

Sequential Learned Linear Predictors for Object Tracking

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Object tracking

- ▶ **Tracking** - iterative estimation of an object pose in a sequence (e.g., 2D position of Basil's head).
- ▶ Trade-off among *accuracy, speed, robustness* is explicitly taken into account.



video: Basil head

Object tracking

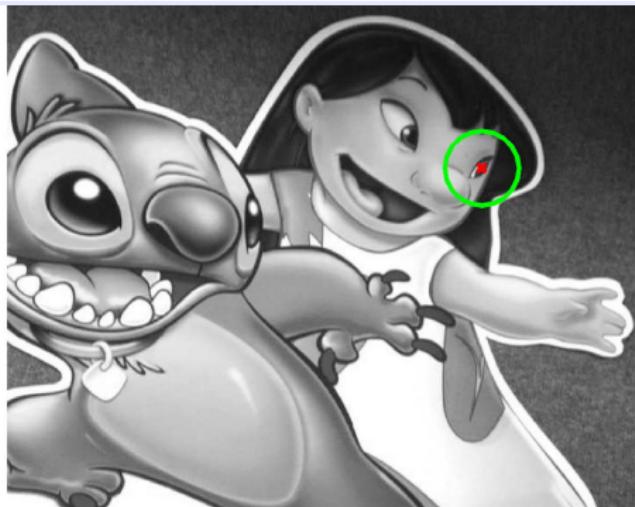
- ▶ **Tracking** - iterative estimation of an object pose in a sequence (e.g., 2D position of Basil's head).
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video: Basil head

Object tracking

- ▶ **Tracking** - iterative estimation of an object pose in a sequence (e.g., 2D position of Basil's head).
- ▶ Trade-off among *accuracy*, *speed*, *robustness* is explicitly taken into account.



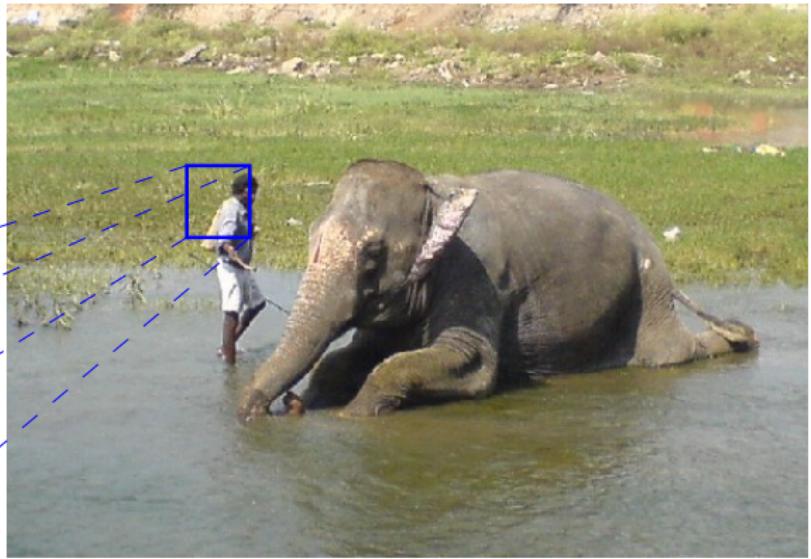
video: Cartoon eye

Tracking

Template



Current image



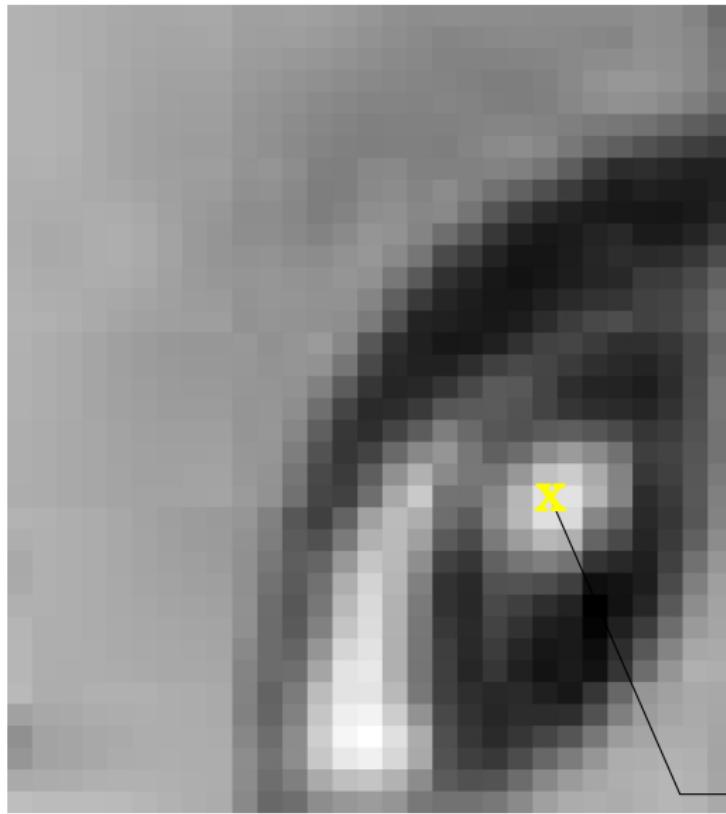
Observation



Motion

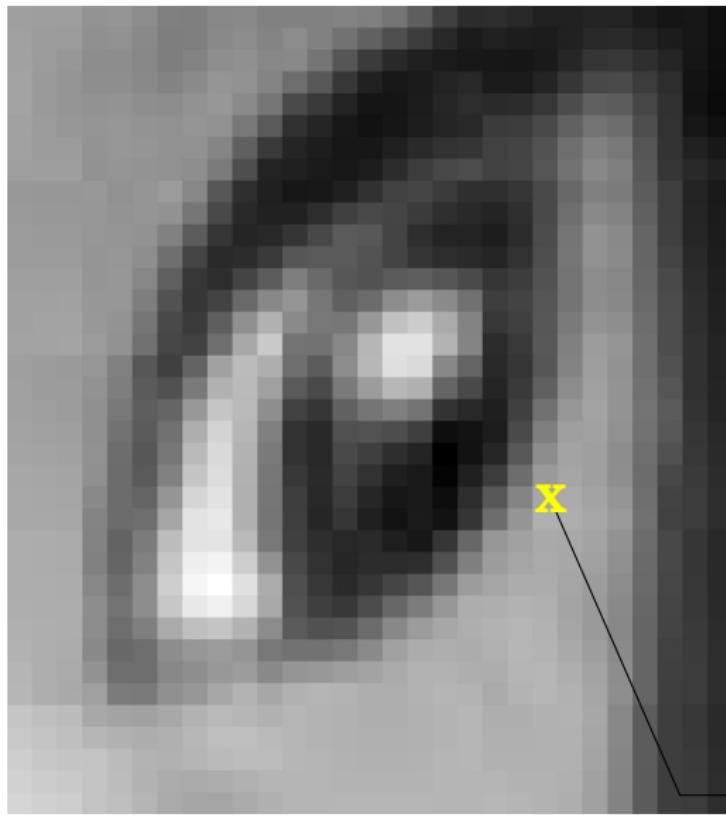
Observation (image) I and template J

Tracking of a single point



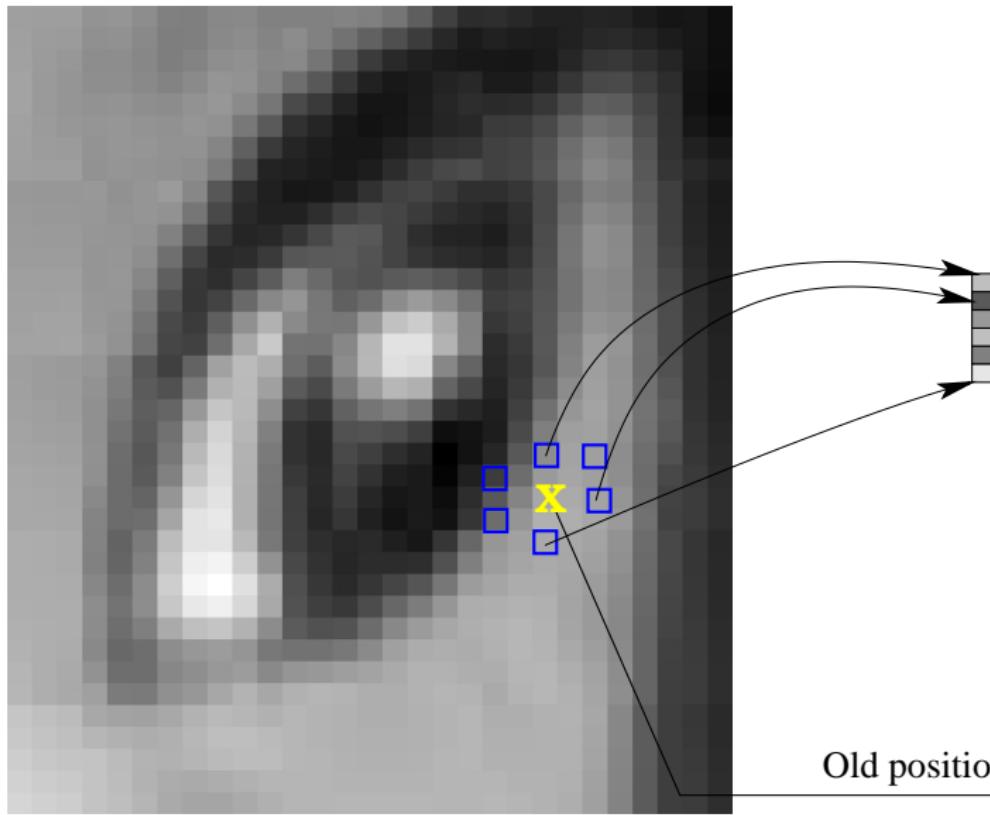
Current position

Tracking of a single point



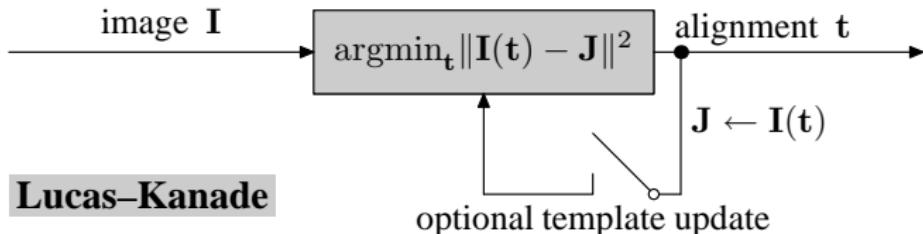
Old position

Tracking of a single point



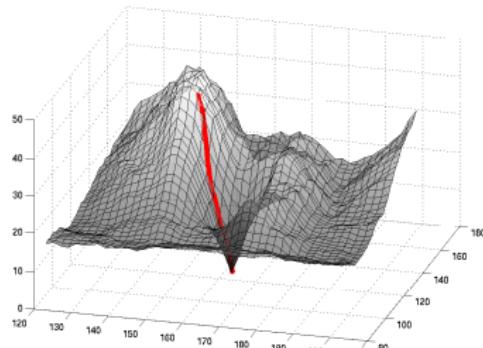
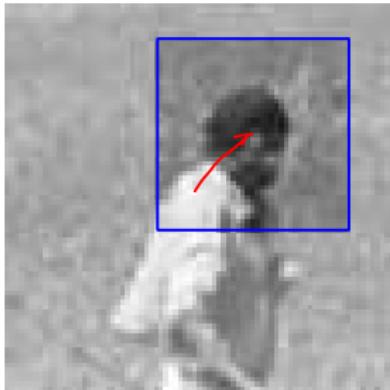
Old position

[Lucas-Kanade1981] - Iterative minimization

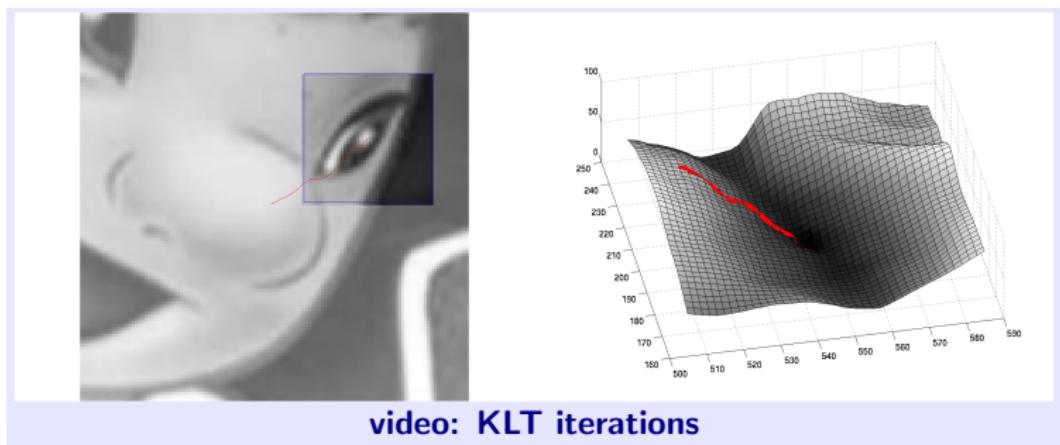
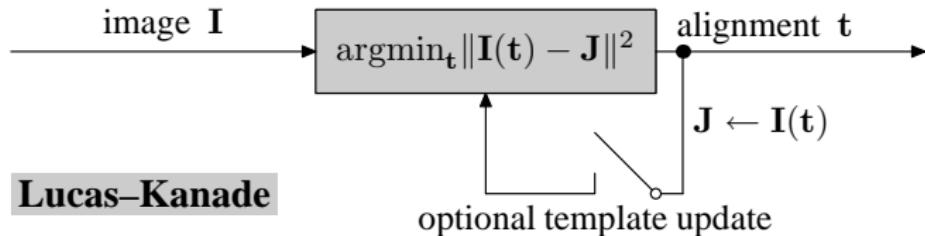


