Sequential Learned Linear Predictors for Object Tracking

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joint work with Karel Zimmermann, Jiří Matas, and David Hurych

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Object tracking

- **Tracking** - iterative estimation of an object pose in a sequence (e.g., 2D position of Basil's head).
- Trade-off among *accuracy, speed, robustness* is explicitly taken into account.

video: Basil head

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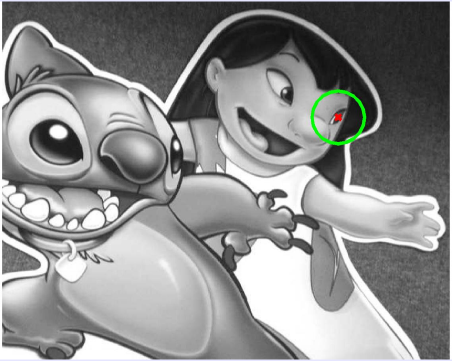
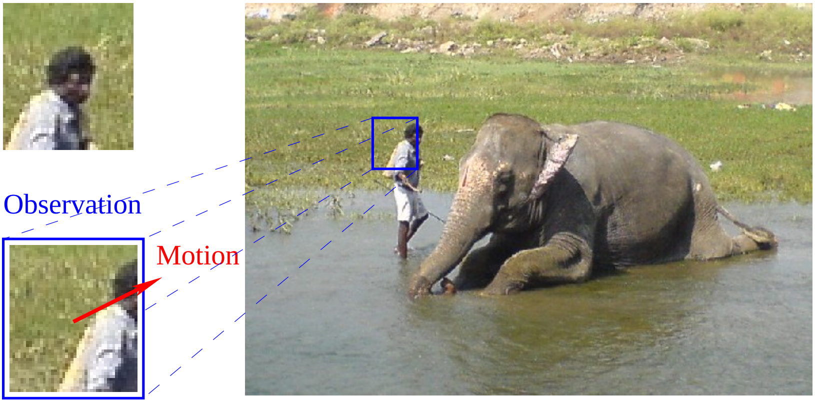


video: Cartoon eye

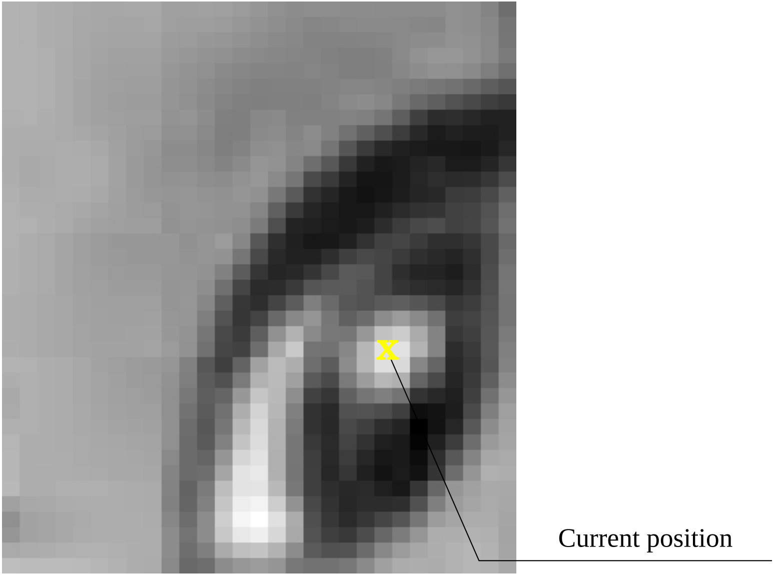
Tracking

Template Current image



Observation (image) **I** and template **J**

Tracking of a single point



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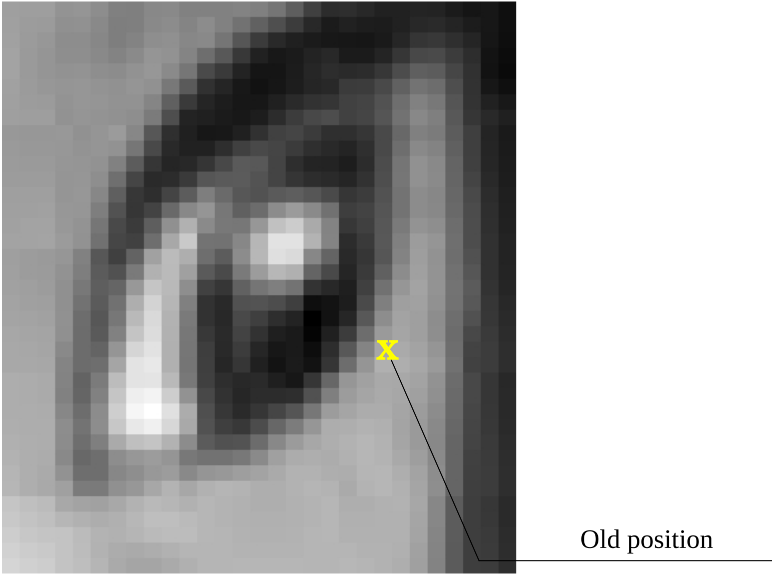

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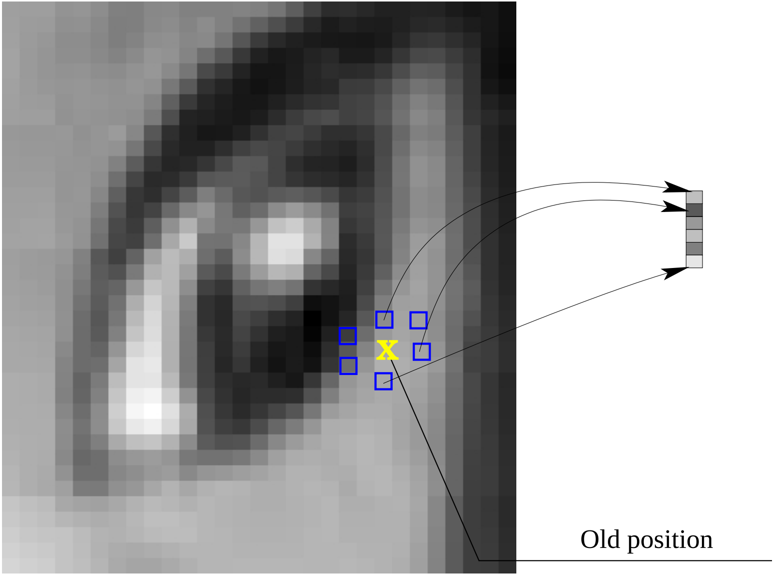
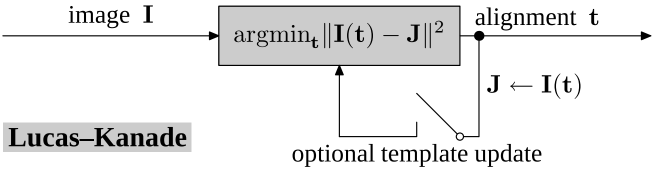
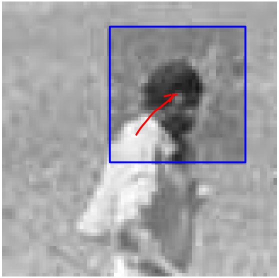
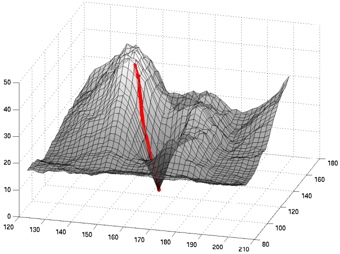


