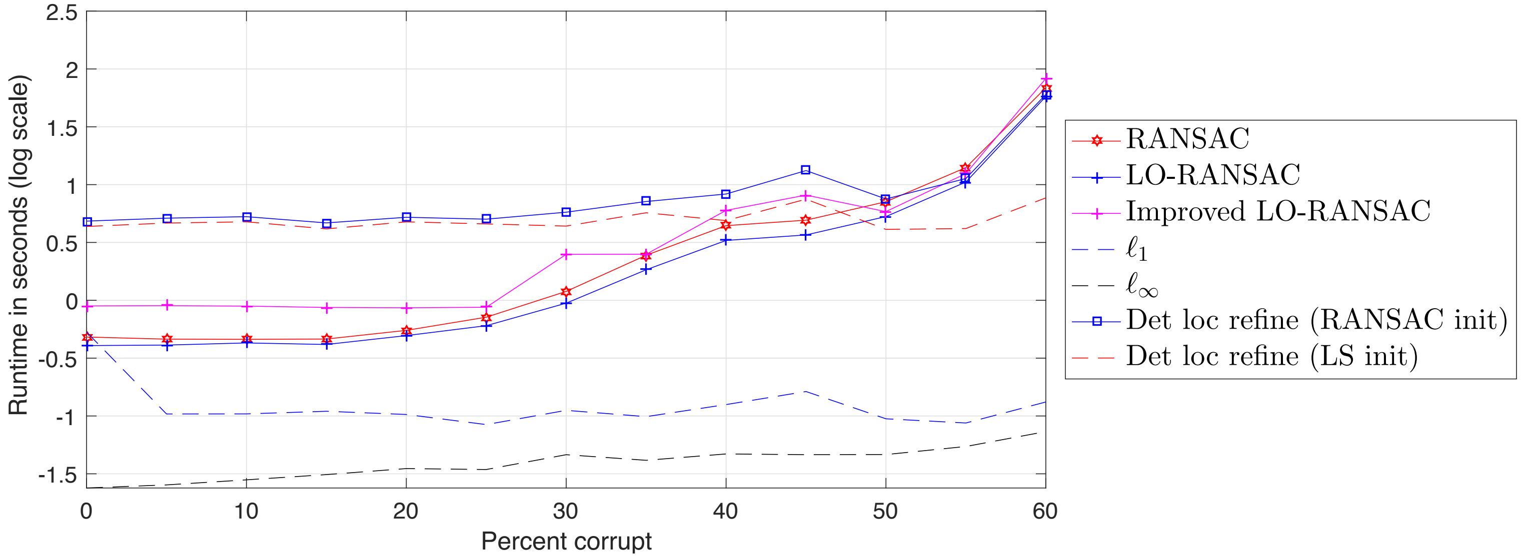
Unbalanced data

- 8 dimensions
- Vary outlier rate
- Record converged consensus and runtime





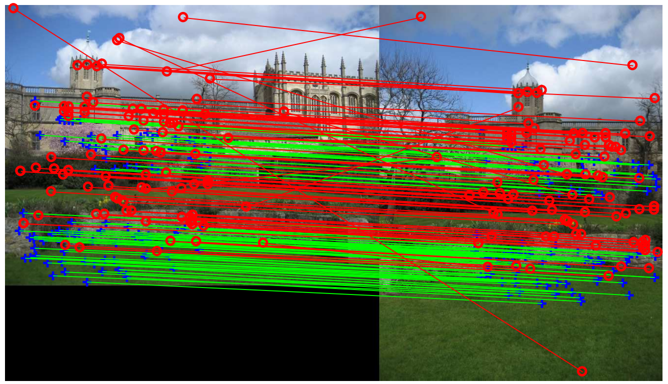
(a)

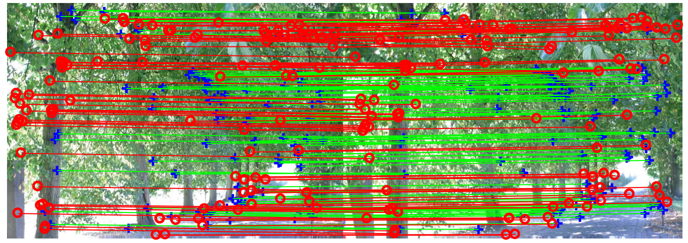


(b)

Deterministic refinement,
initialised using RANSAC.