Lucas-Kanade

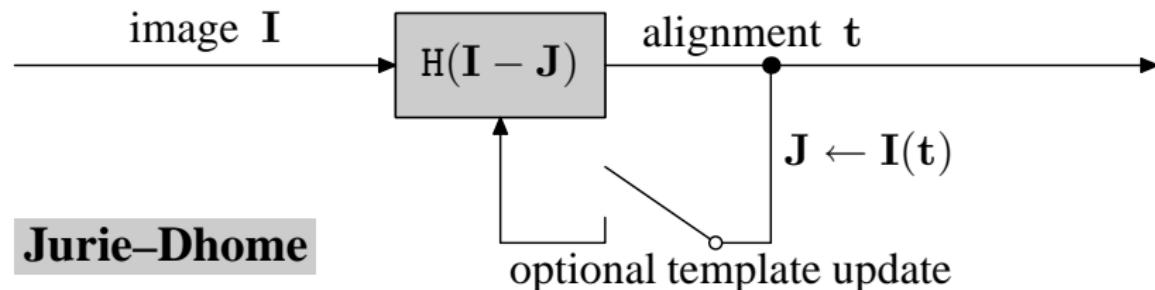
optional template update



[Lucas-Kanade1981] - Iterative minimization

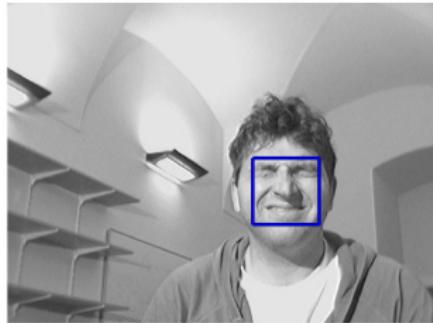


Regression - learn the mapping in advance



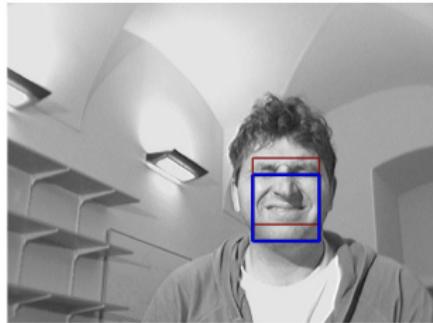
- ▶ [Jurie-BMVC-2002] - learned motion and optional hard template update
- ▶ [Cootes-PAMI-2001] - learned regression during AAM iterations
- ▶ ...

Learning alignment for one predictor



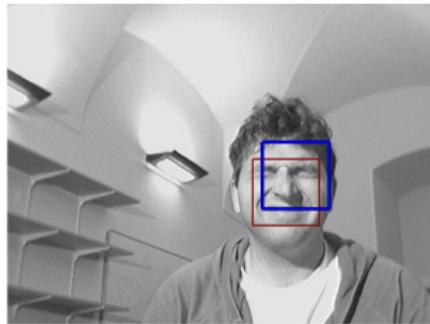
- ▶ $\varphi(\text{[eye]}) = (0, 0)^\top$ ▶ $\varphi(\text{[nose]}) = (0, 0)^\top$
- ▶ $\varphi(\text{[left ear]}) = (-25, 0)^\top$ ▶ $\varphi(\text{[right ear]}) = (-25, 0)^\top$
- ▶ $\varphi(\text{[cheek]}) = (25, -15)^\top$ ▶ $\varphi(\text{[chin]}) = (25, -15)^\top$

Learning alignment for one predictor



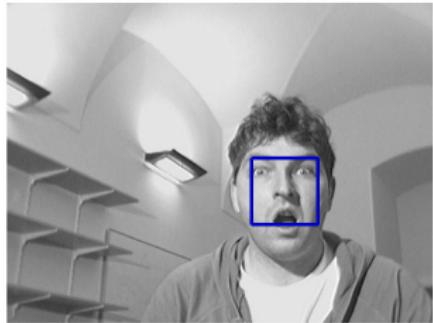
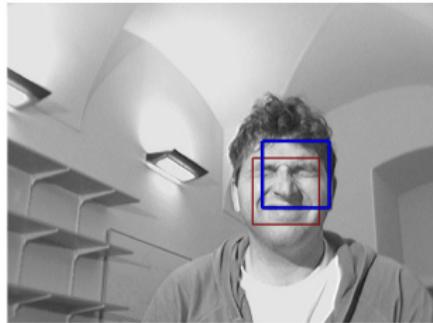
- ▶ $\varphi(\text{[Red Box]}) = (0, 0)^\top$ ▶ $\varphi(\text{[Blue Box]}) = (0, 0)^\top$
- ▶ $\varphi(\text{[Mouth Area]}) = (-25, 0)^\top$ ▶ $\varphi(\text{[Eye Area]}) = (-25, 0)^\top$
- ▶ $\varphi(\text{[Man Face]}) = (25, -15)^\top$ ▶ $\varphi(\text{[Man Face]}) = (25, -15)^\top$

Learning alignment for one predictor



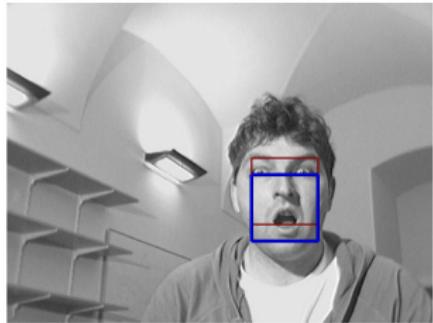
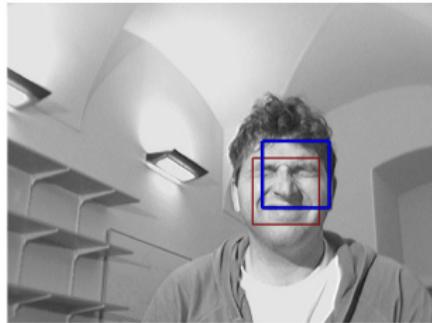
- ▶ $\varphi(\text{[eye]}) = (0, 0)^\top$ ▶ $\varphi(\text{[nose]}) = (0, 0)^\top$
- ▶ $\varphi(\text{[cheek]}) = (-25, 0)^\top$ ▶ $\varphi(\text{[jawline]}) = (-25, 0)^\top$
- ▶ $\varphi(\text{[wrinkle]}) = (25, -15)^\top$ ▶ $\varphi(\text{[forehead]}) = (25, -15)^\top$

Learning alignment for one predictor



- ▶ $\varphi(\text{[Red Box]}) = (0, 0)^\top$ $\varphi(\text{[Blue Box]}) = (0, 0)^\top$
- ▶ $\varphi(\text{[Mouth]}) = (-25, 0)^\top$ $\varphi(\text{[Eyes]}) = (-25, 0)^\top$
- ▶ $\varphi(\text{[Wrinkles]}) = (25, -15)^\top$ $\varphi(\text{[Surprised Face]}) = (25, -15)^\top$

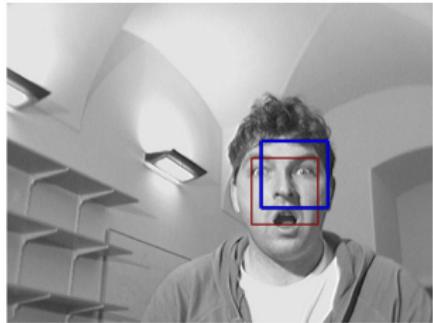
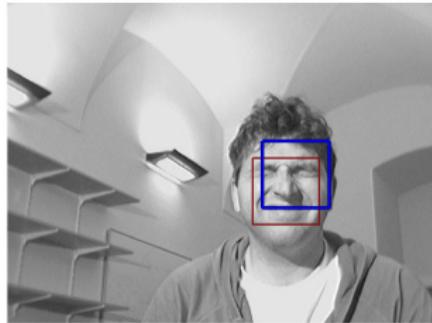
Learning alignment for one predictor



- ▶ $\varphi(\text{[eye]}) = (0, 0)^\top$
- ▶ $\varphi(\text{[smile]}) = (-25, 0)^\top$
- ▶ $\varphi(\text{[frown]}) = (25, -15)^\top$

- ▶ $\varphi(\text{[eye]}) = (0, 0)^\top$
- ▶ $\varphi(\text{[open mouth]}) = (-25, 0)^\top$
- ▶ $\varphi(\text{[closed mouth]}) = (25, -15)^\top$

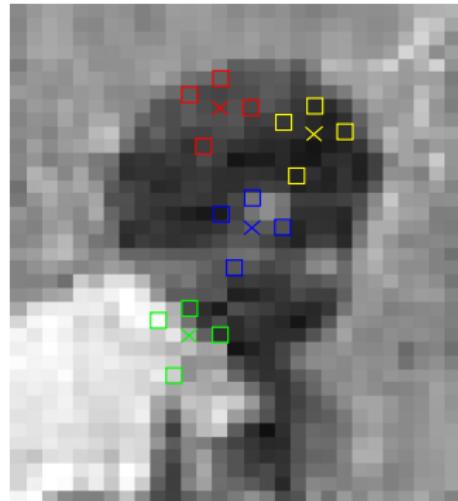
Learning alignment for one predictor



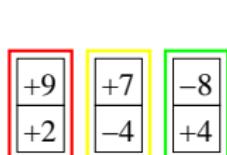
- ▶ $\varphi(\text{[neutral face]}) = (0, 0)^\top$
- ▶ $\varphi(\text{[smile]}) = (-25, 0)^\top$
- ▶ $\varphi(\text{[wrinkled face]}) = (25, -15)^\top$

- ▶ $\varphi(\text{[surprised face]}) = (0, 0)^\top$
- ▶ $\varphi(\text{[open mouth]}) = (-25, 0)^\top$
- ▶ $\varphi(\text{[wide eyes]}) = (25, -15)^\top$

Generating training examples

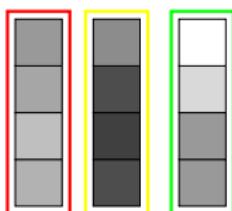


t^i – motion



$I^i = I(x_1, \dots, x_c)$ – observation

generating
examples
↔
displacement
estimation



training set: $T = \begin{bmatrix} +9 & +7 & -8 \\ +2 & -4 & +4 \end{bmatrix}$ $I = \begin{bmatrix} \text{Gray} & \text{Dark Gray} & \text{Light Gray} \\ \text{Gray} & \text{Dark Gray} & \text{Light Gray} \\ \text{Gray} & \text{Dark Gray} & \text{Light Gray} \end{bmatrix}$

Training set: (I, T)

$I = [I^1 - J, I^2 - J, \dots, I^d - J]$ and $T = [t^1, t^2, \dots, t^d]$.

LS learning

$$\varphi^* = \operatorname{argmin}_{\varphi} \sum_{\mathbf{t}} \|\varphi(\mathbf{I}(\mathbf{t} \circ X)) - \mathbf{t}\|^2.$$

Minimizes sum of square errors over all training set. Leads to matrix pseudoinverse computation.

Example for *linear mapping*:

$$H^* = \operatorname{argmin}_H \sum_{i=1}^d \|H(I^i - J) - t^i\|_2^2 = \operatorname{argmin}_H \|HI - T\|_F^2$$

after some derivation

$$H^* = T \underbrace{I^\top (II^\top)^{-1}}_{I^+} = TI^+.$$

LS learning

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Example for *linear mapping*:

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after some derivation

$$\mathbf{H}^* = \mathbf{T} \underbrace{\mathbf{I}^\top (\mathbf{I}\mathbf{I}^\top)^{-1}}_{\mathbf{I}^+} = \mathbf{T}\mathbf{I}^+.$$

LS learning

$$\varphi^* = \operatorname{argmin}_{\varphi} \sum_{\mathbf{t}} \|\varphi(\mathbf{I}(\mathbf{t} \circ X)) - \mathbf{t}\|^2.$$

Minimizes sum of square errors over all training set. Leads to matrix pseudoinverse computation.

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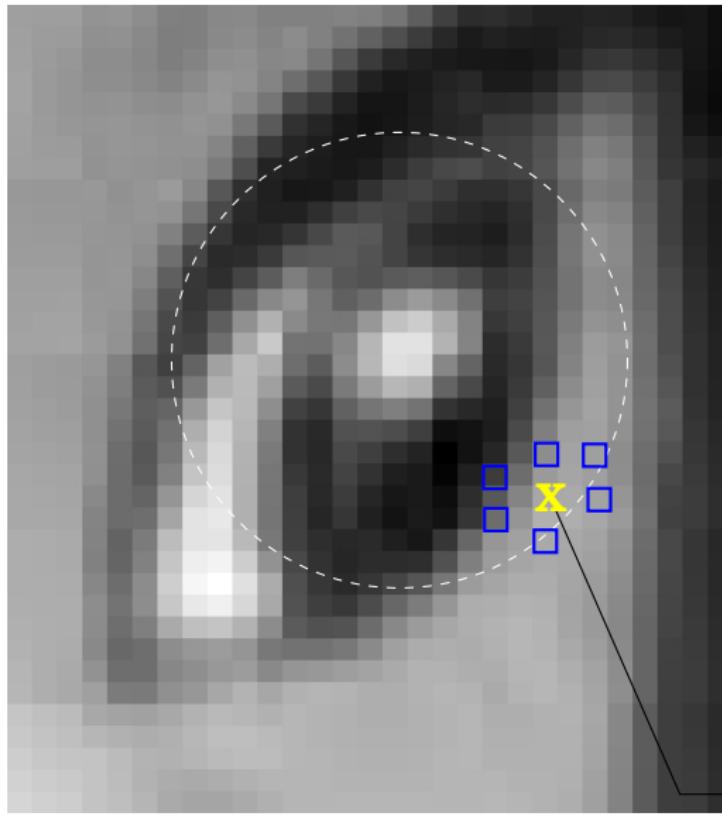
$$H^* = T \underbrace{I^\top (II^\top)^{-1}}_{I^+} = TI^+.$$

Min-max learning

$$\varphi^* = \operatorname{argmin}_\varphi \max_{\mathbf{t}} \|\varphi(\mathbf{l}(\mathbf{t} \circ X)) - \mathbf{t}\|_\infty.$$

Minimizes the *worst case* (the biggest estimation error) in the training set. Leads to linear programming.