[Lucas-Kanade1981] - Iterative minimization







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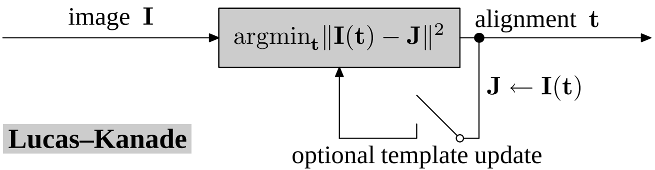
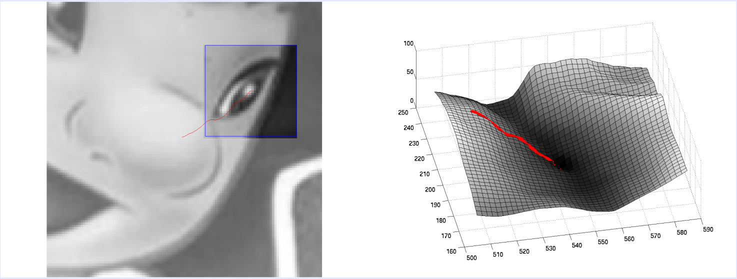
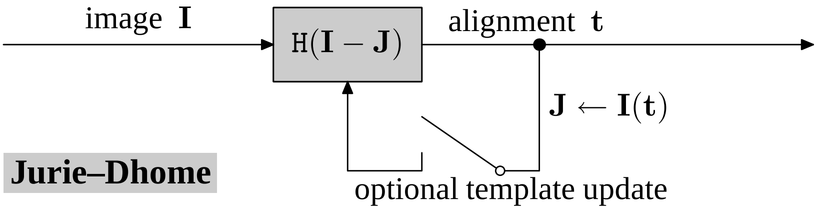




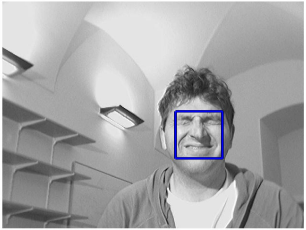
video: KLT iterations

Regression - learn the mapping in advanceo



- [Jurie-BMVC-2002] - learned motion and optional hard template update
- [Cootes-PAMI-2001] - learned regression during AAM iterations
- . . .

Learning alignment for one predictor



- $\varphi($  $) = (0,0)^\top$
- $\varphi(\quad) = (0,0)^\top$
- $\varphi(\quad) = (-25,0)^\top$
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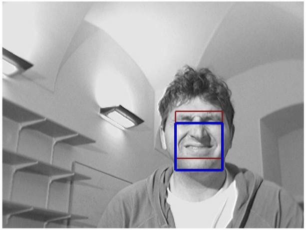

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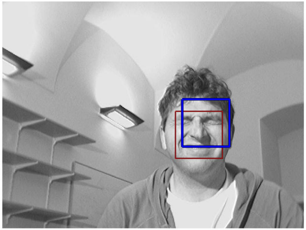

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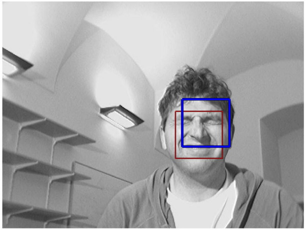

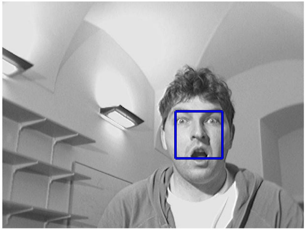

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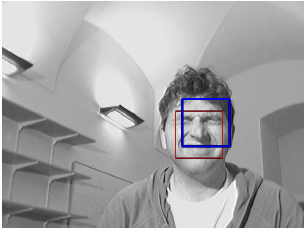

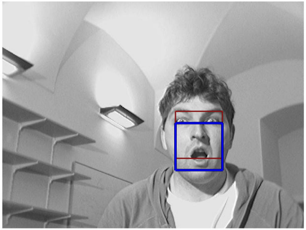

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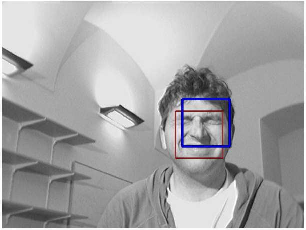

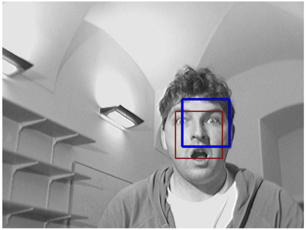

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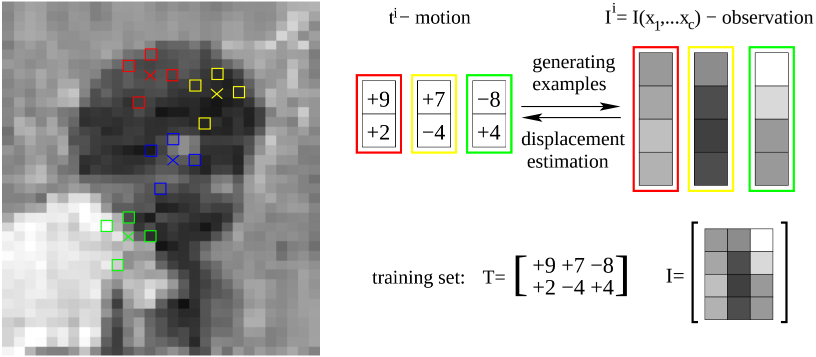
Generating training examples



Training set: $(\mathtt{I}, \mathtt{T})$

$\mathtt{I} = [\mathbf{I}^1 - \mathbf{J}, \mathbf{I}^2 - \mathbf{J}, \ldots \mathbf{I}^d - \mathbf{J}]$ and $\mathtt{T} = [\mathbf{t}^1, \mathbf{t}^2, \ldots \mathbf{t}^d]$.

LS learning

$$\varphi^* = \text{argmin}_\varphi \sum_{\mathbf{t}} \|\varphi\Big(\mathbf{I}(\mathbf{t} \circ X)\Big) - \mathbf{t}\|^2.$$

Minimizes sum of square errors over all training set. Leads to matrix pseudoinverse computation.

Example for *linear mapping*:

$$\mathtt{H}^* = \underset{\mathtt{H}}{\text{argmin}} \sum_{i=1}^{d} \|\mathtt{H}(\mathbf{I}^i - \mathbf{J}) - \mathbf{t}^i\|_2^2 = \underset{\mathtt{H}}{\text{argmin}} \|\mathtt{H}\mathtt{I} - \mathtt{T}\|_F^2$$

after some derivation

$$\mathtt{H}^* = \mathtt{T}\underbrace{\mathtt{I}^\top(\mathtt{I}\mathtt{I}^\top)^{-1}}_{\mathtt{I}^+} = \mathtt{T}\mathtt{I}^+.$$

