Oriented Matching Constraints

Tomáš Werner
Visual Geometry Group
Oxford University
Parks Road
OX13PJ Oxford, U.K.
`http://www.robots.ox.ac.uk/vgg/`

Tomáš Pajdla
Center for Machine Perception
Czech Technical University
Karlovo náměstí 13
12135 Prague, Czech Republic
`http://cmp.felk.cvut.cz`

CONTRIBUTIONS

- Synthesis of theory of chirality and quasi-affine reconstruction [Hartley-IJCV98] and oriented projective geometry [Stolfi-91] represented by Grassmann tensor algebra [Triggs-ICCV95].
- Incorporating orientations of image points and lines into matching constraints.
- Natural inclusion of (central) panoramic cameras.

ORIENTED PROJECTIVE GEOMETRY

- Flats: points, lines, planes, etc.
- All flats are oriented. Point orientation – point seen from front or behind. Line orientation – arrow on the line. Plane orientation – oriented normal. Etc.
- Operations with flats: join, meet, antipode.

Represented by Grassmann tensor algebra.

HIERARCHY OF RECONSTRUCTIONS

Hierarchy of reconstructions from projections $\{\mathbf{x}_n^{A_k}, \mathbf{x}_m^{A_k B_k}\}$ of N points and M lines in K images:

1. **Projective reconstruction (PR).** Usual meaning.

   A set $\{\mathbf{P}_a^{A_k}, \mathbf{x}_n^a, \mathbf{x}_m^{ab}\}$ such that $\mathbf{x}_n^{A_k} \simeq \mathbf{P}_a^{A_k}\mathbf{x}_n^a$ and $\mathbf{x}_n^{A_k B_k} \simeq \mathbf{P}_a^{A_k}\mathbf{P}_b^{B_k}\mathbf{x}_n^{ab}$.
2. **Oriented projective reconstruction (OPR).** PR and scales are positive.

   A set $\{\mathbf{P}_a^{A_k}, \mathbf{x}_n^a, \mathbf{x}_m^{ab}\}$ such that $\mathbf{x}_n^{A_k} \mathrel{\widehat{=}} \mathbf{P}_a^{A_k}\mathbf{x}_n^a$ and $\mathbf{x}_n^{A_k B_k} \mathrel{\widehat{=}} \mathbf{P}_a^{A_k}\mathbf{P}_b^{B_k}\mathbf{x}_n^{ab}$.
3. **Quasi-affine reconstruction (QAR).** OPR and all scene points are in front of a single plane.

   A set $\{\mathbf{P}_a^{A_k}, \mathbf{x}_n^a, \mathbf{x}_m^{ab}, \mathbf{\Omega}_a\}$ such that $\{\mathbf{P}_a^{A_k}, \mathbf{x}_n^a, \mathbf{x}_m^{ab}\}$ is an OPR and $\mathbf{\Omega}_a\mathbf{x}_n^a > 0$.
4. **QAR, cameras preserve orientation (QARC$^+$).** QAR and cameras preserve orientation.

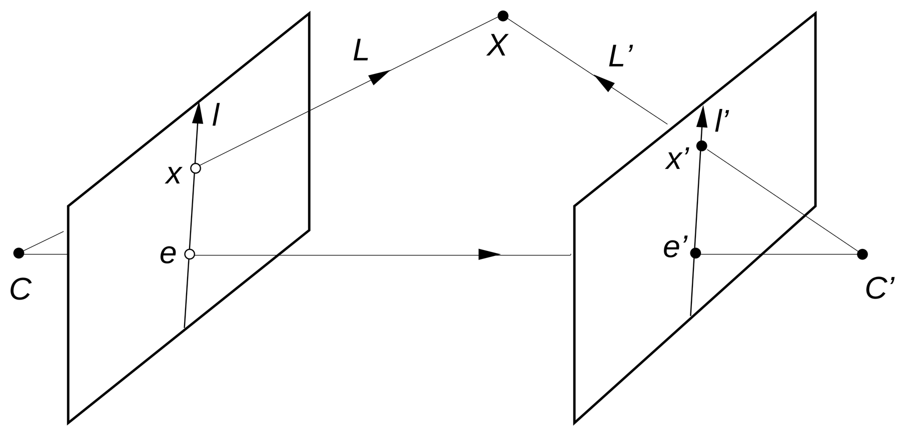
   A set $\{\mathbf{P}_a^{A_k}, \mathbf{x}_n^a, \mathbf{x}_m^{ab}, \mathbf{\Omega}_a\}$ such that $\{\mathbf{P}_a^{A_k}, \mathbf{x}_n^a, \mathbf{x}_m^{ab}, \mathbf{\Omega}_a\}$ is a QAR and $\mathbf{\Omega}_a\mathbf{e}_k^a > 0$.