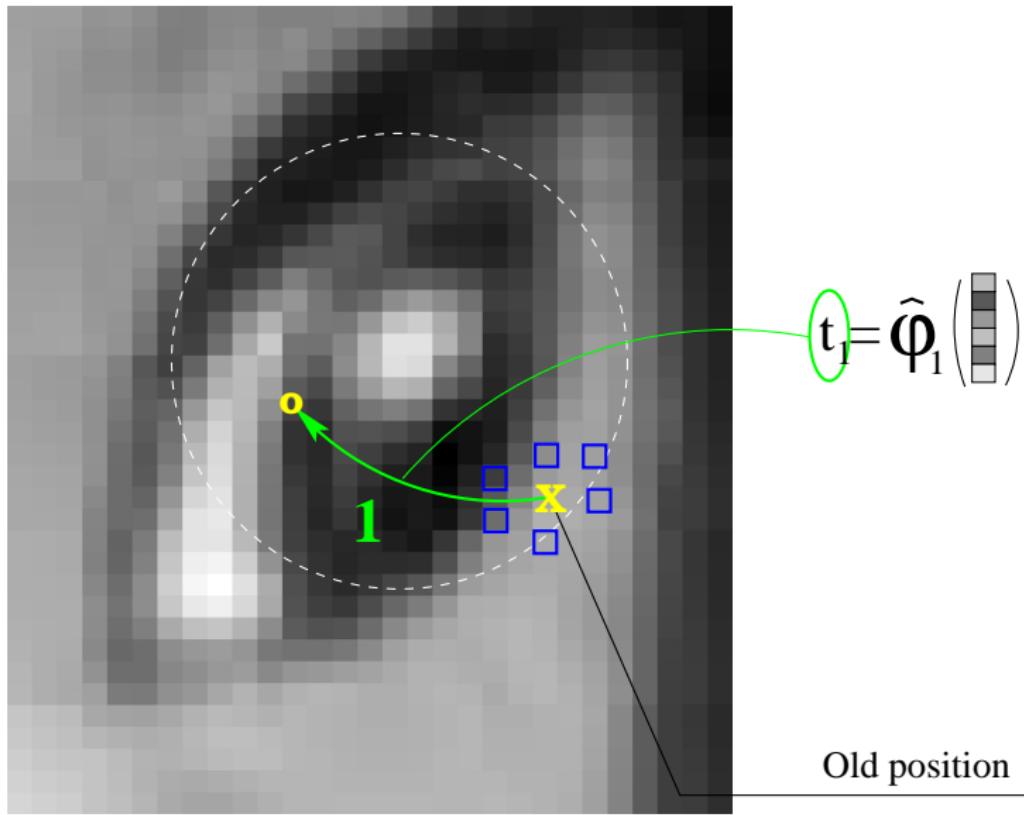
Tracking of a single point by a *sequence* of predictors



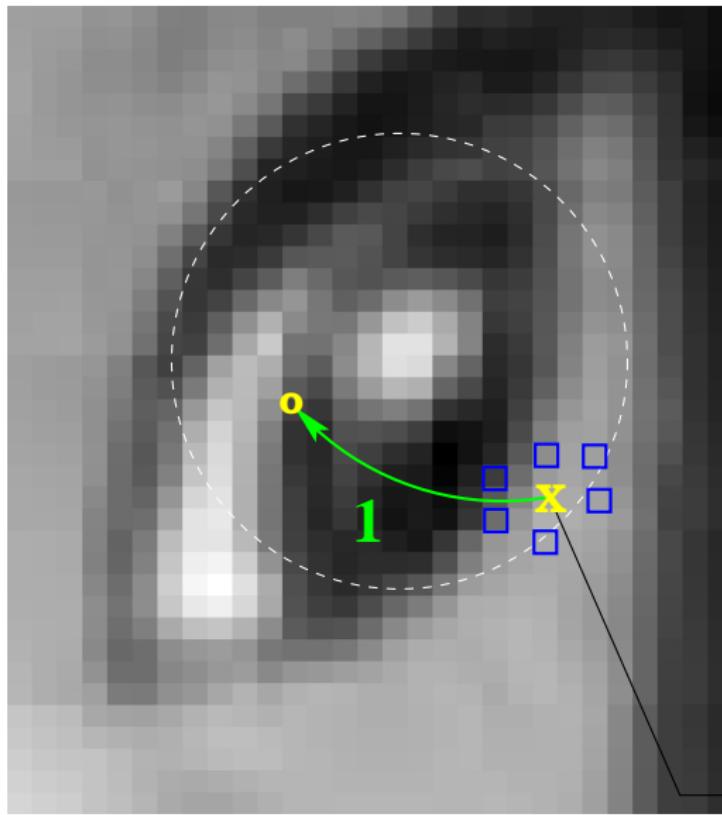
$$t_i = \hat{\Phi}_i \begin{pmatrix} \text{---} \\ \text{---} \\ \text{---} \end{pmatrix}$$

Old position

Tracking of a single point by a sequence of predictors



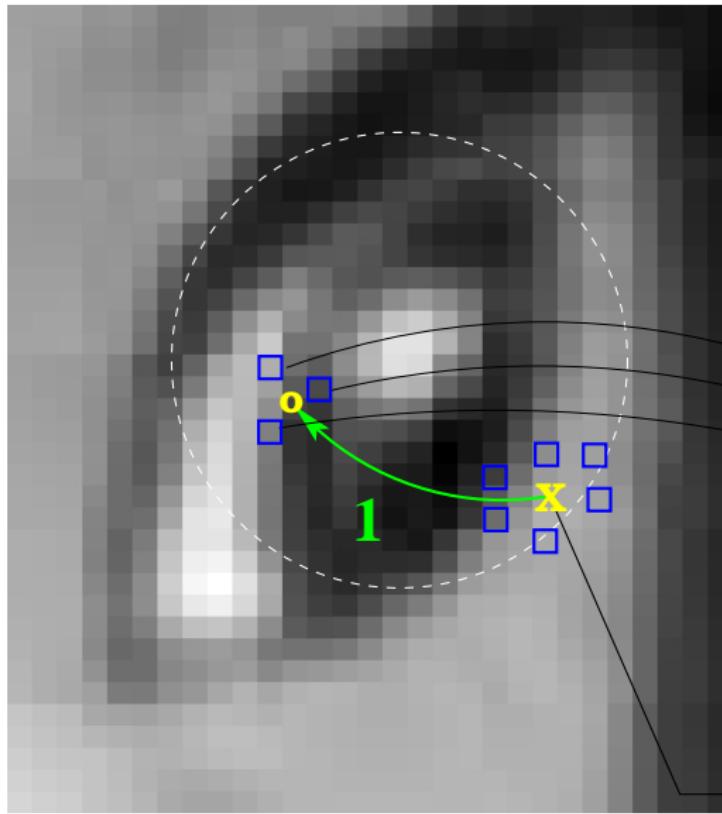
Tracking of a single point by a *sequence* of predictors



$$t_i = \hat{\Phi}_i \begin{pmatrix} \text{---} \\ \text{---} \\ \text{---} \end{pmatrix}$$

Old position

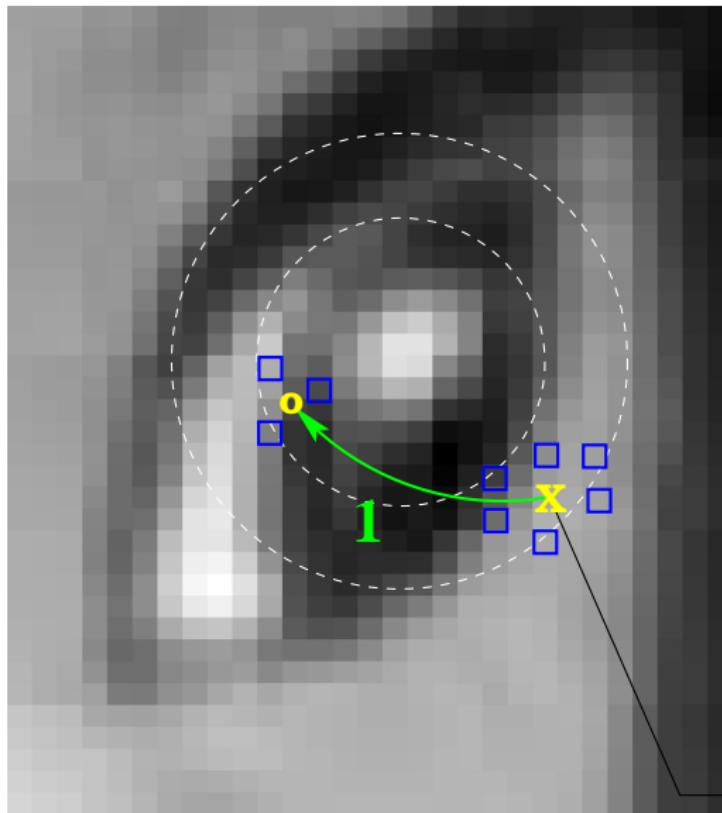
Tracking of a single point by a *sequence* of predictors



$$t_1 = \hat{\Phi}_1 \begin{pmatrix} \text{neighborhood pixels} \end{pmatrix}$$

Old position

Tracking of a single point by a *sequence* of predictors

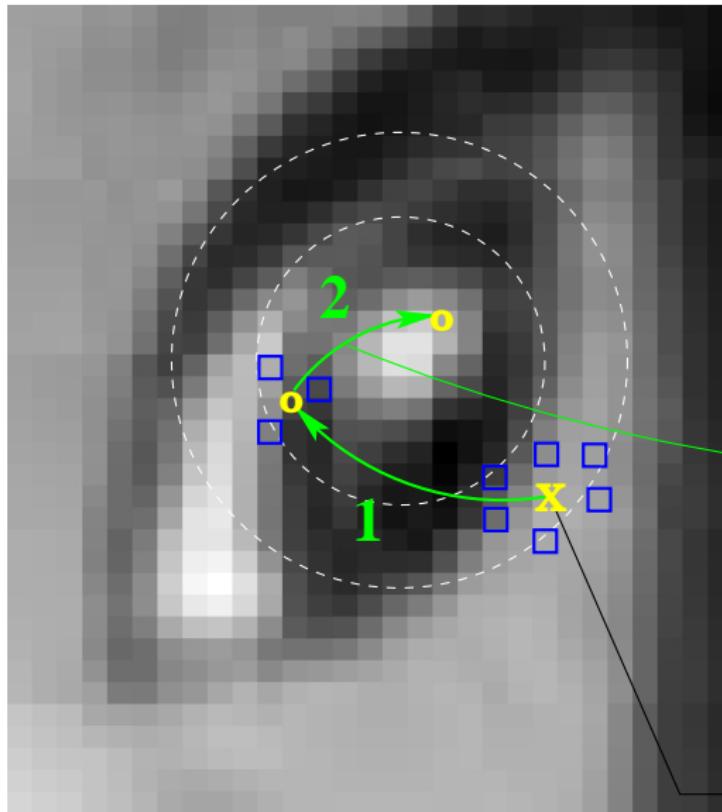


$$t_1 = \hat{\Phi}_1 \begin{pmatrix} \text{blue square} \\ \text{blue square} \\ \text{blue square} \end{pmatrix}$$

$$t_2 = \hat{\Phi}_2 \begin{pmatrix} \text{blue square} \end{pmatrix}$$

Old position

Tracking of a single point by a *sequence* of predictors

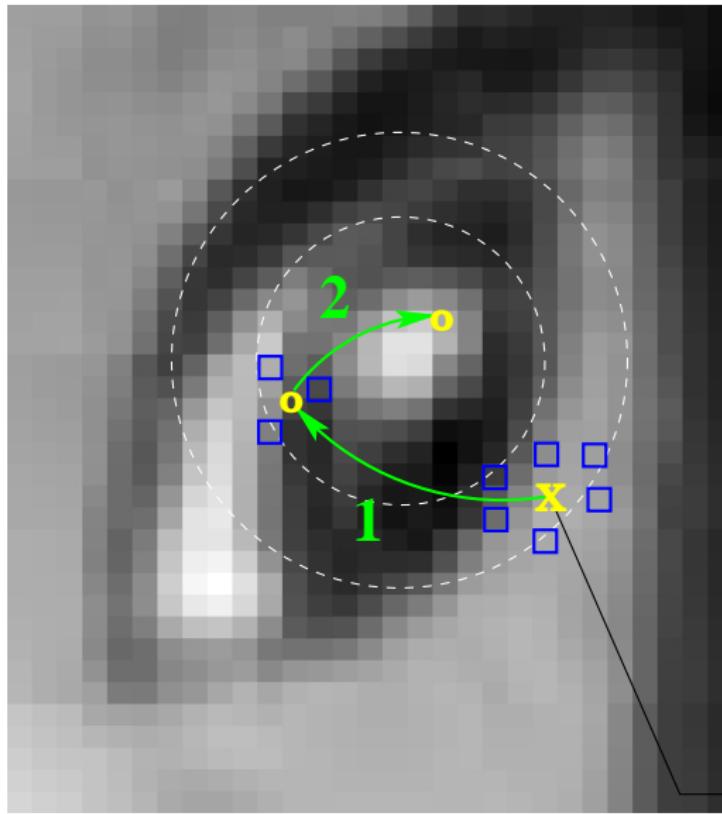


$$t_1 = \hat{\Phi}_1 \left(\begin{array}{c} \text{blue square} \\ \text{blue square} \\ \text{blue square} \end{array} \right)$$

$$t_2 = \hat{\Phi}_2 \left(\begin{array}{c} \text{yellow circle} \\ \text{yellow circle} \\ \text{yellow circle} \end{array} \right)$$

