

# Digital Image Restoration

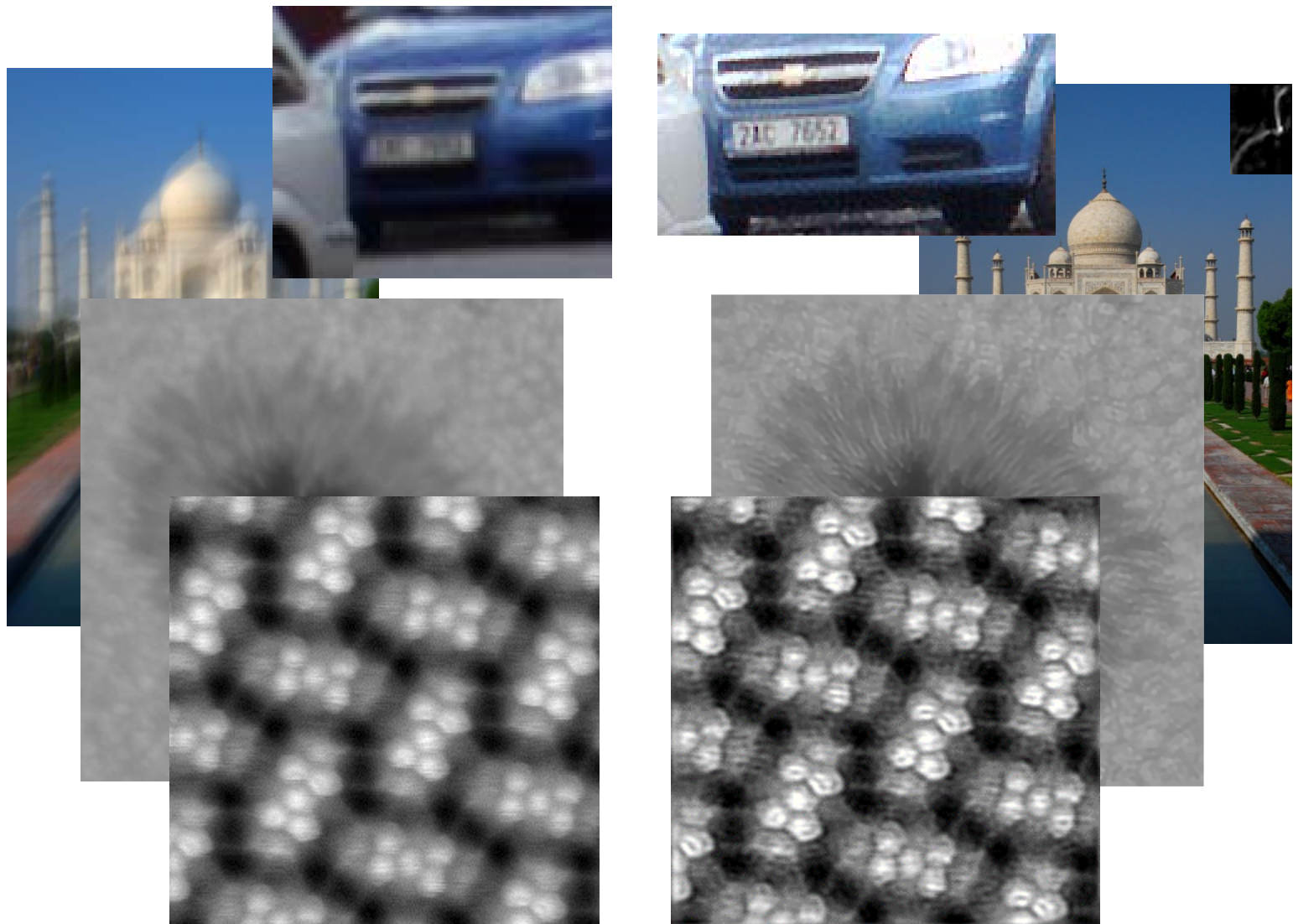
Blur as a chance and not a nuisance

Filip Šroubek

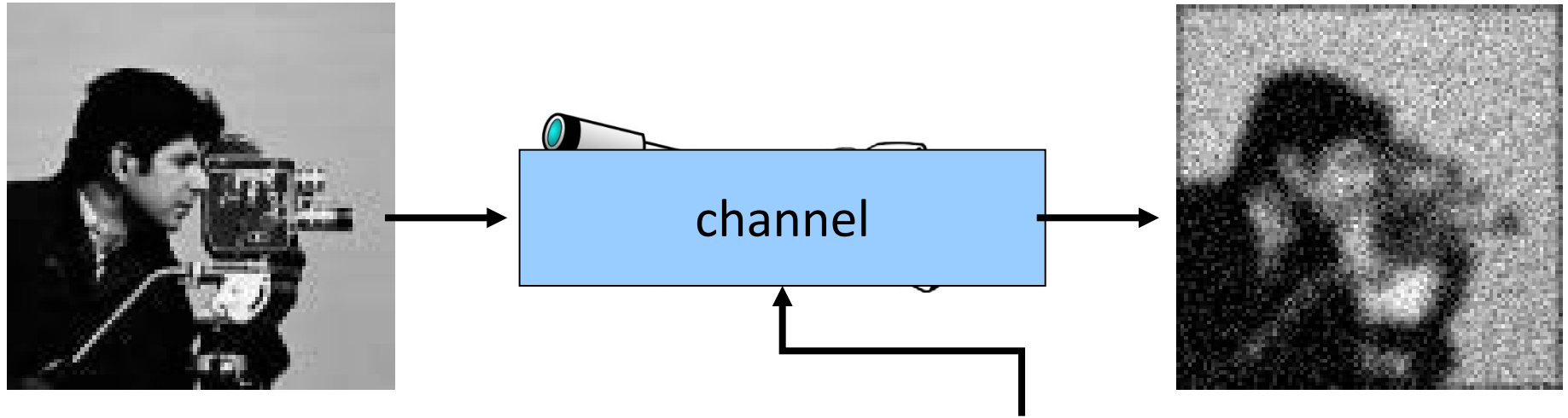
[sroubekf@utia.cas.cz](mailto:sroubekf@utia.cas.cz)

[www.utia.cas.cz](http://www.utia.cas.cz)

Institute of Information Theory and Automation  
Academy of Sciences of the Czech Republic  
Prague



# Image Degradation Model



original image

noise

acquired image

$$[ u * h ]$$

convolution

$$+ n = z$$

# Energy Minimization

$$(\tilde{u}, \tilde{h}) = \arg \min_{u, h} E(u, h)$$

- Alternating Minimization

1. *u*-step:  $\tilde{u} = \arg \min_u E(u, \tilde{h})$

2. *h*-step:  $\tilde{h} = \arg \min_h E(\tilde{u}, h)$

3. *repeat 1 and 2.*

# Energy Minimization

$$E(u, h) = \frac{1}{2} \|(h * u) - z\|^2$$

↙  
Data  
term

- Coupling of  $u$  and  $h$  → **infinite** number of solutions  $(\tilde{u}, \tilde{h})$
- Add regularization → well-posed problem
- restrict solution vs. being general

# Energy Minimization

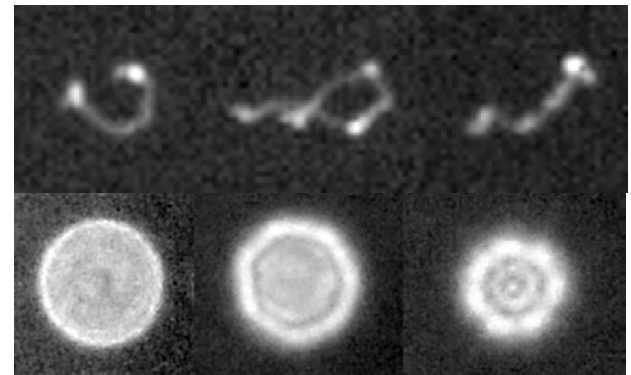
$$E(u, h) = \frac{1}{2} \|(h * u) - z\|^2 + \lambda Q(u) + \gamma R(h)$$

Data term

Blur regularization

- Blur has different shapes
  - Compact support
  - Non-negative
  - Preserve energy

Motion blur



Out-of-focus & Abberations

# Energy Minimization

$$E(u, h) = \frac{1}{2} \|(h * u) - z\|^2 + \lambda Q(u) + \gamma R(h)$$

Data term                      Image regularization

- Enforce image smoothness

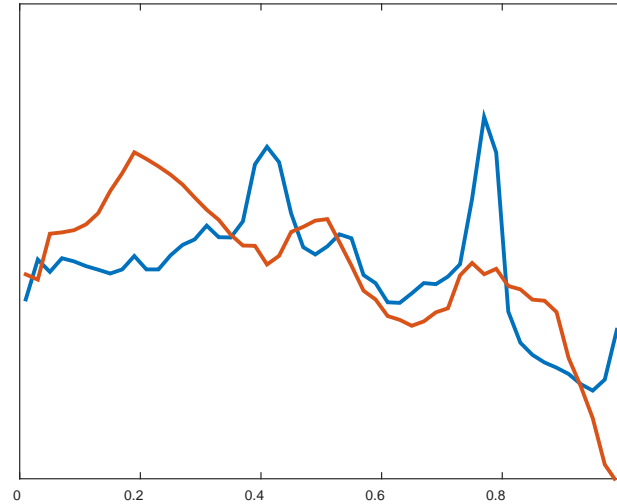
- Wiener:  $Q(u) = \int u(x)^2 dx$

- Tikhonov:  $\int |\nabla u(x)|^2$

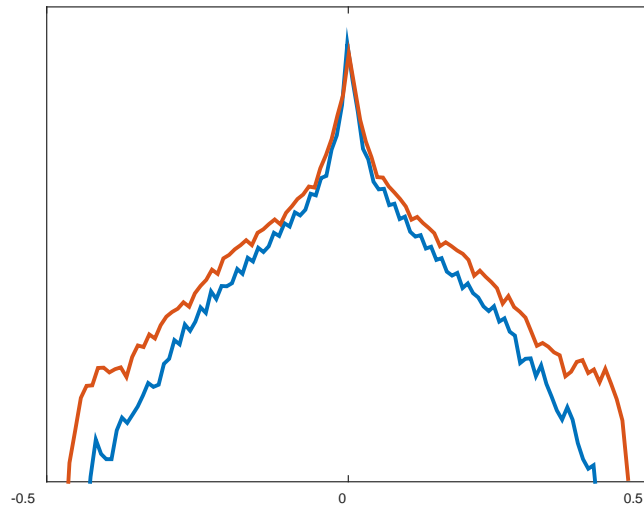
- Total Variation:  $\int |\nabla u(x)|$

