19th Computer Vision Winter Workshop
Zuzana Kúkelová and Jan Heller (eds.)
Křtiny, Czech Republic, February 3–5, 2014

A Novel Fuzzy C-Means Based Defuzzification Approach with an Adapted Minkowski Distance

Abder-Rahman Ali, Adélaïde Albouy-Kissi, Antoine Vacavant,
Manuel Grand-brochier, Jean-Yves Boire

Clermont Université, Université d'Auvergne, ISIT, BP10448, F-63000 Clermont-Ferrand, France
CNRS, UMR6284, BP10448, F-63000 Clermont-Ferrand, France
abder-rahman.ali@etu.udamail.fr, adelaide.kissi@udamail.fr,
antoine.vacavant@udamail.fr, manuel.grand-brochier@udamail.fr,
j-yves.boire@udamail.fr

Abstract *In this paper, we present a novel defuzzification approach by feature distance minimization with an adapted Minkowski distance. The proposed approach aims at providing a segmented image through the defuzzification of the fuzzy partitions of the original image. A mathematical derivation of which best pixel would be added/removed to/from the alpha-region of the crisp set is described. And, an evaluation of the proposed approach is carried out.*

1 Introduction

A gray-scale object may be considered as a fuzzy set, where each pixel is defined with a membership function [1]. The membership values can be of real interest to conduct the segmentation of an image into regions of interest, especially that they are required in fuzzy clustering approaches. During this process, one should integrate more features that will represent these regions by their geometrical properties as shape, area, perimeter, *etc*. Amongst a whole set of *N* possibly informative measurements, feature selection aims at selecting a subset of *n* features from the given set of *N* measurements, where $n<N$ [2].

Preserving some relevant features of the original object in the fuzzy discrete representation aids in the recovering of a crisp object when defuzzifying that representation. Defuzzification by feature distance minimization achieves such preservation of the relevant features in the fuzzy discrete representation. In order to get an output object that resembles the original crisp object, the selection of features that will be included in the defuzzification should take into account their relevance (*i.e.* shape preservation and application), and how well is their preservation in fuzzification. Intuitively, points with high membership degrees to the fuzzy object should be included in its crisp representation, and those with low membership degrees should be assigned to the background. This can be achieved by using the membership degree values of the points as features in the distance measure [3]. Here, one can work on *similarity space* instead of feature space (*i.e.* for the purpose of clustering). Thus, if one can find a similarity measure derived from the object features which is considered appropriate for the problem domain, then a single number can capture the essential closeness of a given pair of objects, and any further analysis can be based only on those numbers [4]. The effectiveness of object recognition is highly dependent on the accurate identification of shapes of clusters. which are determined by the choice of the distance measure [5]. For example, the Euclidean distance is often used to reflect dissimilarity between two patterns and is known to work well when all clusters are spheroids or when all clusters are well separated [6]. The use of the Minkowski dissimilarity measure in the paper was due to its allowance of varying the assumptions of the shape of the clusters by varying the order *m*. The most often used value is $m = 2$ that assumes a circular cluster shape. Using $m = 1$ assumes that the clusters are in the shape of a (rotated) square in two dimensions or a diamond like shape in three or more dimensions. For $m = \infty$, the clusters are assumed to be in the form of a box with sides parallel to the axes [7].

In this paper, we present a novel defuzzification approach by feature distance minimization with an adapted Minkowski distance. The idea is based on the use of floating point search for finding the best crisp set from Fuzzy C-Means clustering using Minkowski distance.

The novelty of the proposed approach lies in the use of a new defuzzification that embeds an improved Minkowski distance, which takes into account both the membership degree values of the elements of the fuzzy set, and the spatial relationships.

The paper is organized as follows: Section 2 gives an overview of different dissimilarity measures, and the reasons behind using Minkowski distance. Section 3 introduces notations used in the paper, fuzzy and crisp sets, and Fuzzy C-Means. In Section 4, we describe the proposed approach, combining Fuzzy C-Means, the new defuzzification process, and an improved Minkowski distance. We evaluate the proposed approach in Section 5, and conclude the paper in Section 6.

2 Dissimilarity Measures

Given two sequences of measurements from $X = \{x_i : i = 1, ..., n\} \in \mathbb{R}^n$ and $Y = \{y_i : i = 1, ..., n\} \in \mathbb{R}^n$ such that *n* is the size of the input image. The dissimilarity between *X* and *Y* is a measure that quantifies the independency between the sequences [8], For the purpose of this paper, we assume that *X* and *Y* represent the fuzzy set and the crisp set, respectively, provided that x_i and y_i represent the membership degrees and the pixel intensities, respectively.

The term *distance* is often used informally to refer to a dissimilarity measure *M* derived from the characteristics describing the objects (*i.e*. Euclidean distance) [9].

A *metric M(X,Y)* is considered a dissimilarity measure if a higher value is produced as corresponding values in *X* and *Y* become less dependent, and which satisfies the following for all $X \in \mathbb{R}^n$ and $Y \in \mathbb{R}^n$ [8]:

- Non-negativity: $M(X, Y) \geq 0$;
- Reflexivity: $M(X, Y) = 0$ *if an only if* $X = Y$;
- Symmetry: $M(X, Y) = M(Y, X)$;
- Triangle inequality: $M(X, Y) + M(Y, Z) \geq M(X, Z)$.

Thus, in order to measure dissimilarity, one of the parameters that can be used is *distance*. This category of measures is known as separability, divergence, or discrimination measures [9].