Old position

Tracking of a single point by a *sequence* of predictors

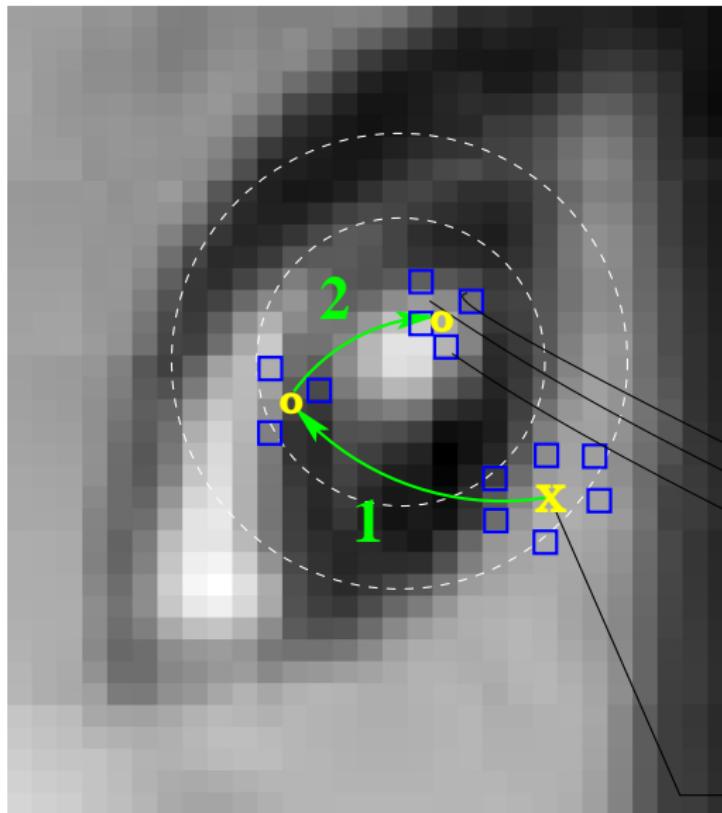


$$t_1 = \hat{\Phi}_1 \left(\begin{array}{c} \text{blue square} \\ \text{blue square} \\ \text{blue square} \end{array} \right)$$

$$t_2 = \hat{\Phi}_2 \left(\begin{array}{c} \text{blue square} \\ \text{blue square} \\ \text{blue square} \\ \text{blue square} \end{array} \right)$$

Old position

Tracking of a single point by a *sequence* of predictors

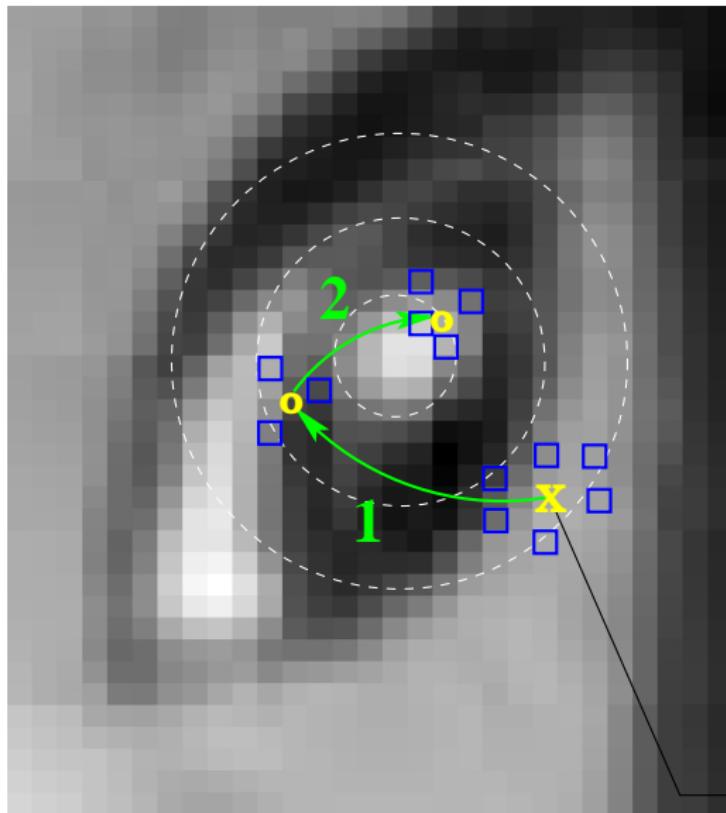


$$t_1 = \hat{\Phi}_1 \left(\begin{array}{c} \text{neighborhood} \\ \text{around } 1 \end{array} \right)$$

$$t_2 = \hat{\Phi}_2 \left(\begin{array}{c} \text{neighborhood} \\ \text{around } 2 \end{array} \right)$$

Old position

Tracking of a single point by a *sequence* of predictors



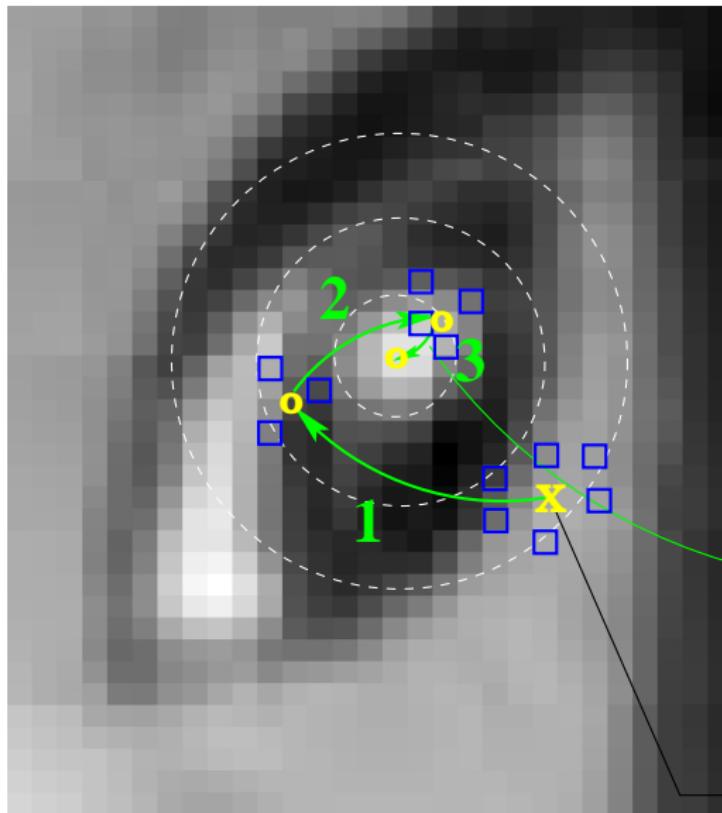
$$t_1 = \hat{\Phi}_1 \left(\begin{array}{c} \\ \\ \end{array} \right)$$

$$t_2 = \hat{\Phi}_2 \left(\begin{array}{c} \\ \\ \end{array} \right)$$

$$t_3 = \hat{\Phi}_3 \left(\begin{array}{c} \\ \\ \end{array} \right)$$

Old position

Tracking of a single point by a *sequence* of predictors



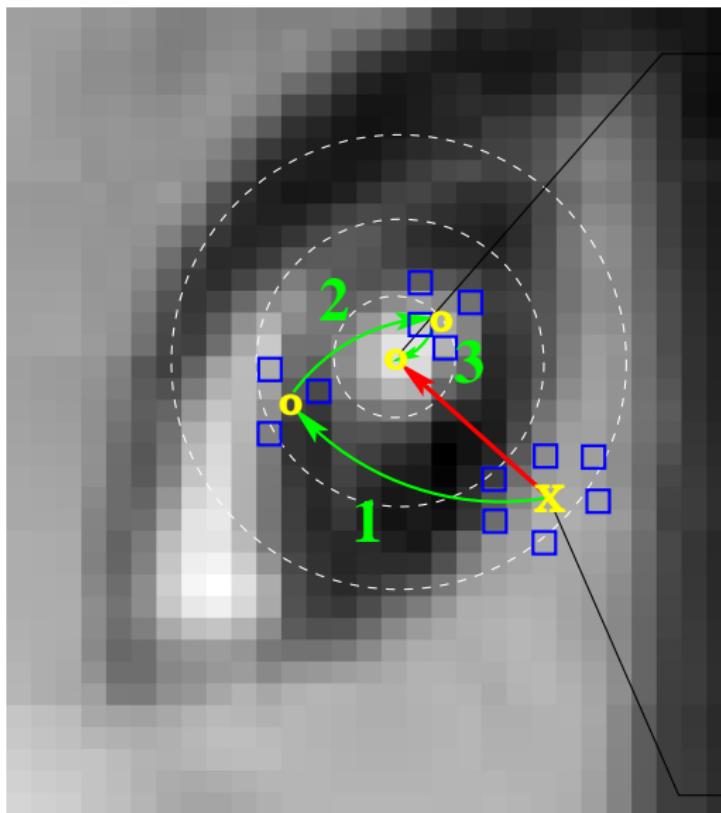
$$t_1 = \hat{\Phi}_1 \left(\begin{array}{c} \text{blue square} \\ \text{blue square} \\ \text{blue square} \end{array} \right)$$

$$t_2 = \hat{\Phi}_2 \left(\begin{array}{c} \text{blue square} \\ \text{blue square} \\ \text{blue square} \end{array} \right)$$

$$t_3 = \hat{\Phi}_3 \left(\begin{array}{c} \text{blue square} \\ \text{blue square} \\ \text{blue square} \end{array} \right)$$

Old position

Tracking of a single point by a *sequence* of predictors



New position

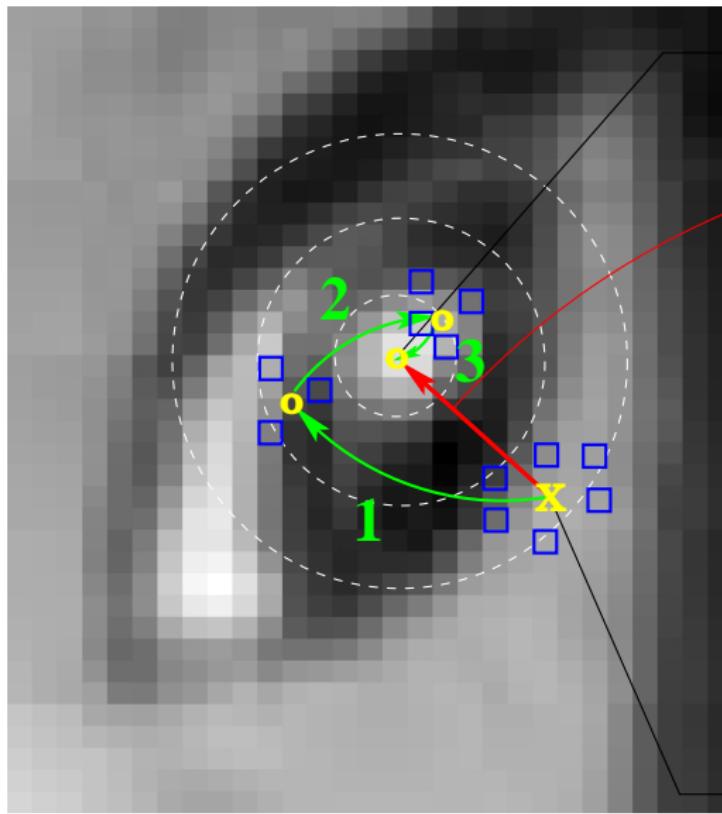
$$t_1 = \hat{\Phi}_1 \left(\begin{array}{c} \text{Support Region} \\ \vdots \end{array} \right)$$

$$t_2 = \hat{\Phi}_2 \left(\begin{array}{c} \text{Support Region} \\ \vdots \end{array} \right)$$

$$t_3 = \hat{\Phi}_3 \left(\begin{array}{c} \text{Support Region} \\ \vdots \end{array} \right)$$