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after some derivation

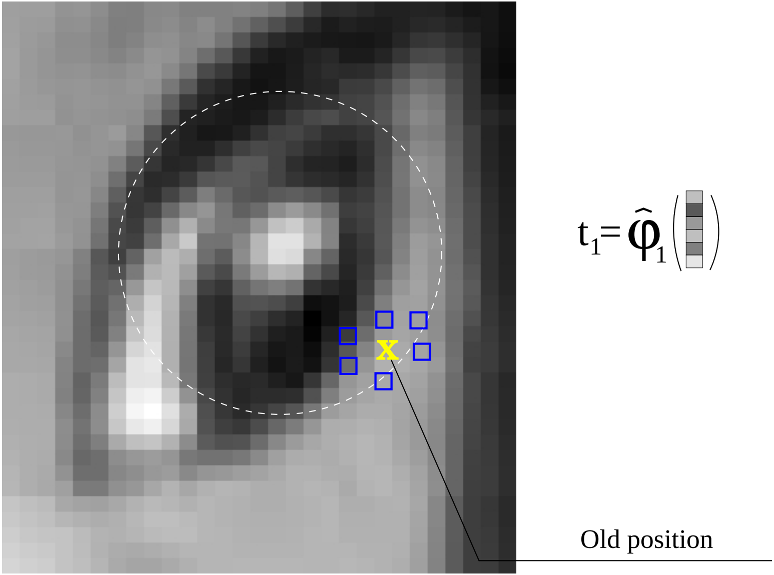
$$\mathtt{H}^* = \mathtt{T}\underbrace{\mathtt{I}^\top(\mathtt{II}^\top)^{-1}}_{\mathtt{I}^+} = \mathtt{TI}^+.$$

Min-max learning

$$\varphi^* = \operatorname{argmin}_\varphi \max_{\mathbf{t}} \|\varphi\Big(\mathbf{I}(\mathbf{t} \circ X)\Big) - \mathbf{t}\|_\infty .$$

Minimizes the *worst case* (the biggest estimation error) in the training set. Leads to linear programming.

Tracking of a single point by a *sequence* of predictors



$$t_1 = \hat{\varphi}_1\left(\;\right)$$

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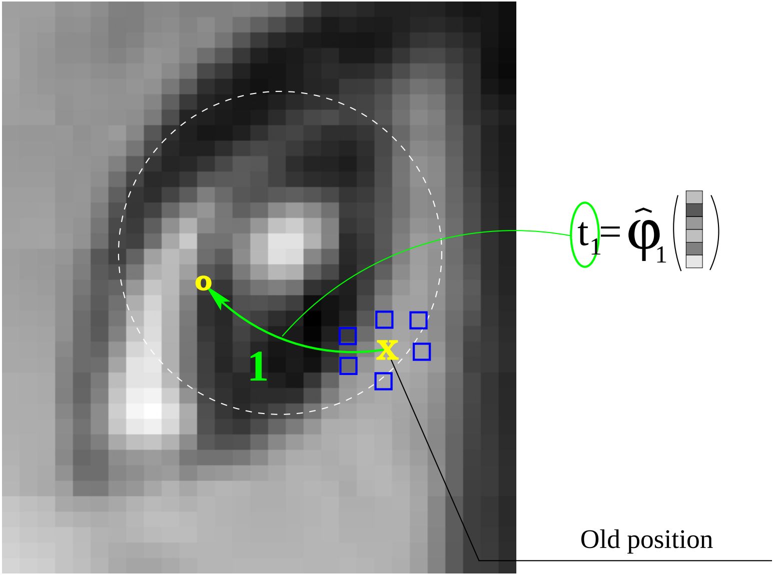

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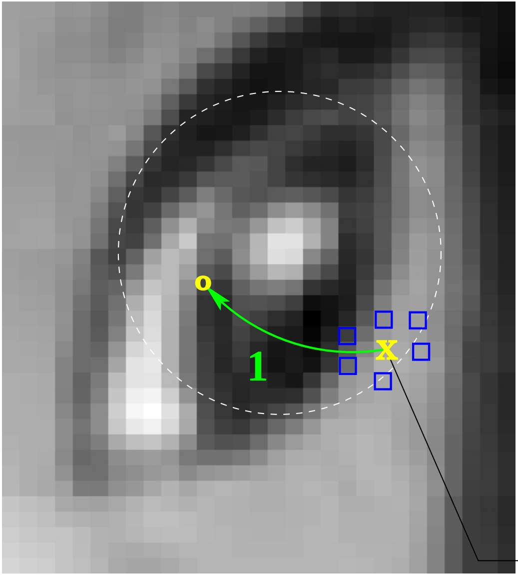

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Old position

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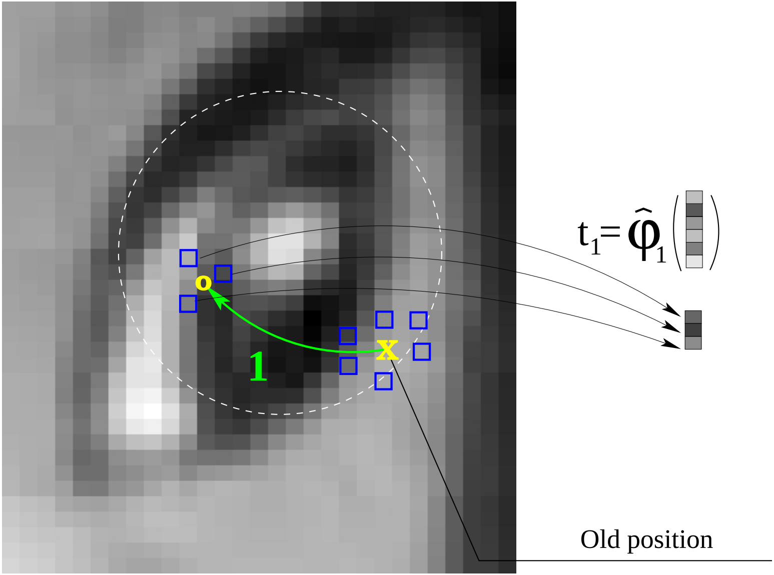

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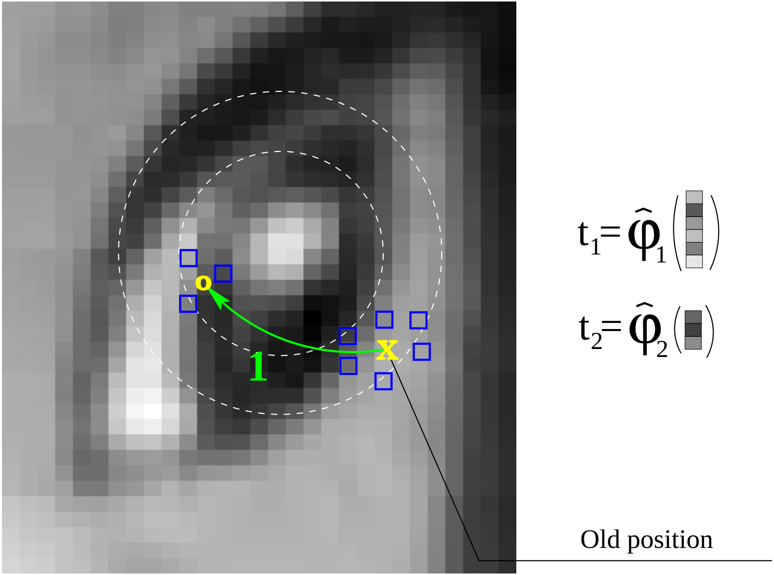
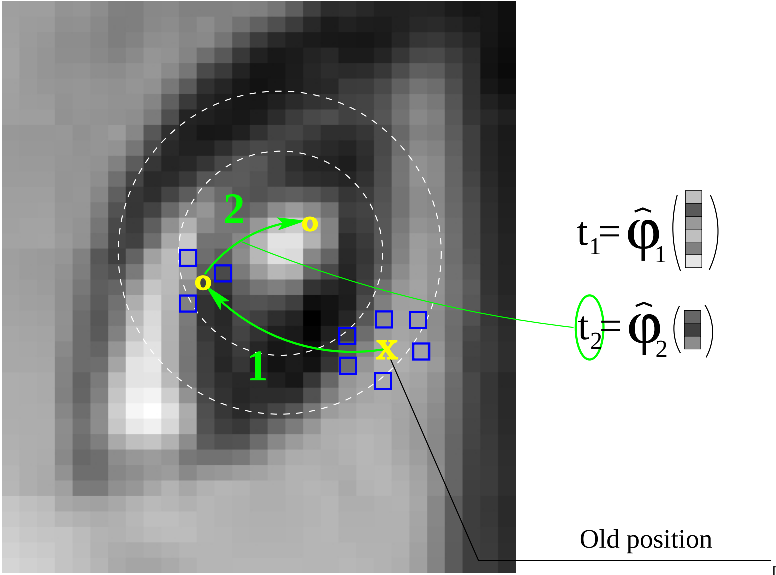