- Non-convex  $L_p$  quasi-norm:  $\int |\nabla u(x)|^p, \quad p < 1$

# Statistics of sharp images



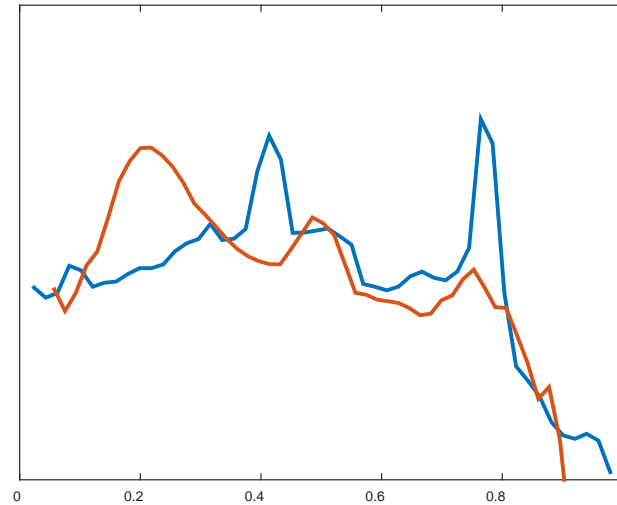
Intensity



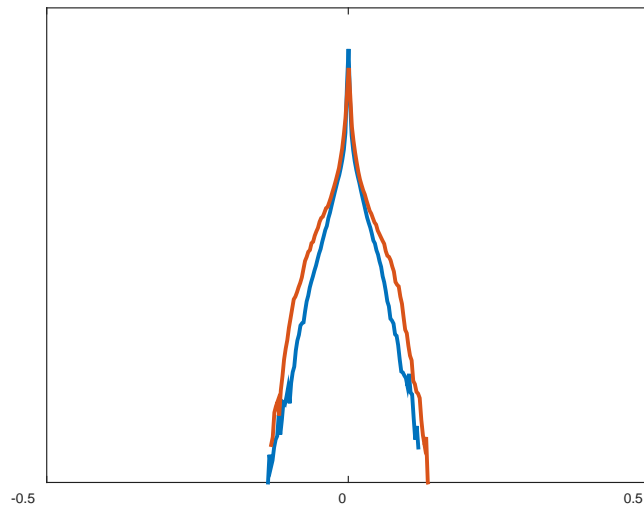
Gradient



# Statistics of blurred images



Intensity



Gradient

# Regularization

Different regularization  $Q(u)$

$$|\nabla u|^2$$

$$|\nabla u|^1$$

$$|\nabla u|^0$$

$u(x)$



$z(x)$



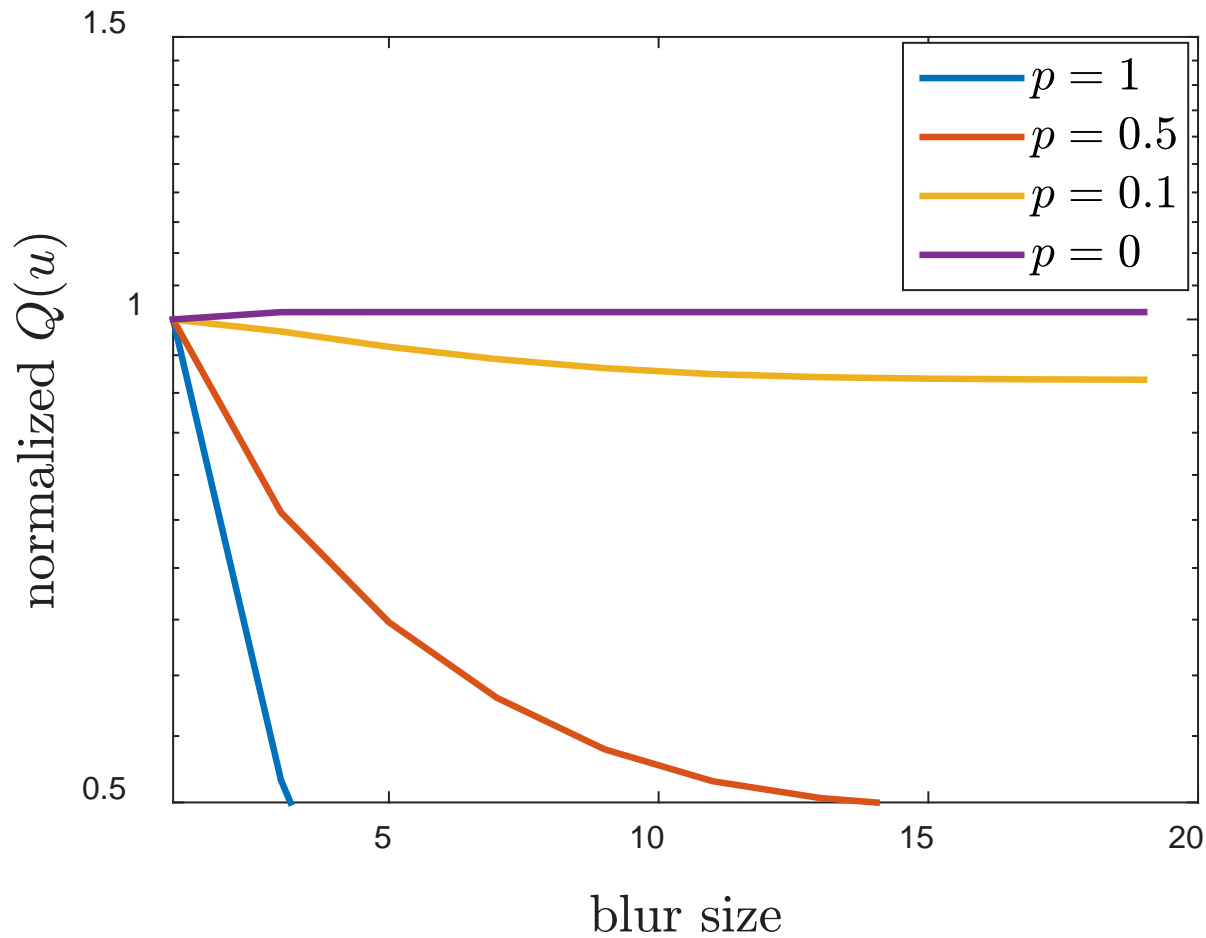
Estimated image  $\tilde{u}(x)$

# Energy Minimization

$$E(u, h) = \frac{1}{2} \|(h * u) - z\|^2 + \lambda Q(u) + \gamma R(h)$$

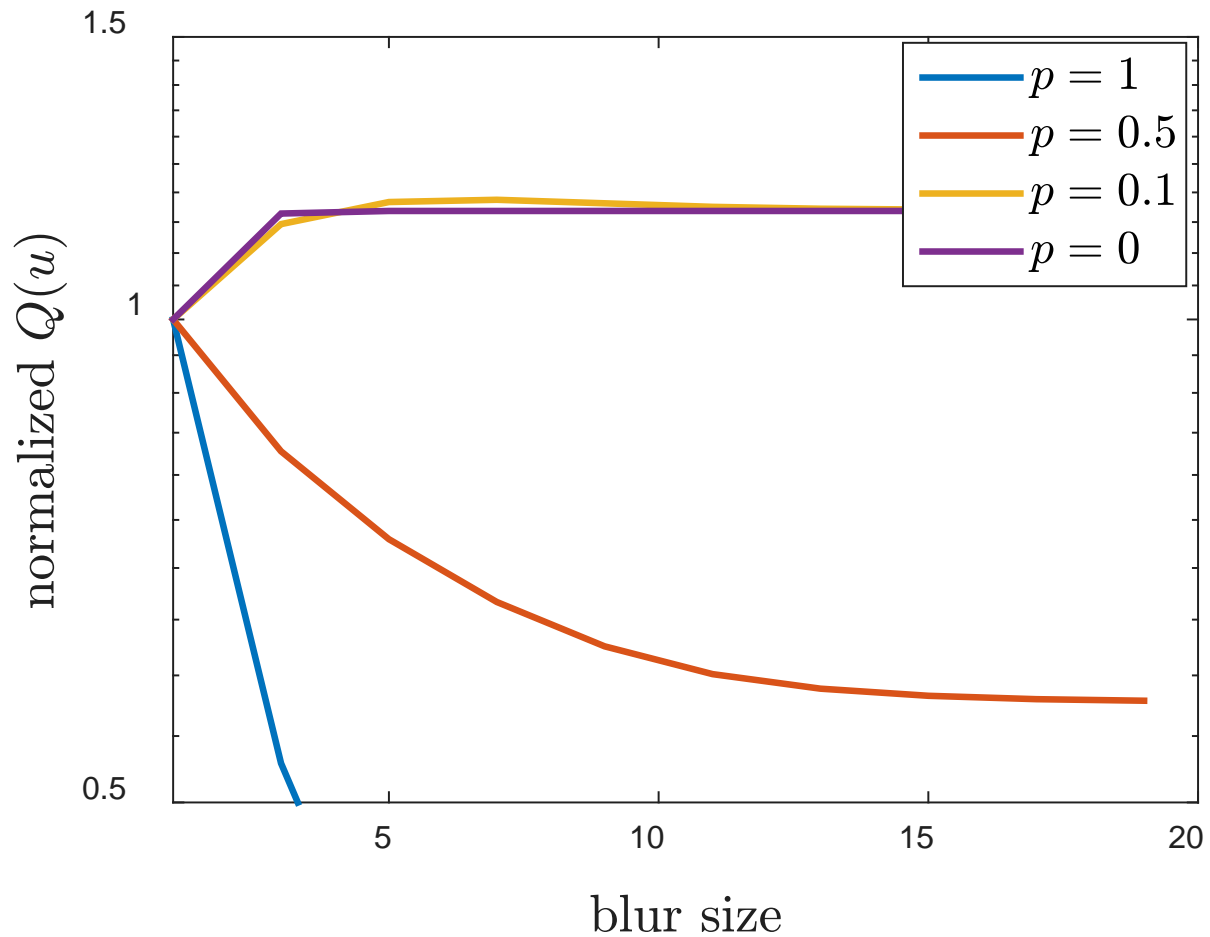
- “**NO-BLUR**” solution:  $\tilde{u}(x) = z(x)$ ,  $\tilde{h}(x) = \delta(x)$
- WHY?
- **Regularization favors blurred images**

# Regularization favors blur



$$Q(u) = \int |\nabla u|^p$$

# Regularization favors blur



$$Q(u) = \int |\nabla u|^p$$

Artificially sparsify images

# Ad-hoc steps!

- To avoid “no-blur” solution:
  - Artificial sharpening
  - Remove spikes
  - Adjusting priors on the fly
  - Hierarchical approach

Chan TIP98  
Shan SigGraph08  
Cho SigGraph09  
Xu ECCV09, CVPR13  
Almeida TIP10  
Krishnan CVPR11  
Zhong CVPR13  
Sun ICCP13  
Michaeli ECCV14  
Perrone PAMI15  
Pan CVPR16

# Artificial Sharpening

Blurred image



Shock filter



Blur prediction  
h-step



Deconvolution  
u-step



Cho et al., SIGGRAPH 2009

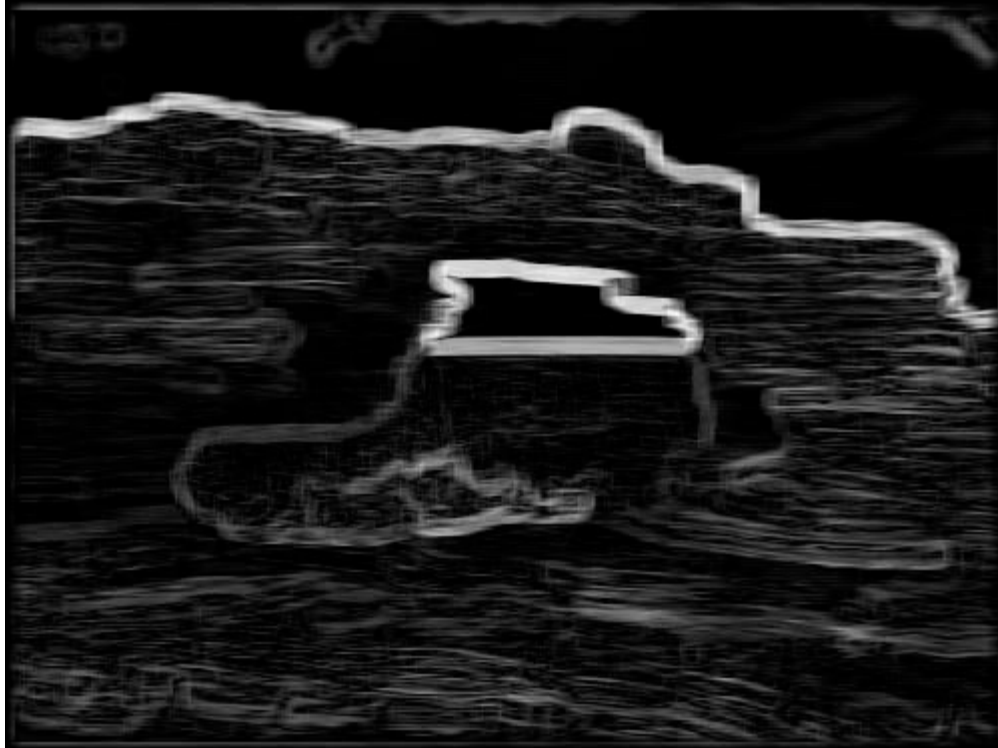
# Remove Spiky Objects



Xu et al., ECCV 2010



# Remove Spiky Objects



Mask out small objects

Xu et al., ECCV 2010

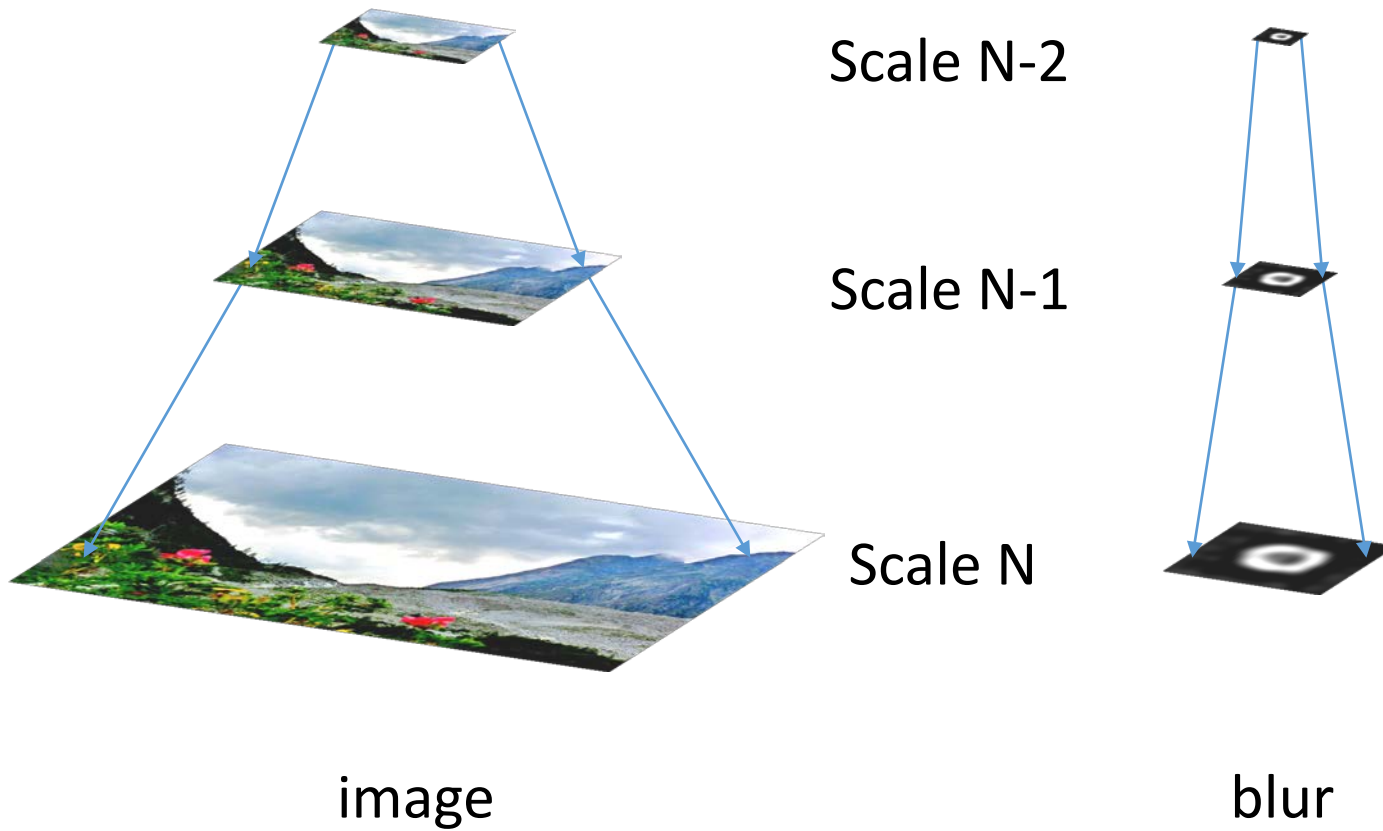
# Remove Spiky Objects



Reconstructed image with  
small objects removed

Xu et al., ECCV 2010

# Hierarchical Deconvolution



# Bayesian Paradigm

$$p(u, h|z) = \frac{p(z|u, h)p(u)p(h)}{p(z)}$$

The diagram shows the Bayesian equation  $p(u, h|z) = \frac{p(z|u, h)p(u)p(h)}{p(z)}$ . The term  $p(u, h|z)$  is enclosed in a green box. The numerator  $p(z|u, h)p(u)p(h)$  is enclosed in a blue and red box. The denominator  $p(z)$  is written below the fraction. Three orange arrows point from the boxes to their respective labels: from the green box to the 'a posteriori distribution' label, from the blue/red box to the 'likelihood' label, and from the denominator  $p(z)$  to the 'a priori distribution' label.