Old position

Tracking of a single point by a *sequence* of predictors



New position

Motion

$$\Phi = (\varphi_1, \varphi_2, \varphi_3)$$

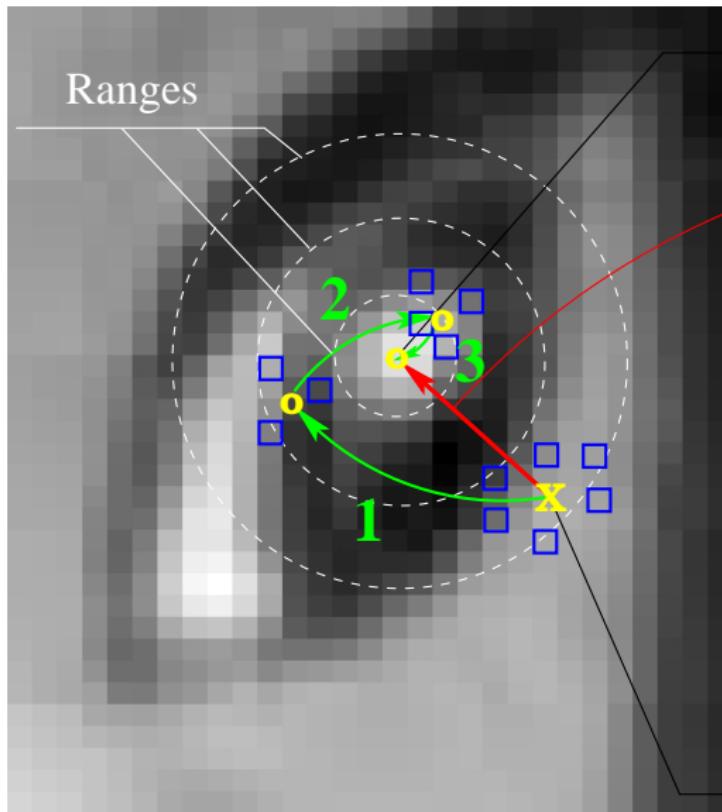
$$t_1 = \hat{\Phi}_1 \left(\begin{array}{c} \text{ } \\ \text{ } \\ \text{ } \end{array} \right)$$

$$t_2 = \hat{\Phi}_2 \left(\begin{array}{c} \text{ } \\ \text{ } \\ \text{ } \end{array} \right)$$

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Old position

Tracking of a single point by a sequence of predictors



New position

Motion

$$\Phi = (\varphi_1, \varphi_2, \varphi_3)$$

$$t_1 = \hat{\Phi}_1 \left(\begin{array}{c} \text{ } \\ \text{ } \\ \text{ } \end{array} \right)$$

$$t_2 = \hat{\Phi}_2 \left(\begin{array}{c} \text{ } \\ \text{ } \\ \text{ } \end{array} \right)$$

$$t_3 = \hat{\Phi}_3 \left(\begin{array}{c} \text{ } \\ \text{ } \\ \text{ } \end{array} \right)$$

Old position

Learning of sequential predictor

- ▶ **Learning** - searching for the sequence with predefined *range*, *accuracy* and minimal *computational cost*.
 - ▶ [Zimmermann-PAMI-2009] - Dynamic programming estimates the optimal sequence of linear predictors.
 - ▶ [Zimmermann-IVC-2009] - Branch & bound search allows for time constrained learning (demo in MATLAB).

[Zimmermann-PAMI-2009] K.Zimmermann, J.Matas, T.Svoboda. Tracking by an Optimal Sequence of Linear Predictors, in *IEEE Transactions on Pattern Analysis and Machine Intelligence*, IEEE computer society, 2009, vol. 31, No 4, pp 677–692.

[Zimmermann-IVC-2009] K.Zimmermann, T.Svoboda, J.Matas. Anytime learning for the NoSLLiP tracker. *Image and Vision Computing*, Elsevier, accepted, available on-line.

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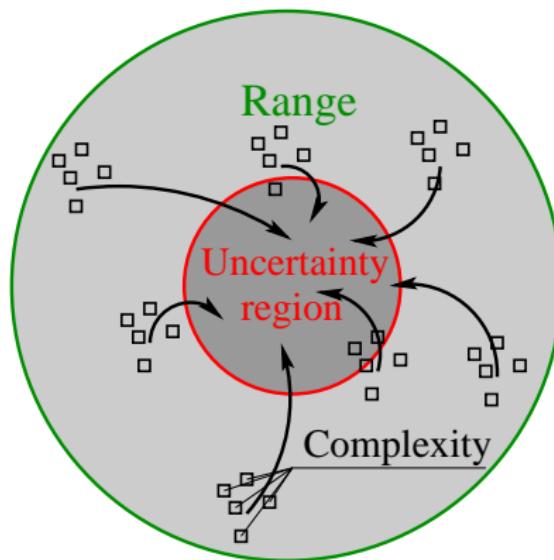
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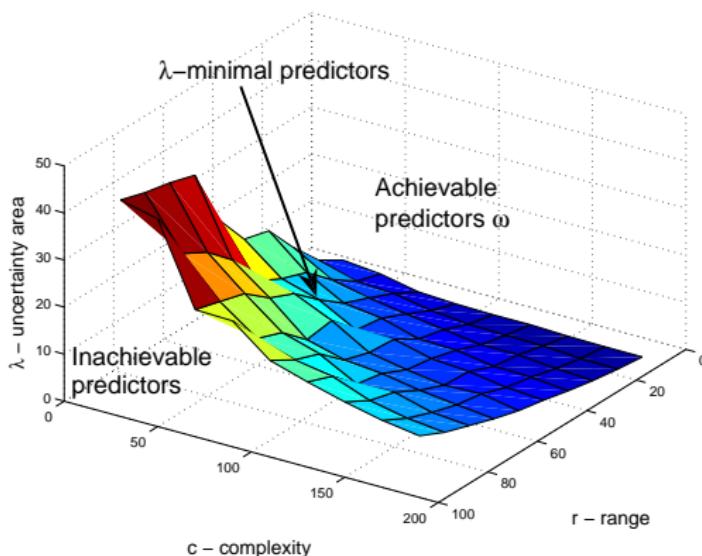
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Learning of the optimal sequence of linear predictors



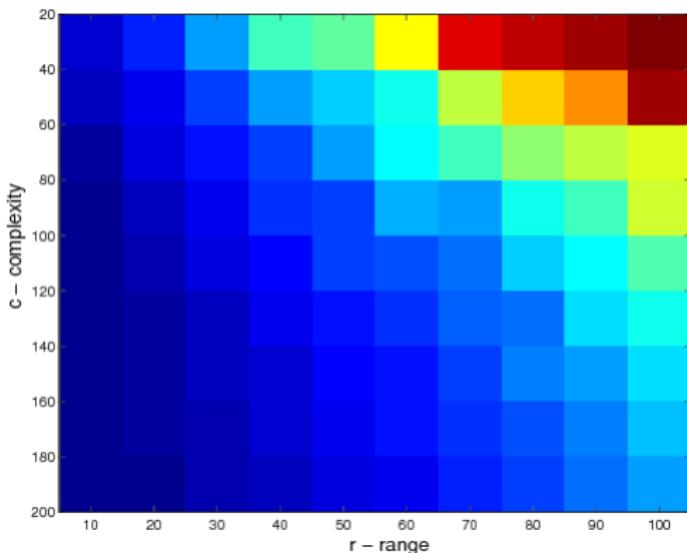
- ▶ **Range:** the set of admissible motions, r .
- ▶ **Complexity:** cardinality of support set, c .
- ▶ **Uncertainty region:** the region within which all predictions lie, λ . Small red circles show acceptable uncertainty.

Learning of the optimal sequence of linear predictors



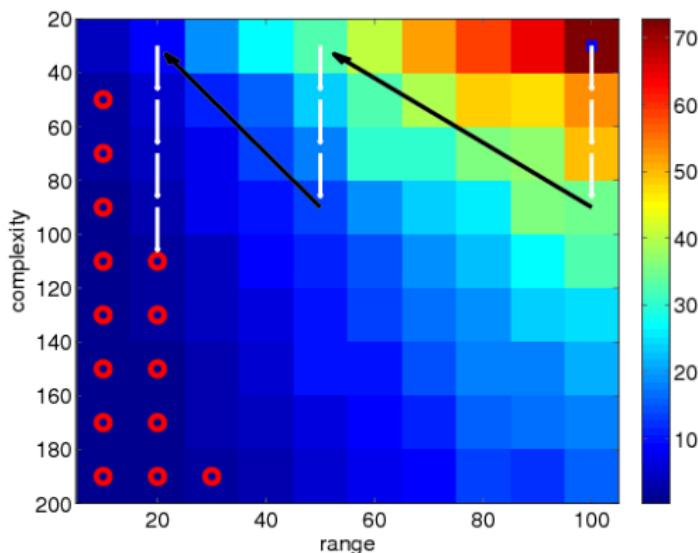
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Learning of the optimal sequence of linear predictors



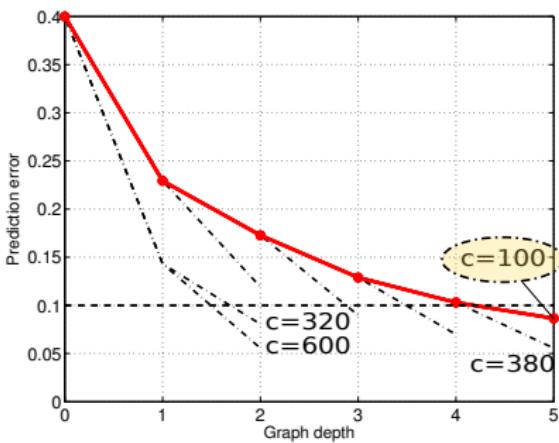
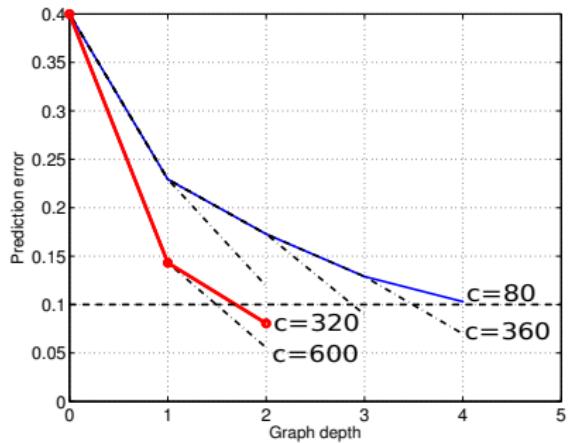
- ▶ **Range:** the set of admissible motions, r .
- ▶ **Complexity:** cardinality of support set, c .
- ▶ **Uncertainty region:** the region within which all predictions lie, λ . Small red circles show acceptable uncertainty.

Learning of the optimal sequence of linear predictors



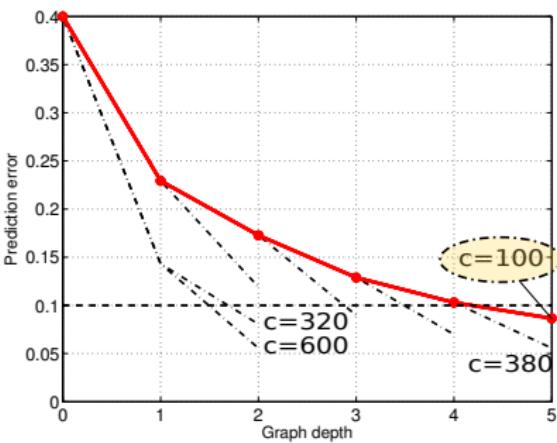
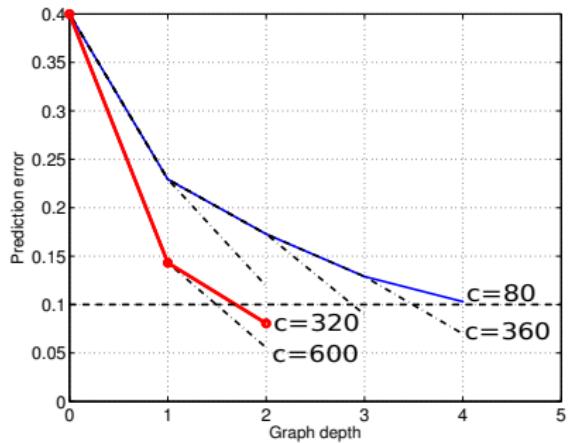
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Branch and Bound



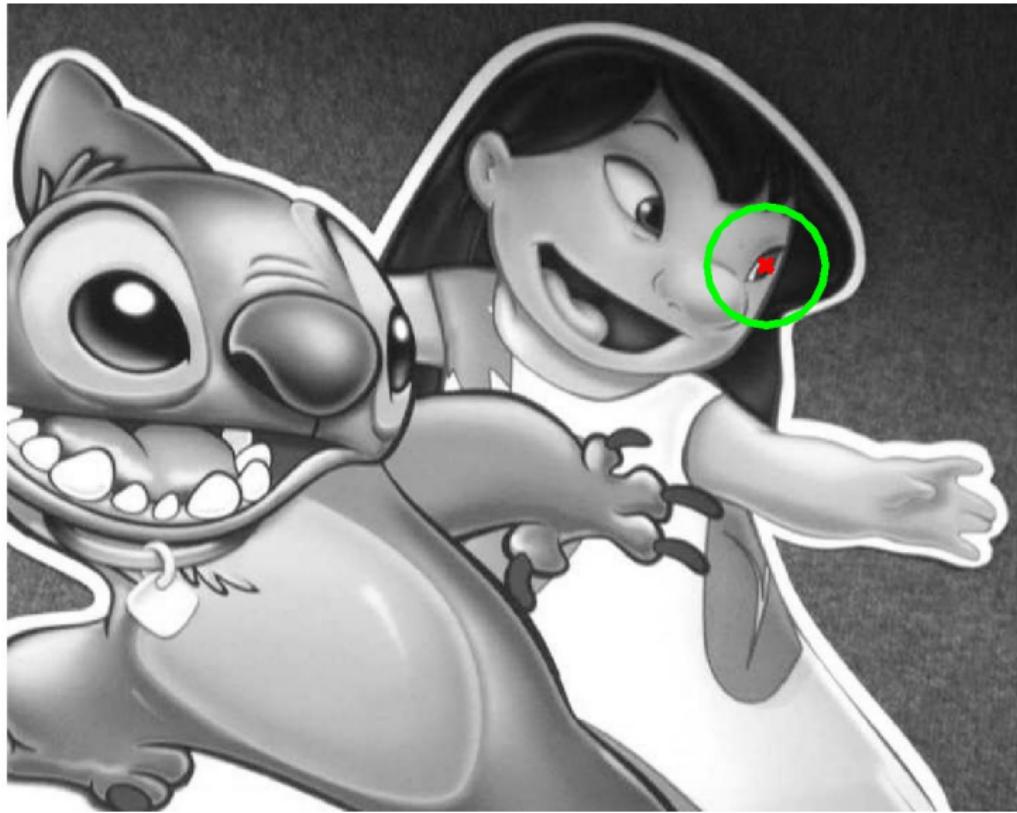
Don't forget to show the live demo!