In clustering analysis, choosing the appropriate dissimilarity measure is required. The most commonly used measures in clustering analysis are: (1) Euclidean distance, (2) Manhattan distance, (3) Minkowski distance, and (4) Mahalanobis distance.

Euclidean distance $d_2(\mathbf{x}_i, \mathbf{x}_j)$, where $\mathbf{x}_i$ and $\mathbf{x}_j$ are p-dimensional features, with $p \in \mathbb{N}_+^*$ is the most popular measure of dissimilarity, and the most common distance metric used. It is the usual manner in which distance is measured in the real world [10], and is defined by [11]:

$$d_2(\mathbf{x}_i, \mathbf{x}_j) = \sqrt{\sum_{k=1}^{p} (\mathbf{x}_{ik} - \mathbf{x}_{jk})^2} \tag{1}$$

Euclidean distance works when each cluster has a shape of a hyper-sphere in space, but, has poor performance when the cluster has a shape of a hyper-ellipsoid [11].

The drawback of Euclidean distance is that it ignores the similarity between attributes, as each attribute is treated as totally different from all other attributes, and does not work well in high dimensions and for categorical variables [10]. In order to overcome this drawback, Gustafon and Kessel distance can be utilized, which is able to discriminate ellipsoidal cluster shapes [4].

Manhattan distance gets its name from the rectangular grid patterns of streets in midtown Manhattan, and is defined by [7]:

$$d_{Manhattan}(\mathbf{x}_i, \mathbf{x}_j) = \sum_{k=1}^{p} |\mathbf{x}_{ik} - \mathbf{x}_{jk}| \tag{2}$$

In some situations, this metric is more preferable to Euclidean distance, since the distance along each axis is not squared, and thus, a large difference in one dimension will not dominate the total distance [7].

Mahalanobis distance is the distance between an observation and the centre for each group in a p-dimensional space defined by *p* variables and their covariance. Thus, a small value of Mahalanobis distance increases the chance of an observation to be closer to the groups center, and the more likely it would be assigned to that group [14]. Mahalanobis distance is defined by [14]:

$$d_{Mahalanobis}(\mathbf{x}_i, \mathbf{x}_j) = \sqrt{(\mathbf{x}_i - \mathbf{x}_j)^T \sum^{-1} \mathbf{x}_i - \mathbf{x}_j} \tag{3}$$

where $\sum^{-1}$ is the inverse covariance matrix.

If there are two non-correlated variables, the Mahalanobis distance between the points of the variables in a 2D scatter plot is the same as the Euclidean distance [15].

Unlike most other distance measures, Mahalanobis distance is not dependent upon the scale on which the variables are measured since it is normalized [14].

Minkowski distance is a generalization of Euclidean and Manhattan distances [12], and is defined by [11]:

$$d_m(\mathbf{x}_i, \mathbf{x}_j) = \sqrt[m]{\sum_{k=1}^{p} |\mathbf{x}_{ik} - \mathbf{x}_{jk}|^m} \tag{4}$$

where *m* is a real number, such that $m \geq 1$. When $m = 1$, it represents the Manhattan distance, and when $m = 2$, it represents the Euclidean distance [13].

Minkowski distance provides a concise, parametric distance function that generalizes many of the distance functions used in the literature. The advantage of using this distance is that mathematical results can be shown for the whole class of distance functions, and the user can adapt the distance function to suit the needs of the application by modifying the Minkowski parameter *m* [12].

3 Background Notations

In this section, the notations of crisp sets, fuzzy sets, core of a fuzzy set, support of a fuzzy set, $\alpha - cut$ of a fuzzy, and Fuzzy C-Means will be explained (sets are usually denoted by upper case letters, and their members by lower case letters [16]).

A crisp set *A* in the universe of discourse *U* has a binary membership function, and thus, has no uncertainty. It is defined as a set of ordered pairs [17]:

$$A = \{(x, \phi_A(x)) \mid x \in U\} \tag{5}$$

where $\phi_A(x)$ is the binary membership function:$\phi_A(x) = 1$ if $x \in A$, and $\phi_A(x) = 0$ if $x \notin A$. $\phi_A(x) \in \{0, 1\}$ [17].

A fuzzy set $A^{\sim}$ in the universe of discourse *U* has a fuzzy membership function, in which fuzziness (uncertainty) exists. It is defined as a set of ordered pairs [17]:

$$A^{\sim} = \{(x, \mu_{A^{\sim}}(x)) \mid x \in U\} \tag{6}$$

where $\mu_{A^{\sim}}$ is the fuzzy membership function that maps *x* to a membership degree between 0 and 1. $\mu_{A^{\sim}}(x) \in [0,1]$ [17].

The *core* of a fuzzy set $A^{\sim}$ in the universe of discourse *U* is a crisp set that contains all the elements of *U* that have membership values in $A^{\sim}$ equal to 1, that is [18]:

$$core\,(A^{\sim}) = \{(x \in U) \mid \mu_{A^{\sim}}\,(x) = 1\} \tag{7}$$

The *support* of a fuzzy set $A^{\sim}$ in the universe of discourse *U* is a crisp set that contains all the elements of *U* that have nonzero membership values in $A^{\sim}$, that is [19]:

$$supp\,(A^{\sim}) = \{(x \in U) \mid \mu_{A^{\sim}}\,(x) > 0\} \tag{8}$$

An $\alpha - cut$ a fuzzy set $A^{\sim}$ is a crisp set $A^{\sim}_{\alpha}$ that contains all the elements in *U* that have membership values in *A* greater than or equal to α, that is [19]:

$$A^{\sim}_{\alpha} = \{x \in U \mid \mu_{A^{\sim}}\,(x) \geq \alpha\} \tag{9}$$

Let $X = \{x_1, ..., x_b, ..., x_n\}$ be a set of *n* objects, and $V = \{v_1, ..., v_b, ..., v_c\}$ be a set of *c* centroids in a *p*-dimensional feature space. The Fuzzy C-Means partitions *X* into *c* clusters by minimizing the following objective function [4]:

$$J = \sum_{j=1}^{n} \sum_{i=1}^{c} (\mathbf{u}_{ij})^m \left\| \mathbf{x}_j - \mathbf{v}_i \right\|^2 \tag{10}$$

where $1 \leq \ m \leq \infty$ is the *fuzzifier*, $\mathbf{v}_i$ is the i^{th} centroid corresponding to cluster β_i, $\mathbf{u}_{ij} \in [0,1]$ is the fuzzy membership of the pattern $\mathbf{x}_j$ to cluster β_i, and $\|.\|$ is the distance norm such that,

$$\mathbf{v}_i = \frac{1}{n_i} \sum_{j=1}^{n} (\mathbf{u}_{ij})^m x_j \quad \text{where} \quad n_i = \sum_{j=1}^{n} (\mathbf{u}_{ij})^m \tag{11}$$

and

$$\mathbf{u}_{ij} = \frac{1}{\sum_{k=1}^{c} \left(\frac{d_{ij}}{d_{kj}}\right)^{\frac{2}{m-1}}} \quad \text{where} \quad d_{ij}^2 = \left\| \mathbf{x}_j - \mathbf{v}_i \right\|^2 \tag{12}$$

FCM starts by randomly choosing *c* objects as centroids (means) of the *c* clusters. Memberships are calculated based on the relative distance (Euclidean distance) of the object $\mathbf{x}_j$ to the centroids using Eq. (12). After the memberships of all objects have been found, the centroids of the clusters are calculated using Eq. (11). The process stops when the centroids from the previous iteration are identical to those generated in the current iteration [4].

4 Defuzzification by Feature Distance Minimization

Defuzzification by distance minimization *D(A)* of a fuzzy set *A* on a reference set *X*, with respect to the distance *d*, is [3]:

$$D\,(A) \in \{C \in P\,(X) \mid d\,(A, C) = min_{B \in P(X)} \ [d\,(A, B)\ ]\} \tag{13}$$

where *P(X)* is the set of crisp subsets of a power set, and *d* is the Minkowski distance between the vector representations of both the fuzzy set and the crisp subset, such that the fuzzy set is represented by its membership values that serve as separate features for every individual pixel [3].

This type of defuzzification of a fuzzy segmented image can be seen as an alternative to crisp segmentation, where, instead of crisp segmentation of gray level images, fuzzy segmentation is performed, and then followed by defuzzification [3].

4.1 Minimizing the distance between fuzzy and crisp sets

Floating search methods - SFFS (Sequential Forward Floating Selection) and SBFS (Sequential Backward Floating Selection) - are considered a development of the *l-r* algorithm, in which the values (features) of *l* and *r* are allowed to *float*, that is, they may change at different stages of the selection procedure [20]. Thus, floating search makes it flexible to change features so as to approximate the optimal solution as much as possible [21].

In *FCM-FloatingSearch* (see algorithm below), the enhancement of the floating search methods is the introduction of the fuzzy membership values and the neighbourhood information. Provided that both adding and removing pixels during region growing is allowed to happen [3].

FCM-FLOATINGSEARCH()

call $fuzzycmeans$;
input $grayscale\ image$;
$fuzzySet \leftarrow fuzzy\ segmented\ image$;
$membershipMatrix \leftarrow \mu$; $/*\ \mu\ is\ the\ degree\ of$
$membership\ */$
$n \leftarrow area(support(fuzzySet)\backslash$
$\alpha - cut(fuzzySet)$;
$C_0 = \alpha - cut(fuzzySet)$;
for $i = 1$ **to** n
  do
    $C_i \leftarrow \phi$; $/*\ empty\ set\ */$
$k \leftarrow 0$;
$/*\ add\ best\ pixel\ that\ minimizes\ the\ distance$
$between\ the\ fuzzy\ set\ and\ the\ crisp\ set\ */$
while $k < n$
  do
    $among\ the\ pixel\ p\ being\ 4 - neighbourhood$
    $of\ C_k\ and\ not\ in\ C_k$;
    if $pixel\ p \in support(fuzzySet)$
      then
        $p \leftarrow degree\ of\ membwership$;
    $select\ the\ pixel\ p\ with\ the\ highest\ degree$
    $of\ membership$; $/*\ minimizes$
    $d_m^{\sim}\,(fuzzySet, C_k \cup \{p\})\ */$
    $C_{new} \leftarrow Ck\ \cup \{p\}$;
    if $C_{k+1} \leftarrow 0 \parallel d_m^{\sim}\,(fuzzySet, C_{new}) <$
      $d_m^{\sim}\,(fuzzySet, C_{k+1})$
      then
        $C_{k+1} \leftarrow C_{new}$;
    $k \leftarrow k + 1$;

```
/* possibly remove pixel */
do
change ← false;
among the pixels p being 4 − neighbourhood of
~ C_k and not in α − cut(fuzzySet); /* ~ C_k is
 the complementof C_k */
if pixels ∈ C_k
   then p ← degree of membership;
select the pixel p with lowest degree of
membership; /* minimizes d~_m(fuzzySet,
C_k\{p}) */
C_new ← C_k \ {p};
k ← k − 1;
change ← true;
while change && k > 0
```

In each iteration of the *FCM-FloatingSearch* algorithm, either for adding or removing a pixel, that selected pixel would be the pixel being a 4-neighbourhood of a crisp set C_k, and the one that minimizes the distance between the fuzzy set (fuzzy partition) and the crisp set (segmented image), including the added pixel in the iteration where we add pixels, and between the fuzzy set and the crisp set excluding the removed pixel in the iteration where we would possibly remove pixels. Fig.1 depicts the *FCM-FloatingSearch* algorithm general process. In sections 4.2 and 4.3, we show what would be considered the best added/removed pixel that minimizes the distance between the fuzzy set and the crisp set.