Real data



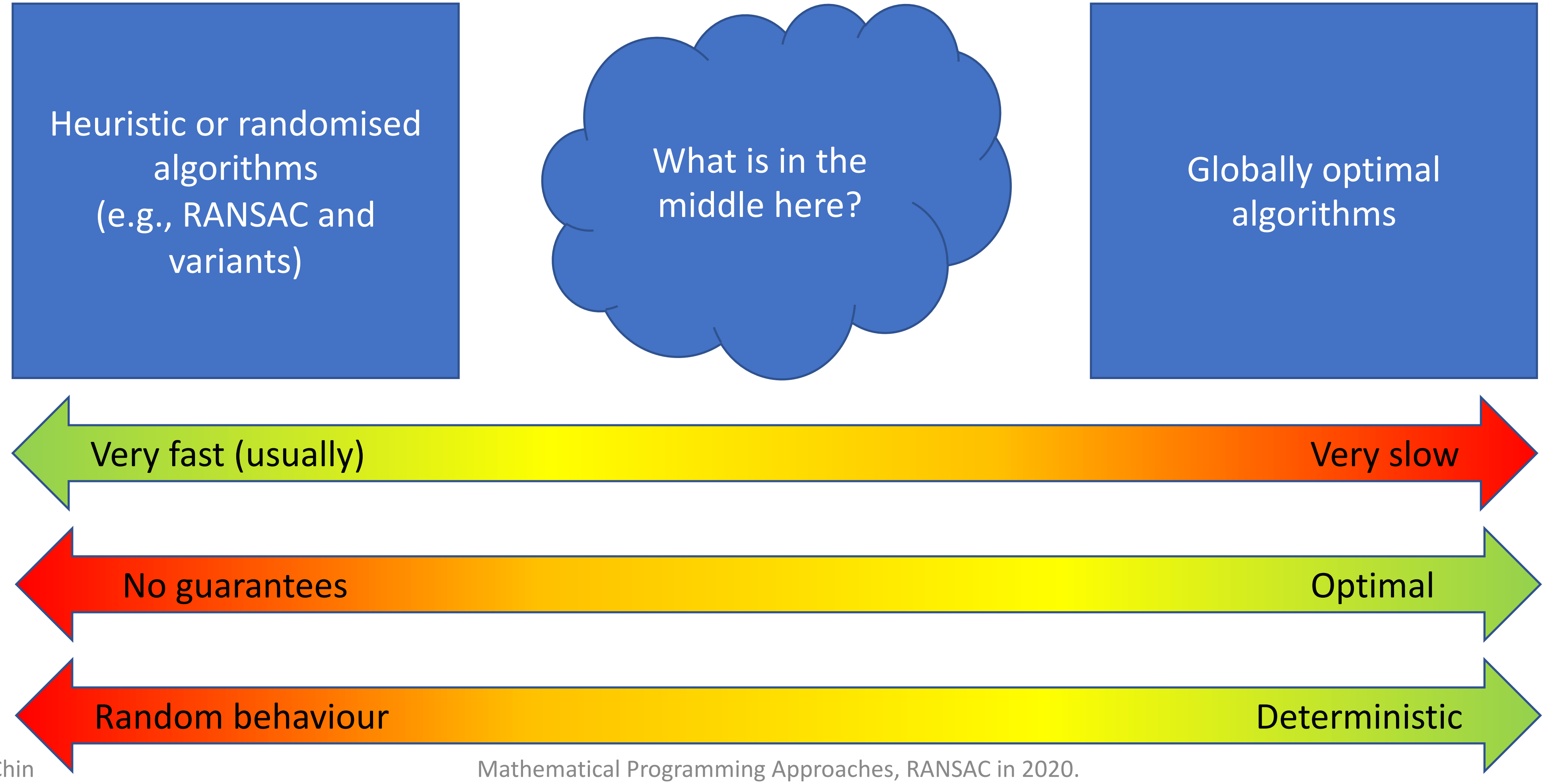


		Method	RS	PS	GMLE	LOR1	LOR2	ℓ_1	ℓ_∞	DLocR
Homography estimation	University Library	$\lvert\mathcal{I}\rvert$	251	269	251	294	294	120	53	**301**
	N = 545	time (s)	0.73	0.62	0.69	1.90	1.89	3.10	2.49	12.76
	Christ Church	$\lvert\mathcal{I}\rvert$	235	236	227	250	246	246	160	**280**
	N = 445	time (s)	0.47	0.47	0.43	1.33	1.61	1.23	2.44	10.37
	Valbonne	$\lvert\mathcal{I}\rvert$	131	134	117	156	136	24	22	**158**
	N = 434	time (s)	3.17	2.39	5.76	3.04	5.80	1.36	1.27	17.20
	Kapel	$\lvert\mathcal{I}\rvert$	163	167	130	167	168	28	161	**170**
	N = 449	time (s)	1.19	1.15	9.89	2.18	2.70	1.62	1.16	8.46
	Invalides	$\lvert\mathcal{I}\rvert$	144	159	140	149	156	84	142	**178**
	N = 413	time (s)	1.36	0.90	1.60	2.17	2.94	1.04	0.71	10.20
Affinity estimation	Bikes	$\lvert\mathcal{I}\rvert$	424	427	425	426	424	387	431	**437**
	N = 557	time (s)	6.09	6.09	5.79	6.28	11.8	1.77	1.77	15.26
	Graff	$\lvert\mathcal{I}\rvert$	126	129	127	134	126	147	274	**276**
	N = 327	time (s)	3.51	3.35	3.14	4.07	6.61	0.99	0.23	5.94
	Bark	$\lvert\mathcal{I}\rvert$	279	288	270	284	279	298	439	**442**
	N = 458	time (s)	4.89	4.93	4.68	5.11	9.54	1.31	0.19	10.19
	Tree	$\lvert\mathcal{I}\rvert$	372	367	371	372	372	377	370	**396**
	N = 568	time (s)	5.70	6.01	5.73	6.93	11.50	4.81	0.81	15.96
	Boat	$\lvert\mathcal{I}\rvert$	476	477	476	477	476	469	464	**483**
	N = 574	time (s)	6.32	6.29	6.02	7.18	12.32	4.12	1.02	14.86

Outline

- What is and isn't fundamentally achievable
- Global algorithms
- Deterministic outlier removal
- Deterministic refinement
- Evaluation
- RANSAC in 2040---Quantum RANSAC?

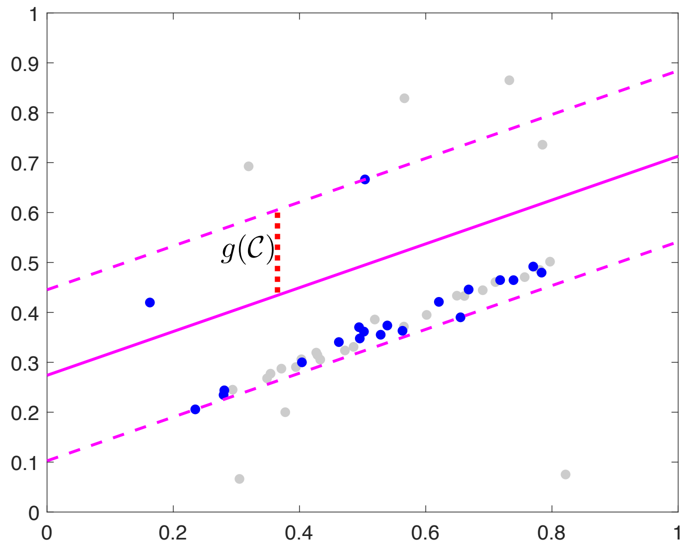
What if we are not restricted by classical computers?

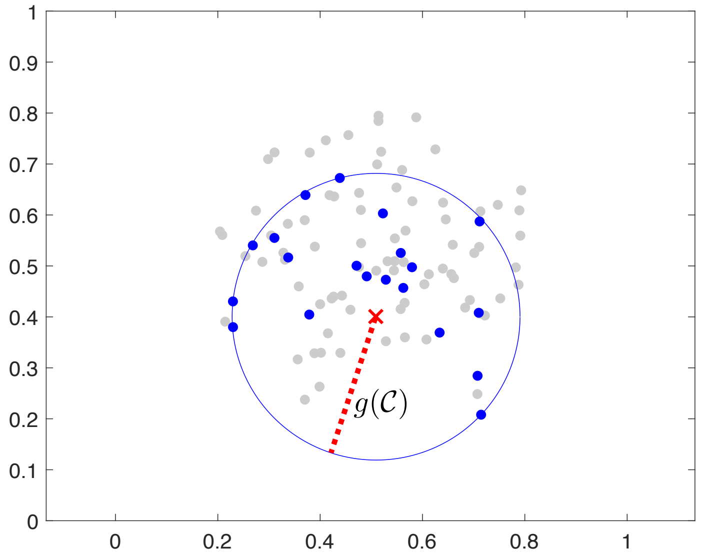


Minimax value

- Let $\mathcal{C} \subseteq \mathcal{D} = \{1, \ldots, N\}$ be a subset of the points, and

$$g(\mathcal{C}) = \min_{\mathbf{x} \in \mathbb{R}^d} \; \max_{i \in \mathcal{C}} \; r_i(\mathbf{x})$$





Boolean feasibility test

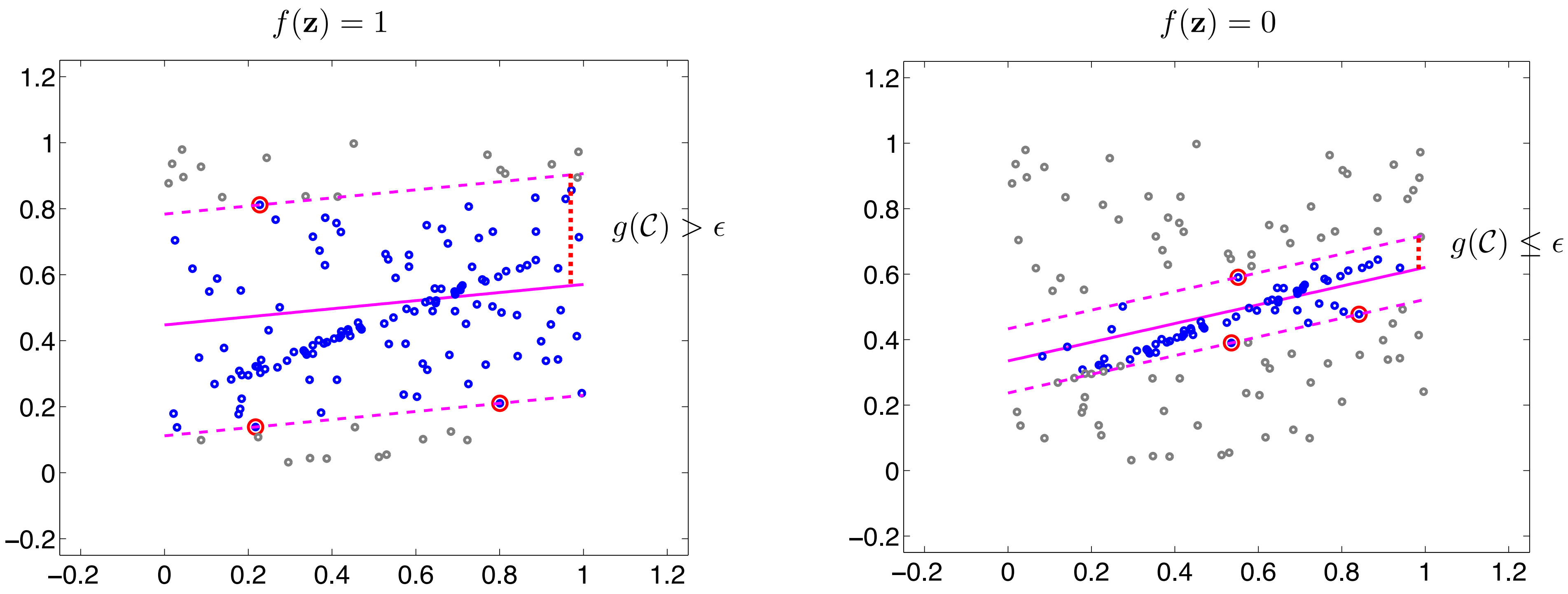
- Let $\mathbf{z} = [z_1, \ldots, z_N] \in \{0, 1\}^N$ be a binary vector that selects subsets of $\mathcal{D}$. The indices selected by a $\mathbf{z}$ is

$$\mathcal{C}_{\mathbf{z}} = \{i \in \mathcal{D} \mid z_i = 1\}$$

- Define Boolean function

$$f(\mathbf{z}) = \begin{cases} 0 & \text{if } g(\mathcal{C}_{\mathbf{z}}) \leq \epsilon; \\ 1 & \text{otherwise.} \end{cases}$$