$t_1 = \hat{\varphi}_1(\cdot)$

$t_2 = \hat{\varphi}_2(\cdot)$

Tracking of a single point by a *sequence* of predictors



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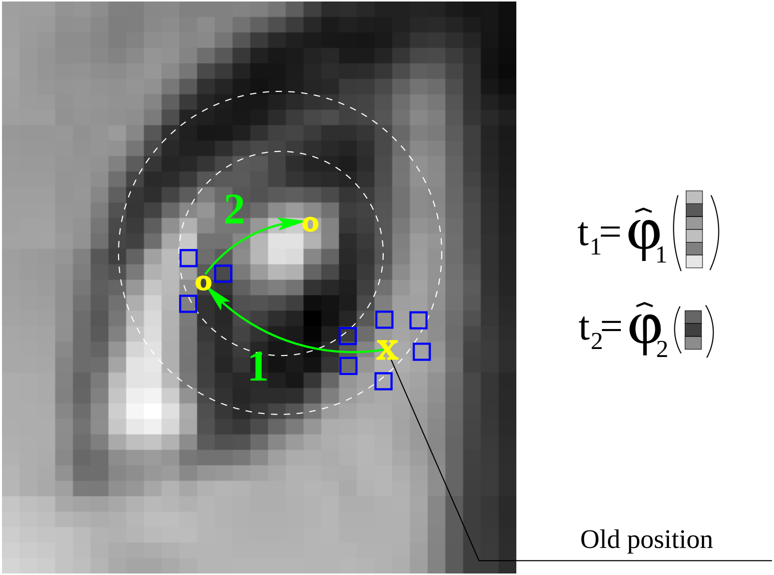

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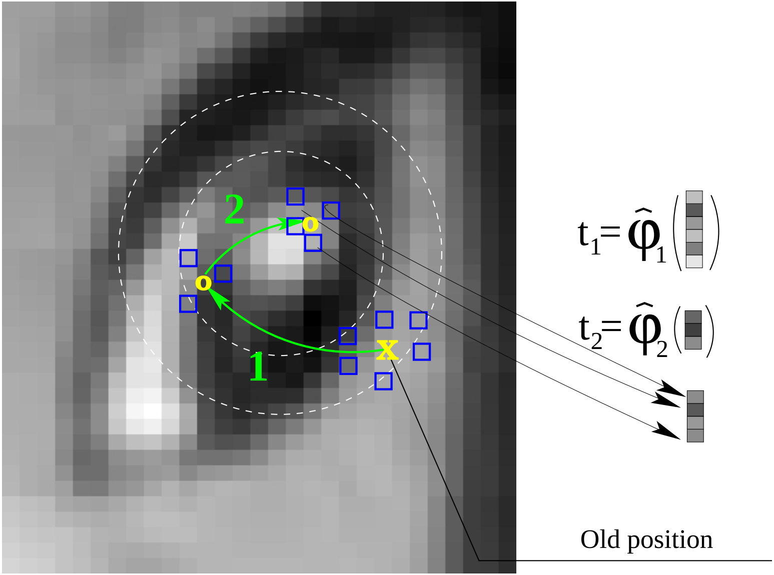

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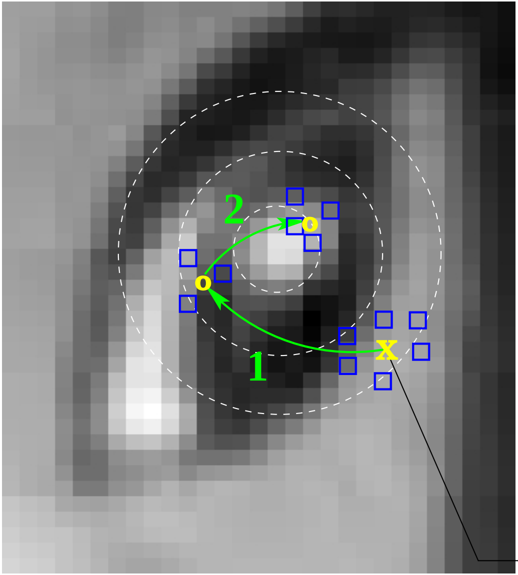

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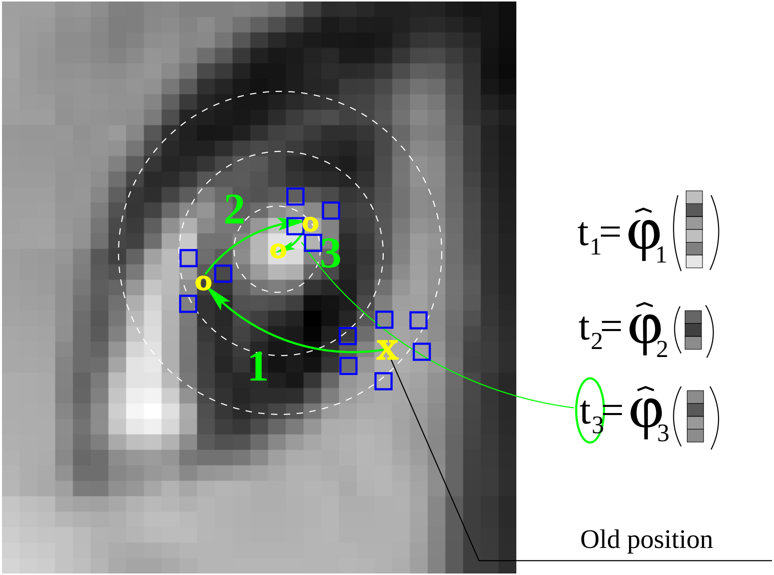

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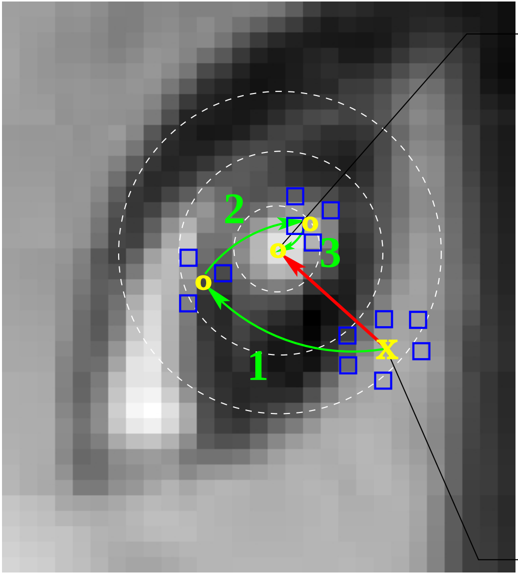