EXISTENCE OF OPR

Formulate matching constraints that imply existence of an OPR:

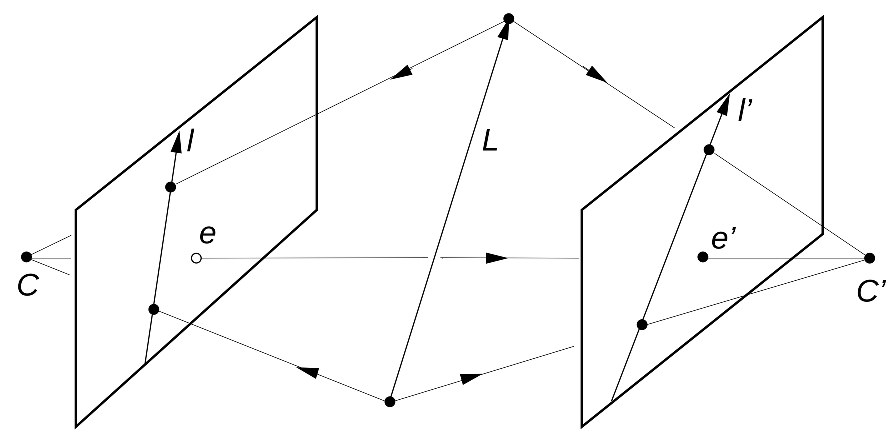
Points in 2 Images

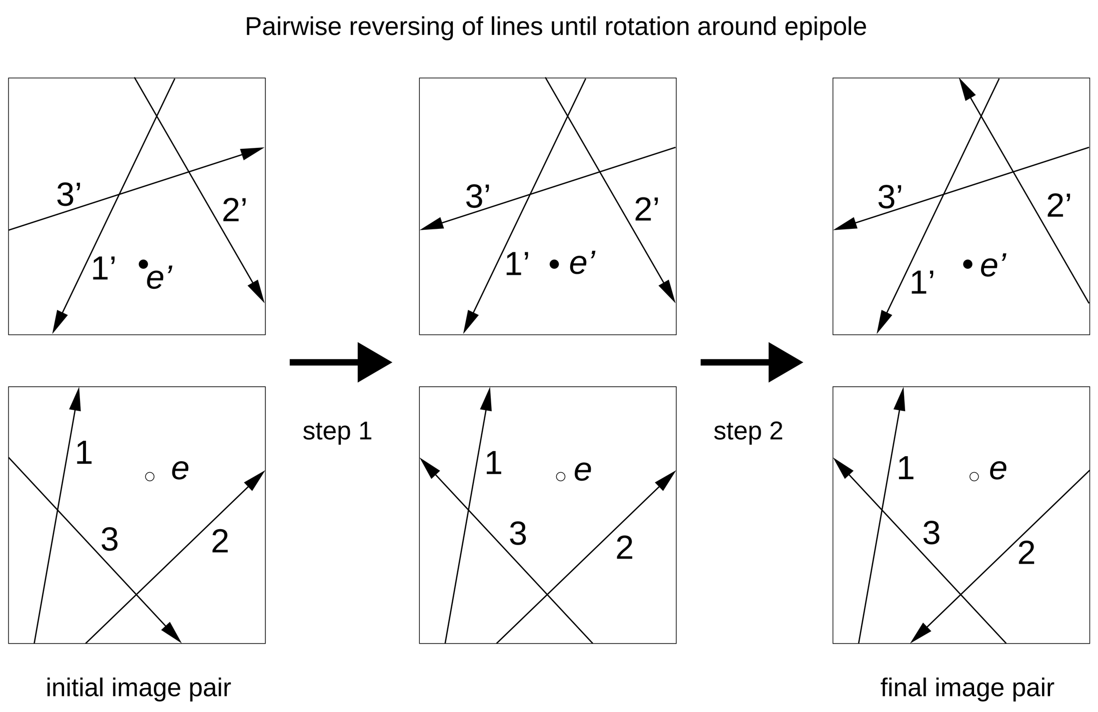
OPR exists $\Leftrightarrow$ $\mathbf{e}_1^{A_2}\mathbf{x}_n^{B_2}\varepsilon_{A_2B_2C_2} \mathrel{\widehat{=}} \mathbf{x}_n^{C_1}\mathbf{F}_{C_1C_2}$



Lines in 2 Images

OPR exists $\Leftrightarrow$ $\mathrm{sgn}(\mathbf{x}_{A_1}^n\mathbf{e}_2^{A_1}) + \mathrm{sgn}(\mathbf{x}_{A_2}^n\mathbf{e}_1^{A_2}) = 0$

