Optic Flow Computation with High Accuracy

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joint work with

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The Optic Flow Problem

- **given:**
  - image sequence \( I(x) \) where \( x = (x, y, t)^\top \)
  - can be Gaussian-smoothed: \( I = K_\sigma \ast I_0 \)

- **wanted:**
  - displacement field (optic flow) \( w = (u, v, 1)^\top \)
  - \( w \) matches object at location \((x, y)\) at time \(t\) to its location \((x+u, y+v)\) at time \(t+1\).

What is Optic Flow Good for?

- extracting motion information e.g. in robotics
- compact coding of image sequences
- related correspondence problems in computer vision:
  e.g. stereo reconstruction and medical image registration
Deformation analysis of plastic foam using an optic flow method. (a) **Top left:** Frame 1 of a deformation sequence. (b) **Top right:** Frame 2. (c) **Bottom left:** Colour-coded displacement field. (d) **Bottom right:** Vector plot of the displacement field.
Pair of stereo images.
Four views of a stereo reconstruction algorithm based on optic flow ideas. **Authors:** Alvarez/Deriche/Sánchez/Weickert (2002)
Variational Optic Flow Methods

- optic flow as minimiser of a suitable energy functional: data constraints plus smoothness constraints
- clear formalism without hidden model assumptions
- rotationally invariant continuous formulations possible
- create dense flow fields
- first model due to Horn and Schunck (1981), but many improvements in the meantime:
  - theoretical foundation (Snyder 1991, W./Schnörr 2000)
- competitive performance
Open Problems

◆ further improvements possible?

◆ some very good methods use strategies that lack theoretical foundation

Goals

◆ presentation of an optic flow algorithm with very good performance

◆ theoretical justification of widely used warping technique

Some Related Work


Outline

- Variational Model
- Algorithmic Aspects
- Relations to Warping
- Evaluation
- Conclusions
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Variational Model (1)

**Basic assumptions**

- **Greyvalue constancy**
  \[ I_w := I(x + w) - I(x) = 0 \]

- **Gradient constancy**
  \[ I_{xw} := \partial_x I(x + w) - \partial_x I(x) = 0 \]
  \[ I_{yw} := \partial_y I(x + w) - \partial_y I(x) = 0 \]

- **Spatio-temporal smoothness**
  \[ |\nabla u|^2 + |\nabla v|^2 = 0 \]
  \[ \nabla = (\partial_x, \partial_y, \partial_t)^\top \]

- **Robustness**
  \[ \Psi(s^2) = \sqrt{s^2 + \epsilon^2} \]

**Energy to minimise:**

\[
E(u, v) = \int_{\Omega} \Psi(I_w^2 + \gamma \cdot (I_{xw}^2 + I_{yw}^2)) \, dx + \alpha \int_{\Omega} \Psi(|\nabla u|^2 + |\nabla v|^2) \, dx
\]
Variational Model (2a)

Minimiser has to fulfill the Euler-Lagrange equations

\[
\alpha \text{ div } (\Psi'(|\nabla u|^2 + |\nabla v|^2) \nabla u) \\
= \Psi'(I_w^2 + \gamma(I_{xw}^2 + I_{yw}^2)) \cdot (I_xI_w + \gamma(I_{xx}I_{xw} + I_{xy}I_{yw}))
\]

\[
\alpha \text{ div } (\Psi'(|\nabla u|^2 + |\nabla v|^2) \nabla v) \\
= \Psi'(I_w^2 + \gamma(I_{xw}^2 + I_{yw}^2)) \cdot (I_yI_w + \gamma(I_{xy}I_{xw} + I_{yy}I_{yw}))
\]

where the indices denote differences or partial derivatives:

\[
I_w := I(x + w) - I(x) \quad I_x := \partial_x I(x + w) \quad I_y := \partial_y I(x + w)
\]

\[
I_{xw} := \partial_x I(x + w) - \partial_x I(x) \quad I_{xx} := \partial_{xx} I(x + w) \quad I_{yy} := \partial_{yy} I(x + w)
\]

\[
I_{yw} := \partial_y I(x + w) - \partial_y I(x) \quad I_{xy} := \partial_{xy} I(x + w)
\]
Minimiser has to fulfill the Euler-Lagrange equations

\begin{align*}
\alpha \text{ div } & (\Psi'(\|\nabla u\|^2 + \|\nabla v\|^2) \nabla u) \\
& = \Psi' \left( I_w^2 + \gamma (I_{xw}^2 + I_{yw}^2) \right) \cdot \left( I_x I_w + \gamma (I_{xx} I_{xw} + I_{xy} I_{yw}) \right) \\
\alpha \text{ div } & (\Psi'(\|\nabla u\|^2 + \|\nabla v\|^2) \nabla v) \\
& = \Psi' \left( I_w^2 + \gamma (I_{xw}^2 + I_{yw}^2) \right) \cdot \left( I_y I_w + \gamma (I_{xy} I_{xw} + I_{yy} I_{yw}) \right)
\end{align*}