New position

$t_1 = \hat{\varphi}_1 \left(\; \right)$

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Old position

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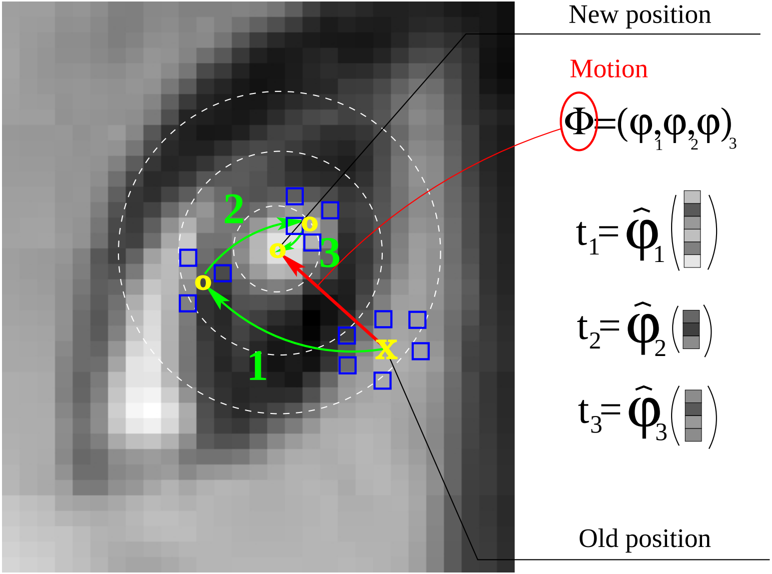

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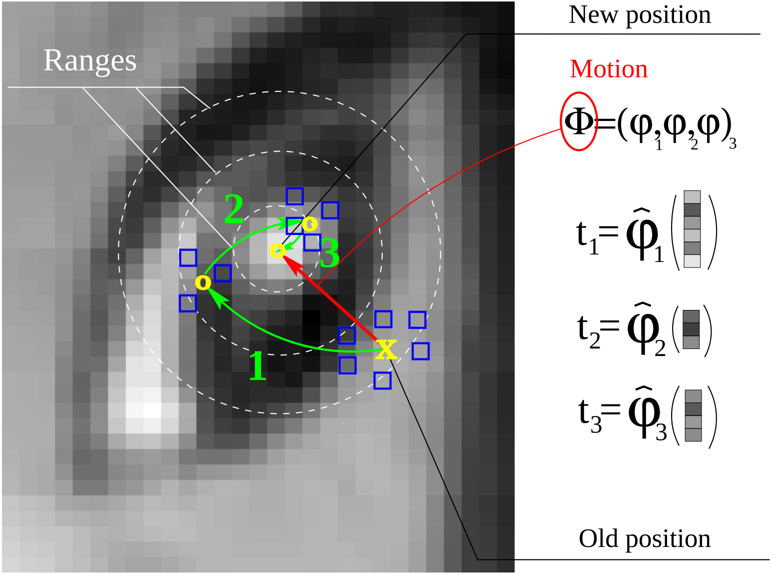

Learning of sequential predictor

- **Learning** - searching for the sequence with predefined *range*, *accuracy* and minimal *computational cost*.
  - [Zimmermann-PAMI-2009] - Dynamic programming estimates the optimal sequence of linear predictors.
  - [Zimmermann-IVC-2009] - Branch & bound search allows for time constrained learning (demo in MATLAB).

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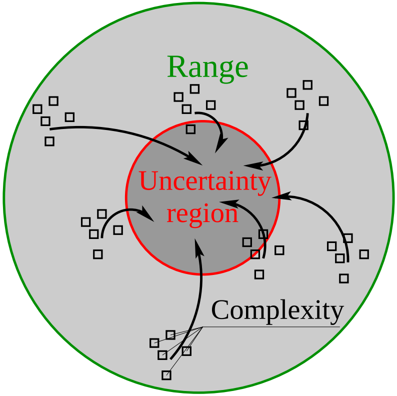
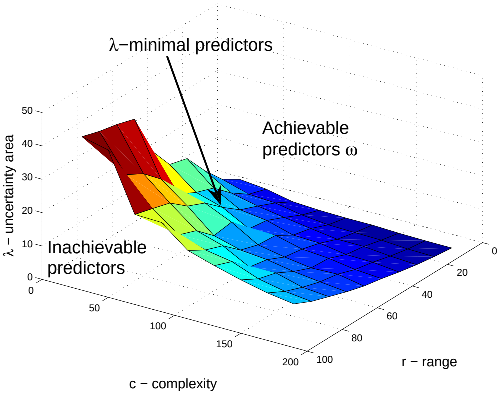


- **Range**: the set of admissible motions, r.
- **Complexity**: cardinality of support set, c.
- **Uncertainty region**: the region within which all predictions lie, λ. Small red circles show acceptable uncertainty.

Learning of the optimal sequence of linear predictors



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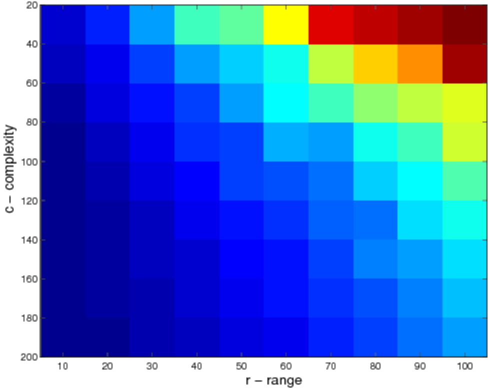

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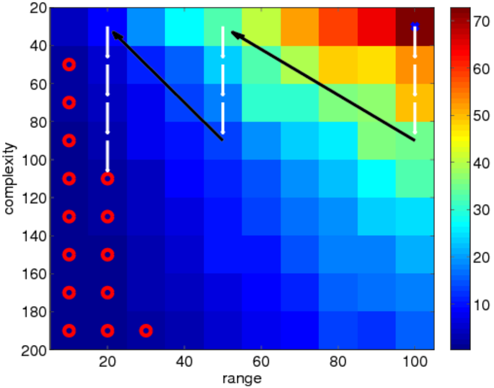

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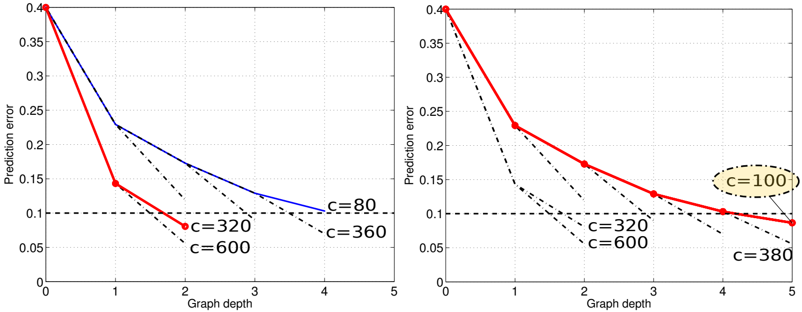
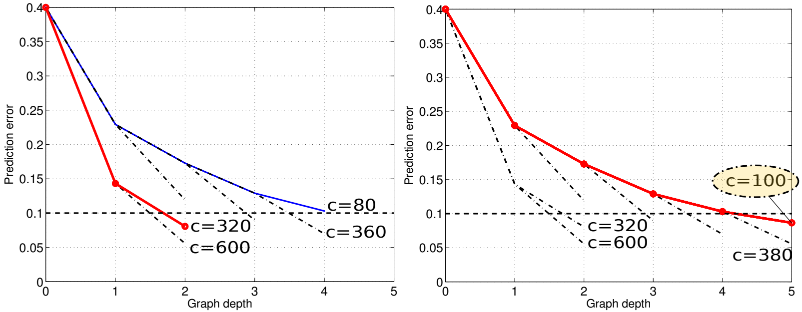
Branch and Bound



Don't forget to show the live demo!

