The Art of Solving Minimal Problems
Tricks session

Laurent Kneip

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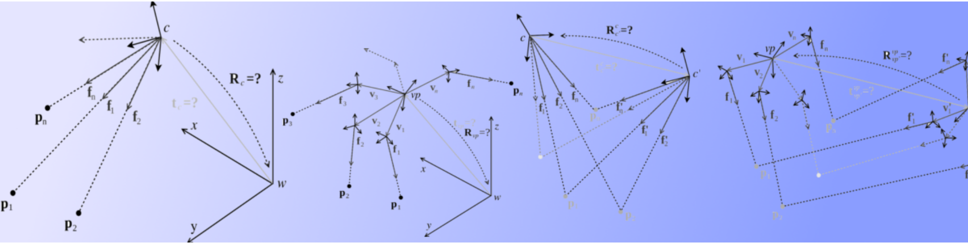
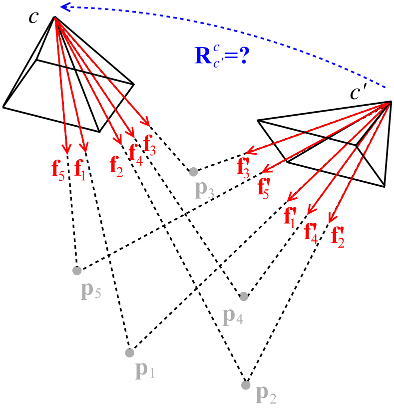
Centre of Excellence for Robotic Vision
The Australian National University



11th December, 2015

Warm start

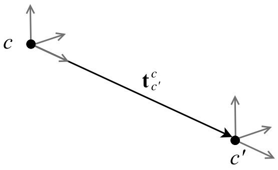
Minimal Relative Rotation with Central Camera



L. Kneip, R. Siegwart, and M. Pollefeys. Finding the Exact Rotation Between Two Images Independently of the Translation. ECCV, Florence, Italy, 2012.

Minimal Relative Rotation with Central Camera

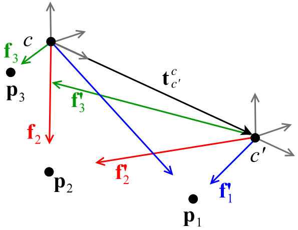
Epipolar Constraint



- Pure translation: epipolar plane normals coplanar
- Goal: constrain rotation

Minimal Relative Rotation with Central Camera

Epipolar Constraint



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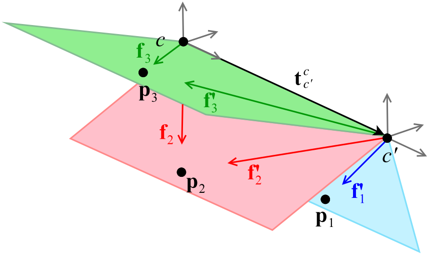

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Minimal Relative Rotation with Central Camera

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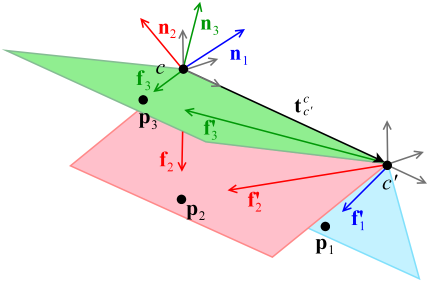

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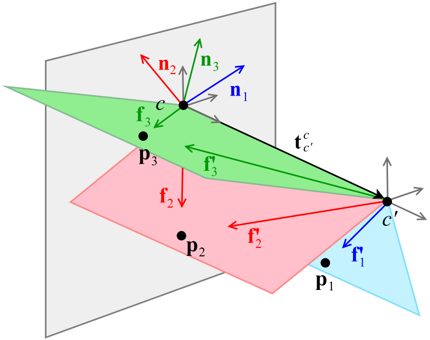

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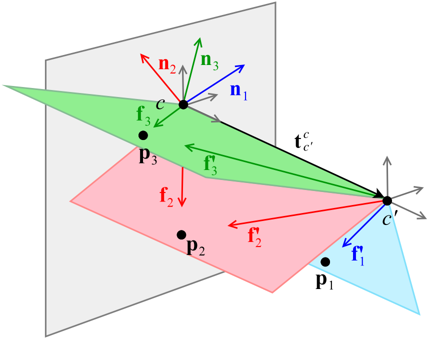