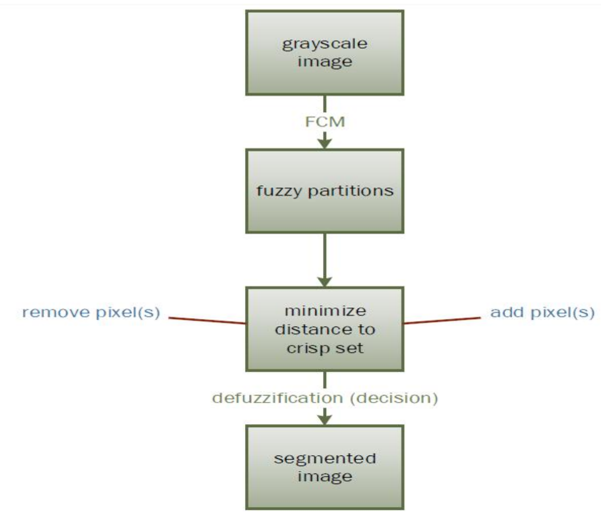

Figure 1: FCM-FloatingSearch

4.2 Adding best pixel

The pixel to be added to the crisp set at each iteration of the floating search algorithm has to meet three conditions: (1) belonging to the support and not the $\alpha - cut$ of the fuzzy set, (2) being a 4-neighbourhood of the crisp set, and (3) minimizing the distance between the fuzzy set and the crisp set including the added pixel.

The Minkowski distance between the fuzzy set f (*i.e.* fuzzy segmented image) and the crisp set C_k at iteration k is:

$$d_m\left(C_k, f\right) = \sqrt[m]{\sum_{i=1}^{length(C_k)} \mid C_k\left(i\right) - f\left(i\right) \mid^m} \tag{14}$$

After adding a pixel p_i to the crisp set, the Minkowski distance between f and the new crisp set $C_{new}\left(C_{k\cup\{p_i\}}\right)$becomes:

$$d_m\left(C_{new}, f\right) = \sqrt[m]{\sum_{i=1}^{length(C_k)} \mid C_{new}\left(i\right) - f\left(i\right) \mid^m} \tag{15}$$

Eq. (15) can be rewritten as:

$$d_m\left(C_{new}, f\right) = \Big(\mid C_{new}\left(p_i\right) - f\left(p_i\right) \mid^m + \sum_{i=1;i\neq p_i}^{length(C_k)} \mid C_k\left(i\right) - f\left(i\right) \mid^m \Big)^{1/m} \tag{16}$$

The difference between C_k and C_{new} is only in the element at location p_i. Thus, in order to include p_i in C_k, $|C_k\left(p_i\right) - f\left(p_i\right)|^m$ would be subtracted from $|C_{new}\left(p_i\right) - f\left(p_i\right)|^m$, and the condition $j \neq p_i$ in the summation of Eq. (16) would be removed.

As the singleton pixel *pi* is added to C_{new}, $C_{new}\left(p_i\right) = 1$, and the corresponding location in $C_k\left(p_i\right) = 0$. By substituting those values , we get:

$$d_m^{\sim}\left(C_{new}, f\right) = \Big(\mid 1 - f(p_i) \mid^m - \mid - f(p_i)\mid^m + \sum_{i=1}^{length(C_k)} \mid C_k\left(i\right) - f\left(i\right) \mid^m \Big)^{1/m} \tag{17}$$

Assuming that $f(p_i)$ represents the degree of membership of some pixel, which, at the same time, represents the feature of that pixel, it can be noticed from Eq. (17) that as the value of $f(p_i)$ increases, the distance value decreases.

So, when selecting the pixel that minimizes the distance between the fuzzy set and the crisp set, the pixel with the *highest* degree of membership is the pixel which will be added.

4.3 Removing pixels

The pixel to be removed from the crisp set at each iteration of the floating search algorithm has to meet three conditions: (1) belonging to the crisp set and not to the α-cut of the fuzzy set, (2) being a 4-neighbourhood of the crisp set complement, and (3) minimizing the distance between the fuzzy set and the crisp set excluding the removed pixel.

After removing a pixel p_i from the crisp set, the Minkowski distance between f and the new crisp set $C_{new}\left(C_{k\setminus\{p_i\}}\right)$ becomes:

$$d_m\left(C_{new}, f\right) = \sqrt[m]{\sum_{i=1}^{length(C_k)} \mid C_{new}\left(i\right) - f\left(i\right) \mid^m} \tag{18}$$

Following the procedure as that in section 4.2, we conclude:

$$d_m^{\sim}(C_{new}, f) = (\mid -f(p_i) \mid^m - |1 - f(p_i)|^m + \sum_{i=1}^{length(C_k)} \mid C_k(i) - f(i) \mid^m)^{1/m} \quad (19)$$

It can be noticed from Eq. (19) that as the value of $f(p_i)$ decreases. the distance value decreases.