Branch and Bound



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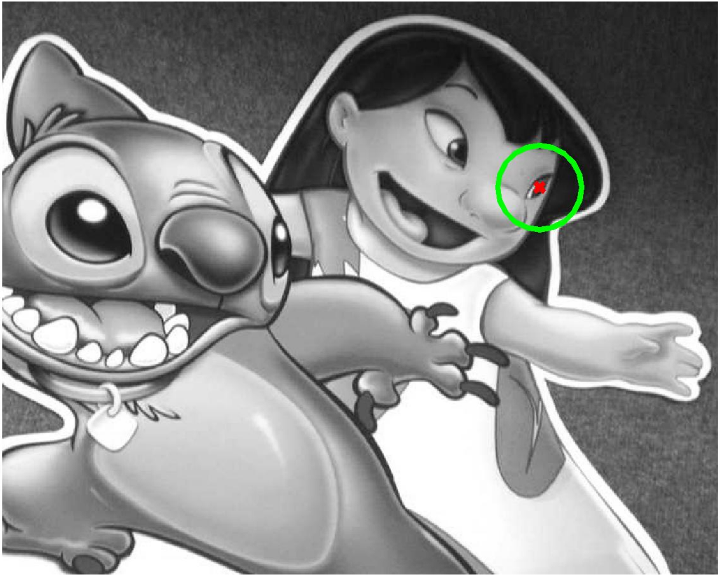
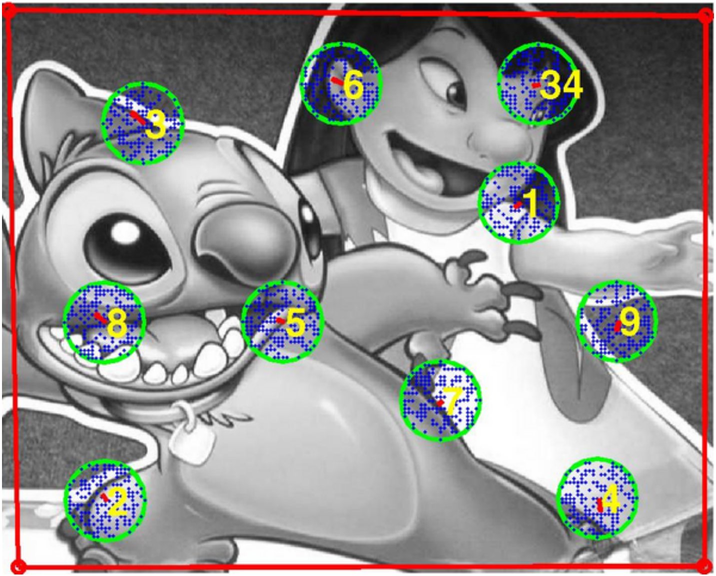
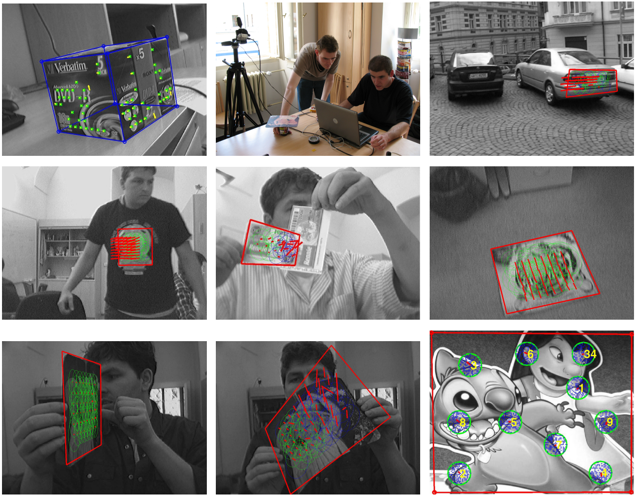
Tracking with one linear predictor.



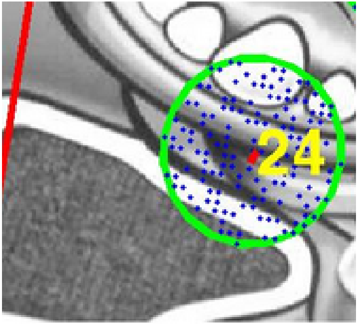
Modeling motion by number of linear predictors.

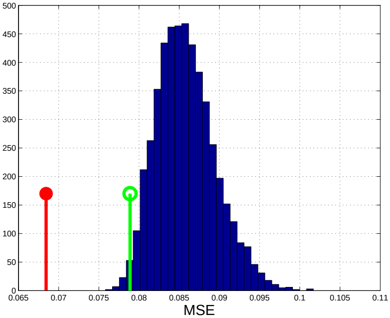


Motion blur, fast motion, views from acute angles and other image distortions.



Support set selection





- Greedy LS selection (red) of an efficient support set.
- Much better than 1%-quantile (green) achieavable by randomized sampling

Tracking of objects with variable appearance

- **Variable appearance** - the way how the object looks like in the camera changes due to illumination, non-rigid deformation, out-of-plane rotation, ...

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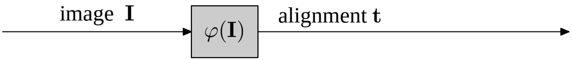
Simultaneous learning of motion and appearance



- Introduce feedback which encodes appearance in a low dimensional space and adjust the predictor.
- Appearance parameters learned in unsupervised way.
- Simultaneous learning of φ and $\gamma \Rightarrow$ appearance encoded in the low dimensional space, which is the most suitable for the motion estimation.

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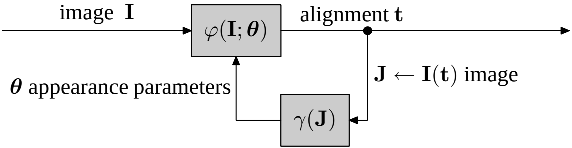

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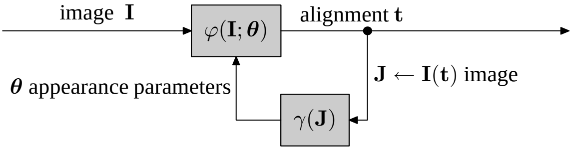

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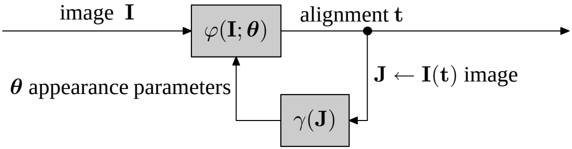