Boolean feasibility test



Influence

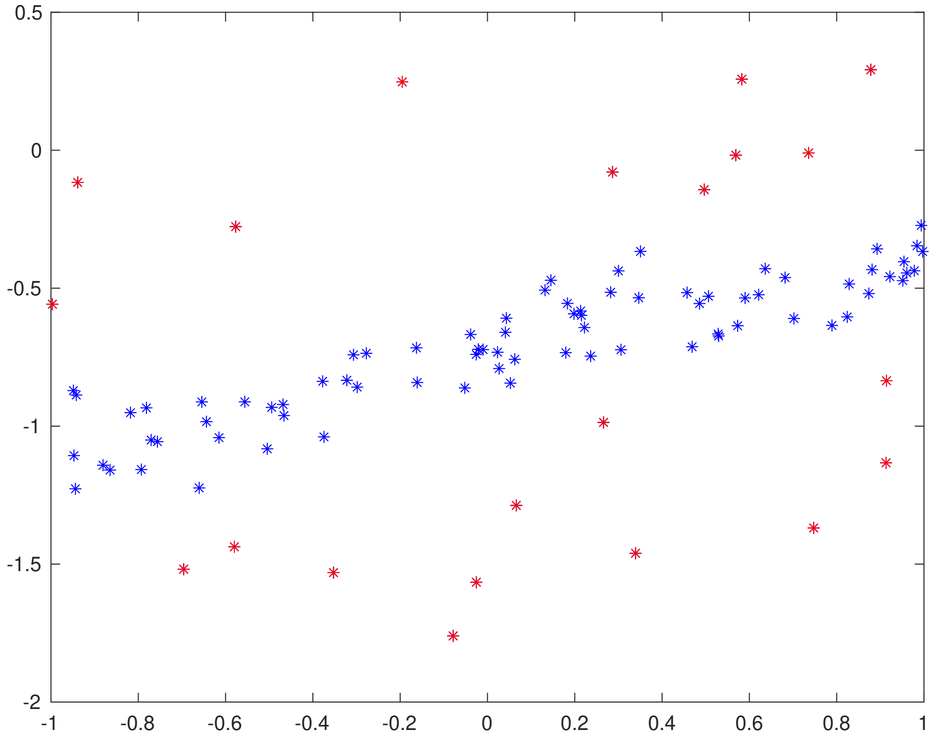
- The influence of the i-th datum is

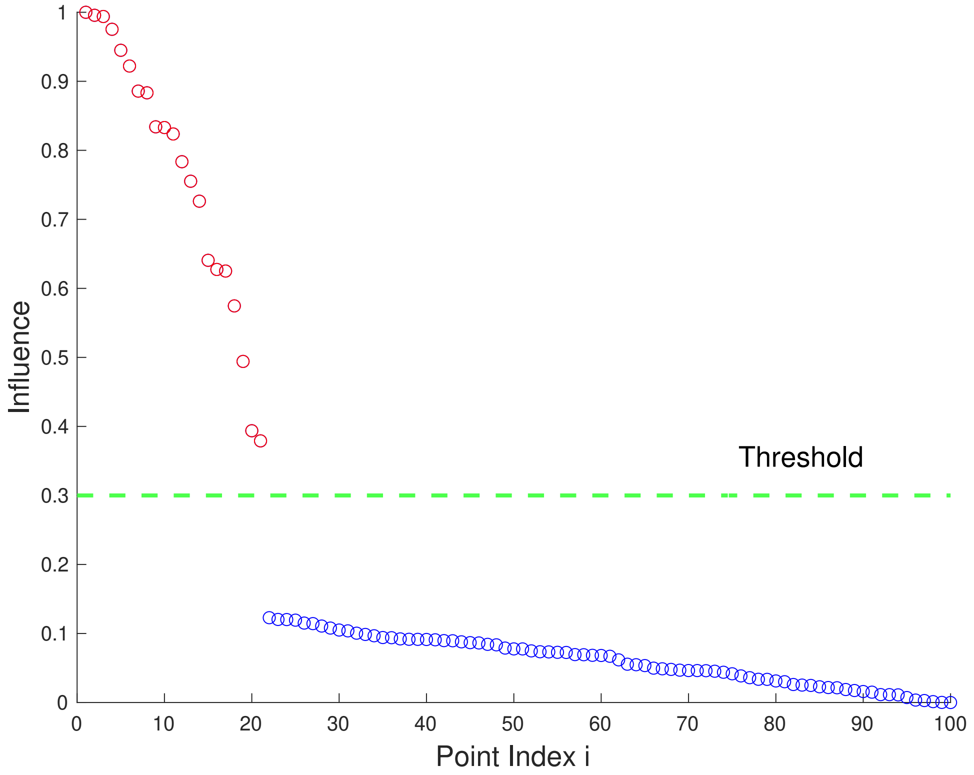
$$\begin{aligned}\alpha_i &= Pr\left[f(\mathbf{z} \oplus \mathbf{e}_i) \neq f(\mathbf{z})\right] \\ &= \frac{1}{2^N}\left|\{\mathbf{z} \in \{0,1\}^N \mid f(\mathbf{z} \oplus \mathbf{e}_i) \neq f(\mathbf{z})\}\right|\end{aligned}$$

i.e., the probability of changing the feasibility of a subset $\mathbf{z}$ by inserting/removing the i-th datum into/from $\mathbf{z}$.

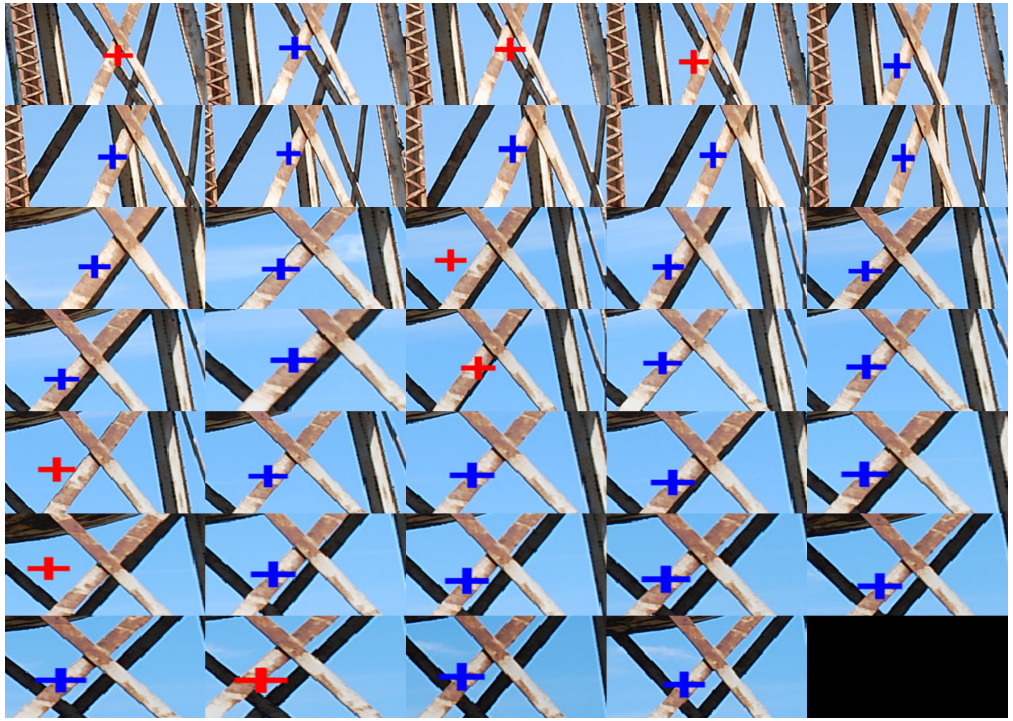
- The probability is taken over all 2^N possible realisations of $\mathbf{z}$.

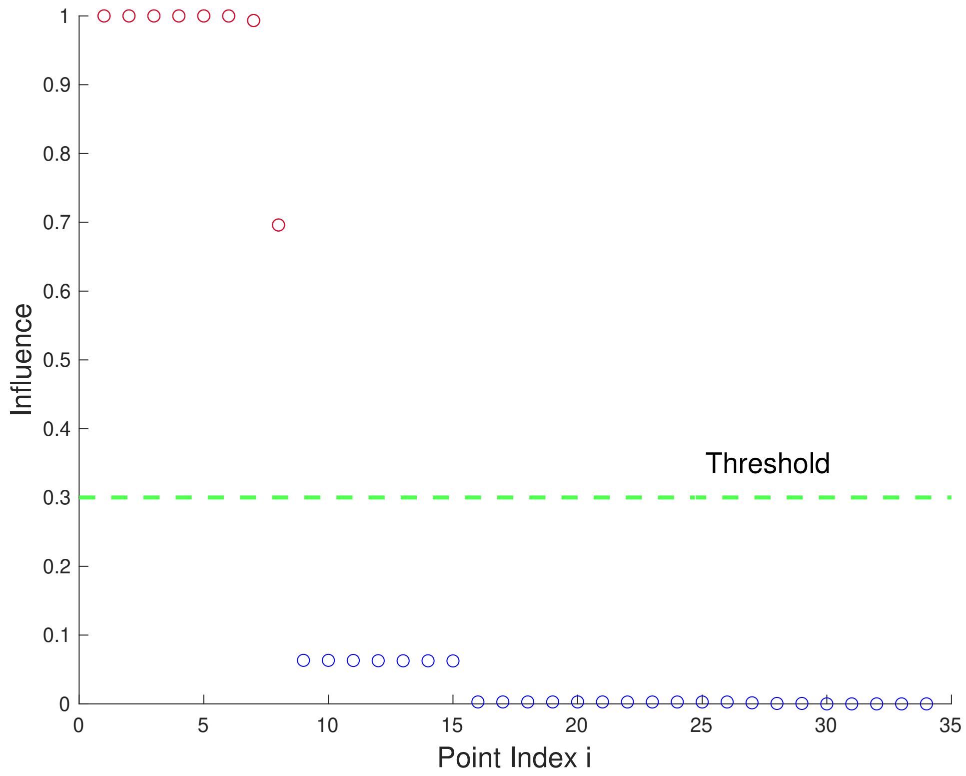
Influence (normalised)