***a posteriori* distribution**  
unknown

**likelihood**  
given by our problem

***a priori* distribution**  
our prior knowledge

- Maximum a posteriori (MAP):  $\max p(u, h|z)$

# Blind deconvolution with MAP

- max *a posteriori* probability  $p(u, h|z)$   
==> min  $-\log p(u, h|z)$

$$-\log p(u, h|z) \propto -\log p(z|u, h) - \log p(u) - \log p(h)$$

- Exponential family

$$E(u, h) = \frac{\lambda}{2} \|u * h - z\|^2 + Q(u) + R(h)$$

**NOTHING NEW!**

# Bayesian Paradigm revisited

- Marginalize the posterior

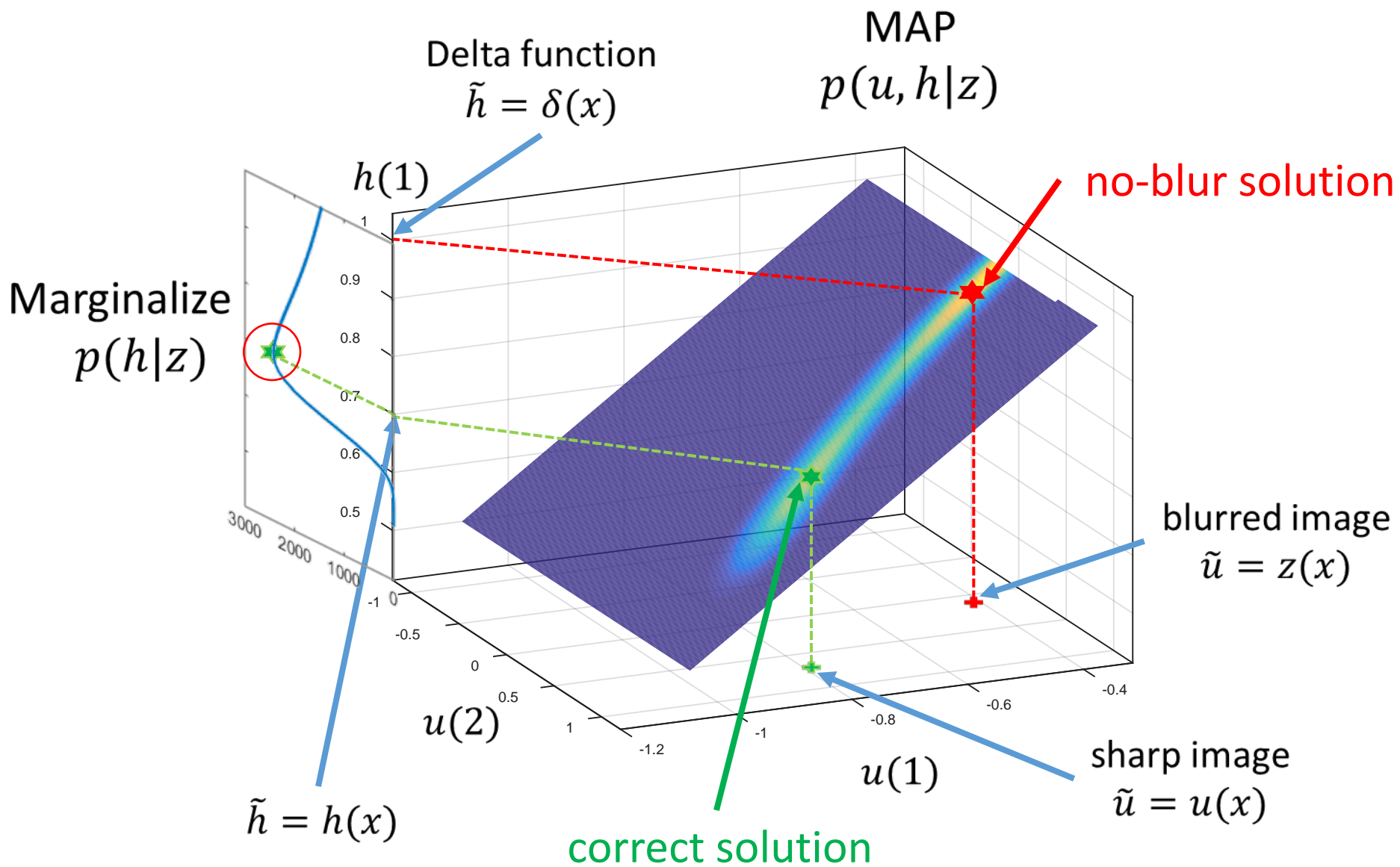
$$p(h|z) = \int p(u, h|z) du$$

- Maximize the marginalized prob.

$$\hat{h} = \arg \max_h p(h|z)$$

- Then maximize the posterior

$$\hat{u} = \arg \max_u p(u, \hat{h}|z)$$



# How to marginalize?

$$p(h|z) = \int p(u, h|z) du$$

- If Gaussian distributions  $\rightarrow$  analytic solution exists in the form of Gaussian distribution
- If not (our case)  $\rightarrow$  approximation
  - Laplace approximation
  - Factorization with Variational Bayes



# Variational Bayes

- Factorization of the posterior

$$p(u, h|z) \approx q(u)q(h)$$

and then marginalization is trivial.

- Every factor  $q$  depends on moments of other variables => must be solved iteratively.

Miskin AICA00  
Fergus SIGGRAPH06  
Tzikas TIP09  
Whyte IJCV12  
Levin PAMI11  
Babacan ECCV12  
Wipf JMLR14

# Marginalization in blind deconvolution



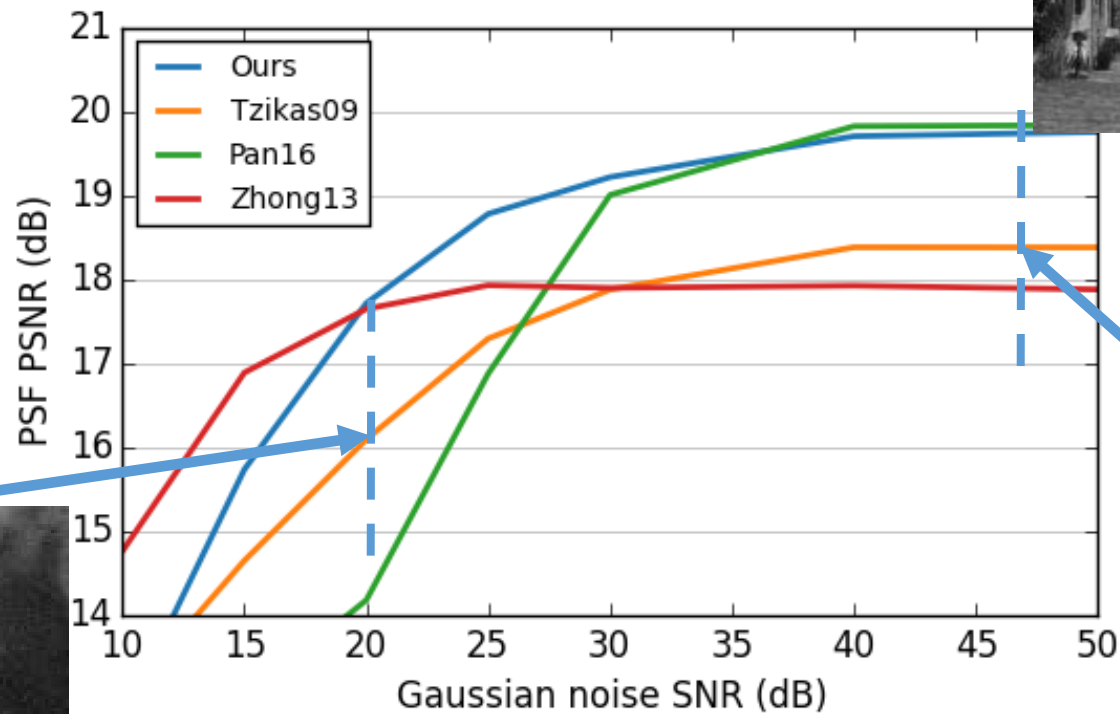
Blurred image  
 $z(x)$



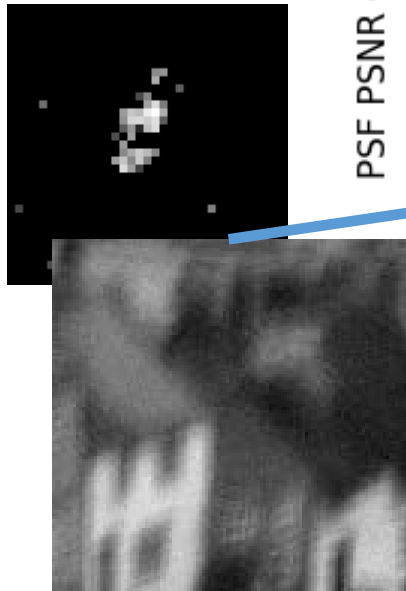
Reconstructed image  
 $\tilde{u}(x)$

# Limitations of BD

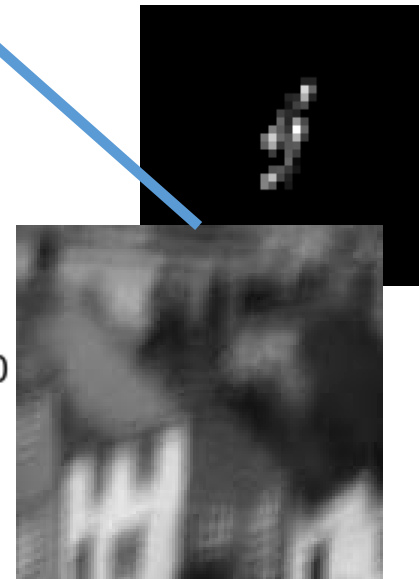
- Gaussian noise



original



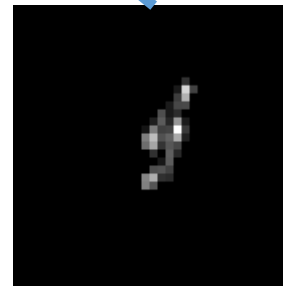
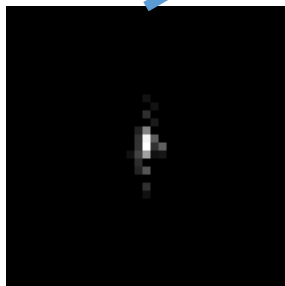
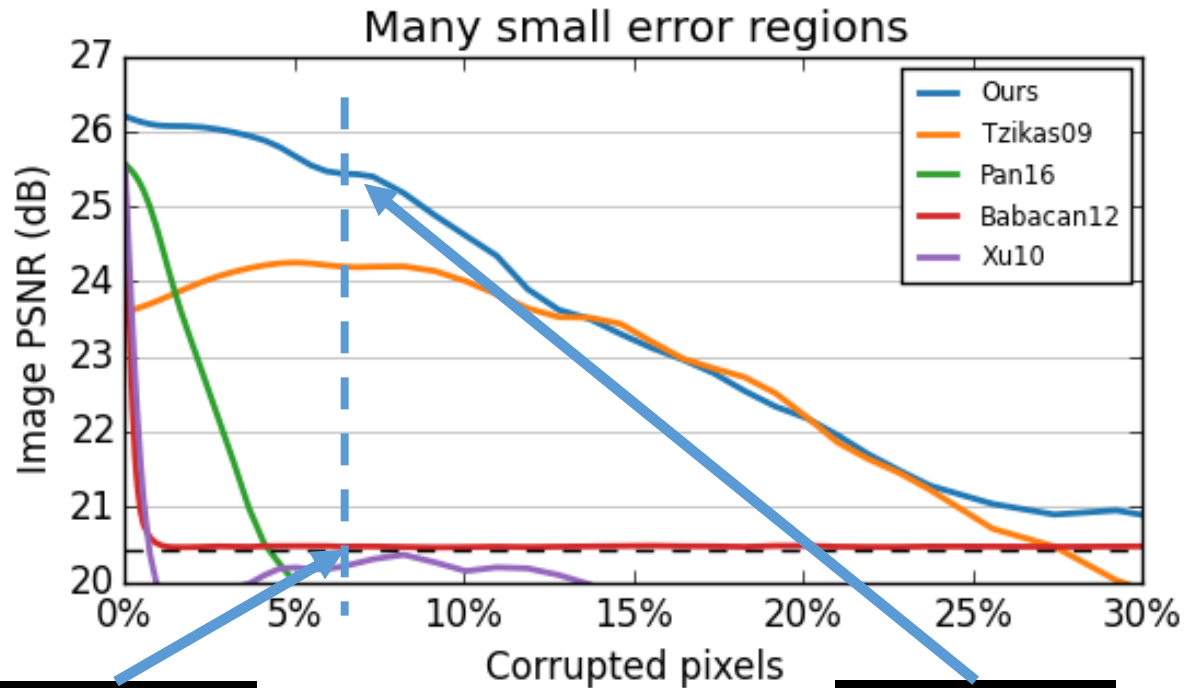
SNR=20dB



