RANSAC
Traditional Approaches

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RANSAC

**In:** $U = \{x_i\}$ set of data points, $|U| = N$

- $f(S) : S \rightarrow p$ function $f$ computes model parameters $p$ given a sample $S$ from $U$
- $\rho(p, x)$ the cost function for a single data point $x$

**Out:** $p^*$, parameters of the model maximizing the cost function

$k := 0$

Repeat until $P\{\text{better solution exists}\} < \eta$ (a function of $C^*$ and no. of steps $k$)

$k := k + 1$

I. Hypothesis

1. select randomly set $S_k \subset U$, sample size $|S_k| = m$
2. compute parameters $p_k = f(S_k)$

II. Verification

1. compute cost $C_k = \sum_{x \in U} \rho(p_k, x)$
2. if $C^* < C_k$ then $C^* := C_k$, $p^* := p_k$
3. end

- Non-uniform sampling
- Additional constraints
- Randomized verification
- Improving precision
- Potential degeneracy tests
Outline

Non-uniform sampling
   NAPSAC
   PROSAC

Local Optimization (model refinement)

Degenerate configurations

Checking the sample

Randomized verification
Sampling Locally

NAPSAC, ...
Local Neighborhoods

Inliers tend to lie on densely populated manifolds

- Draw a first data point at random
- Draw the rest of the sample from some neighborhood of the initial point

Myatt, Torr, Nasuto, Bishop, Craddock: NAPSAC: High Noise, High Dimensional Robust Estimation - It’s in the Bag. BMVC 2002

Ni, Jin, Dellaert: GroupSAC: Efficient consensus in the presence of groupings. ICCV 2009

Barath, Noskova, Ivashechkin, Matas: MAGSAC++, a fast, reliable and accurate robust estimator. CVPR 2020
(Semi-) Local Geometry

Idea: verify a tentative match “+” by comparing neighboring features

Schmid, Mohr: Local Greyvalue Invariants for Image Retrieval. PAMI 1997
PROSAC
Progressive Sample Consensus
Quality of Correspondences

• Not all correspondences are created equally
• Some are better than others – are correct with higher probability
• Some quality is typically used to select tentative correspondences (similarity, first-to-second distance ratio, etc)

$\begin{align*}
 p \left( \begin{array}{c} \text{Image 1} \\ \text{Image 2} 
\end{array} \right) &= ? \\
 p \left( \begin{array}{c} \text{Red}, \text{Red} \\ \text{Blue}, \text{Blue} 
\end{array} \right) &= ? \\
 p \left( \begin{array}{c} \text{Red}, \text{Red} \\ \text{Red}, \text{Red} 
\end{array} \right) &\leq p \left( \begin{array}{c} \text{Red}, \text{Red} \\ \text{Red}, \text{Red} 
\end{array} \right)
\end{align*}$

Order all correspondences according to the quality

1 2 3 4 5 ... N-2 N-1 N
Evaluate “Better” Samples Earlier

Quality of a sample given by its worst correspondence

Draw all the samples first
Reorder with respect to the quality
Evaluate in the order of the sample quality

+ Evaluates “better” samples first
+ If the quality is uninformative ends up with uniform sampling
- Inefficient
PROSAC Sampling

Drawing samples of $m$ correspondences

$L := m$

Let $T_L$ be the expected number of samples with quality $L$

Draw $\lceil T_L \rceil$ samples containing $m-1$ correspondences “better” than $L$ and $L$

Draw uniformly from $(1 \ldots L-1)$

$L := L + 1$
Stopping Criterion of PROSAC

Tentative correspondences commonly selected by thresholding their quality

Stopping criterion of RANSAC is based the fraction of inliers

Choose a threshold on the fly so that the fraction of inliers is maximized – earlier termination

Problems:
small sets of tentative correspondences (eg sample size) will have large inlier ratios – non-randomness
the ordering often introduces a bias (eg correspondences on a fronto-paralell plane match better)
Example