Branch and Bound

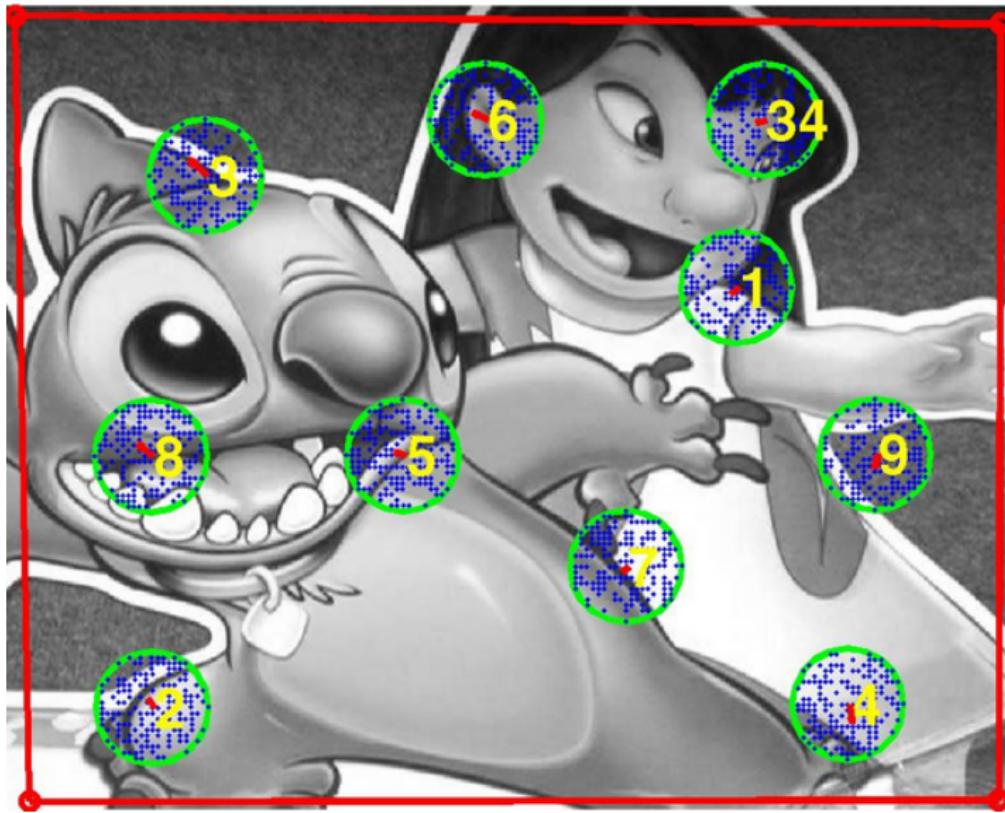


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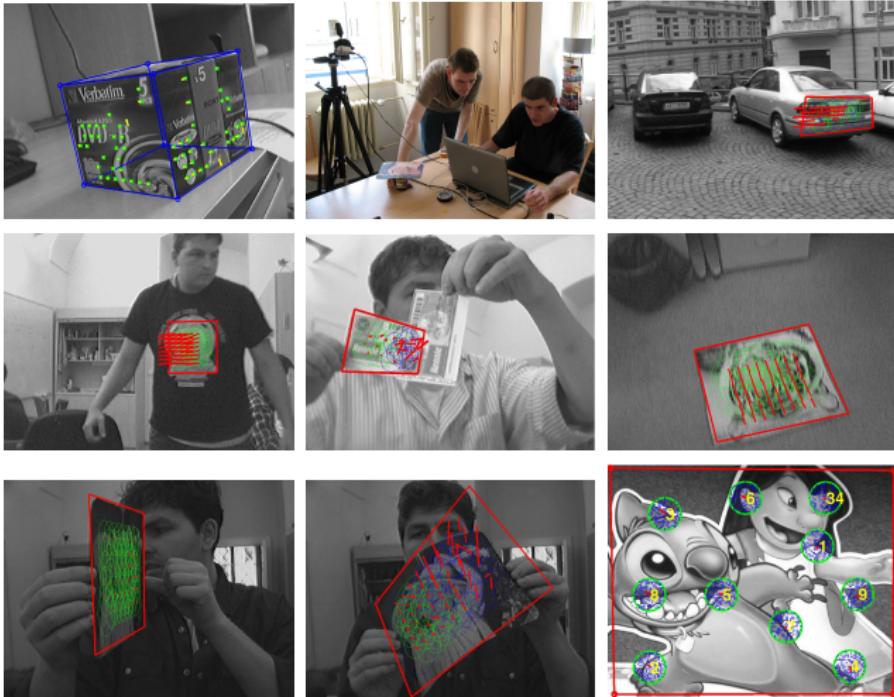
Tracking with one linear predictor.



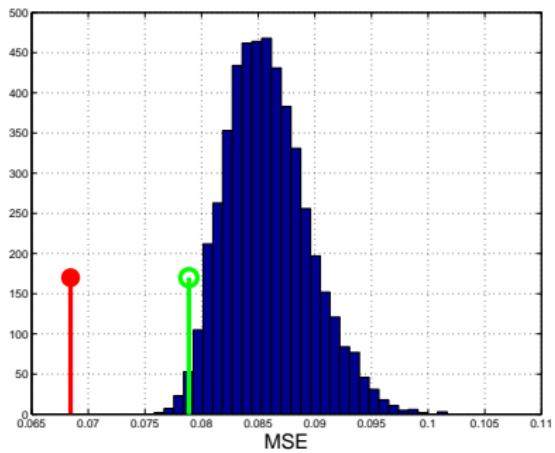
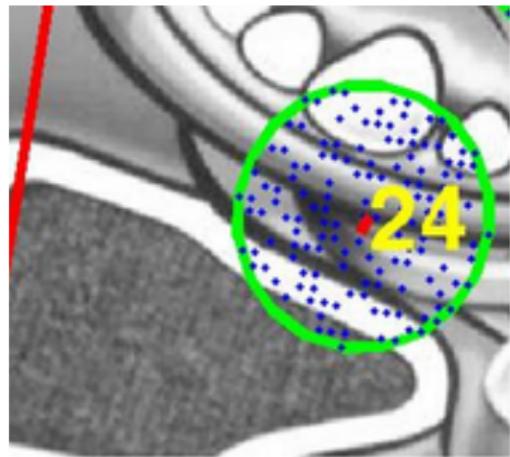
Modeling motion by number of linear predictors.



Motion blur, fast motion, views from acute angles and other image distortions.



Support set selection



- ▶ Greedy LS selection (red) of an efficient support set.
- ▶ Much better than 1%-quantile (green) achievable by randomized sampling

Tracking of objects with variable appearance

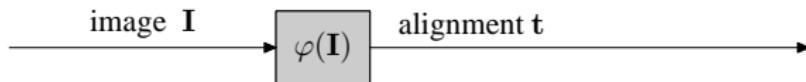
- ▶ **Variable appearance** - the way how the object looks like in the camera changes due to illumination, non-rigid deformation, out-of-plane rotation, ...

Tracking of objects with variable appearance

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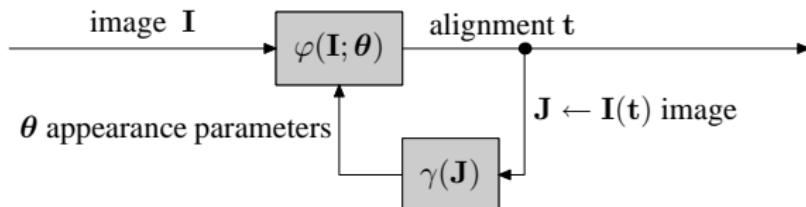


Simultaneous learning of motion and appearance



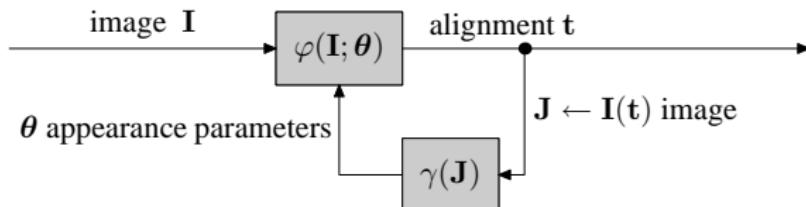
- ▶ Introduce feedback which encodes appearance in a low dimensional space and adjust the predictor.
- ▶ Appearance parameters learned in unsupervised way.
- ▶ Simultaneous learning of φ and $\gamma \Rightarrow$ appearance encoded in the low dimensional space, which is the most suitable for the motion estimation.

Simultaneous learning of motion and appearance



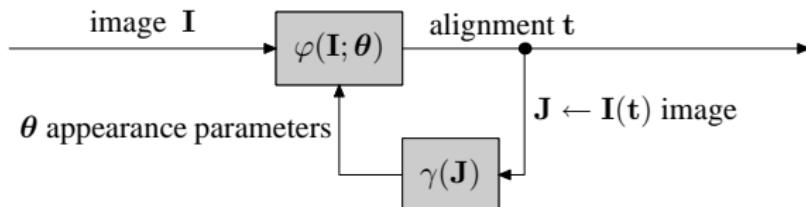
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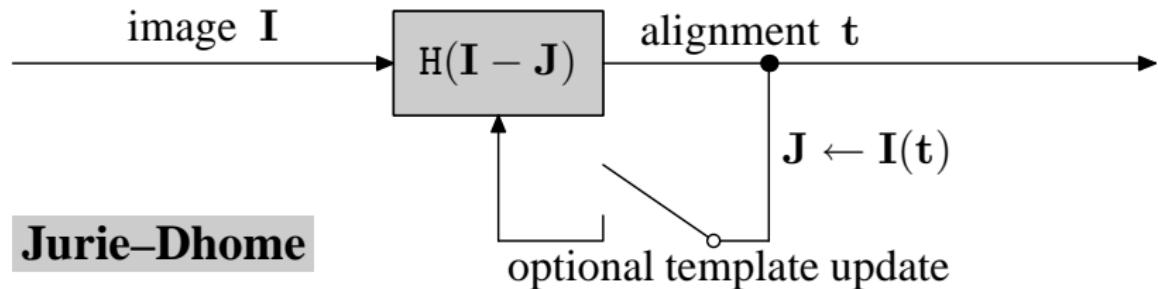
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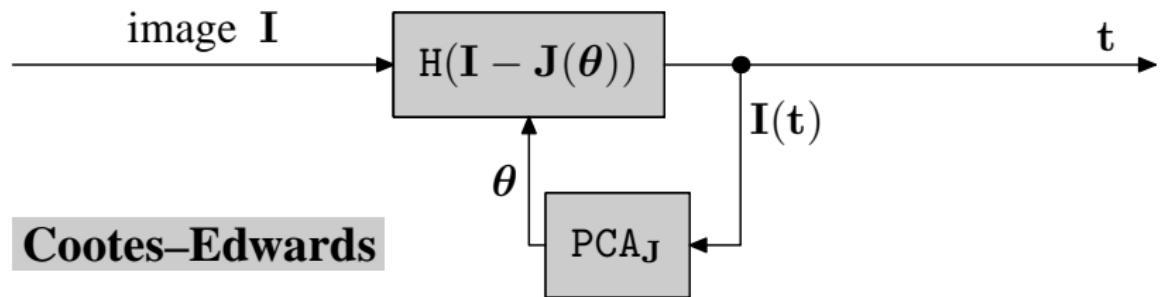


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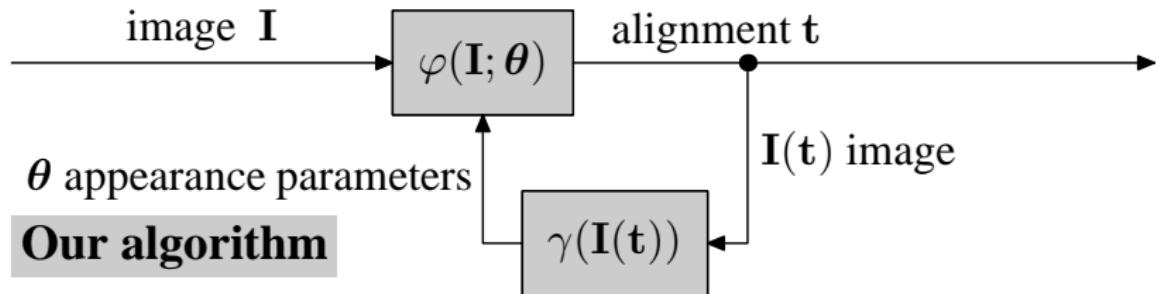
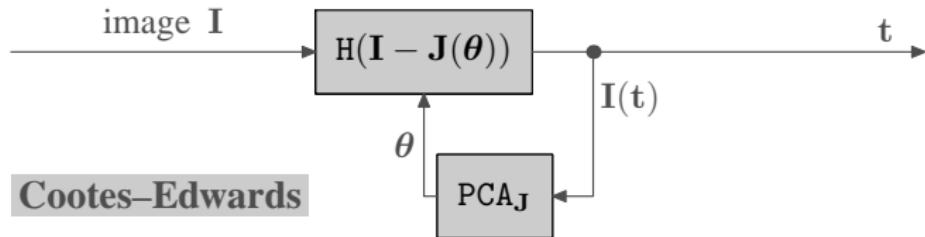
Learning appearance