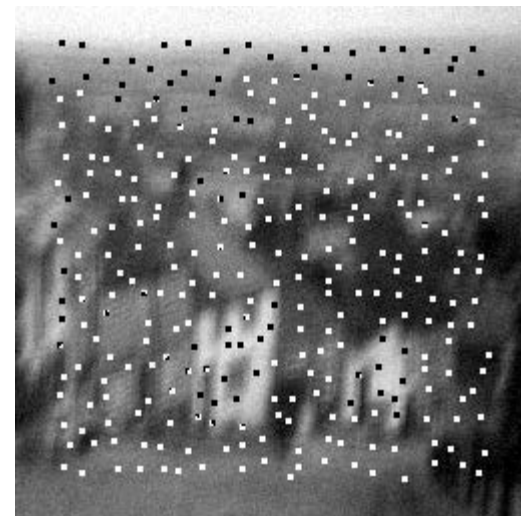
SNR=50dB

# Limitations of BD

- Model discrepancies



original



7% discrepancy

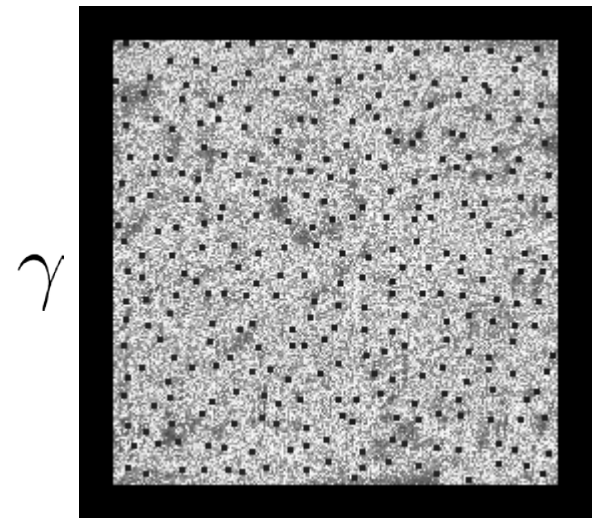
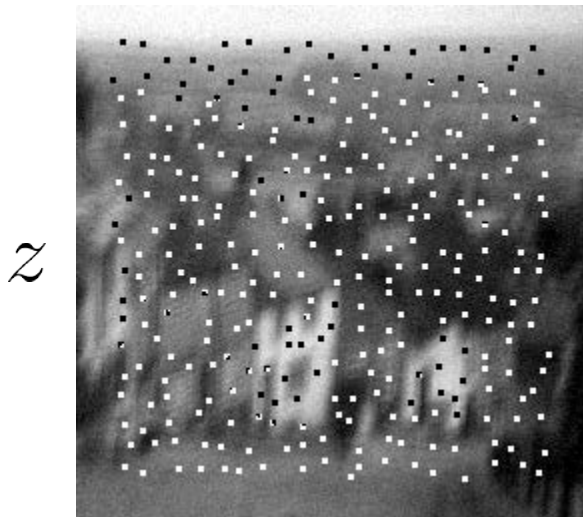
# Automatic Relevance Determination (Students' $t$ -distribution)

- Standard data term

$$\|h * u - z\|^2 = \int ([h * u](x) - z(x))^2$$

replaced by

$$\|h * u - z\|_{\gamma}^2 = \int \gamma(x) ([h * u](x) - z(x))^2$$





with  $\gamma$



without  $\gamma$



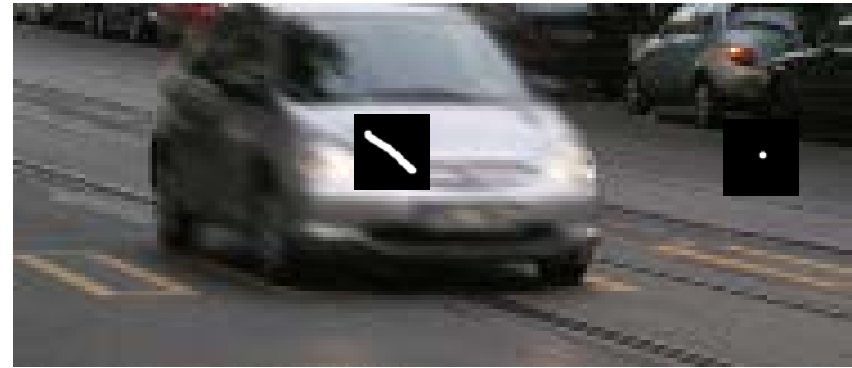
# Space-variant Blur



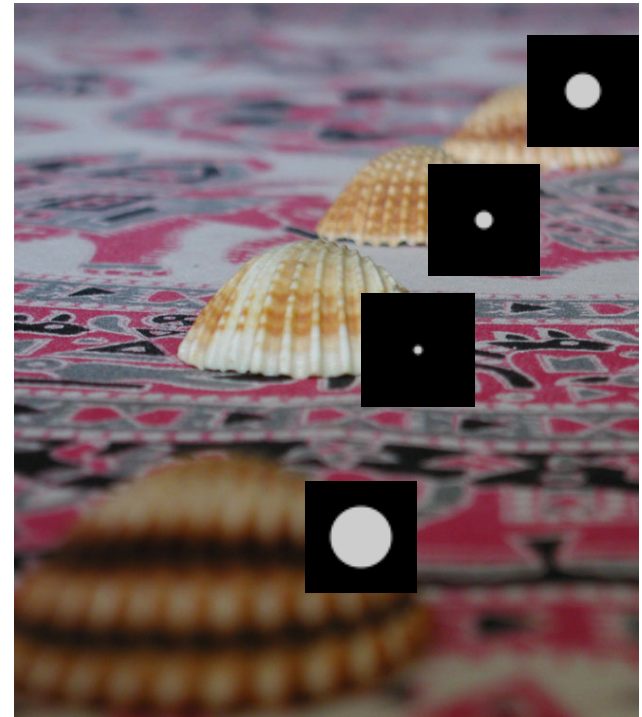
Camera motion



Optical aberrations



Object motion



Scene depth

# Approximation of SV Blur

- Patch-wise convolution

- General SV PSF
- Locally convolution may not hold

Joshi CVPR08, Sorel ICIP09,  
Ji CVPR12, Sun CVPR15

- Parametric model (Blur Basis)

- More accurate
- Model may not hold

Whyte CVPR10, Gupta ECCV10,  
Hirsch ICCV11, Zhang NIPS13

- Conversion to space-invariant

- HW design

Levin SIGGRAPH08, Ben-Ezra PAMI04,  
Tai PAMI10, Wavefront coding

- Local object motion

- Segmentation
- Line blur

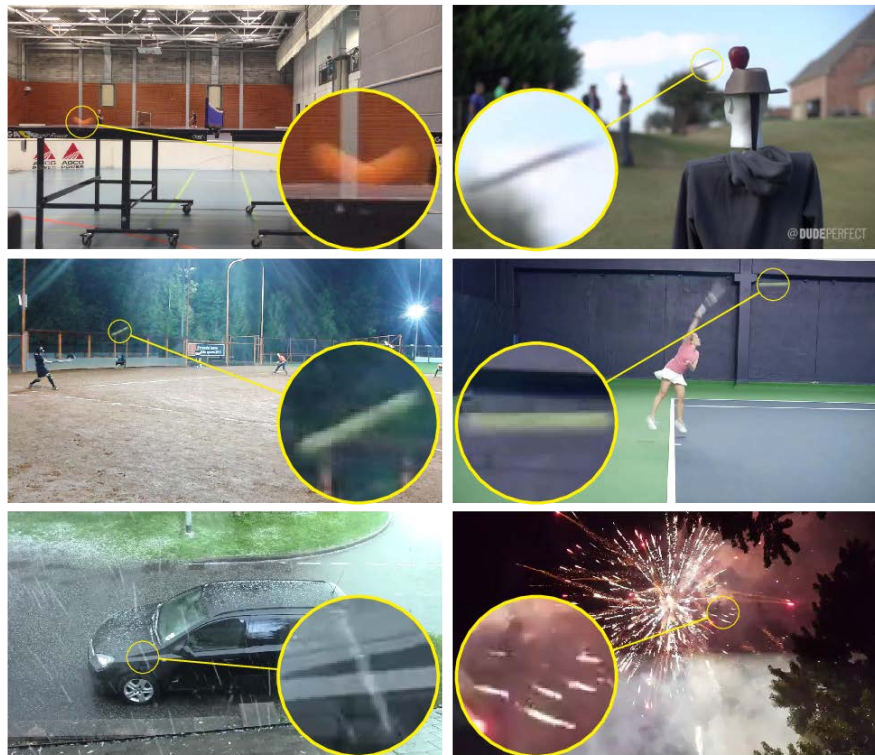
Levin NIPS06, Shan ICCV07, Dai CVPR08,  
Chakrabarti CVPR10, Kim CVPR14



# Fast Moving Objects

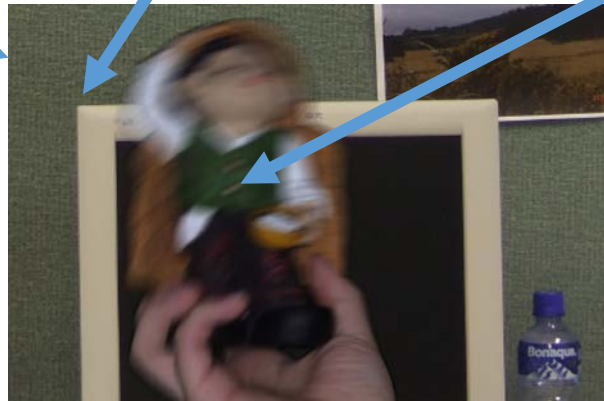
D.R.,J.K.,F.S.,L.N.,J.M.  
arXiv:1611.07889

- FMO moves over a distance exceeding its size within exposure time



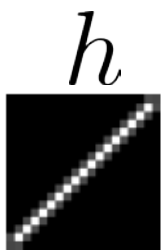
# Formation model

$$z = (1 - h * m) b + h * f$$



$z$

Alpha matting



\*



=



# FMO Appearance Estimation

$$z = (1 - h * m)b + h * f$$

- Assumptions:
  - Background  $b$  is known  $\rightarrow$  previous frame
  - Blur  $h$  is a FMO trajectory  $\rightarrow$  estimated by a tracker

- Optimization problem:

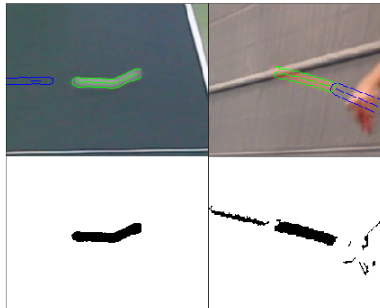
$$E(f, m, h) = \|(1 - h * m)b + h * f - z\|_1 + \lambda Q(f)$$

- Alternating minimization w.r.t.  $f, m, h$
- Initial estimation of  $h$  is critical  $\rightarrow$  novel tracker designed for FMOs

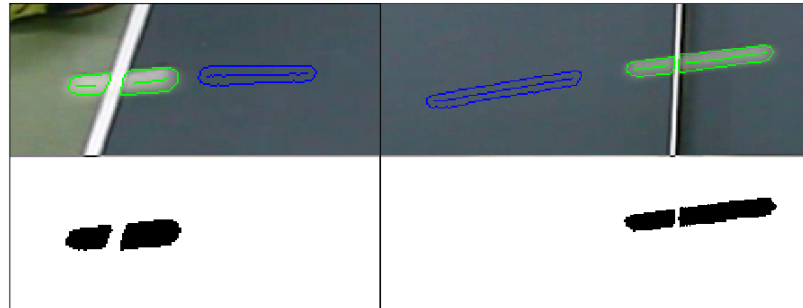
# FMO Localization

- The pipeline consists of three stages:

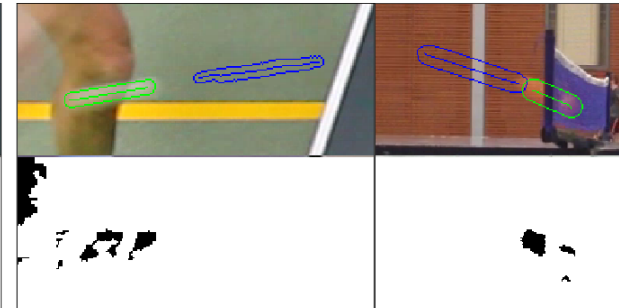
1) Explorer



2) Detector



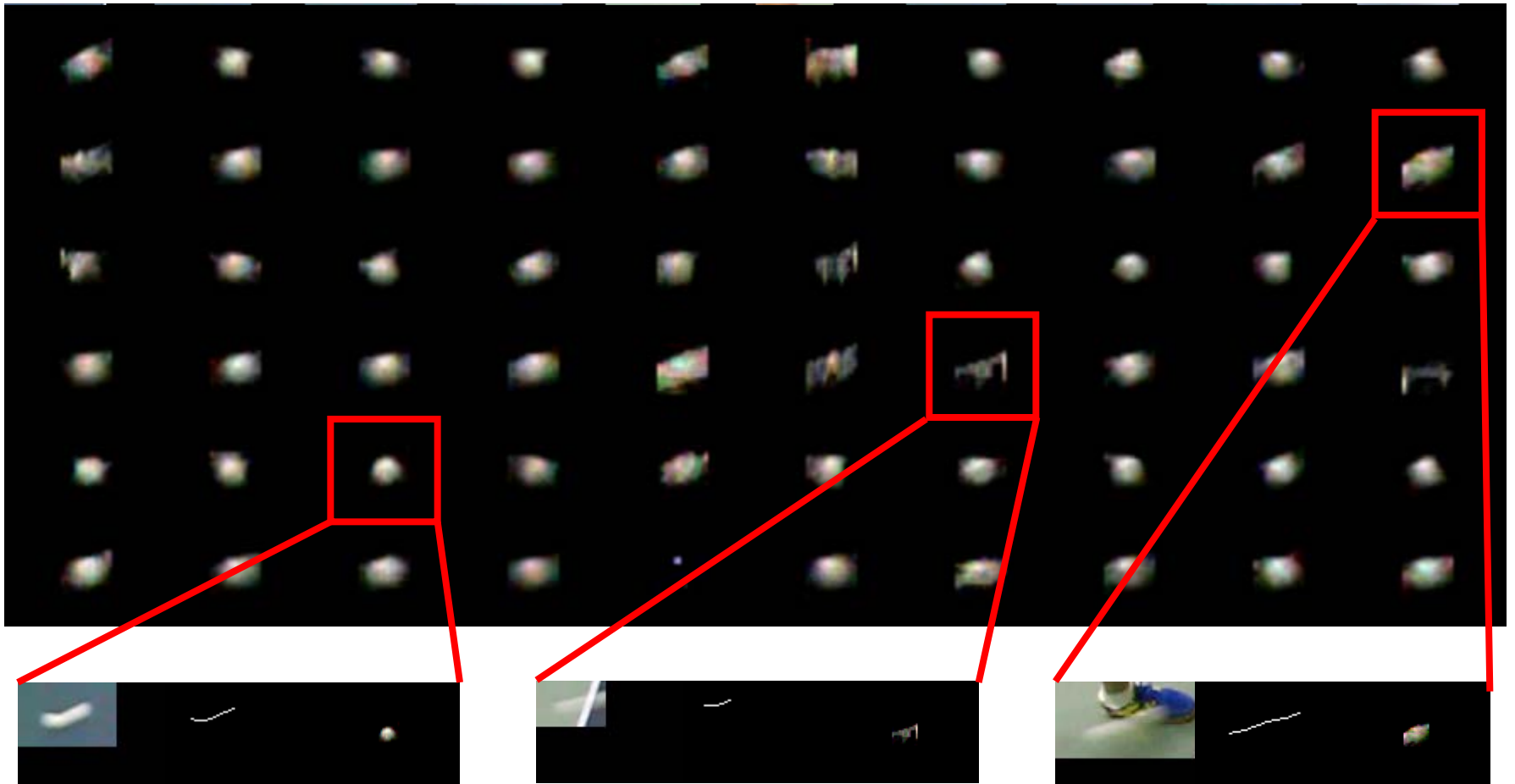
3) Tracker



# Input FMO Video



# FMO Tracking + Reconstruction



# Temporal SR - Table tennis

Original sequence

# Temporal SR - Darts

Original sequence



# Rotating FMOs

- Simple convolution

$$z = (1 - h * m)b + h * f$$

replaced by space-variant convolution

$$z = (1 - \mathcal{H}m)b + \mathcal{H}f$$

$\mathcal{H}$  is a function of trajectory and angular velocity

$f$  is a 3D object (sphere)

- Gradient descent w.r.t  $f$
- Exhaustive search w.r.t angular velocity

# Temporal SR with Rotation



Input video frame



Estimated  
high-speed video

# Limitations

- Poor collaboration between tracking and restoration
  - Estimated blur and FMO appearance improve tracking
- Unstable FMO appearance estimation
  - Combine multiple video frames
- Track multiple FMOs

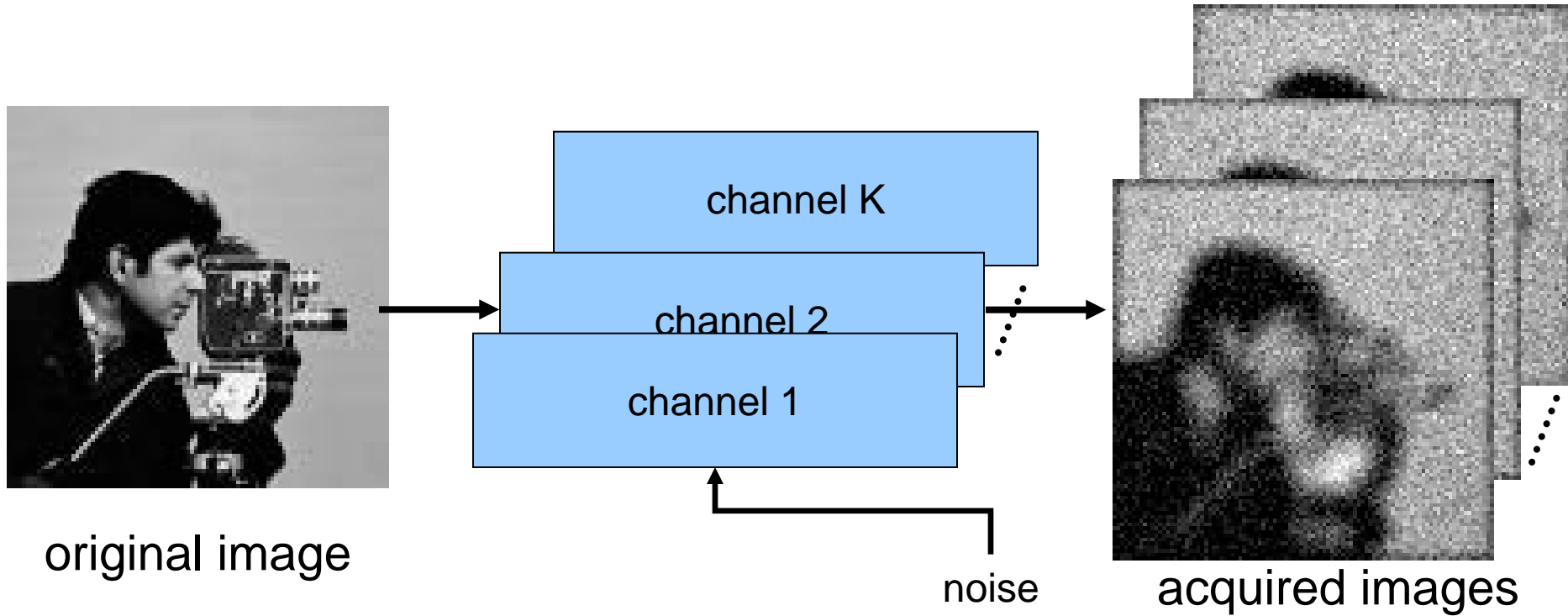


**Thank You**

**For Your Attention**



# Multichannel Model



$$[u * h_k] + n_k = z_k$$

# Multichannel Model

- Acquisition model:

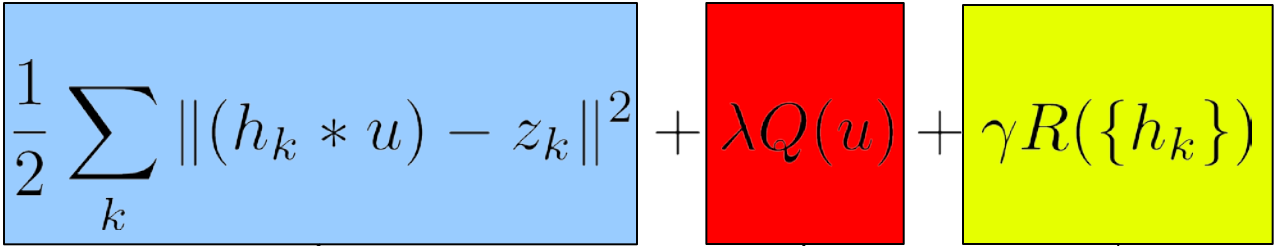
$$z_1 = (h_1 * u) + n_1$$

$$\vdots$$

$$z_K = (h_K * u) + n_K$$

- Optimization problem

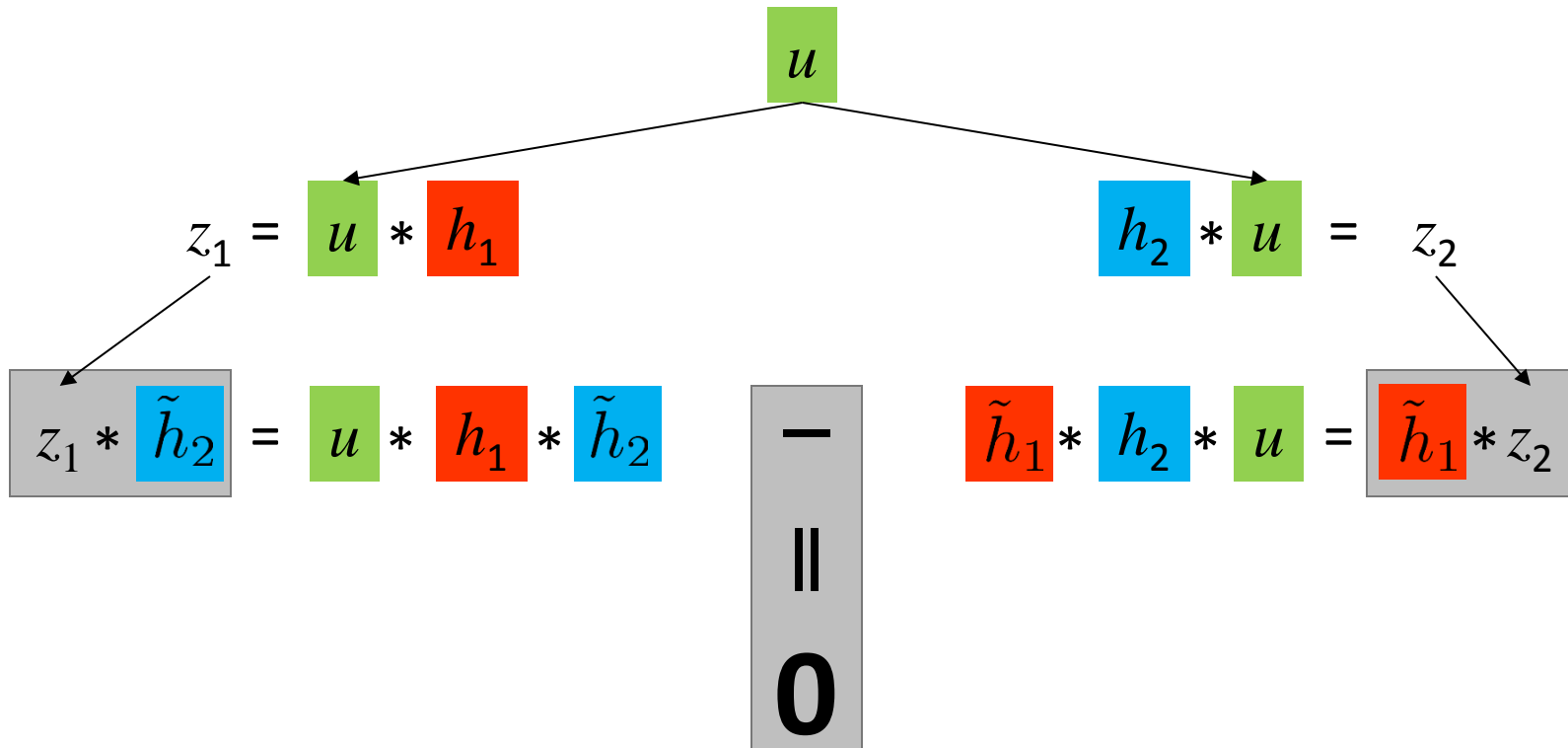
$$E(u, \{h_k\}) = \frac{1}{2} \sum_k \|(h_k * u) - z_k\|^2 + \lambda Q(u) + \gamma R(\{h_k\})$$



Data term                      Image regularization term                      Blur Regularization term

Gaussian noise L2 norm                       $Q(u) = \int \phi(|\nabla u(x)|) dx$

# Multichannel Blur Regularization



$$R(\{h_k\}) = \frac{1}{2} \sum_{1 \leq p, q \leq P} \|z_p * h_q - z_q * h_p\|^2$$