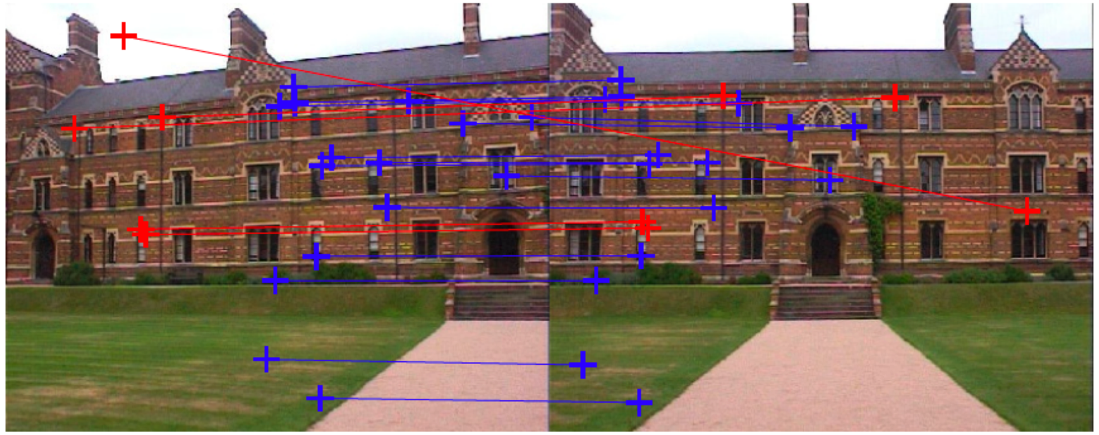


Influence (normalised)

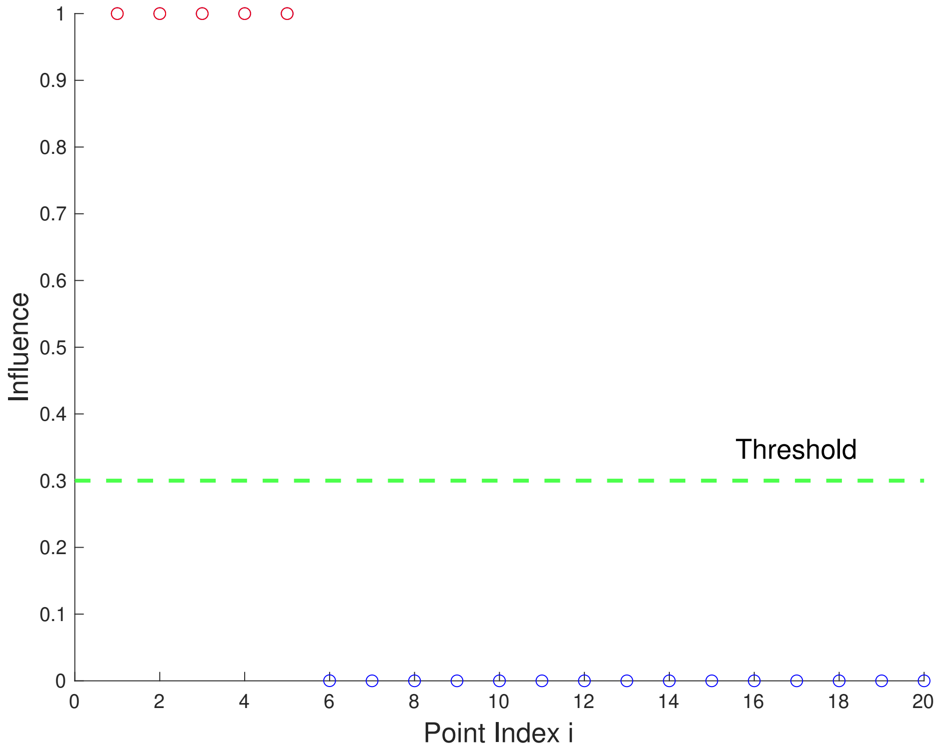




Influence (normalised)



(e) Feature correspondences across two views.



(f)

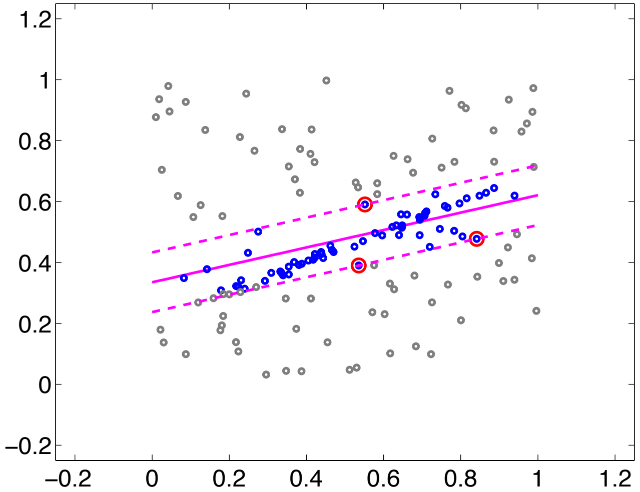
Monotonicity

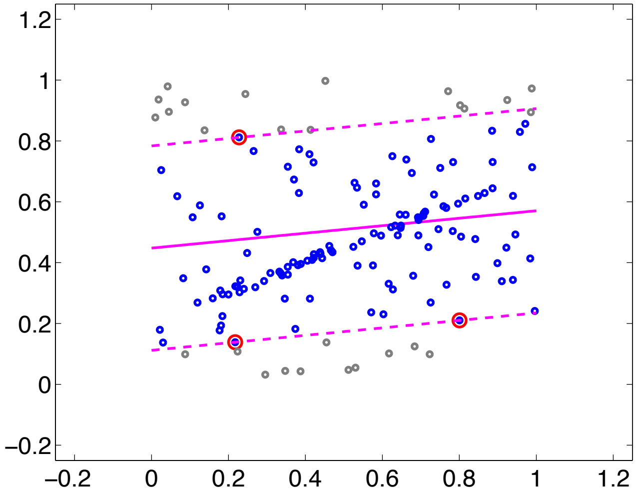
- The Boolean function f is monotonic because

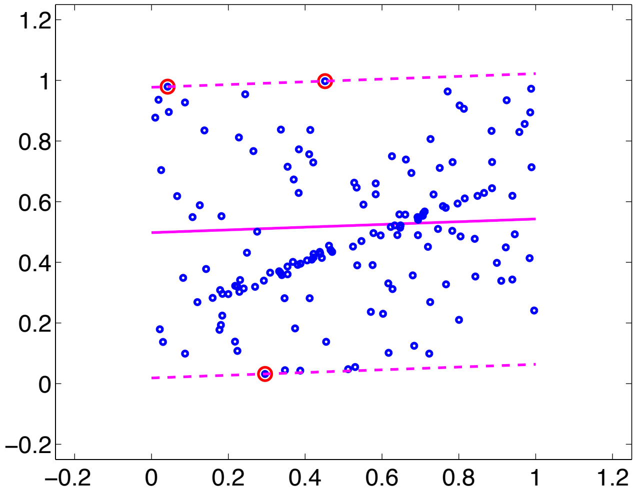
$$\mathcal{B} \subseteq \mathcal{C} \subseteq \mathcal{D} \implies g(\mathcal{B}) \leq g(\mathcal{C}) \leq g(\mathcal{D})$$

- The influence is closely related to the Fourier transform of a Monotone Boolean function.