So, when selecting the pixel that minimizes the distance between the fuzzy set and the crisp set, the pixel with the *lowest* degree of membership is the pixel which will be removed.

4.4 Novelty of the proposed distance

The proposed distance, in contrast to other distance measures, takes into account the *membership degree* values of the elements in the fuzzy set, in addition to the *spatial relationship* (neighbourhood information). It is also very flexible in trying to minimize the distance, as the approach tries to enhance the result when adding/removing pixels.

Example: $A = \{1.3, 5.4, 3.7, 2.1, 4.5\}$; $B = \{6.8, 2.3, 7.9, 10.1, 3.7\}$; $C = \{5.3,\ 9.4,\ 3.1,\ 4.8,\ 9.9\}$; *(assumptions:* $m = 2$, $degree\ of\ membership\ when\ adding\ pixel = 1$, $degree\ of\ membership\ when\ removing\ pixel = 0$*)*

For comparing the result of the proposed distance with the Euclidean distance, Manhattan distance, and the classical Minkowski distance, two approaches can be used (Since we need to know the covariance of the data for Mahalanobis distance, which is not always available, Mahalanobis distance was not used in this example as it is beyond its scope) :

(i) Calculate the distance between the *median* of the two sets:

d_2 (A,B) = 3.1
$d_{Manhattan}$ (A,B) = 3.1
d_m (A,B) = 3.1
$d_m^{\sim}$(A,B) = 2,93

(ii) Find the average of the distances between each pair:

d_2 (A,B) = 4.32
$d_{Manhattan}$ (A,B) = 4.32
d_m (A,B) = 4.32
$d_m^{\sim}$(A,B) = 4.12

It can be noticed that the proposed distance gives the lowest value among the other distances, which is an important characteristic in *FCM-FloatingSearch*.

The proposed distance can be considered as a *metric*, as it complies with the metric postulates mentioned in *section 2*, that is, *non-negativity*, *reflexivity*, *symmetry*, and *triangle inequality*. Provided that, in the case of reflexivity, the fuzzy set is considered a crisp set as they are equal, and can be treated as the classical Minkowski distance (fuzzy membership values omitted).

- Non-negativity: $d_m^{\sim}$(A,B) = 11.00
- Reflexivity: $d_m^{\sim}$(A, A) = 0
- Symmetry: $d_m^{\sim}$(A,B) = $d_m^{\sim}$(B,A) = 11.00
- Triangle inequality: $d_m^{\sim}$(A,B) + $d_m^{\sim}$(B,C) $\{22.88\} >$ $d_m^{\sim}$(A,C) $\{8.23\}$

5 Evaluation

In this section, we will quantitatively evaluate our segmentation results produced by the *FCM-FloatingSearch* algorithm. We will follow a low level supervised evaluation criteria, that is, the segmentation output is what would only be considered in the evaluation, and the original image information will not be taken into account. Such assessment of the quality of segmentation is achieved by comparing the segmentation output to a ground truth [24].

5.1 Image database

An image database [25] composed of synthetic images having a ground truth was used. The database[1] includes 8400 images. Images used in the study were specifically extracted from the *B0U* group which is composed of 100% textured regions.

5.2 Influence of the FCM fuzzifier

The fuzzifier $m \in [1, +\infty)$ has a significant impact on the performance of FCM. It controls the amount of fuzziness of the final C-partition in the FCM algorithm [22]. Pal and Bezdek suggested that the value of *m* is probably in the interval [1.5, 2.5] [23]. Huang et al [22] suggest that the range of values of *m* that create significant changes in the FCM membership values, and which are considered as effective boundaries for the level of fuzziness, is approximately [1.4, 2.6]. They also recommend that an analyst should not be concerned about the changes of the membership values outside of these boundaries, as they encapsulate the uncertainty associated with the level of fuzziness parameter. For *m*, most researchers adopt $m = 2$ when performing the FCM algorithm [22]. Although, if we take the average of the above two suggested boundaries: (1.5+2.5)/2=2 and (1.4+2.6)/2=2. Thus, the fuzziness parameter value chosen in this paper is 2.

5.3 Segmentation results

Images extracted from the *B0U* group of the database were segmented according to the *FCM-FloatingSearch* process shown in Fig. 1. In order to analyze the segmentation results (quality, accuracy, extracted information, ...*etc*), nine observation criteria have been used [26]: *Precision*, *Recall*, and the *Dice index* (or *F-measure*), which characterize the overall quality of the segmentation area. The *Manahattan* (or *Matching*) *index* gives the ability to study the similarity rate of the entire image. The *Jaccard* (or *Tanimoto*) *index* studies the similarity rate between two segmentation areas. The above criteria are based on statistical tests of true or false positives, denoted TP and FP, respectively. And, true or false negatives, denoted, TN and FN, respectively.

[1] http://www.ecole.ensicaen.fr/~rosenber/ressources_UK.html

Precision and *Recall* are defined by:

$$Precision = \frac{TP}{TP + FP} \quad and \quad Recall = \frac{TP}{TP + FN} \tag{20}$$

The *Dice index* is defined by:

$$Dice\ index = 2.0 \times \frac{Precision \times Recall}{Precision + Recall} \tag{21}$$

The *Manhattan index* is defined by:

$$Manhattan\ index = \frac{TP + TN}{TP + FP + TN + FN} \tag{22}$$

and, the *Jaccard index* is defined by:

$$Manhattan\ index = \frac{TP}{TP + FP + FN} \tag{23}$$

The other criteria are: *Hamming measure*, which calculates the number of disparities between two images, and is defined by:

$$M_H\left(I_1 \Rightarrow I_2\right) = n - \sum_{R_2 \in I_2} max_{R_1 \in I_1} \left|R_2 \cap R_1\right| \tag{24}$$

where R_1 and R_2 are segmentation areas in the images I_1 and I_2, respectively. And, n is the number of pixels of one image.