Epipolar geometry estimation

Executed on all TC

<table>
<thead>
<tr>
<th>Background</th>
<th>(N = 783, \varepsilon = 79%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(I)</td>
</tr>
<tr>
<td>PROSAC</td>
<td>617</td>
</tr>
<tr>
<td>RANSAC</td>
<td>617</td>
</tr>
</tbody>
</table>

Executed on outliers to the background model

<table>
<thead>
<tr>
<th>Mug</th>
<th>(N = 166, \varepsilon = 31%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(I)</td>
</tr>
<tr>
<td>PROSAC</td>
<td>51.6</td>
</tr>
<tr>
<td>RANSAC</td>
<td>52.3</td>
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</table>

Stopping length

Inlier ratio

Too small set of TC – could be random
LO-RANSAC
Locally Optimized RANSAC
RANSAC Recap

Select minimal sample at random
Calculate model parameters that fit the data in the sample
Calculate error function for each data point
Select data that support current hypothesis

Repeat sampling
RANSAC Recap

Repeat sampling

… until you hit an **ALL INLIER** sample

Get inliers consistent with the model

Make sure that there is no better model in the data

This is also **ALL INLIER** sample!
All-inlier Sample

Not every all-inlier sample gives a model consistent with all inliers

Lower number of inliers is detected

RANSAC needs more samples

Post-processing of RANSAC inliers (least squares) often leads to a reasonable model
Local Optimization (LO)

Idea:
Insert optimization step into the hypothesis-verify loop

Use inliers of the LO step to determine the best model

Execute LO for so-far-the-best samples

\[
\sum_{l=1}^{k} P_l = \sum_{l=1}^{k} \frac{1}{l} \leq \int_{1}^{k} \frac{1}{x} \, dx + 1 = \log k + 1
\]

The LO step can be relatively complex, eg:
- least squares fit
- iteratively re-weighted least squares
- inlier re-sampling (inner RANSAC)
- graph-cup

Chum, Matas, Kittler: Locally Optimized RANSAC. DAGM 2003
Stability of LO-RANSAC

Ideally, inliers are always detected as inliers, outliers as outliers

Ideally, inliers have probability one, outliers probability zero

Lebeda, Matas, Chum: Fixing the Locally Optimized RANSAC. BMVC 2012
Stability of LO-RANSAC

Lebeda, Matas, Chum: Fixing the Locally Optimized RANSAC. BMVC 2012
Hierarchical Model Estimation

Sample to estimate an imprecise / approximate model
In LO step, use inliers to the sampled model to estimate the full model

**SPEED-UP** by reducing the SAMPLE SIZE

**EG from 3 Local Affine Frame (LAF) correspondences**

Each region provides 3 points
3 LAFs determine EG
Points are close to each other (low precision)

**Epipolar geometry through Affine Epipolar Geometry**

**LO-RANSAC:**
1. Approximated by affine EG only 2 LAF correspondences needed
2. In LO step, estimate the full EG from multiple correspondences

*Chum, Matas, Obdržálek: Enhancing RANSAC by Generalized Model Optimization. ACCV 2004*

*Pritts, Chum, Matas: Approximate Models for Fast and Accurate Epipolar Geometry Estimation. IVCNZ 2013*
## How Many Samples?

### $I / N [%]$

<table>
<thead>
<tr>
<th>Size of the sample $m$</th>
<th>15%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>70%</th>
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<tr>
<td>2</td>
<td>132</td>
<td>73</td>
<td>32</td>
<td>17</td>
<td>10</td>
<td>4</td>
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<tr>
<td>4</td>
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<td>1871</td>
<td>368</td>
<td>116</td>
<td>46</td>
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<tr>
<td>7</td>
<td>$1.75 \cdot 10^6$</td>
<td>$2.34 \cdot 10^5$</td>
<td>$1.37 \cdot 10^4$</td>
<td>1827</td>
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<td>35</td>
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<tr>
<td>8</td>
<td>$1.17 \cdot 10^7$</td>
<td>$1.17 \cdot 10^6$</td>
<td>$4.57 \cdot 10^4$</td>
<td>4570</td>
<td>765</td>
<td>50</td>
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<tr>
<td>12</td>
<td>$2.31 \cdot 10^{10}$</td>
<td>$7.31 \cdot 10^8$</td>
<td>$5.64 \cdot 10^6$</td>
<td>$1.79 \cdot 10^5$</td>
<td>$1.23 \cdot 10^4$</td>
<td>215</td>
</tr>
<tr>
<td>18</td>
<td>$2.08 \cdot 10^{13}$</td>
<td>$1.14 \cdot 10^{13}$</td>
<td>$7.73 \cdot 10^9$</td>
<td>$4.36 \cdot 10^7$</td>
<td>$7.85 \cdot 10^5$</td>
<td>1838</td>
</tr>
<tr>
<td>30</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$1.35 \cdot 10^{16}$</td>
<td>$2.60 \cdot 10^{12}$</td>
<td>$3.22 \cdot 10^9$</td>
<td>$1.33 \cdot 10^5$</td>
</tr>
<tr>
<td>40</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$2.70 \cdot 10^{16}$</td>
<td>$3.29 \cdot 10^{12}$</td>
<td>$4.71 \cdot 10^6$</td>
</tr>
</tbody>
</table>