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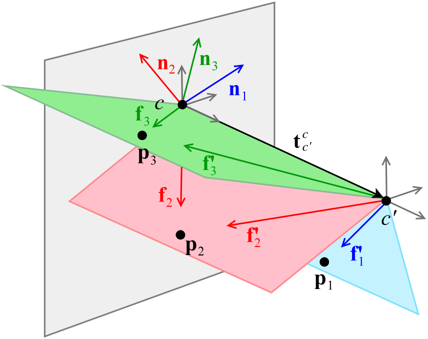

- Pure translation: epipolar plane normals coplanar
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Minimal Relative Rotation with Central Camera

Translation Independent Computation of Rotation

- Idea: Unrotate $\mathbf{f}'_i$ such that $\mathbf{n}_i$ coplanar
- Epipolar plane normal: $\mathbf{n}_i = \mathbf{f}_i \times \mathbf{R}^{c}_{c'} \mathbf{f}'_i$
- Coplanarity given if three $\mathbf{n}_i$ linearly dependent
  - Analyze determinant of 3×3 matrix with three $\mathbf{n}_i$
- 3 DoF ⇒ 3 constraints needed

Formulation 2: Minimal relative rotation

$$\begin{cases} |(\mathbf{f}_1 \times \mathbf{R}^{c}_{c'} \mathbf{f}'_1)\ (\mathbf{f}_2 \times \mathbf{R}^{c}_{c'} \mathbf{f}'_2)\ (\mathbf{f}_3 \times \mathbf{R}^{c}_{c'} \mathbf{f}'_3)| = 0 \\ |(\mathbf{f}_1 \times \mathbf{R}^{c}_{c'} \mathbf{f}'_1)\ (\mathbf{f}_2 \times \mathbf{R}^{c}_{c'} \mathbf{f}'_2)\ (\mathbf{f}_4 \times \mathbf{R}^{c}_{c'} \mathbf{f}'_4)| = 0 \\ |(\mathbf{f}_1 \times \mathbf{R}^{c}_{c'} \mathbf{f}'_1)\ (\mathbf{f}_2 \times \mathbf{R}^{c}_{c'} \mathbf{f}'_2)\ (\mathbf{f}_5 \times \mathbf{R}^{c}_{c'} \mathbf{f}'_5)| = 0 \end{cases}$$

- Solving Gröbner basis ⇒ **20 rotation matrices**
  - 8000 code lines

Minimal Relative Rotation with Central Camera

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Minimal Relative Rotation with Central Camera

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Minimal Relative Rotation with Central Camera

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Minimal Relative Rotation with Central Camera

Warm Start

- A first solution:
  - Standard Parametrization:

$$\mathbf{R}_{c'}^{c} = \begin{pmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3 \end{pmatrix} = \begin{pmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \mathbf{r}_3 \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

  - Equations:

$$\begin{cases}
|(\mathbf{f}_1 \times \mathbf{R}_{c'}^{c}\mathbf{f}_1') \; (\mathbf{f}_2 \times \mathbf{R}_{c'}^{c}\mathbf{f}_2') \; (\mathbf{f}_3 \times \mathbf{R}_{c'}^{c}\mathbf{f}_3')| & = & 0 \\
|(\mathbf{f}_1 \times \mathbf{R}_{c'}^{c}\mathbf{f}_1') \; (\mathbf{f}_2 \times \mathbf{R}_{c'}^{c}\mathbf{f}_2') \; (\mathbf{f}_4 \times \mathbf{R}_{c'}^{c}\mathbf{f}_4')| & = & 0 \\
|(\mathbf{f}_1 \times \mathbf{R}_{c'}^{c}\mathbf{f}_1') \; (\mathbf{f}_2 \times \mathbf{R}_{c'}^{c}\mathbf{f}_2') \; (\mathbf{f}_5 \times \mathbf{R}_{c'}^{c}\mathbf{f}_5')| & = & 0 \\
|(\mathbf{f}_1 \times \mathbf{R}_{c'}^{c}\mathbf{f}_1') \; (\mathbf{f}_3 \times \mathbf{R}_{c'}^{c}\mathbf{f}_3') \; (\mathbf{f}_4 \times \mathbf{R}_{c'}^{c}\mathbf{f}_4')| & = & 0 \\
|(\mathbf{f}_1 \times \mathbf{R}_{c'}^{c}\mathbf{f}_1') \; (\mathbf{f}_3 \times \mathbf{R}_{c'}^{c}\mathbf{f}_3') \; (\mathbf{f}_5 \times \mathbf{R}_{c'}^{c}\mathbf{f}_5')| & = & 0 \\
|(\mathbf{f}_1 \times \mathbf{R}_{c'}^{c}\mathbf{f}_1') \; (\mathbf{f}_4 \times \mathbf{R}_{c'}^{c}\mathbf{f}_4') \; (\mathbf{f}_5 \times \mathbf{R}_{c'}^{c}\mathbf{f}_5')| & = & 0 \\
|(\mathbf{f}_2 \times \mathbf{R}_{c'}^{c}\mathbf{f}_2') \; (\mathbf{f}_3 \times \mathbf{R}_{c'}^{c}\mathbf{f}_3') \; (\mathbf{f}_4 \times \mathbf{R}_{c'}^{c}\mathbf{f}_4')| & = & 0 \\
|(\mathbf{f}_2 \times \mathbf{R}_{c'}^{c}\mathbf{f}_2') \; (\mathbf{f}_3 \times \mathbf{R}_{c'}^{c}\mathbf{f}_3') \; (\mathbf{f}_5 \times \mathbf{R}_{c'}^{c}\mathbf{f}_5')| & = & 0 \\
|(\mathbf{f}_2 \times \mathbf{R}_{c'}^{c}\mathbf{f}_2') \; (\mathbf{f}_4 \times \mathbf{R}_{c'}^{c}\mathbf{f}_4') \; (\mathbf{f}_5 \times \mathbf{R}_{c'}^{c}\mathbf{f}_5')| & = & 0 \\
|(\mathbf{f}_3 \times \mathbf{R}_{c'}^{c}\mathbf{f}_3') \; (\mathbf{f}_4 \times \mathbf{R}_{c'}^{c}\mathbf{f}_4') \; (\mathbf{f}_5 \times \mathbf{R}_{c'}^{c}\mathbf{f}_5')| & = & 0 \\
\mathbf{r}_1 \mathbf{R}_{c'}^{c\,T} & = & \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \\
\mathbf{r}_2 \begin{pmatrix} \mathbf{r}_2^T & \mathbf{r}_3^T \end{pmatrix} & = & \begin{pmatrix} 1 & 0 \end{pmatrix} \\
\mathbf{r}_3 \mathbf{r}_3^T & = & 1
\end{cases}$$

Minimal Relative Rotation with Central Camera

Warm Start

- Macaulay output

{2}(9)mmmmmmmmm{3}(28)mmmmmmmmmmmmmmmmmmmmmmmmmmmm{4}(122)mmmomomoooooooomommoo
ooooomoooooooomooooommmmmmommmoommmmmmmmoooooooooooomoomoommmoommmmmmmmmmmmmmoooo
mmmmmmmmmoooo{5}(330)ooooooooooooooooooooooooooooomoomoooooooooooommmmmmmmmmmoo
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ooooooooooooooooooooooooooooooooooooooooooo{6}(141)oooooooooooooooooooo
oooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooo
oooooooooooooooooooooooooooooooooooooooooo
number of (nonminimal) gb elements = 131
number of monomials = 3294
#reduction steps = 42277
#spairs done = 3866
ncalls = 11190
nloop = 46519
nsaved = 130

Minimal Relative Rotation with Central Camera

Warm Start

- Trick: add more equations!

$$
\begin{cases}
|(\mathbf{f}_1 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_1)\ (\mathbf{f}_2 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_2)\ (\mathbf{f}_3 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_3)| & = & 0 \\
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|(\mathbf{f}_1 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_1)\ (\mathbf{f}_2 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_2)\ (\mathbf{f}_5 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_5)| & = & 0 \\
|(\mathbf{f}_1 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_1)\ (\mathbf{f}_3 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_3)\ (\mathbf{f}_4 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_4)| & = & 0 \\
|(\mathbf{f}_1 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_1)\ (\mathbf{f}_3 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_3)\ (\mathbf{f}_5 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_5)| & = & 0 \\
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|(\mathbf{f}_2 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_2)\ (\mathbf{f}_3 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_3)\ (\mathbf{f}_4 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_4)| & = & 0 \\
|(\mathbf{f}_2 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_2)\ (\mathbf{f}_3 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_3)\ (\mathbf{f}_5 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_5)| & = & 0 \\
|(\mathbf{f}_2 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_2)\ (\mathbf{f}_4 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_4)\ (\mathbf{f}_5 \times \mathbf{R}^{c}_{c'}\mathbf{f}'_5)| & = & 0 \\
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\mathbf{r}_1 \mathbf{R}^{c\ T}_{c'} & = & \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \\
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\mathbf{r}_3 \mathbf{r}_3^T & = & 1 \\
\mathbf{c}_1^T \mathbf{R}^{c}_{c'} & = & \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \\
\mathbf{c}_2^T \begin{pmatrix} \mathbf{c}_2 & \mathbf{c}_3 \end{pmatrix} & = & \begin{pmatrix} 1 & 0 \end{pmatrix} \\
\mathbf{c}_1 \times \mathbf{c}_2 - \mathbf{c}_3 & = & \mathbf{0} \\
\mathbf{c}_2 \times \mathbf{c}_3 - \mathbf{c}_1 & = & \mathbf{0} \\
\mathbf{c}_3 \times \mathbf{c}_1 - \mathbf{c}_2 & = & \mathbf{0}
\end{cases}
$$