David Suter, Ruwan Tennakoon, Erchuan Zhang, Tat-Jun Chin, Alireza Bab-Hadiashar. Monotone Boolean Functions, Feasibility/Infeasibility, LP-type problems and MaxCon. CoRR abs/2005.05490 (2020)

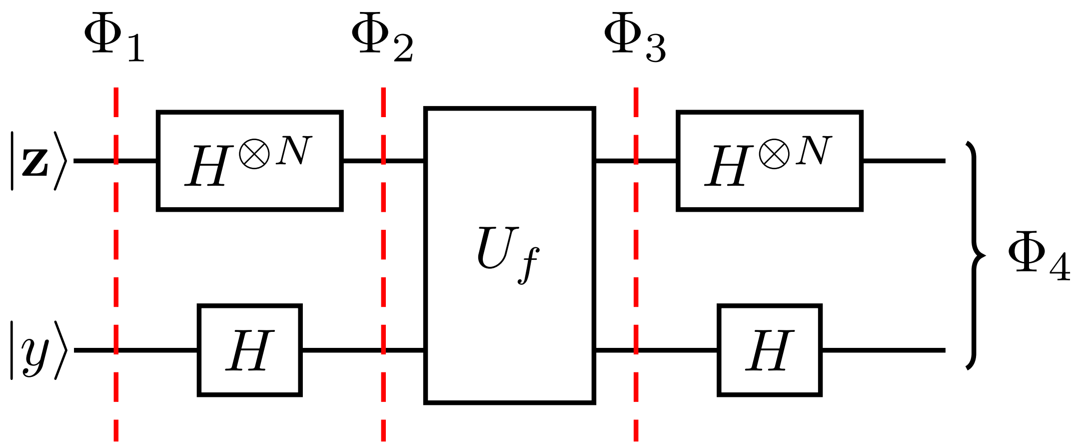




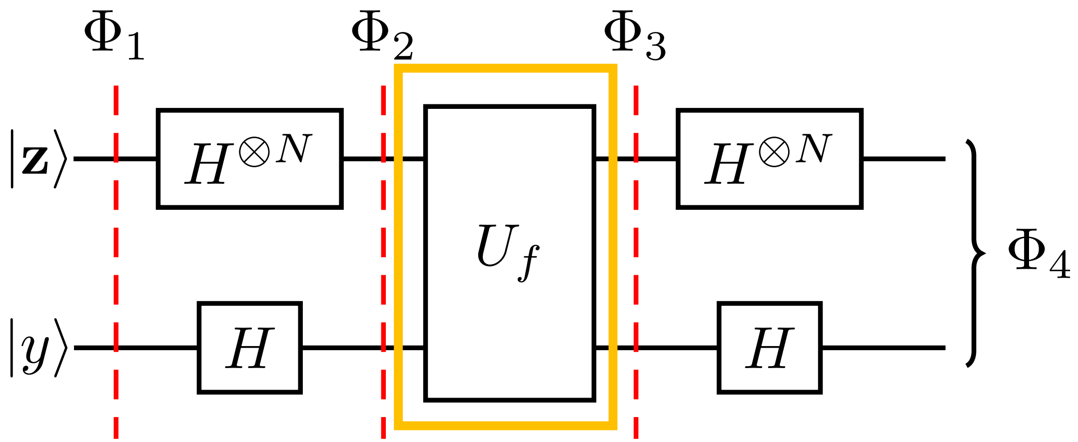


Influence computation

- Exact computation is not possible except for very small N.
- Lets try a quantum approach…
- This is the Bernstein-Vazirani (BV) quantum circuit:



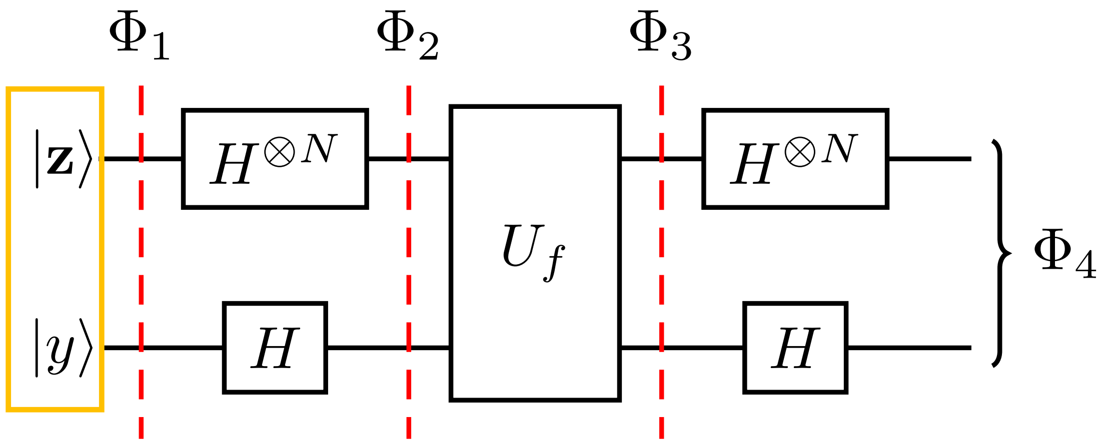
Influence computation



- The data and Boolean function f is "implemented" in the main block.
- For quasiconvex residuals, f can be evaluated efficiently, i.e., in polynomial time ➔ there is an equivalent quantum implementation that is efficient, involving polynomial number of quantum gates.

Michael A. Nielsen and Isaac L. Chuang. *Quantum computation and quantum information*. Cambridge University Press, 2010.

Quantum operations



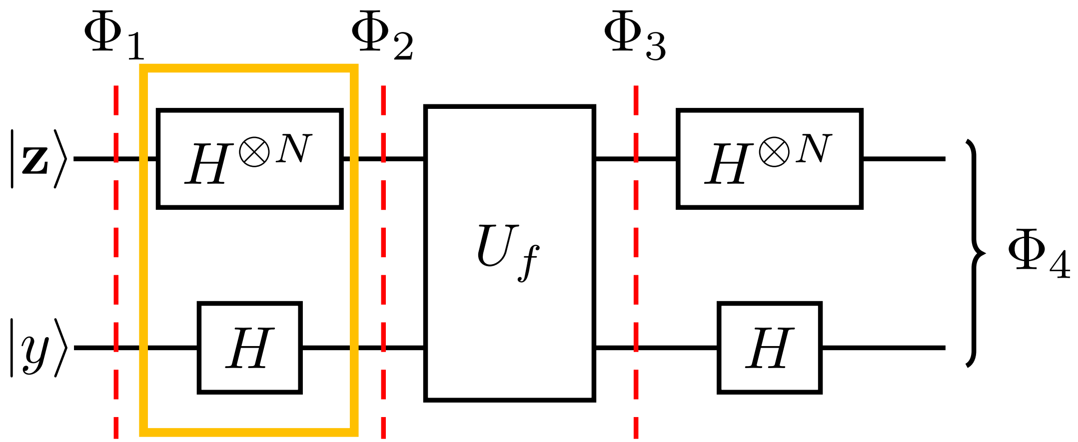
- Initialise N + 1 qubits

$$|\mathbf{z}\rangle = |\mathbf{0}\rangle \qquad |y\rangle = |1\rangle$$

hence

$$\Phi_1 = |\mathbf{0}\rangle\,|1\rangle$$

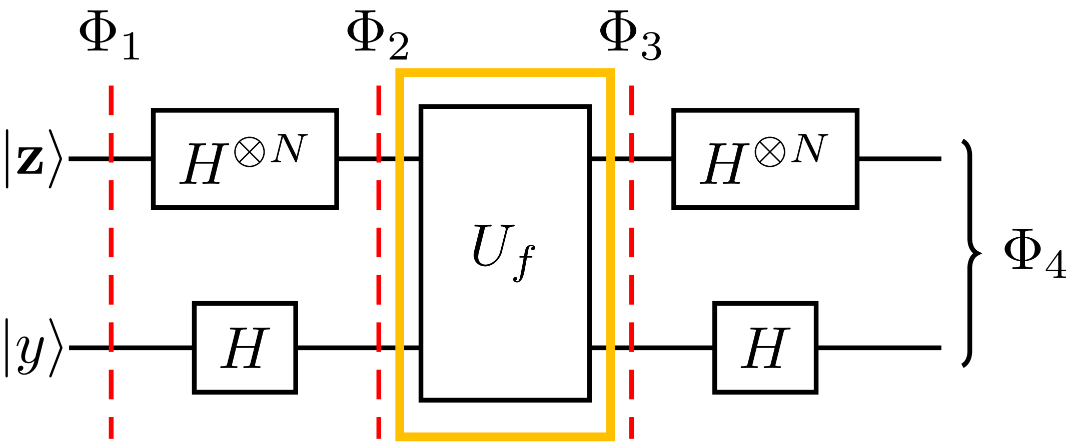
Quantum operations



- Apply N+1 Hadamard gates $H^{\otimes(N+1)}$ to put the quantum state in superposition

$$
\begin{aligned}
\Phi_2 &= H^{\otimes(N+1)}\Phi_1 \\
&= \frac{1}{\sqrt{2^N}} \sum_{\mathbf{t}\in\{0,1\}^N} |\mathbf{t}\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}
\end{aligned}
$$