The *mean absolute distance* (*MAD*), which analyzes the contour points, and thus, the shape of the segmentation, is defined by:

$$MAD\left(R_1, R_2\right) = \frac{1}{M} \sum_{m=1}^{M} \left\|x_m - y_m\right\| \tag{25}$$

where x_m and y_m are contour points of R_1 and R_2, respectively.

And, the *structural similarity* (*SSIM*) for the extracted structural information, is defined by:

$$SSIM\left(R_1, R_2\right) = \frac{\left(2m_1m_2 + k_1\right)\left(2cov_{1,2} + k_2\right)}{\left(m_1^2 + m_2^2 + k_1\right)\left(\sigma_1^2 + \sigma_2^2 + k_2\right)} \tag{26}$$

where m_1 and m_2 are the average values of R_1 and R_2. σ_1^2 and σ_2^2 are the variance, $cov_{1,2}$ is the covariance. k_1 and k_2 are two coefficients proportional to the dynamic range of the pixel values.

Fig. 2 shows that we can obtain a perfect defuzzification, similar to that proposed in [3], when the input image is fuzzified.

Fig. 3 shows the results of the proposed approach, applied on a specific image, using different $\alpha - cut$ values, along with its fuzzy partition.

Fig. 4 shows results of applying the proposed approach on more complex images with different number of textures (*B0UnR*, where *n* represents the number of textures), along with the $\alpha - cut$ vlaues being used.

Fig. 5 shows a cell image without a ground truth.

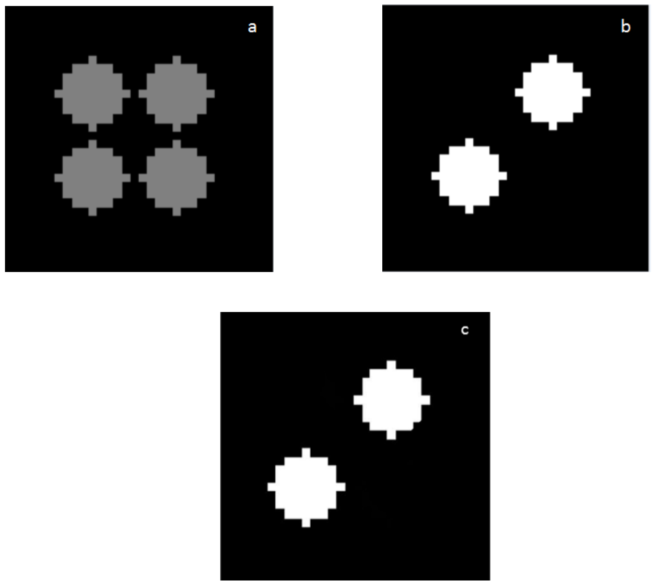

Figure 2: (a) digital disk synthetic image; (b) result of the approach in [2]; (c) result of the proposed approach

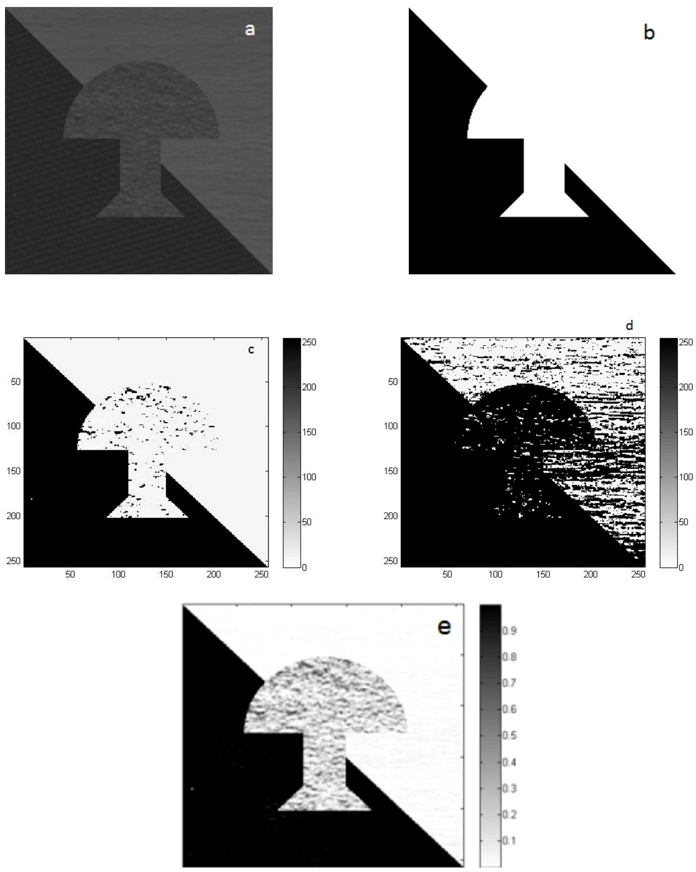

Figure 3: Results of applying the FCM-FloatingSearch process: (a) three-textured image; (b) ground truth of (a); (c) defuzzification with $\alpha - cut = 0.5$ of (a); (d) defuzzification with $\alpha - cut : \ = 1$ of (a); and (e) fuzzy partition of (a).