Minimal Relative Rotation with Central Camera

Warm Start

- New Macaulay output

```
{2}(20)mmmmmmmmmmmmmmmmmmmm{3}(74)oooooooooooooooooooooooooooooooooooooooo
oooooooooooooooooooooooommmmmmmmmm{4}(62)mmmmmmmmmmmmmmmmmmmmommmmmmooooooo
oooooooooooooooooooooooooooo{5}(144)ooooooooooooooooooooooooooooooooooooooo
ooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooo
oooooooooooooooooooooooooooooo
number of (nonminimal) gb elements = 56
number of monomials = 873
#reduction steps = 12993
#spairs done = 1261
ncalls = 3059
nloop = 13367
nsaved = 63
```

Minimal Relative Rotation with Central Camera

Warm Start: Comparison

{2}(9)mmmmmmmmm{3}(28)mmmmmmmmmmmmmmmmmmmmmmmm{4}(122)mmmomomoooooooomommoo
oooomoooooooomoooommmmmmommmoommmmmmmmooooooooomoomoommmoommmmmmmmmmmmoooo
mmmmmmmmoooo{5}(330)oooooooooooooooooooooooooooomoomooooooooooommmmmmmmmmoo
ooommmmmmmmmmmmmooooooooooooooooooooooooooooooooooooooooooooooooooooooooooo
ooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooo
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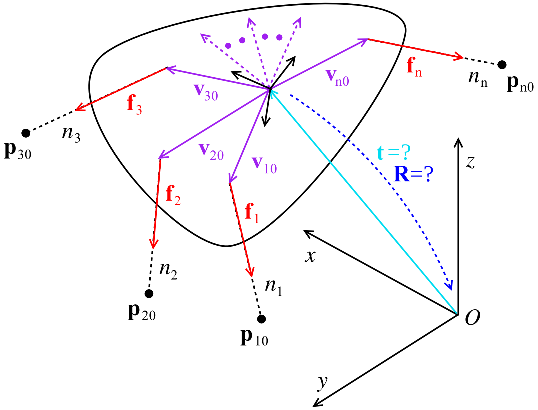
Absolute Pose with Non-Central Camera

n-Point Case: NPnP



L.Kneip, H. Li, and Y. Seo. UPnP: An optimal O(n) solution to the absolute pose problem with universal applicability.

ECCV, Zurich, Switzerland, 2014.

Absolute Pose with Non-Central Camera

n-Point Case: NPnP

- Problem formulation:

$$\begin{aligned} \text{minimize } E &= \sum_i \epsilon_i = \tilde{\mathbf{s}}^T \left\{ \sum_i \begin{bmatrix} \mathcal{A}_i^T \mathcal{A}_i & \mathcal{A}_i^T \beta_i \\ \beta_i^T \mathcal{A}_i & \beta_i^T \beta_i \end{bmatrix} \right\} \tilde{\mathbf{s}} = \tilde{\mathbf{s}}^T \mathbf{M} \tilde{\mathbf{s}}. \\ \text{subject to } \mathbf{q}^T \mathbf{q} &= 1 \end{aligned} \tag{1}$$

- Lagrange multiplier method:

$$\begin{aligned} \begin{pmatrix} \frac{\partial E}{\partial q_1} \\ \vdots \\ \frac{\partial E}{\partial q_4} \end{pmatrix} &= -\lambda \cdot \begin{pmatrix} 2q_1 \\ \vdots \\ 2q_4 \end{pmatrix} \\ \mathbf{q}^T \mathbf{q} &= 1 \end{aligned}$$

  - **80 solutions**

Absolute Pose with Non-Central Camera

n-Point Case: NPnP

- Assume ideal case:
  - Some stationary points of E fulfil norm constraint exactly!
  - Set Lagrange multiplier λ to 0⇒**16 solutions**!

$$\begin{aligned} \begin{pmatrix} \frac{\partial E}{\partial q_1} \\ \vdots \\ \frac{\partial E}{\partial q_4} \end{pmatrix} &= \mathbf{0} \\ \mathbf{q}^T\mathbf{q} &= 1 \end{aligned}$$