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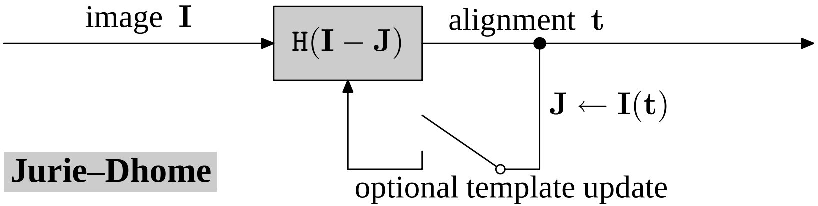
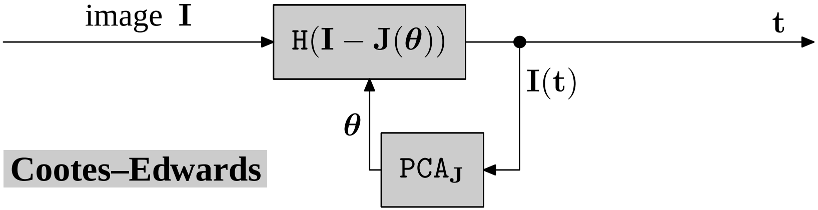


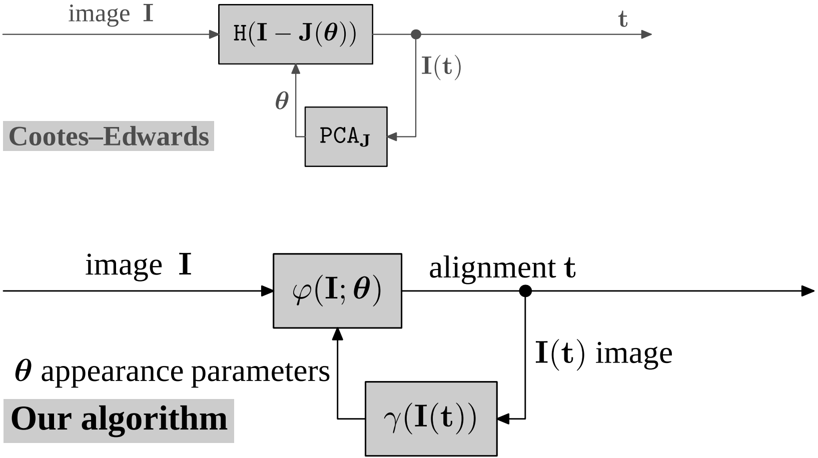
Jurie–Dhome

Learning appearance



Cootes–Edwards

Learning appearance – our approach



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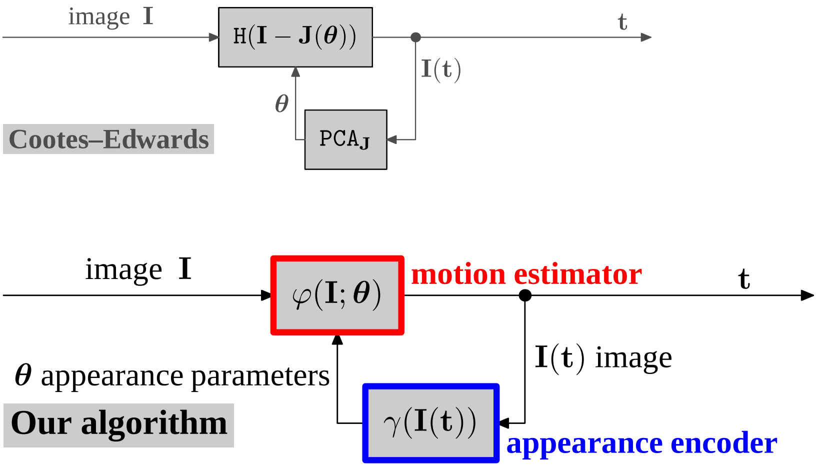
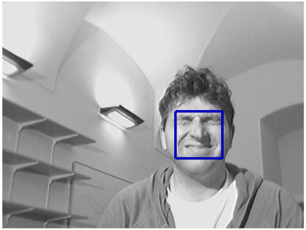
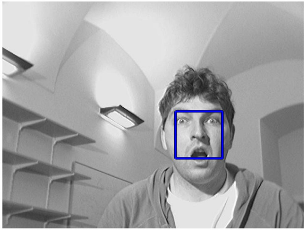


Learning the appearance encoder γ

- Current appearance encoded in low-dim parameters.





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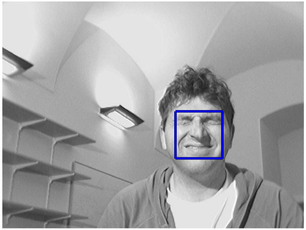

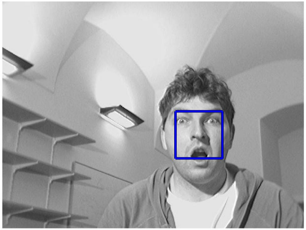

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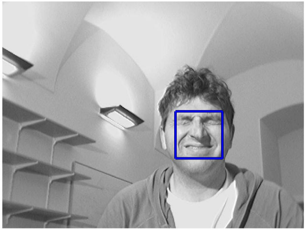

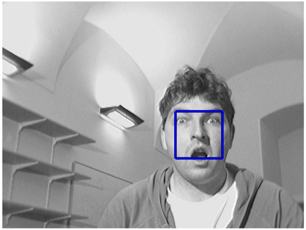

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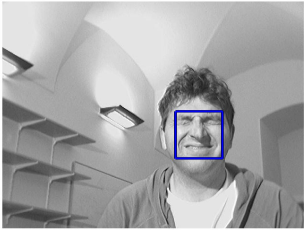
Learning the tracker $\varphi(\mathbf{I};\boldsymbol{\theta})$



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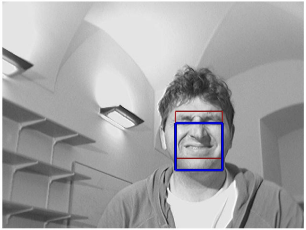

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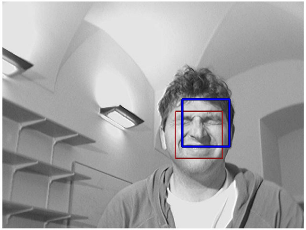

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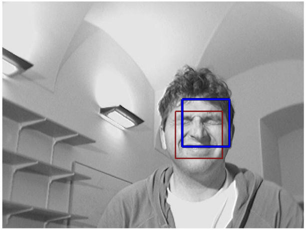

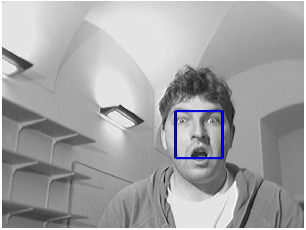

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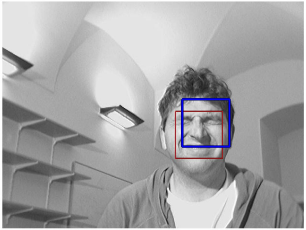

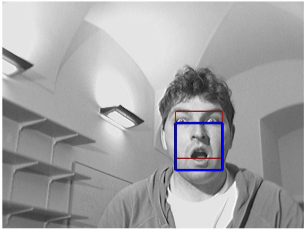

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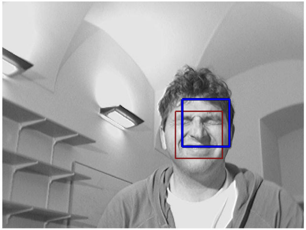

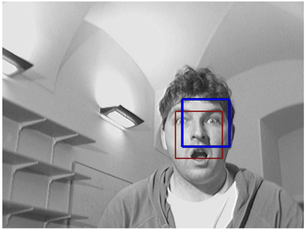

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Simultaneous learning of φ and γ