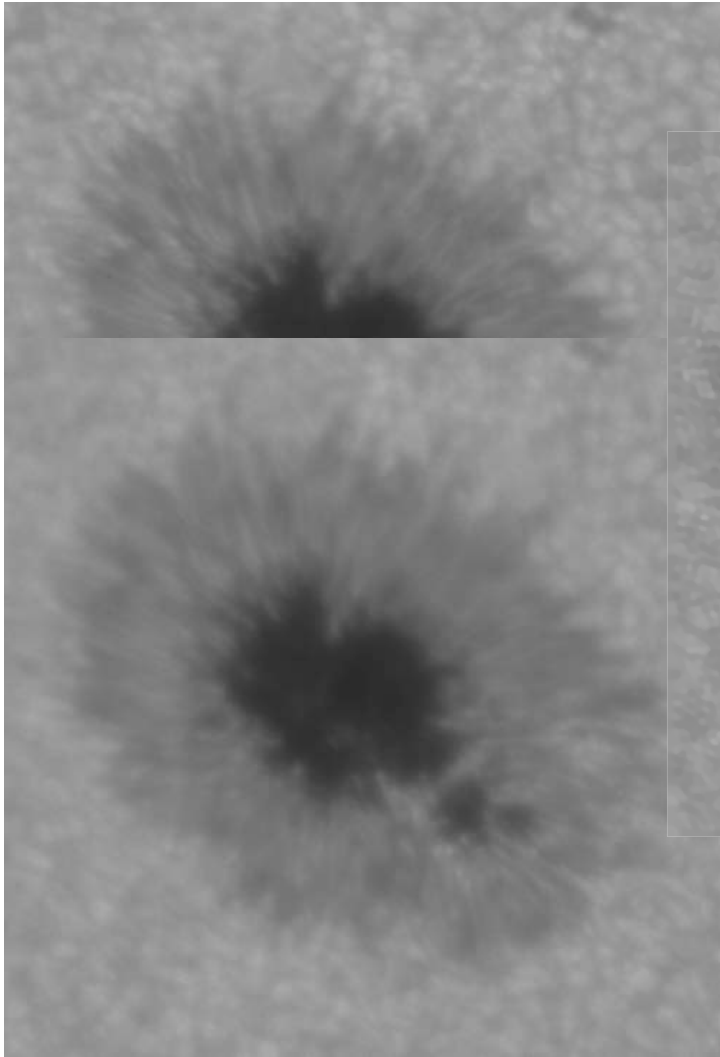


# Camera motion

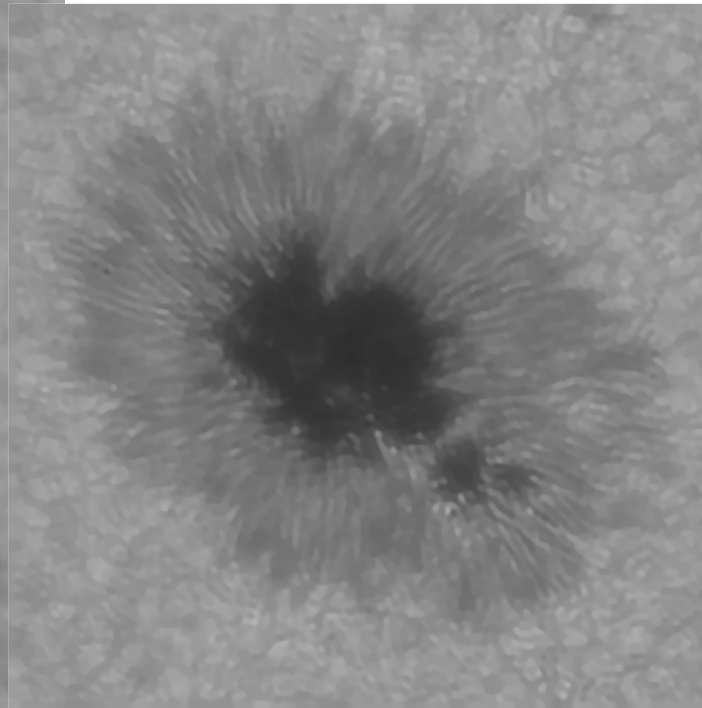


# Astronomical images

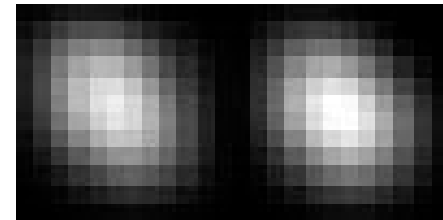
Degraded images



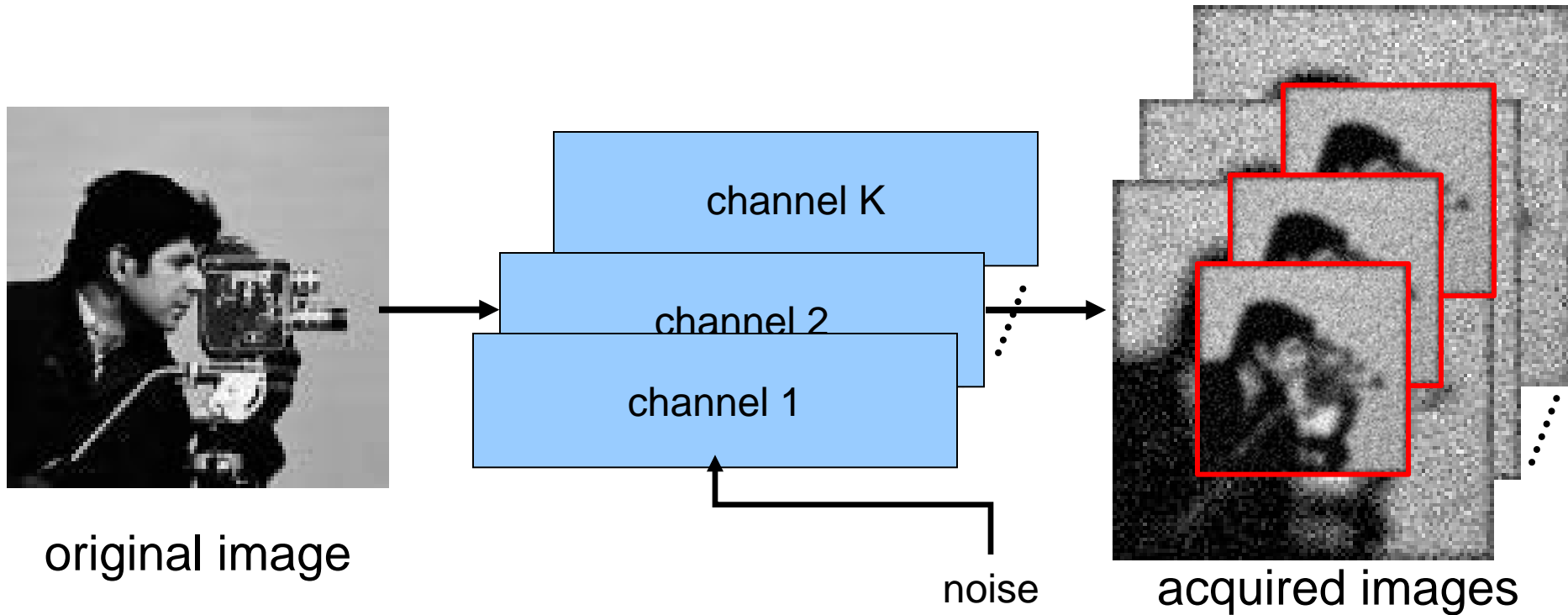
Reconstructed image



PSF estimation



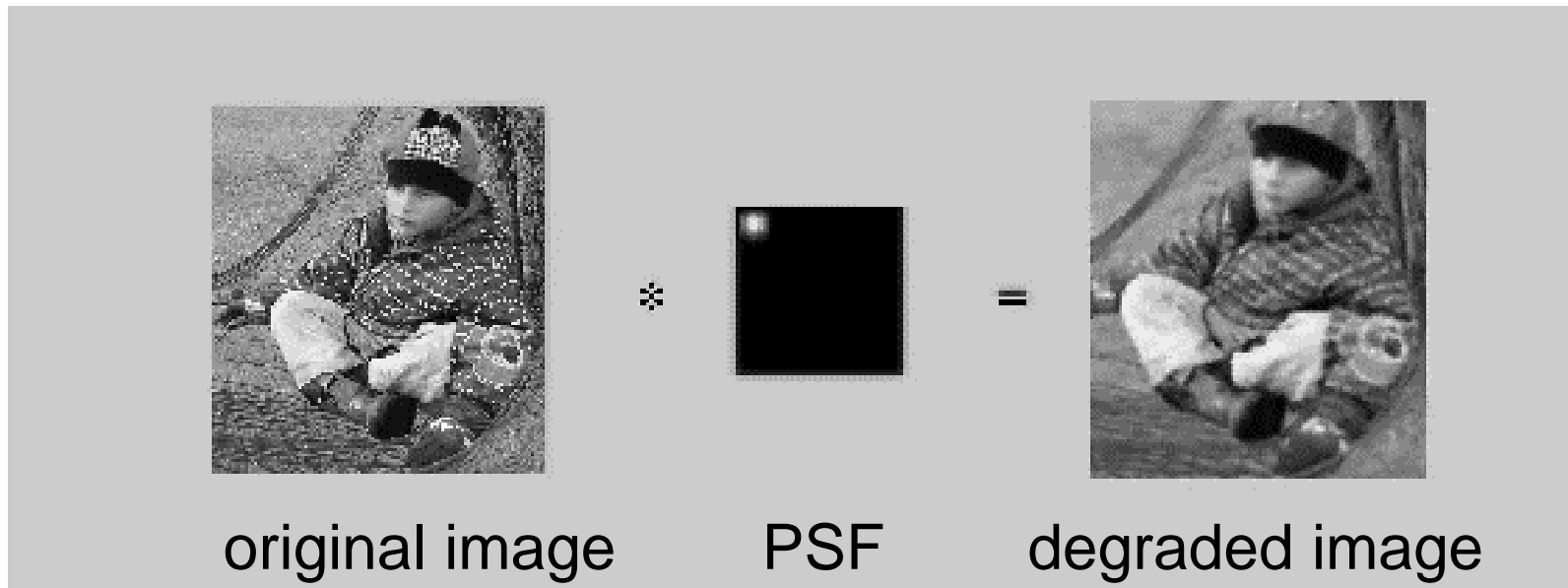
# MC Model with Decimation



$$\mathbf{D} [u * h_k] + n_k = z_k$$

$$\min_{u, h} E(u, h) = \min_{u, h} \frac{1}{2} \sum_k \| \mathbf{D}(h_k * u) - z_k \|^2 + \lambda Q(u) + \gamma R(h)$$

# Robustness to misalignment



$$D(u * h_k)[\tau_k(x)] + n_k(x) = z_k(x)$$

$$D(u * g_k)(x) + n_k(x) = z_k(x)$$



# Super-resolution



rough registration



Superresolved image (2x)



Optical zoom (ground truth)