*affine EG 2 LAF + LO*

EG 7pt algorithm
Degenerate Configurations
Data with Outliers
Degenerate Data with Outliers

Infinite number of models passes through a degenerate configuration

Sample from the degenerate configuration and outlier(s) is a problem

It has higher than random support (the degenerate whole of configuration)
The Dominant Plane Problem

Given many (almost) coplanar points, a few points off the plane, and (possibly) many outliers estimate the epipolar geometry (EG), if possible.

RANSAC run with 95% confidence actually finds the inliers on the lamppost in only 17% of executions.
Dealing with Degenerate Configurations

Model with high number of inliers found
Dealing with Degenerate Configurations

Frahm, Pollefeys: RANSAC for (Quasi-)Degenerate Data (QDEGSAC). CVPR 2006
DEGENSAC

Assumption: Degenerate configuration hit by an all-outlier model - unlikely

The sample spans the degenerate configuration

Analyze the sample and the inliers for degenerate configuration – deterministic (no new RANSAC)

Chum, Werner, Matas: Epipolar Geometry Estimation Unaffected by the Dominant Plane. CVPR 2005
Dominant Plane

RANSAC draws minimal samples of 7 correspondences to hypothesize the epipolar geometry.

When dominant plane is present, samples with more than 4 coplanar correspondences often appear.

7 or 6 coplanar correspondences: the sample is consistent with a family of fundamental matrices. This case is easily be detected.

It was shown that the epipolar geometry hypothesized from 5 coplanar points from the dominant plane and 2 off the plane (a so called H-degenerate sample) has a large RANSAC support.

If at least one of the off-plane correspondences is an outlier, the EG is incorrect.

*Chum, Werner, Matas*: Epipolar Geometry Estimation Unaffected by the Dominant Plane. CVPR 2005
Different Solutions by RANSAC

Similar number of inliers, very different geometries

- Manually selected correspondences
DEGENSAC

Core of the algorithm:

1. Draw samples of 7 correspondences and estimate 1-3 fundamental matrices by the 7-point algorithm
2. Test samples with the largest support so far for H-degeneracy
3. When H-degeneracy was detected, use plane-and-parallax algorithm

Note: the plane-and-parallax needs to draw samples of only 2 correspondences to hypothesize EG, therefore its complexity is negligible compared to RANSAC where 7 correspondences are drawn into a sample

*Irani, Anadan*: Parallax geometry of pairs of points for 3D scene analysis. ECCV 1997
Stability of the Results

How many times will a correspondence be labeled as an inlier, if we run the experiment 100 times?