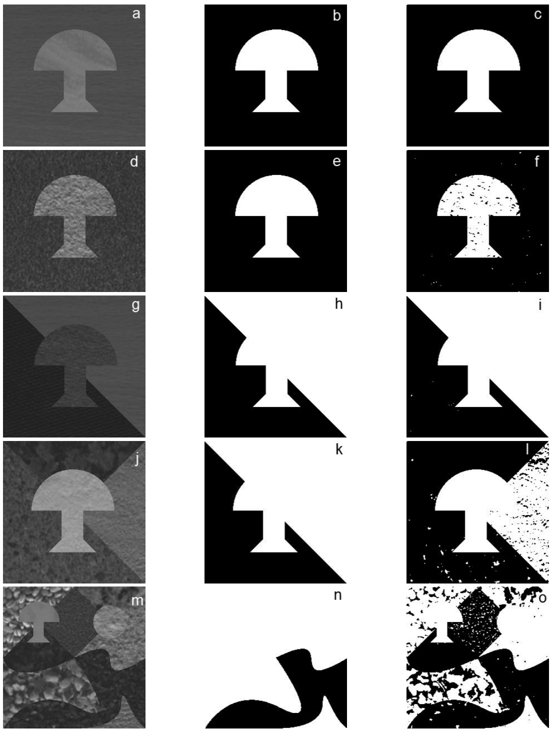

Figure 4: (a) B0U2R_1 image; (b) ground truth of (a) ; (c) defuzzification $\alpha - cut : 0.5$ of (a) ; (d) B0U2R_5 image; (e) ground truth of (d); (f) defuzzification $\alpha - cut : 0.5$ of (d); (g) B0U3R_26 image; (h) ground truth of (g); (i) defuzzification $\alpha - cut : 0.1$ of (g); (j) B0U4R_59 image; (k) ground truth of (j); (l) defuzzification $\alpha - cut : 0.1$ of (j); (m) B0U10R_4 image ; (n) ground truth of (m); (o) defuzzification $\alpha - cut : 0.1$ of (m)

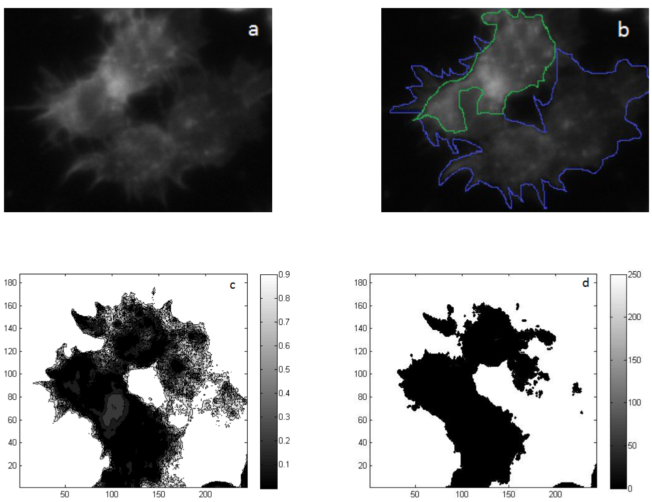

Figure 5: (a) cell image; (b) rough sketch of core $\alpha - cut : \ = 1$ (green border) and support (blue border); fuzzy partition of (a); defuzzification with $\alpha - cut = 0.5$ of (a).

	B0U2R_1	B0U2R_5	B0U3R_26	B0U4R_59	B0U10R_4
Precision	100%	99.09%	100%	65.64%	47.05%
Recall	100%	99.91%	99.96%	98.69%	98.64%
Dice	100%	99.50%	99.98%	78.84%	63.71%
Jaccard	100%	99.01%	99.96%	65.08%	46.75%
Manhattan	100%	99.18%	99.98%	78.51%	71.54%
Hamming	1	536	11	14083	18651
Coeff. Vinet	0.002%	0.82%	0.02	21.49%	28.46%
MAD	0	8.10	0.62	17.69	23.79
SSIM	0.9997	0.9118	0.9951	0.6315	0.5185

Table 1: Segmentation results (evaluation) of FCM-FloatingSearch

Tab. 1 summarizes segmentation results (evaluation) of the proposed approach. Dice index emphasizes the quality of the segmentation. The Mean Absolute Distance highlights the shape of the segmentation, and the SSIM highlights the quantity of extracted information. Jaccard and Manhattan indices highlight the problems of sub- or over-segmentation. The number of Hamming is the number of disparity between the segmentation and ground truth. Thus, as can be noticed from the table, *FCM-FloatingSearch* provides good segmentation results for the first three images despite noise or other factors. For the remaining two images, although over-segmentation is introduced, *FCM-FloatingSearch* extracts almost all of the information corresponding to the ground truth. Over-segmentation could be solved by extracting the relevant ROI (Region of Interest) as a preprocessing step to *FCM-FloatingSearch*. That is, selecting the best crisp set manually.

6 Conclusion

In this paper, we have presented a novel defuzzification approach, based on Fuzzy C-Means, with an adapted Minkowski distance. For an image without ground truth, the algorithm removes additional unnecessary data, providing a more clearer segmentation. It can be noticed that the choice of a proper $\alpha - cut$ value is essential to the defuzzification output, provided that it is not necessary that the highest $\alpha - cut$ value implies a better defuzzification. Thanks to the ground truth, FCM-FloatingSearch was able to be evaluated for the ability of detecting that ground truth. For images with ground truth, and of different textures, FCM-FloatingSearch was able to provide good segmentation despite noise and other factors. And, where over-segmentation occurred, the proposed approach was able to extract almost all the information corresponding to the ground truth. This can be enhanced by selecting the best crisp set manually. The adapted Minkowski distance proposed in the paper, which took into account the fuzzy membership degrees and the neighbourhood information, showed better results compared to other distance measures. A prospect for this work is to combine the algorithm proposed with different fuzzifier values in FCM. Also, since the defuzzification in this paper was based on one cluster, a future work would be to adapt the algorithm for a number of clusters higher than 2 (foreground/background) as a first step, and then, explore the possibility to integrate the proposed defuzzification approach for other clustering methods such as RFCM, Competitive Agglomerative Clustering, ... *etc*.