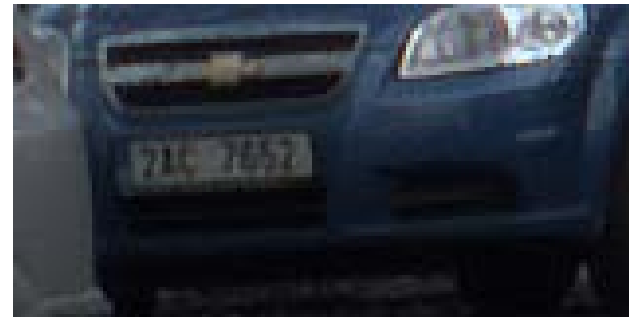
8 images



original



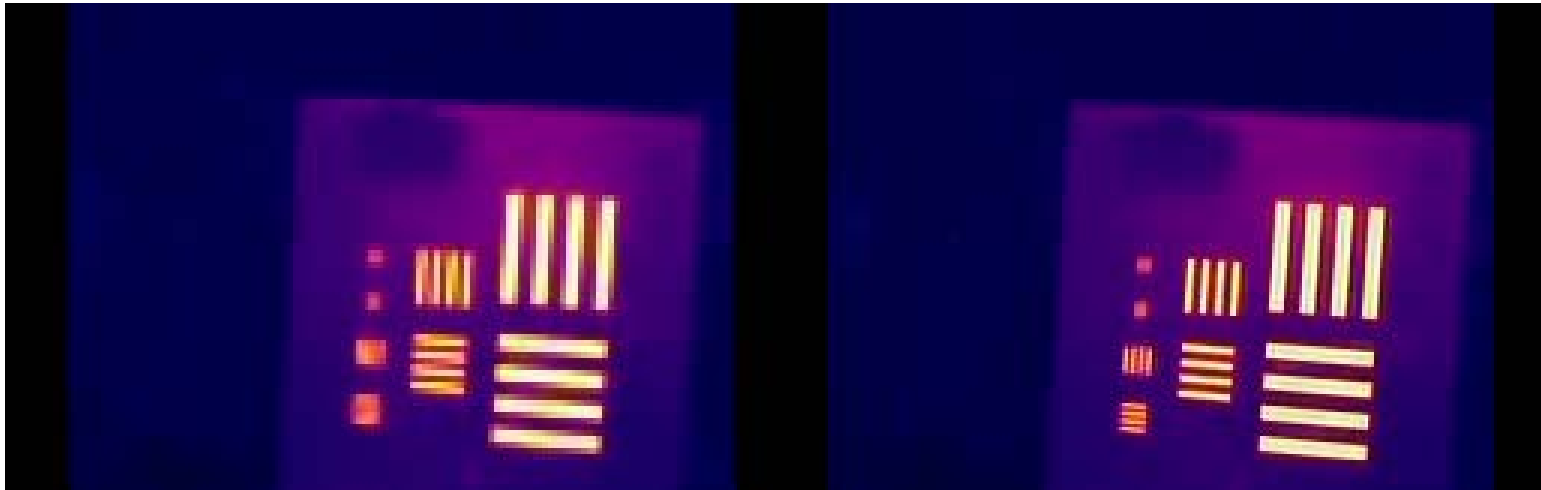
SR 2x



SR 3x



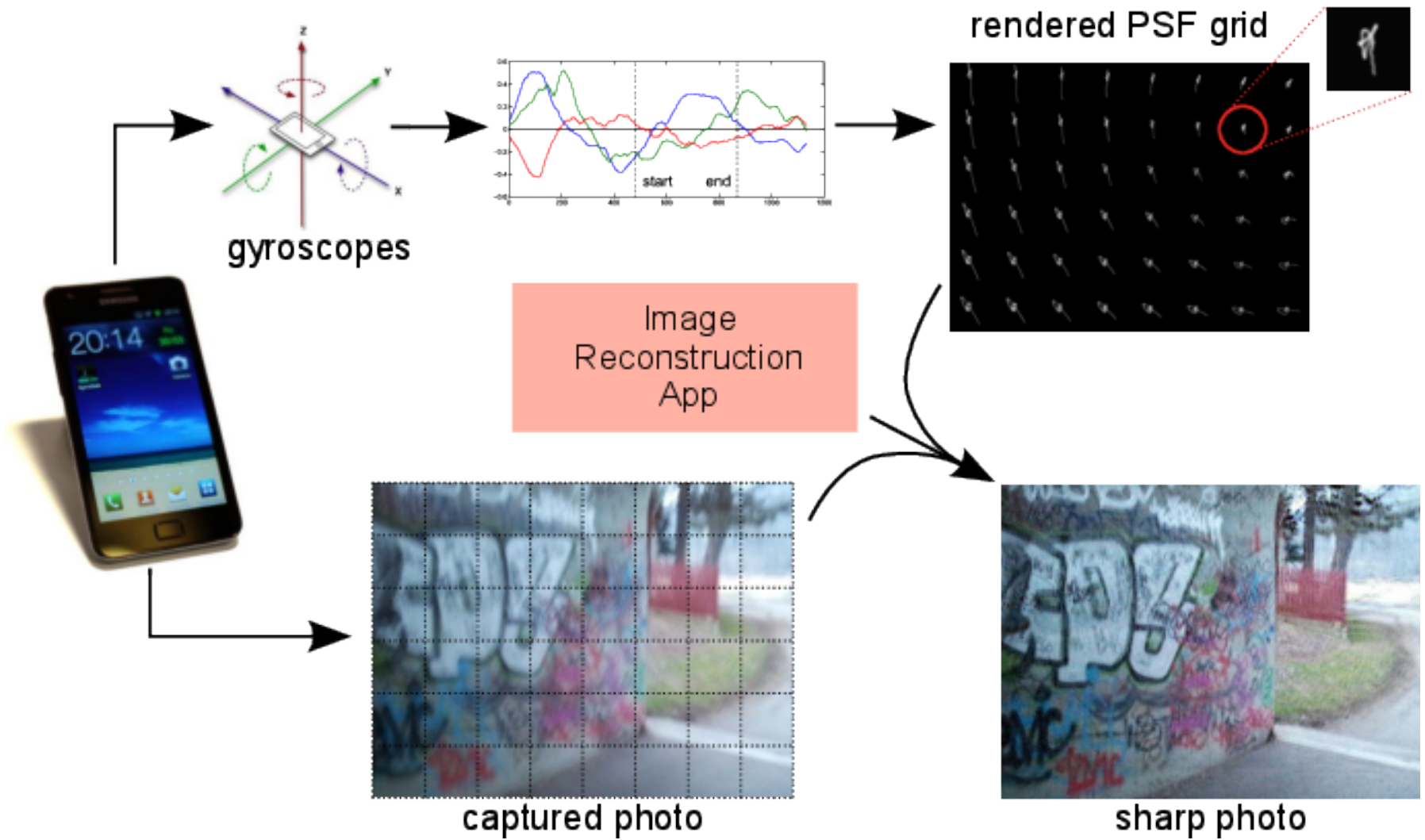
# Video Super-resolution



Input video

Super-resolution

# Embedded Systems





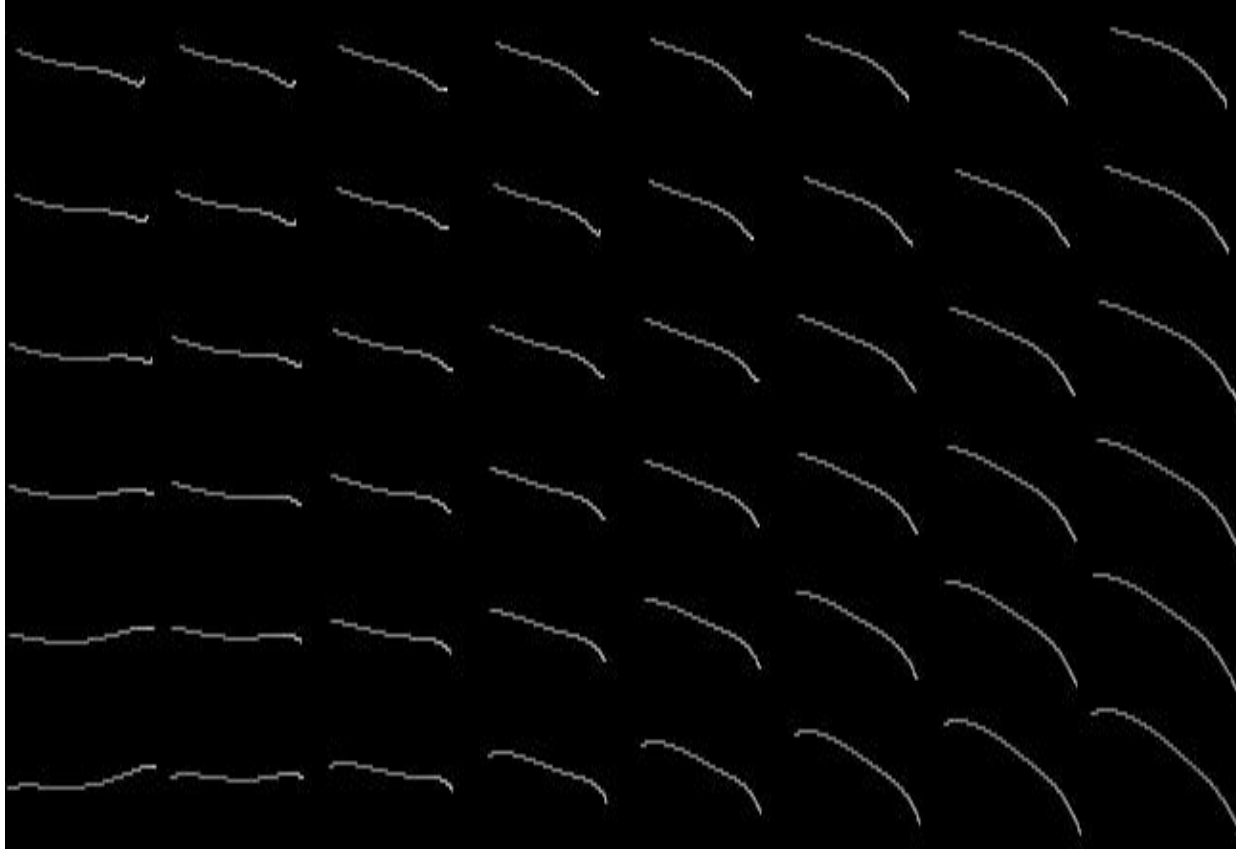


# Acquired blurred image





# Blur estimation

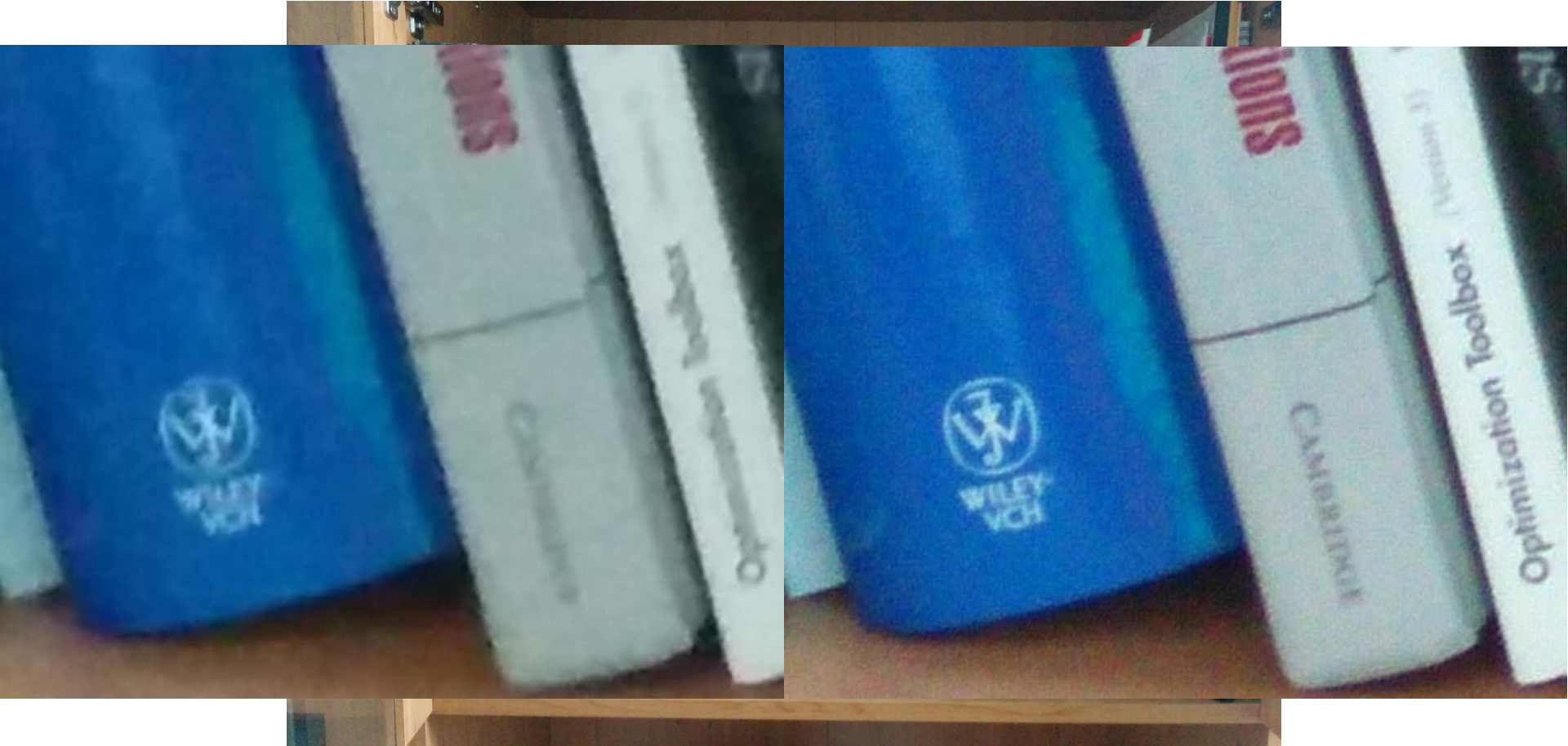




# Patch-wise Deconvolution



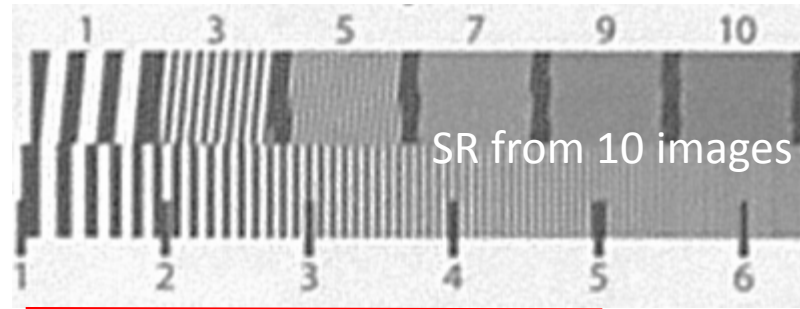
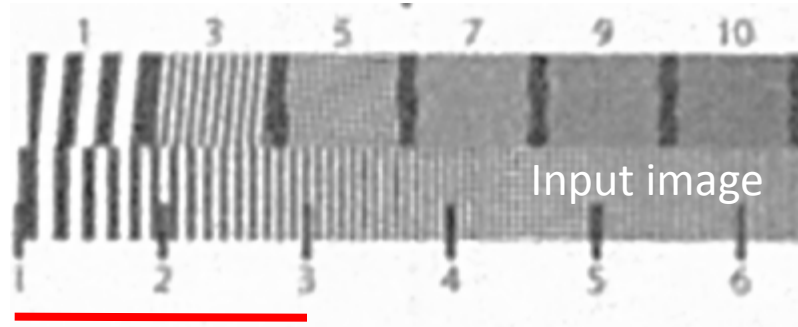
# Super-resolution