Quantum operations



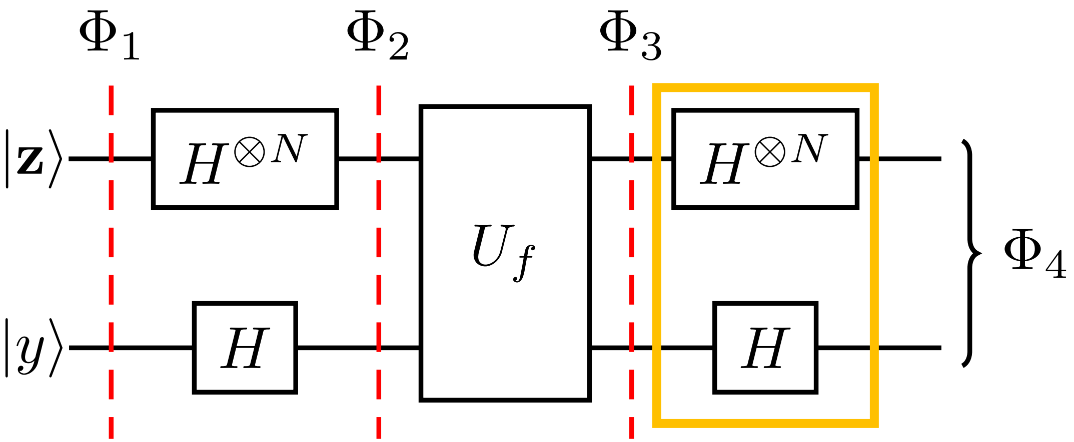
- The Boolean function as a quantum gate behaves as

$$U_f \, |\mathbf{z}\rangle \, |y\rangle = |\mathbf{z}\rangle \, |y \otimes f(\mathbf{z})\rangle$$

- Applying the quantum gate on the superposed state

$$\begin{aligned} \Phi_3 &= U_f \Phi_2 \\ &= \frac{1}{\sqrt{2^N}} \sum_{\mathbf{t} \in \{0,1\}^N} (-1)^{f(\mathbf{t})} \, |\mathbf{t}\rangle \, \frac{|0\rangle - |1\rangle}{\sqrt{2}} \end{aligned}$$

Quantum operations



- Applying the Hadamard gates again

$$
\begin{aligned}
\Phi_4 &= H^{\otimes(N+1)}\Phi_3 \\
&= \frac{1}{2^N} \sum_{\mathbf{s}\in\{0,1\}^N} \sum_{\mathbf{t}\in\{0,1\}^N} (-1)^{f(\mathbf{t})+\mathbf{s}\cdot\mathbf{t}} |\mathbf{s}\rangle\, |1\rangle
\end{aligned}
$$

Output of quantum algorithm

- Focussing on the top-N qubits, we have

$$\sum_{\mathbf{s} \in \{0,1\}^N} I(\mathbf{s}) \left|\mathbf{s}\right\rangle$$

where $I(\mathbf{s}) := \sum_{\mathbf{t} \in \{0,1\}^N} (-1)^{f(\mathbf{t}) + \mathbf{s} \cdot \mathbf{t}}$.

Theorem 1. *Let* $\mathbf{s} = [s_1, \dots, s_N] \in \{0,1\}^N$. *Then*

$$\alpha_i = \sum_{s_i = 1} I(\mathbf{s})^2.$$

In one evaluation of f, we compute all the influences exactly!

The classical algorithm needs to evaluate the function 2^N times.

Tat-Jun Chin, David Suter, Shin Fang Ch'ng and James Quach. Quantum Robust Fitting. CoRR abs./2006.06986 (2020)

Output of quantum algorithm

- Unfortunately, limitations by nature allows us to access the information by quantum measurements only.
- We measure the top-N qubits of Φ_4 in the standard basis: with probability

$$Pr(s_i = 1) = \sum_{s_i=1} I(\mathbf{s})^2 = \alpha_i$$

  we obtain the value of 1 for the i-th qubit.
- As soon as a measurement is conducted, the quantum state Φ_4 collapses.

Quantum influence computation

- We thus invoke the BV algorithm and measurement M times and collect the “statistics” of the measurement.

Algorithm 2 Quantum algorithm to compute influence.

Require: N input data points $\mathcal{D}$, inlier threshold ϵ, number of iterations M.

for $m = 1, \ldots, M$ **do**
  $\mathbf{s}^{[m]} \leftarrow$ Run BV algorithm with $\mathcal{D}$ and ϵ and measure top-N qubits in standard basis.
end for
for $i = 1, \ldots, N$ **do**
  $\hat{\alpha}_i \leftarrow \frac{1}{M} \sum_{m=1}^{M} s_i^{[m]}$.
end for
return $\{\hat{\alpha}_i\}_{i=1}^{N}$.

Algorithm 1 Classical algorithm to compute influence.

Require: N input data points $\mathcal{D}$, combinatorial dimension k, inlier threshold ϵ, number of iterations M.

for $m = 1, \ldots, M$ **do**
  $\mathbf{z}^{[m]} \leftarrow$ Randomly choose k-tuple from $\mathcal{D}$.
  for $i = 1, \ldots, N$ **do**
    if $f(\mathbf{z}^{[m]} \oplus \mathbf{e}_i) \neq f(\mathbf{z}^{[m]})$ **then**
      $X_i^{[m]} \leftarrow 1$.
    else
      $X_i^{[m]} \leftarrow 0$.
    end if
  end for
end for
for $i = 1, \ldots, N$ **do**
  $\hat{\alpha}_i \leftarrow \frac{1}{M} \sum_{m=1}^{M} X_i^{[m]}$.
end for
return $\{\hat{\alpha}_i\}_{i=1}^{N}$.

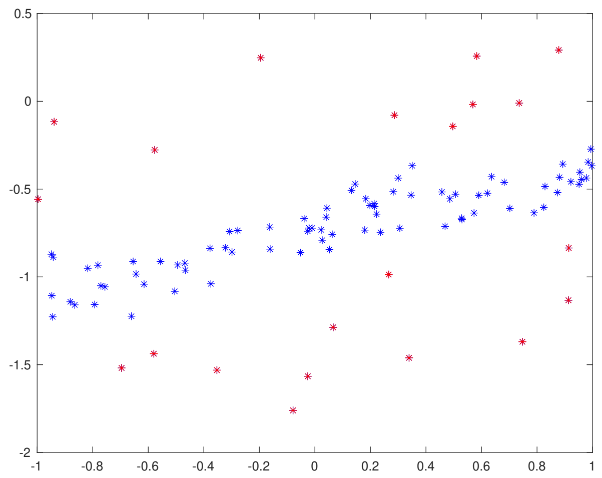
- Compared to the classical algorithm, the quantum algorithm is N times faster.

Analysis

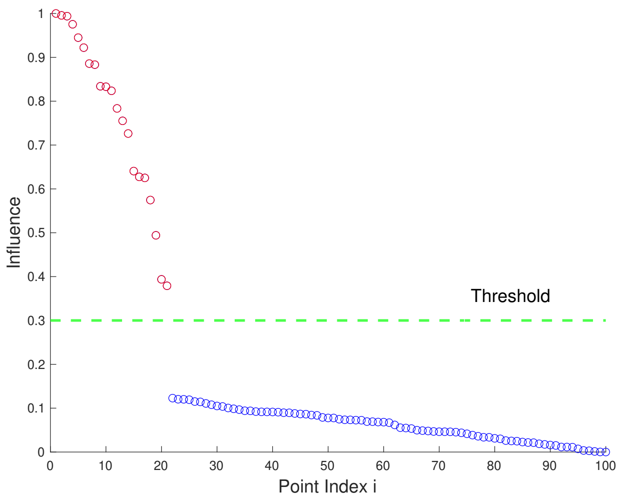
- By regarding each quantum measurement as sampling from a Bernoulli random variable with mean α_i, we can use Hoeffding's inequality to bound the error:

$$Pr(|\hat{\alpha}_i - \alpha_i| < \delta) > 1 - 2e^{-2M\delta^2}$$

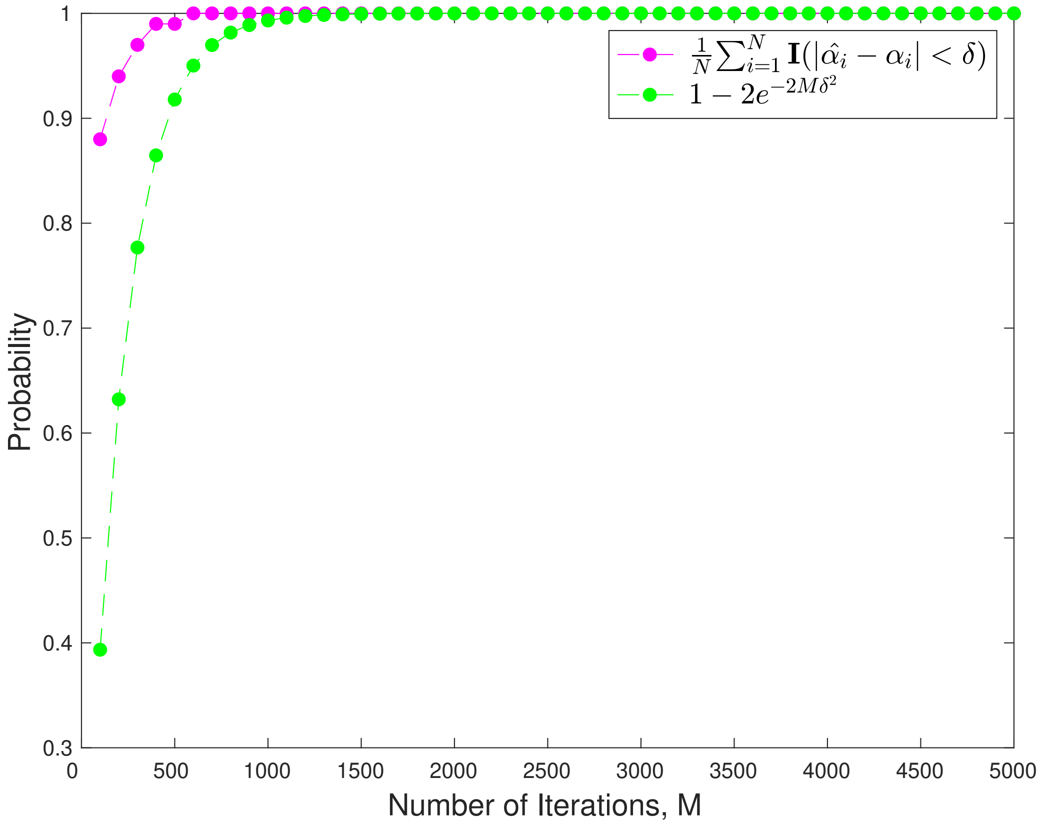
Tat-Jun Chin, David Suter, Shin Fang Ch'ng and James Quach. Quantum Robust Fitting. CoRR abs./2006.06986 (2020)