References

[1] Judith Prewitt. *Object Enhancement and Extraction*. Picture Process Psychopict, pages 75-194, 1970.

[2] Pavel Pudil, Jana Novovičová, and Josef Kittler. *Floating Search Methods in Feature Selection*. Pattern Recognition Letters, volume 15, pages 1119-1125, 1994.

[3] Nataša Sladoje, Jokaim Lindblad, and Ingela Nyström. *Defuzzification of Spatial Fuzzy Sets by Feature Distance Minimization*. Image and Vision Computing, volume 29, no. 2-3, pages 127-141, 2011.

[4] Pradipta Maji and Sankar Pal. *Maximum Class Separability for Rough-Fuzzy C-Means Based Brain MR Image Segmentation*. T. Rough Sets, 9, pages 114-134, 2008.

[5] Lisa Serir, Emmanuel Ramasso, and Noureddine Zerhouni. E2GK: *Evidential Evolving Gustafsson-Kessel Algorithm for Data Streams Partitioning Using Belief Functions*. ECSQARU, pages 326-337, 2011.

[6] Alexander Strehl and Joydeep Ghosh. *Relationship-based Clustering and Visualization for High-dimensional Data Mining*. INFORMS Journal on Computing, 15(2), pages 208 230, 2003.

[7] Michael Berry and Gordon S. Linoff . *Data Mining Techniques: For Marketing, Sales and Customer Relationship Management, 2nd Edition*. Wiley Publishing, 2009.

[8] Ardeshir Goshtasby. *Image Registration: Principles, Tools and Methods*. Advances in Computer Vision and Pattern Recognition, Springer, ISBN 978-1-4471-2457-3, pp. I-XVIII, 1-441, 2012.

[9] David Hand, Heikki Mannila, and Padhraic Smyth. *Principles of Data Mining*. MIT Press, 2001.

[10] Levent Ertöz, Michael Steinbach, and Vipin Kumar. *Finding Clusters of Different Sizes, Shapes, and Densities in Noisy, High Dimensional Data*. Clustering and Information Retrieval, pages 83-104, 2003.

[11] Sergiy Butenko, W Art Chaovalitwongse, and Panos M Pardalos. *Clustering Challenges in Biological Networks*. World Scientific, 2009.

[12] Patrick Groenen,Uzay Kaymak, and Joost Rosmalen. *Fuzzy Clustering with Minkowski Distance Functions*. Econometric Institute Report, 2006.

[13] Jiawei Han, Micheline Kamber, and Jian Pei. *Data Mining: Concepts and Techniques, 3rd Edition*. Elsevier, 2011.

[14] Geoffrey McLachlan. *Mahalanobis Distance*. Reasonance 4, pages 20-26, 1999.

[15] Paul Lewicki and Thomas Hill. *Statistics: Methods and Applications : a Comprehensive Reference for Science, Industry, and Data Mining*. StatSoft Inc, 2006.

[16] A.K. Sharma. *Introduction To Set Theory*. Discovery Publishing House, 2010.

[17] Hossein Bidgoli. The Internet Encyclopedia: Volume 2. John Wiley Sons, 2004.

[18] M. Ganesh. *Introduction to Fuzzy Sets And Fuzzy Logic*.PHI Learning Pvt. Ltd, 2006.

[19] Li-Xin Wang. *A Course in Fuzzy Systems and Control*. Prentice Hall, 1997.

[20] test Andrew Webb and Keith Copsey. *Statistical Pattern Recognition*. John Wiley Sons, 2011.

[21] Huan Liu and Hiroshi Motoda. *Feature Extraction Construction and Selection: A Data Mining Perspective*. Springer, 1998.

[22] Ming Huang, Zhixun Xia, Hongbo Wang, Qinghua Zeng, and Qian Wang. *The range of the value for the fuzzifier of the fuzzy c-means algorithm*. Pattern Recognition Letters, volume 33, issue 16, pages 2280-2284, 2012.

[23] Nikhil Pal and James Bezdek. *On Cluster Validity for the Fuzzy C-Means Model*. IEEE Trans. Fuzzy Syst 3, pages 370-379, 1995.

[24] M. Beauchemin, K. Thomson, and G. Edwards. *On the Hausdorff Distance Used For the Evaluation of Segmentation Results*. CJRS, volume 24(1), pages 38, 1998.

[25] Sebastien Chabrier, Bruno Emile, Christophe Rosenberger, and Helene Laurent. *Unsupervised Performance Evaluation of Image Segmentation*. EURASIP Journal on Applied Signal Processing, Special Issue on Performance Evaluation in Image Processing, 1-12, 2006.

[26] Manuel Grand-Brochier, Antoine Vacavant, Guillaume Cerutti, Kvin Bianchi and Laure Tougne. *Comparative Study of Segmentation Methods for Tree Leaves Extraction*. In ACM ICVS 2013, Workshop: VIGTA, Saint Petersburg, Russia, 2013.