JPEG

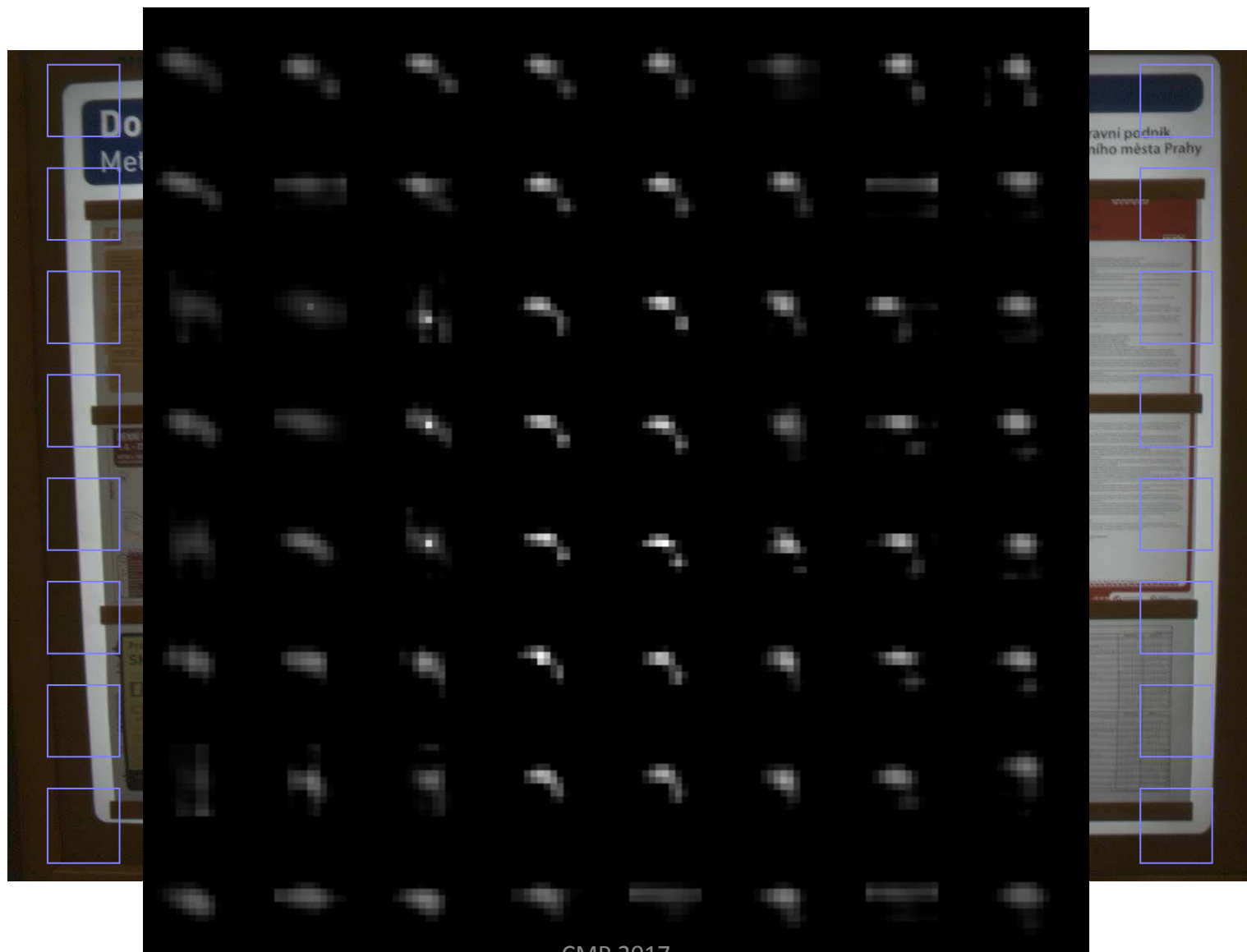
SR

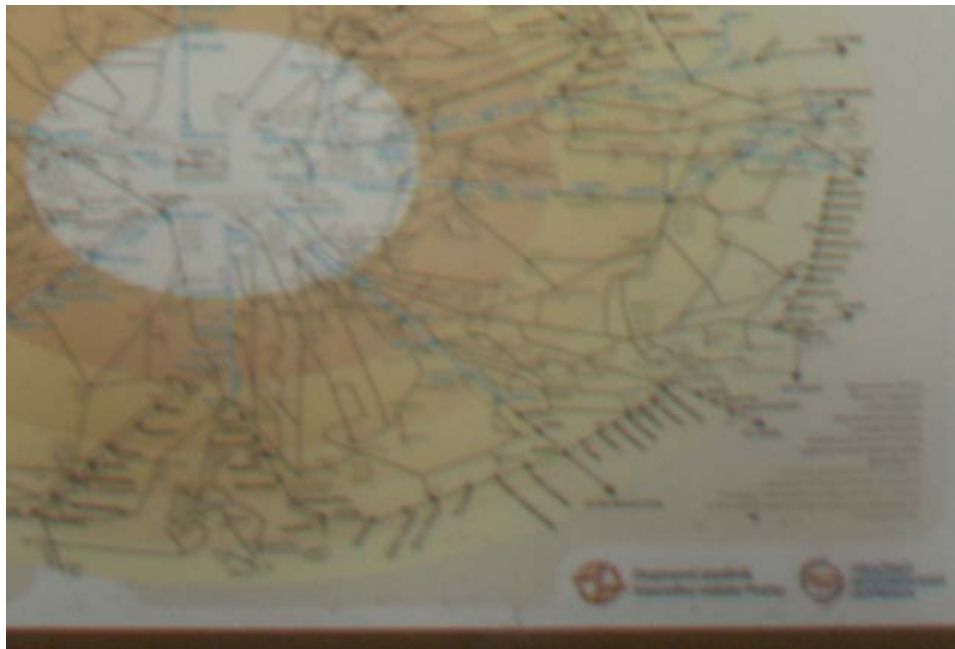
# Embedded Super-Resolution





# Patch-wise restoration





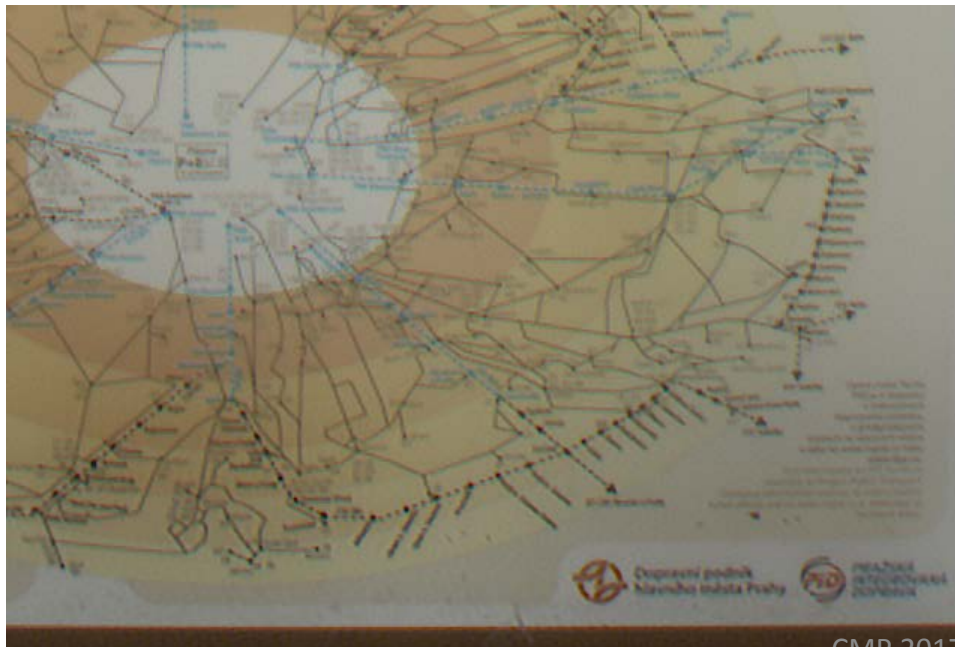
### Jízdenky a kupony MHD

Prague public transport tickets and passes

Typ jízdenky / Typ kuponu	Dospělý	Dítě	Junior	Student	Senior
1 hodina	18 Kč	9 Kč	18 Kč	18 Kč	9 Kč
2 hodiny	26 Kč	13 Kč	26 Kč	26 Kč	13 Kč
1 den	100 Kč	50 Kč	100 Kč	100 Kč	50 Kč
2 dny	330 Kč	•	330 Kč	330 Kč	•
5 dní	500 Kč	•	500 Kč	500 Kč	•
30 dní / měsíční	550 Kč	130 Kč	260 Kč	260 Kč	250 Kč
90 dní / čtvrtletní	1 480 Kč	360 Kč	720 Kč	720 Kč	660 Kč
365 dní / roční	4 750 Kč	•	•	•	•

\* Dítě může sdílet místo s dospělým. Dítě musí být doprovázeno platícím dospělým.  
 • Dítě může sdílet místo s dospělým. Dítě musí být doprovázeno platícím dospělým.

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 • Dítě může sdílet místo s dospělým. Dítě musí být doprovázeno platícím dospělým.



### Jízdenky a kupony MHD

Prague public transport tickets and passes

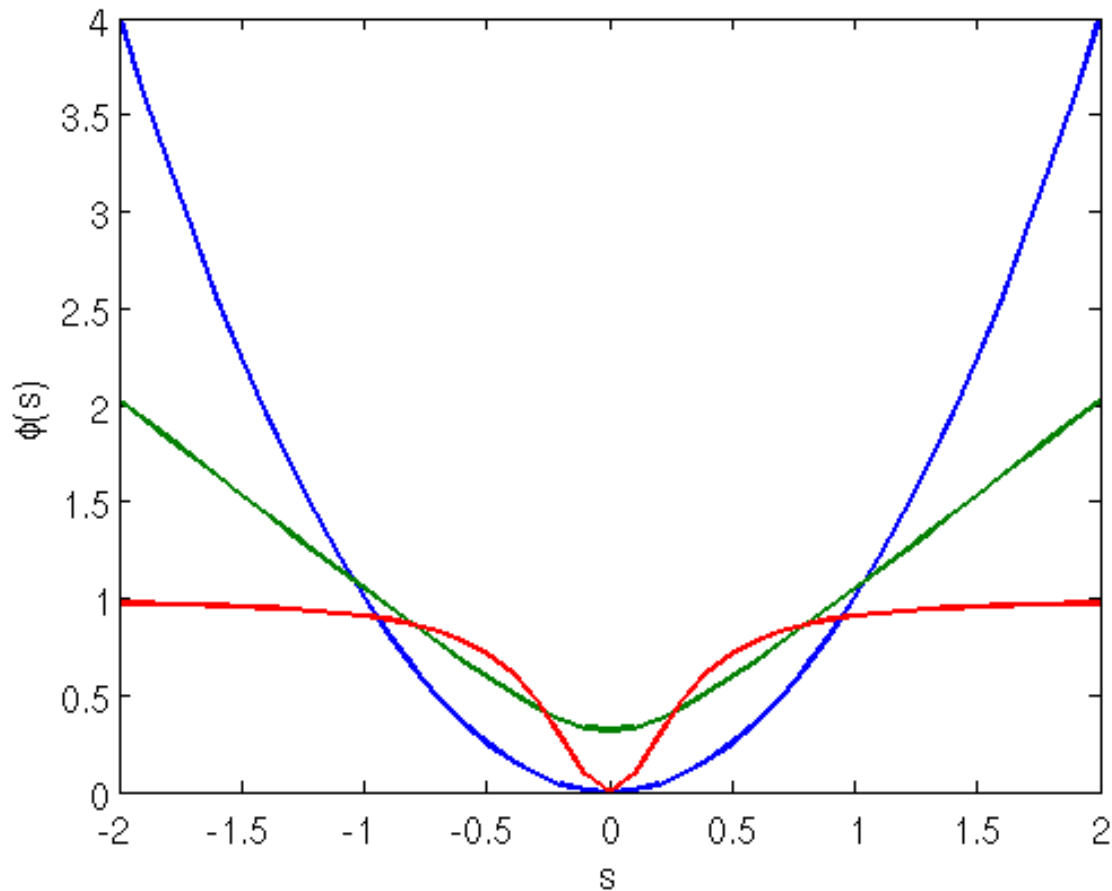
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90 dní / čtvrtletní	1 480 Kč	360 Kč	720 Kč	720 Kč	660 Kč
365 dní / roční	4 750 Kč	•	•	•	•

\* Dítě může sdílet místo s dospělým. Dítě musí být doprovázeno platícím dospělým.  
 • Dítě může sdílet místo s dospělým. Dítě musí být doprovázeno platícím dospělým.

\* Dítě může sdílet místo s dospělým. Dítě musí být doprovázeno platícím dospělým.  
 • Dítě může sdílet místo s dospělým. Dítě musí být doprovázeno platícím dospělým.

\* Dítě může sdílet místo s dospělým. Dítě musí být doprovázeno platícím dospělým.  
 • Dítě může sdílet místo s dospělým. Dítě musí být doprovázeno platícím dospělým.

# Regularization



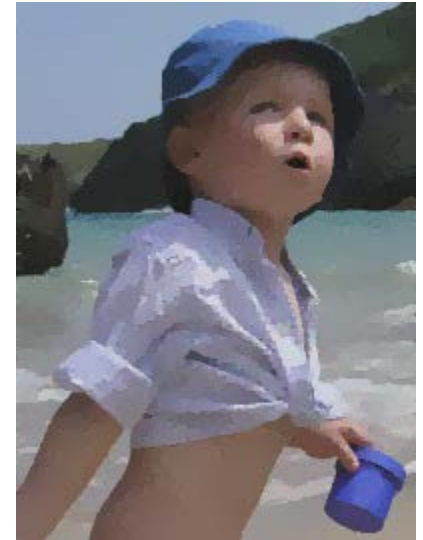
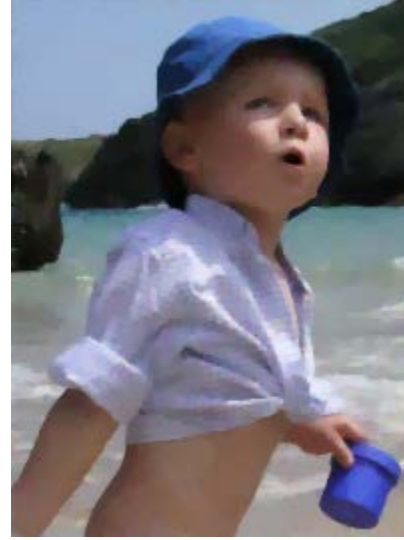
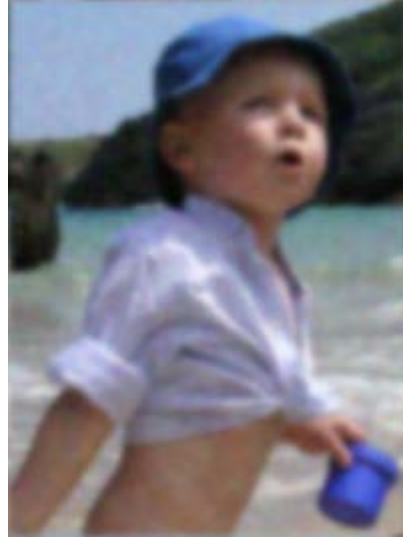
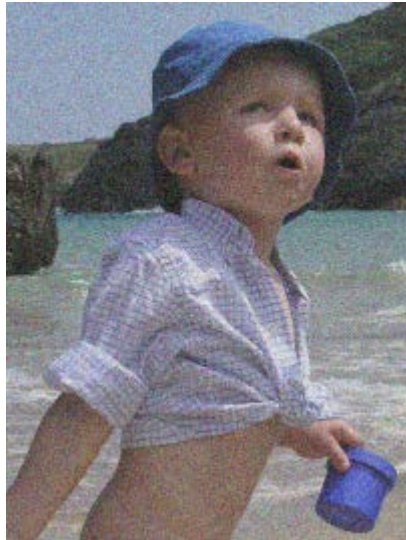
Tikhonov ...  $|s|^2$

TV ...  $|s| \approx \sqrt{|s|^2 + \epsilon}$

$l_0$ -norm ...  $|s|_0 \approx \frac{|s|^2}{|s|^2 + \epsilon}$



# Adjusting priors



$$u \quad Q(u) = \sum_i |\nabla u_i|^2$$

$$\sum_i |\nabla u_i|^1$$

$$\sum_i |\nabla u_i|^{0.5}$$

- Start with an overestimated noise level and slowly decrease it to the correct level.
- Start with  $p \ll 1$  and slowly increase it to  $p=1$ .