Off the plane correspondences

RANSAC

DEGENSAC
Non-minimal Samples

**Finding lines in 3D via plane fitting**
(the line being a degenerate configuration of the plane)

**Assumption:** Outliers are uniformly distributed in 3D

**Algorithm:**
- Draw samples of three points & fit a plane to them
- Calculate the support of the plane
- Check the three lines defined by the points of samples with the largest support so far

**Advantage:** it is more likely to draw at least 2 inliers out of 3 than 2 out of 2 when fitting a line

**Disadvantage:** does not work if all points occupy a single plane
  (all outlier model - off the line - hits the degenerate configuration)
Non-minimal Samples for Homography

Finding homographies via EG fitting

Ratio of the number of samples needed to estimate homography by drawing samples of four and seven correspondences respectively

Probability of drawing 4 inlierers

\[ P_1 = \varepsilon_H^4 \]

Probability of drawing a 7-tuple with at least 5 inliers

\[ P_2 = \sum_{i=5}^{7} \binom{7}{i} \varepsilon_H^i (1 - \varepsilon_H)^{7-i} \]
Sample Checks

Oriented Constraints
Constraints

Constraints on the model parameters that do not reduce the model complexity exist in some problems. Model complexity - the number of data points that define the model ‘uniquely’ (sample size) e.g. line segment of a fixed length (2 points are still needed)

Possible approaches:

Sample so that only hypotheses satisfying the constraints are generated
   - may be time demanding or even intractable

Ignore additional constraints
   - typically works OK since models that are not allowed do not have sufficient support

Check the constraints before verification step
   - can save a lot of time on verification
   - e.g. line segment of a fixed length, sampled points are further than the length
Oriented Constraints for Two-View Geometry

Cameras can only observe points in front of the camera – rays are half-lines


Laveau, Faugeras: 3-D scene representation as a collection of images. ICPR 1994

Werner, Pajdla: Oriented matching constraints. BMVC 2001
Oriented Constraints

Homography

Epipolar Geometry

Need to preserve convexity
Compare signs of two determinants (four times)
Can be performed prior to computing the H matrix

Estimate F using (2) – up to 3 solutions
Test all 7 points in the sample using (1)
Takes 27 – 81 floating point operations
Discards 5% – 45% of models

Chum, Werner, Matas: Epipolar Geometry Estimation via RANSAC Benefits from the Oriented Epipolar Constraint. ICPR 2004
Randomized Verification
Time Complexity of RANSAC

I. Hypothesis

1. select randomly set $S_k \subset U$, sample size $|S_k| = m$
2. compute parameters $p_k = f(S_k)$

II. Verification

3. compute cost $C_k = \sum_{x \in U} \rho(p_k, x)$
4. if $C^* < C_k$ then $C^* := C_k$, $p^* := p_k$

Overall time spent in $k$ samples

\[ t = k(t_M + \overline{m}_S \overline{N}) \]
Randomized Verification

I. Hypothesis

(1) select randomly set \( S_k \subset U \), sample size \(|S_k| = m\)

(2) compute parameters \( p_k = f(S_k) \)

II.a Pre-verification

(3) reject weak hypotheses after a few randomly selected verification

II.b Verification

(4) compute cost \( C_k = \sum_{x \in U} \rho(p_k, x) \)

(5) if \( C^* < C_k \) then \( C^* := C_k, p^* := p_k \)

Overall time spent in \( k \) samples

\[ t = k(t_M + \overline{m}_S t_V) \]
Pre-verification Errors

False positive

Incorrect model is not rejected
Decreases efficiency (increases the average number of verifications)

False negative

Correct model is rejected with probability $\alpha$
Decreases confidence in the solution (good model could have been discarded)

Increase # sample by factor $1 / (1 - \alpha)$ to guarantee RANSAC confidence in the solution

$$\frac{k}{1 - \alpha} (t_M + \bar{m}_S t_V) < k(t_M + \bar{m}_S N)$$

Efficient randomized verification strategy
RANSAC with the $T_{d,d}$ Test

Verify $d << N$ data points, reject the model if not all $d$ data points are consistent with the model.

For most problems and inlier ratios, the optimal value is $d = 1$, that is $T_{1,1}$.