- **Learning = minimization of the least-squares error**

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Linear mapping

- $\gamma(\mathbf{J}) : \boldsymbol{\theta} = \mathtt{G}\mathbf{J}$
- $\varphi(\mathbf{I}, \boldsymbol{\theta}) : \mathbf{t} = (\mathrm{H}_0 + \theta_1 \mathrm{H}_1 + \cdots + \theta_n \mathrm{H}_n)\mathbf{I}$
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φ – motion (geometry mapping), γ – appearance mapping

color encodes criterion value $e(\varphi, \gamma)$

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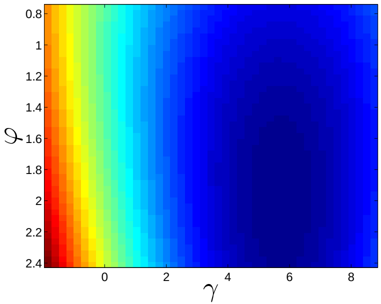

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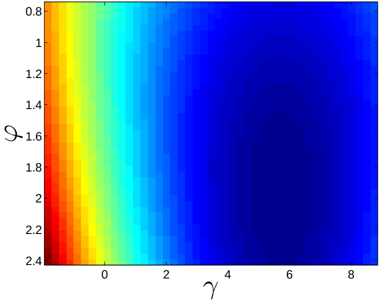

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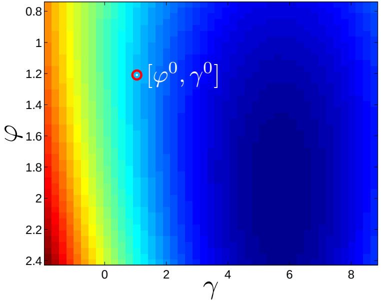

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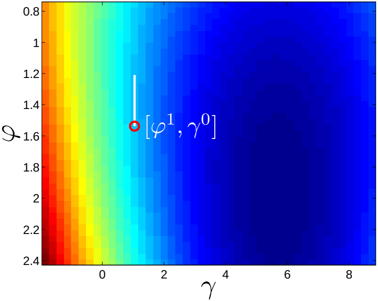

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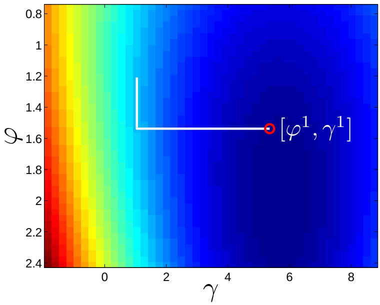

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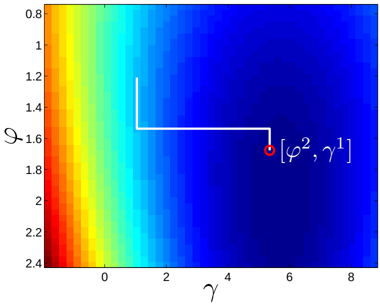

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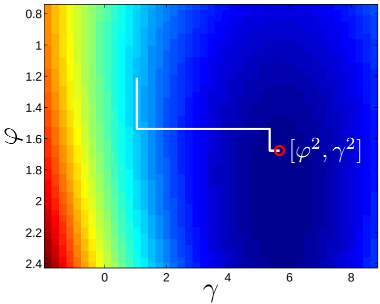

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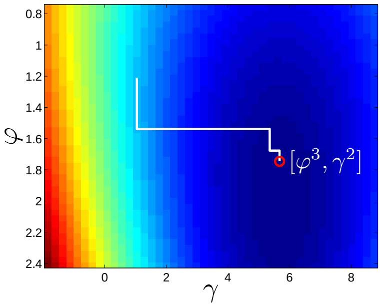

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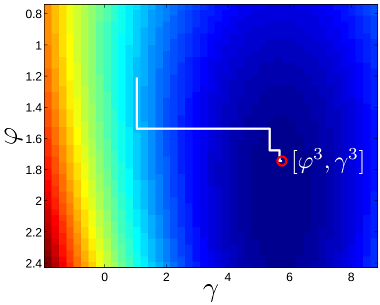

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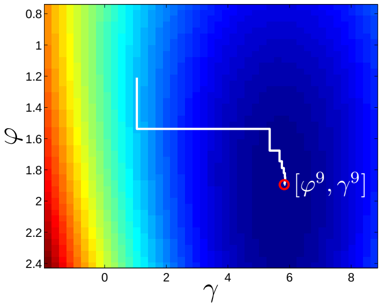

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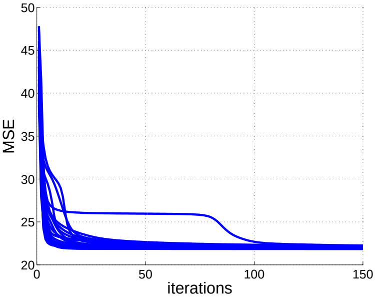


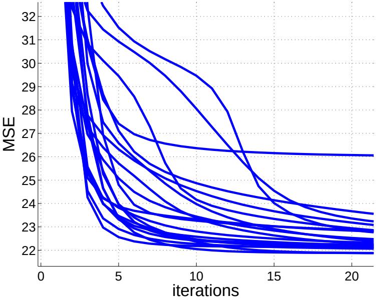
- Global optimality for linear φ, γ experimentally shown.

Algorithm: iterative minimization of criterion $e(\varphi, \gamma)$

φ – motion (geometry mapping), γ – appearance mapping

color encodes criterion value $e(\varphi, \gamma)$

- Iterative minimization:
  - initialization $\gamma^0 = \mathrm{rand}$
  - $\varphi^1 = \arg\min_{\varphi} e(\varphi, \gamma^0)$
  - $\gamma^1 = \arg\min_{\gamma} e(\varphi^1, \gamma)$
  - $\varphi^2 = \arg\min_{\varphi} e(\varphi, \gamma^1)$
  - $\gamma^2 = \arg\min_{\gamma} e(\varphi^2, \gamma)$
  - $\varphi^3 = \arg\min_{\varphi} e(\varphi, \gamma^2)$
  - $\gamma^3 = \arg\min_{\gamma} e(\varphi^3, \gamma)$
  - until convergence reached



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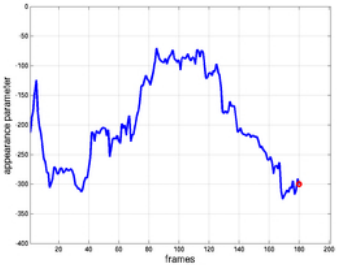
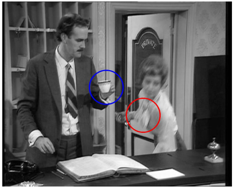
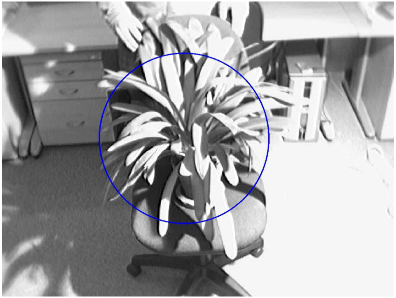
Simultaneous learning of motion and appearance





Experiments - videos II



Conclusions

- Learnable and very efficient tracking of objects with variable appearance.
- Accuracy, speed, robustness explicitly taken into account.
- Simultaneous learning motion and appearance.

Limitations

- Small, thin objects intractable.

Data, papers, various implemenations freely available at
http://cmp.felk.cvut.cz/demos/Tracking/linTrack/

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