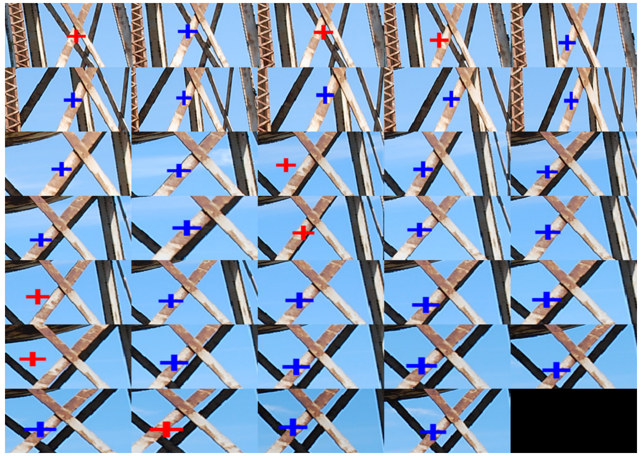


Exact influences

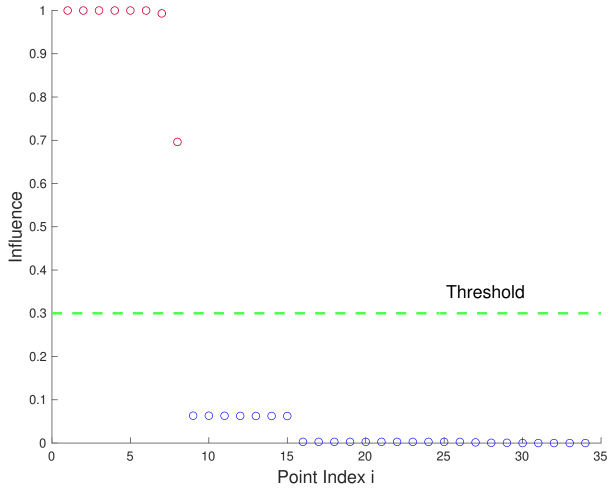


Error of approximate influences over M iterations vs probabilistic bound.

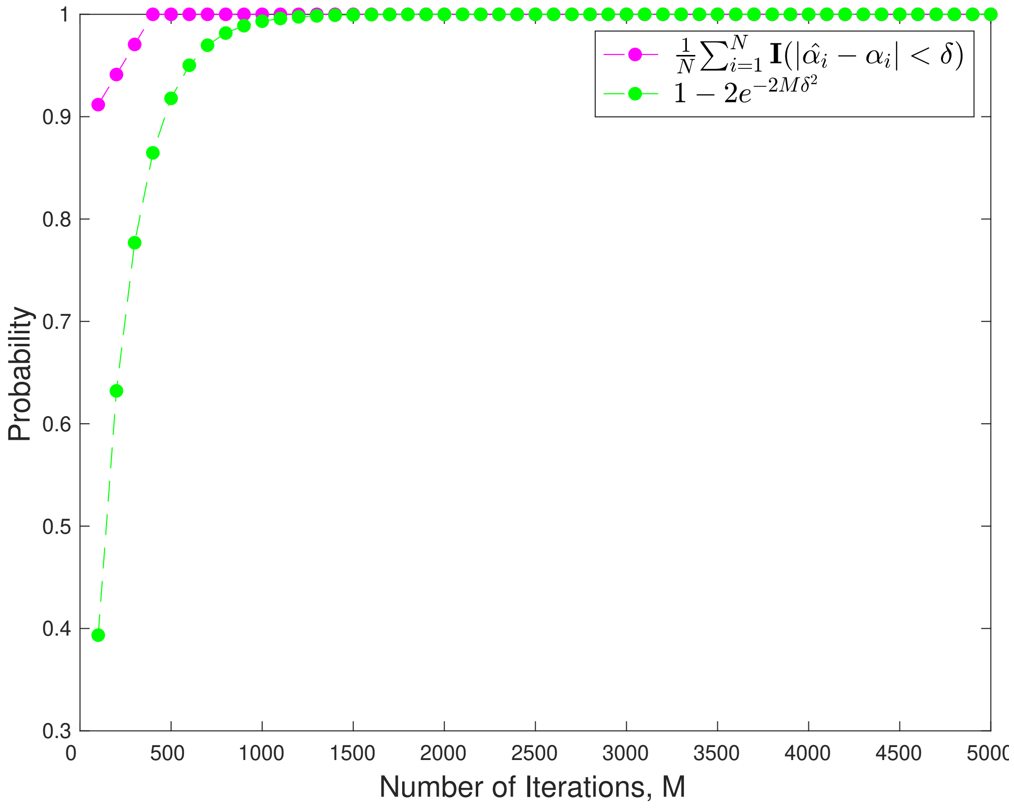


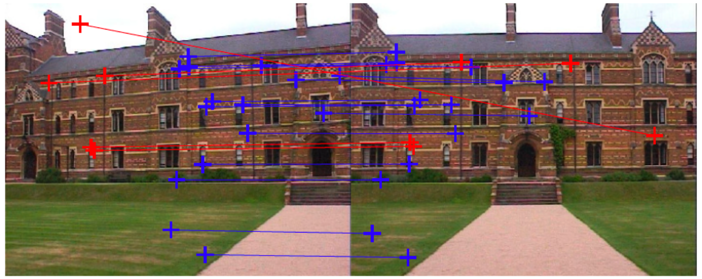


Exact influences



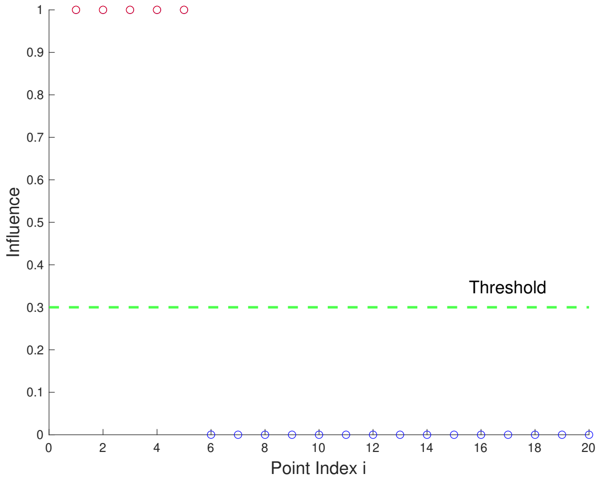
Error of approximate influences over M iterations vs probabilistic bound.