<table>
<thead>
<tr>
<th>d</th>
<th>samples</th>
<th>models</th>
<th>tests</th>
<th>inliers</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1866</td>
<td>4569</td>
<td>6821</td>
<td>600</td>
</tr>
<tr>
<td>1</td>
<td>4717</td>
<td>11536</td>
<td>16311</td>
<td>600</td>
</tr>
<tr>
<td>2</td>
<td>11849</td>
<td>28962</td>
<td>33841</td>
<td>600</td>
</tr>
</tbody>
</table>

Synthetic experiment on 1500 correspondences, 40% inliers, 30 repetitions

*Chum, Matas:* Randomized RANSAC with $T_{d,d}$ Test. BMVC 2002
Bail out Test

After each verification, check the current number of inliers $I_n$ against the expected number of inliers of a good model

Exact probabilities intractable

Approximate by a Normal distribution

\[ I_n \sim N(\mu, \sigma^2) = N\left(n\varepsilon_i, n\varepsilon_i(1 - \varepsilon_i)\left(\frac{N - n}{N - 1}\right)\right) \]

Test with a threshold $I_n^{\text{min}} = \left[ n\varepsilon_i - z_c \sigma \right]$, where $z_c$ is a quantile of the Normal distribution

*Capel*: An Effective Bail-Out Test for RANSAC Consensus Scoring. BMVC 2005
WaldSAC: Optimal Randomized Verification

Sequential decision making

- Accept
- Take another observation
- Reject
- Discard the model
- Never taken – always verify all
- Verify next data point

A. Wald: Sequential Analysis. Dover, 1947

Optimal strategy in sequential decision making is based on sequential probability ratio test (SPRT)

Single parameter $A$:
- Probability of rejecting a good model: $\alpha = 1/A$
- Average number of measurements before rejection: $C \log(A)$

$t(A) = \frac{k}{(1 - 1/A)}(t_M + \bar{m}C \log A)$

Chum, Matas: Optimal Randomized RANSAC. TPAMI 2008

Randomized verification:

Experimental Comparison

Synthetic data, 30% inliers, varying N, epipolar geometry estimation

SPRT* optimal setting, fraction of inliers known beforehand
SPRT estimated fraction of inliers updated on the fly
Experimental Comparison

Synthetic data, 30% inliers, varying N, epipolar geometry estimation
Experimental Comparison

Synthetic data, 30% inliers, varying N, epipolar geometry estimation
Problem: how to meaningfully parametrize high-dimensional models such as fundamental matrix even homography is problematic – image points can be sent to infinity (or beyond)

Korman, Litman: Latent RANSAC. CVPR 2018
Fixed Time Budget

Real time system
Task: find the best model within given fixed time (not a confidence in the solution)
Solution: fixed number of hypotheses and fixed number of verifications

Idea: Better hypotheses are likely to be also better on a subset of data

Animation courtesy of David Nistér
Preemptive RANSAC

Observations

Animation courtesy of David Nistér

Nister: Preemptive RANSAC for Live Structure and Motion Estimation. ICCV 2003
Conclusions

Non-uniform sampling can speed-up RANSAC significantly
  - Locality
  - Quality

Local Optimization stabilizes the results and reduces the number of samples
  - Approximate models bring significant speed-up

Degenerate Configurations
  - Avoid wrong models with high number of inliers
  - Efficiently detect models as degenerate configurations of higher dimensional models

Checking samples of additional constraints can avoid the verification
  - Oriented constraints in two-view problems

Randomized verification
  - Speed-up for problems with large number of correspondences

Most of the algorithms implemented in USAC

Thanks!
References

**Barath, Noskova, Ivashechkin, Matas**: MAGSAC++, a fast, reliable and accurate robust estimator. CVPR 2020

**Capel**: An Effective Bail-Out Test for RANSAC Consensus Scoring. BMVC 2005

**Chum, Matas**: Optimal Randomized RANSAC. TPAMI 2008

**Chum, Matas**: Matching with PROSAC - Progressive Sampling Consensus. CVPR 2005

**Chum, Matas, Obdrzalek**: Enhancing RANSAC by Generalized Model Optimization, ACCV 2004

**Chum, Matas, Kittler**: Locally Optimized RANSAC. DAGM 2003

**Chum, Matas**: Randomized RANSAC with $T_d,d$ Test. BMVC 2002

**Chum, Werner, Matas**: Epipolar Geometry Estimation Unaffected by the Dominant Plane. CVPR 2005

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**P. M. Lee**: Sequential Probability Ratio Test. Univ. of York, 2007
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