where the indices denote differences or partial derivatives:

\begin{align*}
I_w & := I(x + w) - I(x) & I_x & := \partial_x I(x + w) & I_y & := \partial_y I(x + w) \\
I_{xw} & := \partial_x I(x + w) - \partial_x I(x) & I_{xx} & := \partial_{xx} I(x + w) & I_{yy} & := \partial_{yy} I(x + w) \\
I_{yw} & := \partial_y I(x + w) - \partial_y I(x) & I_{xy} & := \partial_{xy} I(x + w)
\end{align*}

\[ S(w) = D(w) \]
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Problem 1: Local Minima

- energy functional $E(u, v)$ is not convex
- reason: terms involving $I(x + w)$
- should not be linearised for large displacements
- numerical algorithms may yield suboptimal local minima of $E(u, v)$, if initialisation is not chosen properly

Solution: Initialisation by Coarse-to-Fine Strategy

- downsample problem in a full pyramid
- start with zero displacement at coarsest scale
- solve Euler-Lagrange equations $S(w) = D(w)$
- use resulting flow field as initialisation at next finer scale
Problem 2: Solve Euler-Lagrange Equations

- discretise $S(w) = D(w)$ by finite differences
- yields large nonlinear system of equations
- nonlinearity caused by $I(x + w)$ and nonlinear penaliser $\Psi$

Solution

- nonlinear system is simplified by
  - two nested fixed point iterations
  - linearisation of $I(x + w)$
- leads to large linear system of equations
- can be solved by iterative methods such as SOR
Detailed Structure on the Linear System

Resulting linear system for $du^{k,l+1}$, $dv^{k,l+1}$:

$$\alpha \ \text{div} \left( (\Psi')^{k,l}_{Smooth} \nabla (u^k + du^{k,l+1}) \right)$$

$$= \ (\Psi')^{k,l}_{Data} \cdot \left( I^k_x (I^k_z + I^k_x du^{k,l+1} + I^k_y dv^{k,l+1}) \right)$$

$$+ \ \gamma \left( I^k_{xx} (I^k_x du^{k,l+1} + I^k_{xy} dv^{k,l+1}) + I^k_{xy} (I^k_y + I^k_{xy} du^{k,l+1} + I^k_{yy} dv^{k,l+1}) \right)$$

$$\alpha \ \text{div} \left( (\Psi')^{k,l}_{Smooth} \nabla (v^k + dv^{k,l+1}) \right)$$

$$= \ (\Psi')^{k,l}_{Data} \cdot \left( I^k_y (I^k_z + I^k_x du^{k,l+1} + I^k_y dv^{k,l+1}) \right)$$

$$+ \ \gamma \left( I^k_{xy} (I^k_x du^{k,l+1} + I^k_{xy} dv^{k,l+1}) + I^k_{yy} (I^k_y + I^k_{xy} du^{k,l+1} + I^k_{yy} dv^{k,l+1}) \right)$$
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Warping

- widely used for optic flow computation with large displacements (e.g. Anandan 1989, Black/Anandan 1996, Mémin/Pérez 1998)
- downsample image data
- solve problem at coarse scale
- use this flow field at next finer scale: warp image in order to compensate for this estimated motion
- solve modified problem (with other image data) at finer scale
- continue until finest scale reached
- sum up optic flow contributions from all scales
- successful in practice, but no theoretical justification!
relations to warping (2)

we have proven equivalence between

- our numerical method for minimising a simplified energy $E(u, v)$ by coarse-to-fine flow initialisations and nested fixed point iterations
- warping method (nested problems with motion-compensated image data) of Mémin / Pérez

they lead to the same linear system of equations.

this explains the success of warping:

warping has a sound theory as a numerical algorithm for minimising a single energy functional!
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Evaluation (1)

Yosemite Sequence

- Synthetic sequence
  \((316 \times 252 \times 15)\)

- Known ground truth between frame 8 and frame 9

Sequence

Ground Truth

Computed Flow
Qualitative Evaluation

Vector plot of the optic flow field for the Yosemite sequence with clouds. (a) **Left:** Ground truth. (b) **Right:** Computed flow.
Vector plot of the optic flow field for the Yosemite sequence **without** clouds. (a) **Left:** Ground truth. (b) **Right:** Computed flow.
Quantitative Evaluation

- Comparison to the best results from literature
- Average angular errors (AAE) and standard deviations (STD) for the Yosemite sequence with clouds:

<table>
<thead>
<tr>
<th>Technique</th>
<th>AAE</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anandan 1989</td>
<td>13.36°</td>
<td>15.64°</td>
</tr>
<tr>
<td>Nagel 1983</td>
<td>10.22°</td>
<td>16.51°</td>
</tr>
<tr>
<td>Uras et al. 1988</td>
<td>8.94°</td>
<td>15.61°</td>
</tr>
<tr>
<td>Alvarez et al. 2000</td>
<td>5.53°</td>
<td>7.40°</td>
</tr>
<tr>
<td>Weickert et al. 2003</td>
<td>5.18°</td>
<td>8.68°</td>
</tr>
<tr>
<td>Mémin/Pérez 1998</td>
<td>4.69°</td>
<td>6.89°</td>
</tr>
<tr>
<td><strong>our method</strong></td>
<td><strong>1.94°</strong></td>
<td><strong>6.02°</strong></td>
</tr>
</tbody>
</table>
For the Yosemite sequence **without** clouds, even better results are possible:

<table>
<thead>
<tr>
<th>Technique</th>
<th>AAE</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black/Anandan 1996</td>
<td>4.56°</td>
<td>4.21°</td>
</tr>
<tr>
<td>Ju <em>et al.</em> 1996</td>
<td>2.16°</td>
<td>2.00°</td>
</tr>
<tr>
<td>Bab-Hadiashar/Suter 1997</td>
<td>2.05°</td>
<td>2.92°</td>
</tr>
<tr>
<td>Lai/Vemuri 1998</td>
<td>1.99°</td>
<td>1.41°</td>
</tr>
<tr>
<td>Mémin/Pérez 1998</td>
<td>1.58°</td>
<td>1.21°</td>
</tr>
<tr>
<td>Weickert <em>et al.</em> 2003</td>
<td>1.46°</td>
<td>1.50°</td>
</tr>
<tr>
<td>Farnebäck 2003</td>
<td>1.14°</td>
<td>2.14°</td>
</tr>
</tbody>
</table>

**our method**                  | 0.98°| 1.17°|
Robustness under Noise

- Added Gaussian noise with zero mean and different standard deviations $\sigma_n$.

- Results for Yosemite sequence with clouds:

<table>
<thead>
<tr>
<th>$\sigma_n$</th>
<th>AAE</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.94°</td>
<td>6.02°</td>
</tr>
<tr>
<td>10</td>
<td>2.50°</td>
<td>5.96°</td>
</tr>
<tr>
<td>20</td>
<td>3.12°</td>
<td>6.24°</td>
</tr>
<tr>
<td>30</td>
<td>3.77°</td>
<td>6.54°</td>
</tr>
<tr>
<td>40</td>
<td>4.37°</td>
<td>7.12°</td>
</tr>
</tbody>
</table>

- Average angular error for $\sigma_n = 40$ outperforms all other methods with $\sigma_n = 0$!
Frame 8 of the Yosemite sequence with clouds. (a) **Left:** Original. (b) **Right:** Gaussian noise with standard deviation $\sigma_n = 40$ added.
Robustness under Parameter Variations

- Three intuitive parameters:
  - $\sigma$: Gaussian presmoothing of the input data
  - $\alpha$: weight of smoothness term
  - $\gamma$: weight of gradient constancy term

- Parameter variation for the Yosemite sequence with clouds:

  \[
  \begin{array}{cccc}
  \sigma & \alpha & \gamma & \text{AAE} \\
  0.8 & 80 & 100 & 1.94^\circ \\
  0.4 & " & " & 2.10^\circ \\
  1.6 & " & " & 2.04^\circ \\
  0.8 & 80 & 100 & 1.94^\circ \\
  " & 40 & " & 2.67^\circ \\
  " & 160 & " & 2.21^\circ \\
  0.8 & 80 & 100 & 1.94^\circ \\
  " & " & 50 & 2.07^\circ \\
  " & " & 200 & 2.03^\circ \\
  \end{array}
  \]

- Deviations from the optimum by a factor 2 hardly influence the result.
Real-World Data

- Real-world image sequence "Ettlinger Tor" by Nagel (512 × 512 × 50)

Sequence

Computed Flow
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Summary

- **novel model**
  - gradient constancy assumption within energy functional
  - combines many successful features in a single functional

- **novel theory**
  - postpone all linearisations to the numerical scheme
  - numerical scheme based on two nested iterations
  - warping theoretically justified as a special numerical approximation

- **excellent results**
  - angular errors belong to smallest in the literature
  - robust under parameter variations
  - highly robust under noise
Ongoing Work

- alternative data terms
- correspondence between data and smoothness terms
- automatised selection of smoothing parameters \( \sigma, \alpha \)
- more efficient numerics: PCG, multigrid, domain decomposition
- novel warpings inspired from suitable numerics?

Message

- It is advantageous to combine transparent continuous modelling with consistent numerics.
- Good performance and deeper theoretical understanding are not contradictive: They are two sides of the same medal.
Thank you very much!

more informations:
www.mia.uni-saarland.de