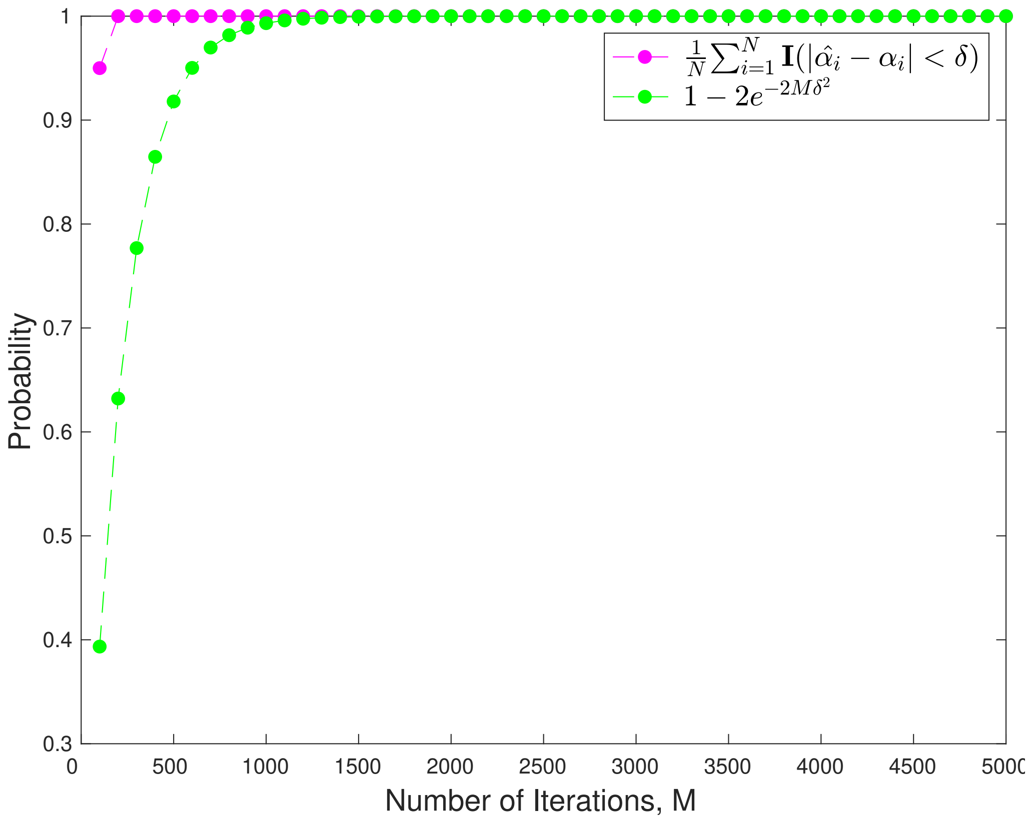
(e) Feature correspondences across two views.

Exact influences



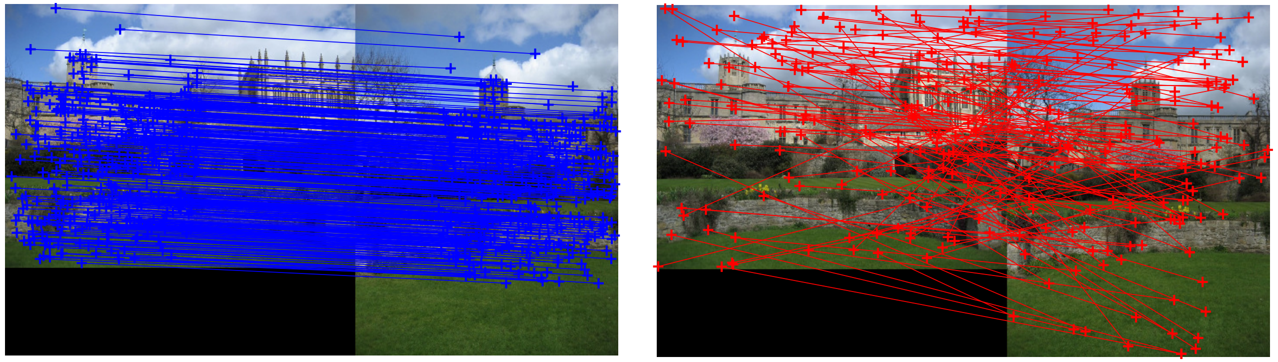
(f)

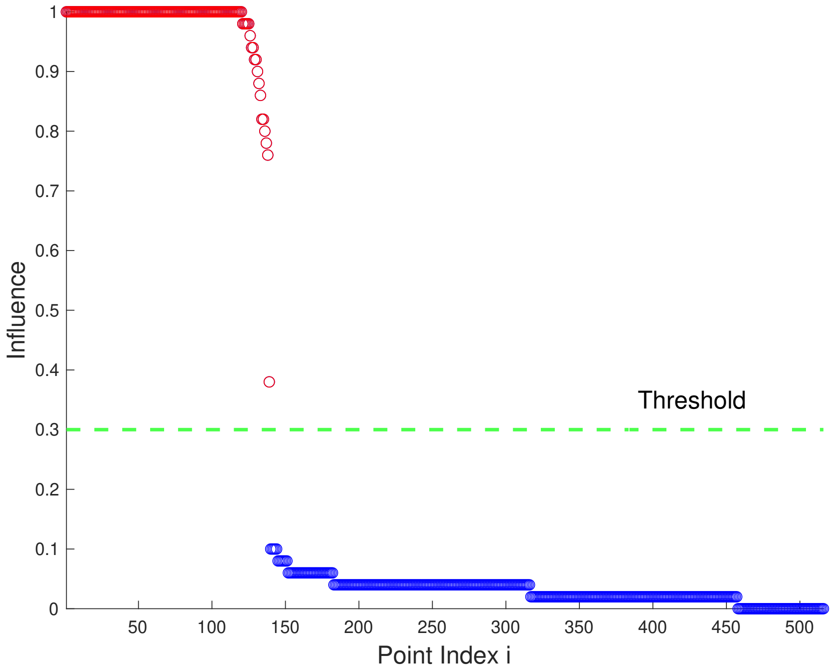
Error of approximate influences over M iterations vs probabilistic bound.



Quantum speed-up

- Homography estimation instance with N = 516 correspondences. The quantum approach will speed-up the (approximate) influence computation by a factor of 516!





Quantum Robust Fitting

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arXiv:2006.06986v1 [cs.CV] 12 Jun 2020

Abstract

Many computer vision applications need to recover structure from imperfect measurements of the real world. The task is often solved by robustly fitting a geometric model onto noisy and outlier-contaminated data. However, recent theoretical analyses indicate that many commonly used formulations of robust fitting in computer vision are not amenable to tractable solution and approximation.

In this paper, we explore the usage of quantum computers for robust fitting. To do so, we examine and establish the practical usefulness of a robust fitting formulation inspired by Fourier analysis of Boolean functions. We then investigate a quantum algorithm to solve the formulation and analyse the computational speed-up possible over the classical algorithm. Our work thus proposes one of the first quantum treatments of robust fitting for computer vision.

1. Introduction

Curve fitting is vital to many computer vision capabilities [14]. We focus on the special case of "geometric" curve fitting [16], where the curves of interest derive from the fundamental constraints that govern image formation and the physical motions of objects in the scene. Geometric curve fitting is conducted on visual data that is usually contaminated by outliers, thus necessitating robust fitting.

To begin, let $\mathcal{M}$ be a geometric model parametrised by a vector $\mathbf{x} \in \mathbb{R}^d$. For now, we will keep $\mathcal{M}$ generic; specific examples will be given later. Our aim is to fit $\mathcal{M}$ onto N data points $\mathcal{D} = \{\mathbf{p}_i\}_{i=1}^N$, i.e., estimate $\mathbf{x}$ such that $\mathcal{M}$ describes $\mathcal{D}$ well. To this end, we employ a residual function

$$r_i(\mathbf{x}) \tag{1}$$

which gives the nonnegative error incurred on the i-th datum $\mathbf{p}_i$ by the instance of $\mathcal{M}$ that is defined by $\mathbf{x}$. Ideally we would like to find an $\mathbf{x}$ such that $r_i(\mathbf{x})$ is small for *all* i.

However, if $\mathcal{D}$ contains outliers, there are no $\mathbf{x}$ where all $r_i(\mathbf{x})$ can be simultaneously small. To deal with outliers, computer vision practitioners often maximise the consensus

$$\Psi(\mathbf{x}) = \sum_{i=1}^N \mathbb{I}(r_i(\mathbf{x}) \leq \epsilon), \tag{2}$$

of $\mathbf{x}$, where ϵ is a given inlier threshold, and $\mathbb{I}$ is the indicator function that returns 1 if the input predicate is true and 0 otherwise. Intuitively, $\Psi(\mathbf{x})$ counts the number of points that agree with $\mathbf{x}$ up to threshold ϵ, which is a robust criterion since points that disagree with $\mathbf{x}$ (the outliers) are ignored [9]. The maximiser $\mathbf{x}^*$, called the maximum consensus estimate, agrees with the most number of points.

To maximise consensus, computer vision practitioners often rely on randomised sampling techniques, i.e., RANSAC [12] and its variants [20]. However, random sampling cannot guarantee finding $\mathbf{x}^*$ or even a satisfactory alternative. In fact, recent analysis [7] indicates that there are no efficient algorithms that can find $\mathbf{x}^*$ or bounded-error approximations thereof. In the absence of algorithms with strong guarantees, practitioners can only rely on random sampling methods [12, 20] with supporting heuristics to increase the chances of finding good solutions.

Robust fitting is in fact intractable in general. Beyond maximum consensus, the fundamental hardness of robust criteria which originated in the statistics community (e.g., least median squares, least trimmed squares) have also been established [4]. Analysis on robust objectives (e.g., minimally trimmed squares) used in robotics [23] also point to the intractability and inapproximability of robust fitting.

In this paper, we explore a robust fitting approach based on "influence" as a measure of outlyingness recently introduced by Suter et al. [22]. Specifically, we will establish

- The practical usefulness of the technique;
- A probabilistically convergent classical algorithm; and
- A quantum algorithm to speed up the classical method, thus realising quantum robust fitting.

Watch this space

by T.-J. Chin

References (incomplete, apologies for missing some work)

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