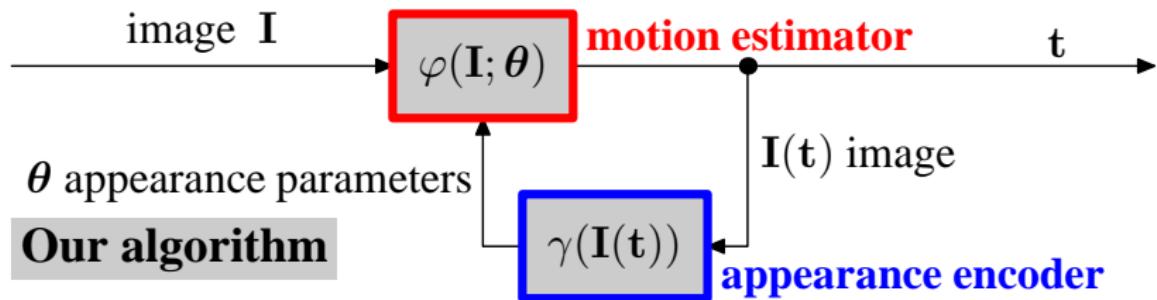
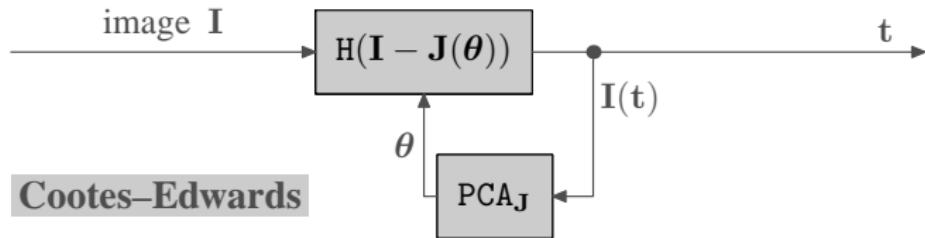
Learning appearance



Learning appearance – our approach

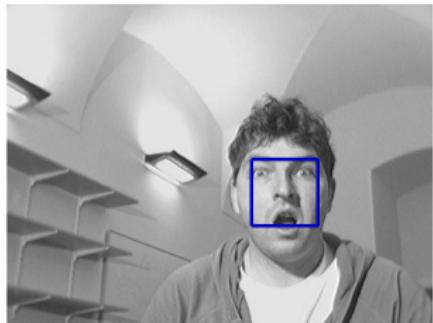
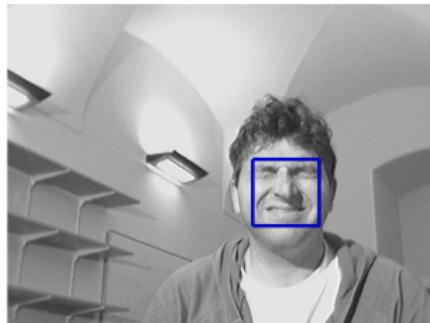


Learning appearance – our approach



Learning the appearance encoder γ

- ▶ Current appearance encoded in low-dim parameters.

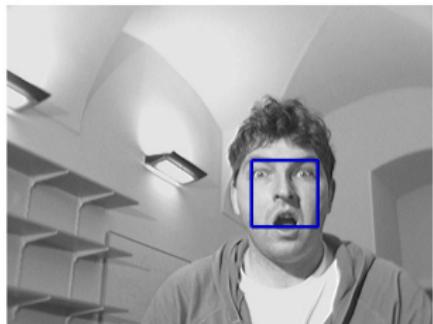
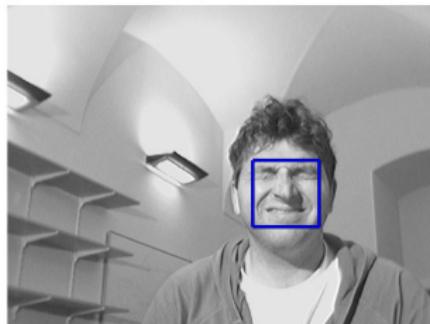


$$\gamma(\quad) = \theta_1$$

$$\gamma(\quad) = \theta_2$$

Learning the appearance encoder γ

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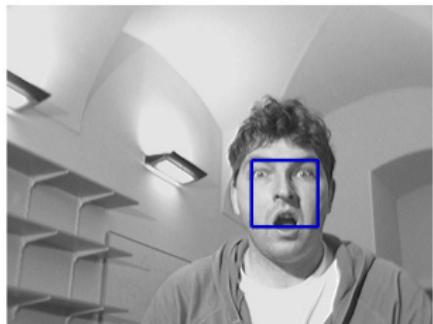
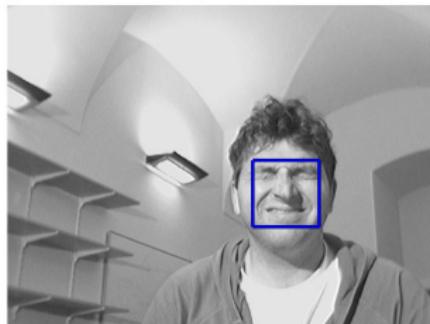


▶ $\gamma(\text{[face]}) = \theta_1$

▶ $\gamma(\text{[open mouth]}) = \theta_2$

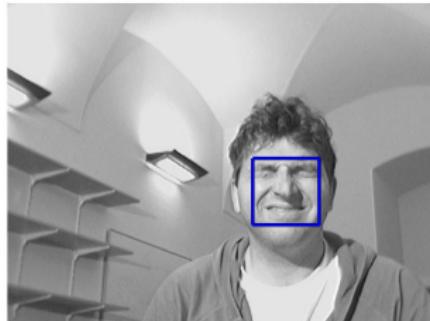
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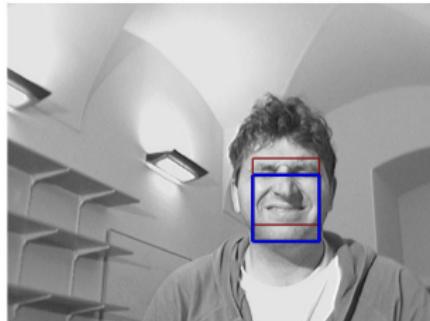
- ▶ $\gamma(\text{[face]}) = \theta_1$
- ▶ $\gamma(\text{[face]}) = \theta_2$

Learning the tracker $\varphi(\mathbf{l}; \theta)$



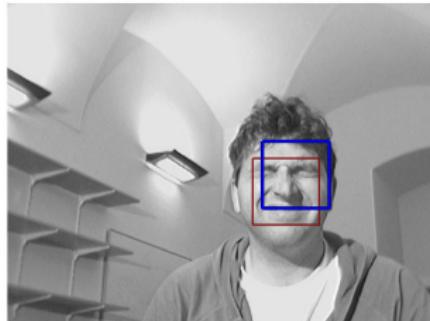
- ▶ $\varphi(\text{[face]}; \theta_1) = (0, 0)^\top$ ▶ $\varphi(\text{[eyes]}; \theta_2) = (0, 0)^\top$
- ▶ $\varphi(\text{[nose]}; \theta_1) = (-25, 0)^\top$ ▶ $\varphi(\text{[mouth]}; \theta_2) = (-25, 0)^\top$
- ▶ $\varphi(\text{[cheek]}; \theta_1) = (25, -15)^\top$ ▶ $\varphi(\text{[chin]}; \theta_2) = (25, -15)^\top$

Learning the tracker $\varphi(\mathbf{l}; \theta)$



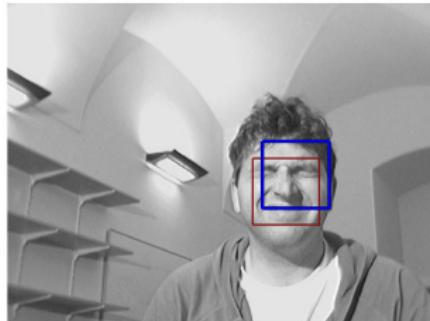
- ▶ $\varphi(\text{[face]}; \theta_1) = (0, 0)^\top$ ▶ $\varphi(\text{[eyes]}; \theta_2) = (0, 0)^\top$
- ▶ $\varphi(\text{[smile]}; \theta_1) = (-25, 0)^\top$ ▶ $\varphi(\text{[mouth]}; \theta_2) = (-25, 0)^\top$
- ▶ $\varphi(\text{[head]}; \theta_1) = (25, -15)^\top$ ▶ $\varphi(\text{[face]}; \theta_2) = (25, -15)^\top$

Learning the tracker $\varphi(\mathbf{l}; \theta)$



- ▶ $\varphi(\text{[eyes]}; \theta_1) = (0, 0)^\top$ ▶ $\varphi(\text{[mouth]}; \theta_2) = (0, 0)^\top$
- ▶ $\varphi(\text{[smile]}; \theta_1) = (-25, 0)^\top$ ▶ $\varphi(\text{[neutral]}; \theta_2) = (-25, 0)^\top$
- ▶ $\varphi(\text{[frown]}; \theta_1) = (25, -15)^\top$ ▶ $\varphi(\text{[neutral]}; \theta_2) = (25, -15)^\top$

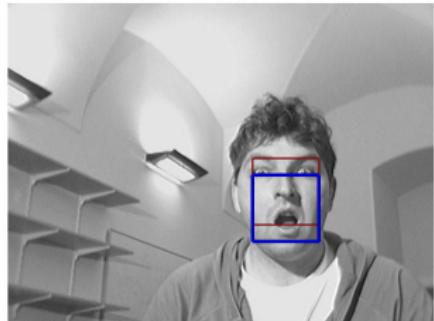
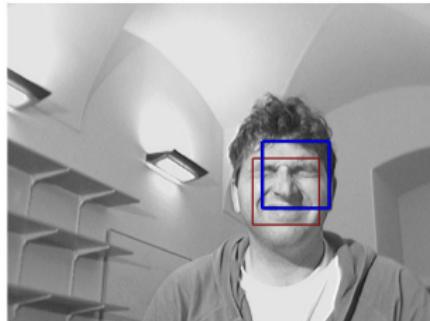
Learning the tracker $\varphi(\mathbf{l}; \theta)$



- ▶ $\varphi(\text{[smiling face]}; \theta_1) = (0, 0)^\top$
- ▶ $\varphi(\text{[smiling mouth]}; \theta_1) = (-25, 0)^\top$
- ▶ $\varphi(\text{[frowning face]}; \theta_1) = (25, -15)^\top$

- ▶ $\varphi(\text{[surprised face]}; \theta_2) = (0, 0)^\top$
- ▶ $\varphi(\text{[surprised mouth]}; \theta_2) = (-25, 0)^\top$
- ▶ $\varphi(\text{[neutral face]}; \theta_2) = (25, -15)^\top$

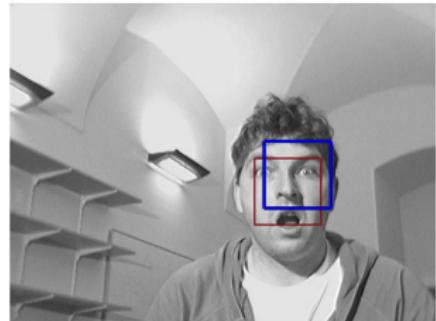
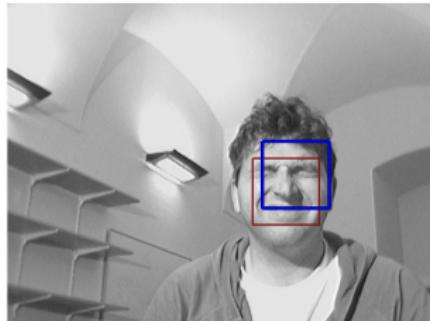
Learning the tracker $\varphi(\mathbf{l}; \theta)$



- ▶ $\varphi(\text{皱眉}; \theta_1) = (0, 0)^\top$
- ▶ $\varphi(\text{张嘴}; \theta_1) = (-25, 0)^\top$
- ▶ $\varphi(\text{皱眉}; \theta_1) = (25, -15)^\top$

- ▶ $\varphi(\text{张嘴}; \theta_2) = (0, 0)^\top$
- ▶ $\varphi(\text{张嘴}; \theta_2) = (-25, 0)^\top$
- ▶ $\varphi(\text{皱眉}; \theta_2) = (25, -15)^\top$

Learning the tracker $\varphi(\mathbf{l}; \theta)$



- ▶ $\varphi(\text{[smile]}; \theta_1) = (0, 0)^\top$
- ▶ $\varphi(\text{[neutral]}; \theta_1) = (-25, 0)^\top$
- ▶ $\varphi(\text{[frown]}; \theta_1) = (25, -15)^\top$

- ▶ $\varphi(\text{[surprise]}; \theta_2) = (0, 0)^\top$
- ▶ $\varphi(\text{[neutral]}; \theta_2) = (-25, 0)^\top$
- ▶ $\varphi(\text{[surprise]}; \theta_2) = (25, -15)^\top$

Simultaneous learning of φ and γ

- ▶ Learning = minimization of the least-squares error

$$(\varphi^*, \gamma^*) = \arg \min_{\varphi, \gamma} \left[\varphi(\text{face 1}; \gamma(\text{face 1})) - (0, 0)^\top \right]^2 +$$
$$\left[\varphi(\text{face 2}; \gamma(\text{face 2})) - (-25, 0)^\top \right]^2 +$$
$$\left[\varphi(\text{face 3}; \gamma(\text{face 3})) - (25, -15)^\top \right]^2 +$$
$$\left[\varphi(\text{face 4}; \gamma(\text{face 4})) - (0, 0)^\top \right]^2 +$$
$$\left[\varphi(\text{face 5}; \gamma(\text{face 5})) - (-25, 0)^\top \right]^2 +$$
$$\left[\varphi(\text{face 6}; \gamma(\text{face 6})) - (25, -15)^\top \right]^2$$

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$$\left[\varphi(\text{open mouth}; \gamma(\text{open mouth})) - (-25, 0)^\top \right]^2 +$$
$$\left[\varphi(\text{look right}; \gamma(\text{look right})) - (25, -15)^\top \right]^2$$

Linear mapping

- ▶ $\gamma(\mathbf{J}) : \boldsymbol{\theta} = \mathbf{G}\mathbf{J}$
- ▶ $\varphi(\mathbf{I}, \boldsymbol{\theta}) : \mathbf{t} = (\mathbf{H}_0 + \theta_1 \mathbf{H}_1 + \cdots + \theta_n \mathbf{H}_n) \mathbf{I}$
- ▶ Criterion is sum of squares of bilinear functions.

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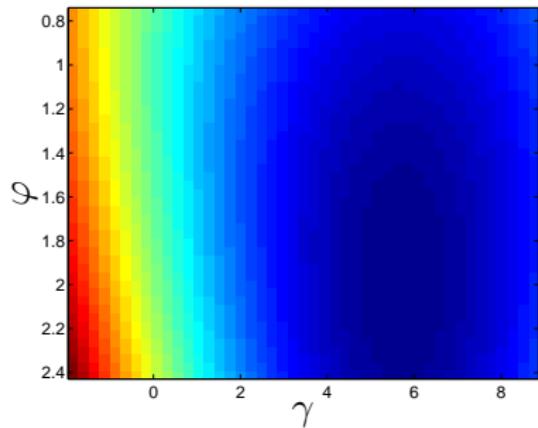
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Algorithm: iterative minimization of criterion $e(\varphi, \gamma)$

φ – motion (geometry mapping), γ – appearance mapping

color encodes criterion value $e(\varphi, \gamma)$

- ▶ Iterative minimization:
 - ▶ initialization $\gamma^0 = \text{rand}$
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- ▶ Global optimality for linear φ, γ experimentally shown.

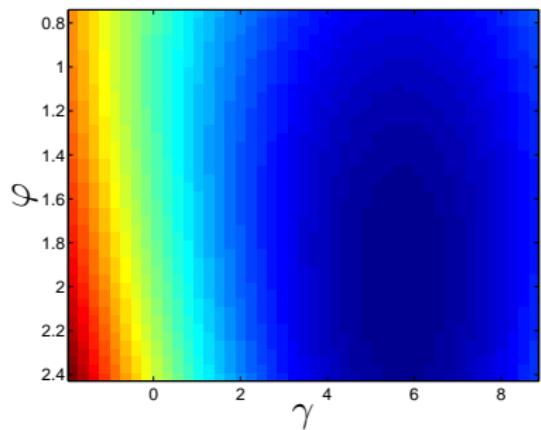
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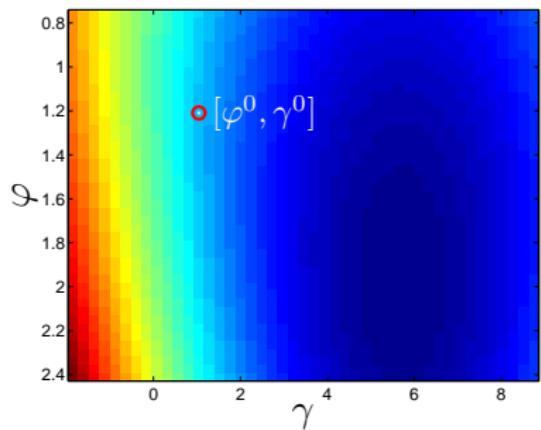
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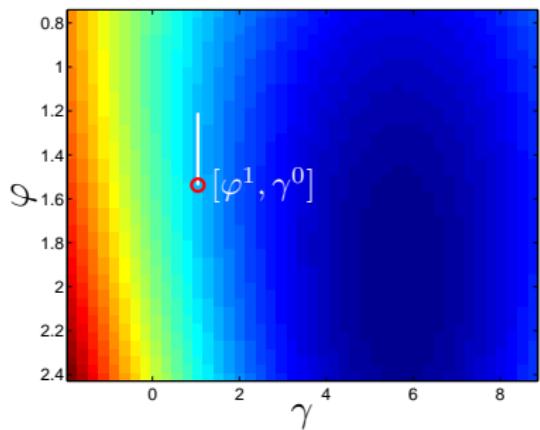
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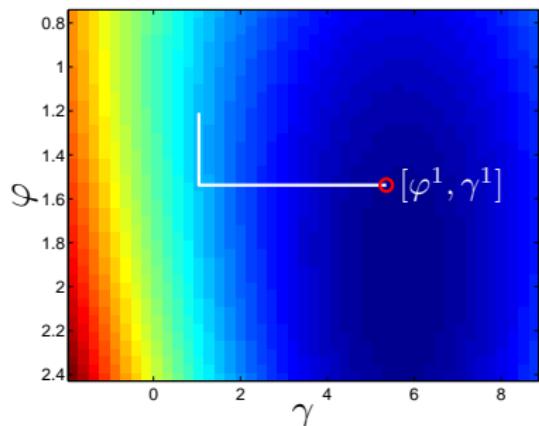
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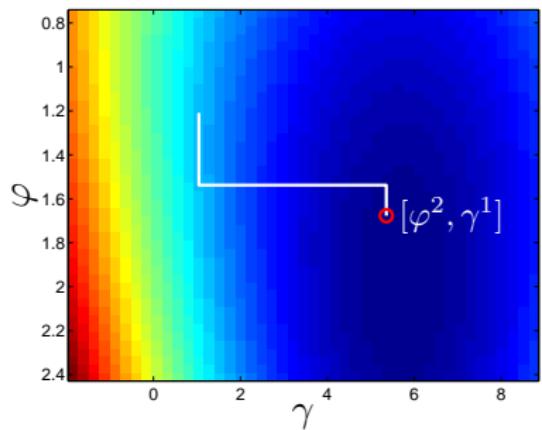
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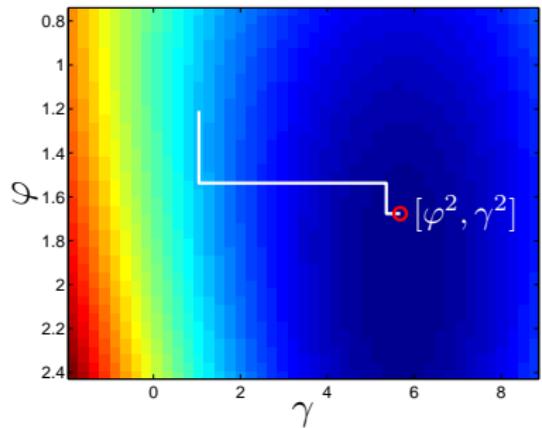
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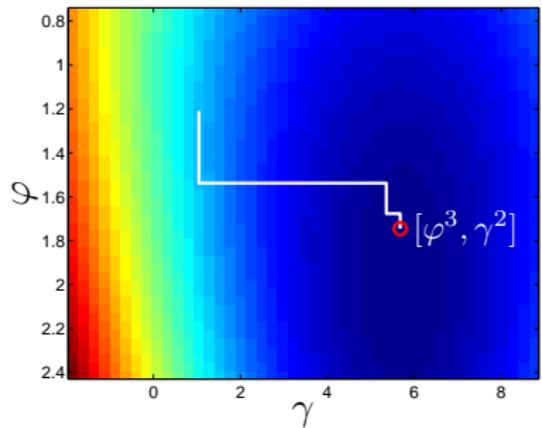
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 - ▶ $\varphi^2 = \arg \min_{\varphi} e(\varphi, \gamma^1)$
 - ▶ $\gamma^2 = \arg \min_{\gamma} e(\varphi^2, \gamma)$
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 - ▶ $\gamma^3 = \arg \min_{\gamma} e(\varphi^3, \gamma)$
 - ▶ until convergence reached



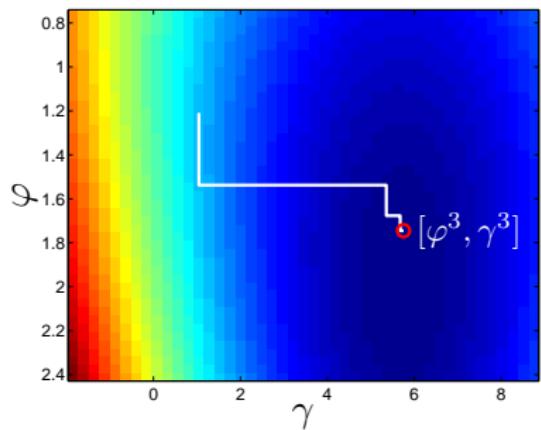
- ▶ Global optimality for linear φ, γ experimentally shown.

Algorithm: iterative minimization of criterion $e(\varphi, \gamma)$

φ – motion (geometry mapping), γ – appearance mapping

color encodes criterion value $e(\varphi, \gamma)$

- ▶ Iterative minimization:
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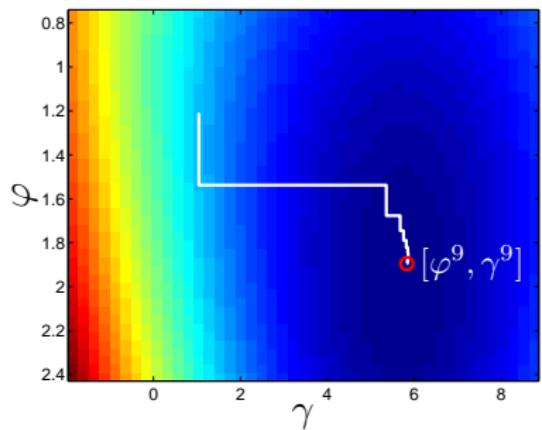
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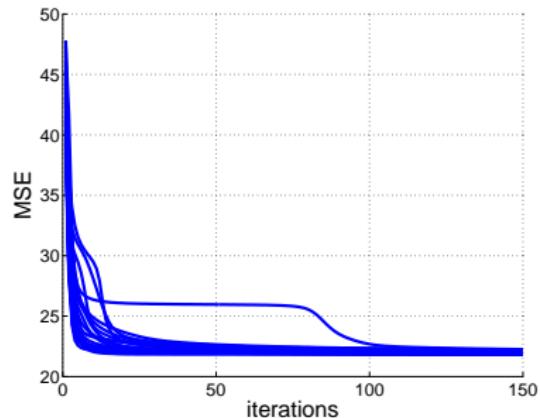
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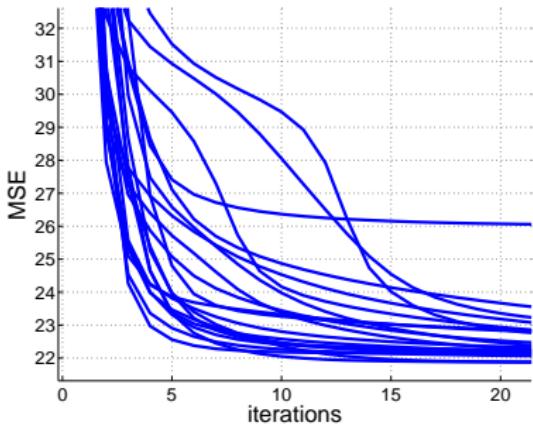
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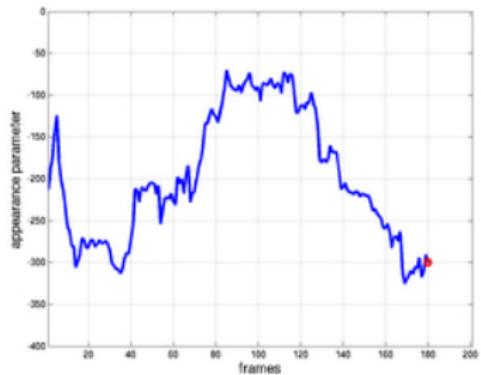
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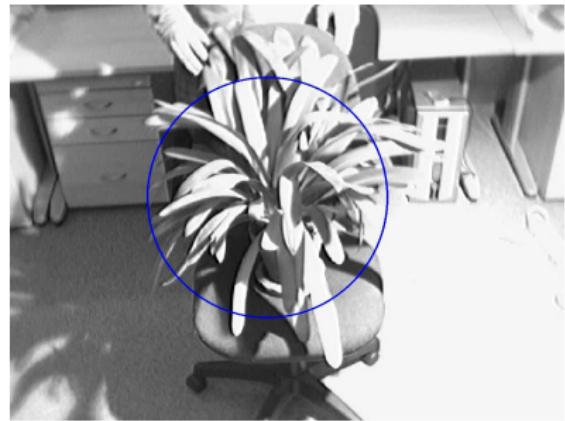
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Simultaneous learning of motion and appearance



Experiments - videos II



Conclusions

- ▶ Learnable and very efficient tracking of objects with variable appearance.
- ▶ Accuracy, speed, robustness explicitly taken into account.
- ▶ Simultaneous learning motion and appearance.

Limitations

- ▶ Small, thin objects intractable.

Data, papers, various implementations freely available at
<http://cmp.felk.cvut.cz/demos/Tracking/linTrack/>

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