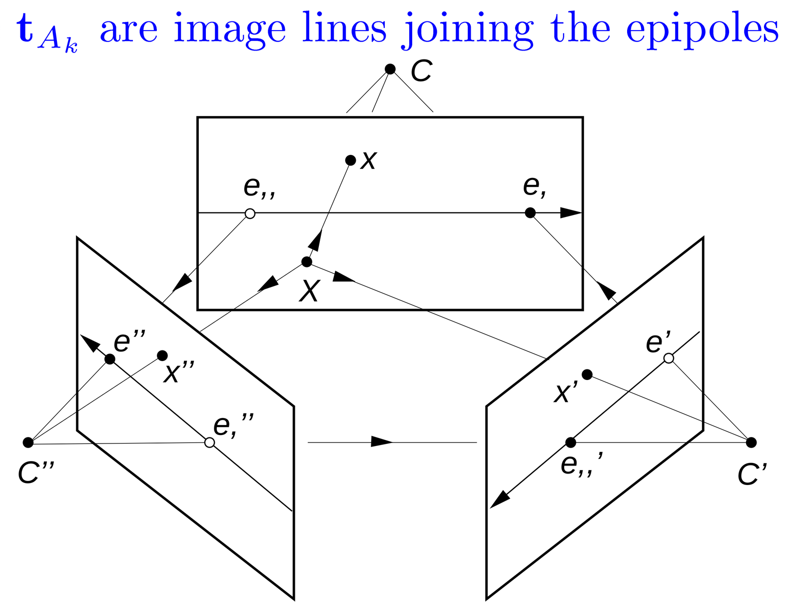




NOTATION: $\simeq$ equality up to a non-zero scale, $\mathrel{\widehat{=}}$ equality up to a positive scale, $\mathbf{e}_k^a$ (oriented) camera center, $\mathbf{e}_l^{A_k}$ epipole)

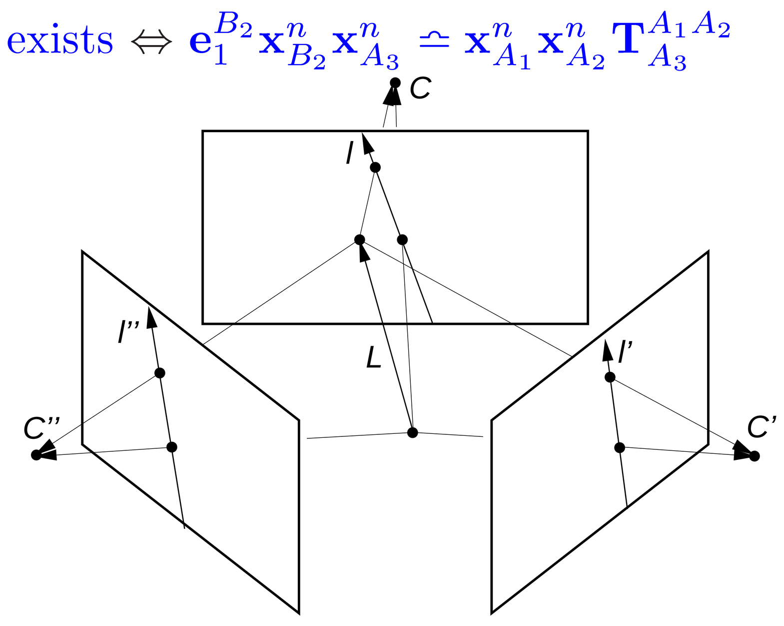
Points in 3 Images

OPR exists $\Rightarrow$
$\mathrm{sgn}(\mathbf{t}_{A_1}\mathbf{x}_n^{A_1}) = \mathrm{sgn}(\mathbf{t}_{A_2}\mathbf{x}_n^{A_2}) = \mathrm{sgn}(\mathbf{t}_{A_3}\mathbf{x}_n^{A_3})$,
where $\mathbf{t}_{A_k}$ are image lines joining the epipoles



Lines in 3 Images

OPR exists $\Leftrightarrow$ $\mathbf{e}_1^{B_2}\mathbf{x}_{B_2}^n\mathbf{x}_{A_3}^n \mathrel{\widehat{=}} \mathbf{x}_{A_1}^n\mathbf{x}_{A_2}^n\mathbf{T}_{A_3}^{A_1A_2}$



Joint Image Matrix

OPR exists $\Leftrightarrow$ $\exists\rho_n^k > 0:\ \mathrm{rank}\,(\rho_n^k\mathbf{x}_n^{A_k}) \le 4$,
where $(\rho_n^k\mathbf{x}_n^{A_k})$ is joint image matrix.

EXISTENCE OF QAR/QARC$^+$

Formulate matching constraints that imply existence of QAR/QARC$^+$. Results for special cases of image points:
Points in joint image matrix:
QAR exists $\Leftrightarrow$
$\exists\rho_n^k > 0$ rank $(\rho_n^k\mathbf{x}_n^{A_k}) \le 4$ and $\exists\mathbf{\Omega}_\alpha : \mathbf{\Omega}_\alpha\mathbf{x}^\alpha > 0$

Using camera directionality:

OPR exists and 1+ cameras are directional $\Rightarrow$ QAR exists

OPR exists, 2 cameras, 1+ cameras directional $\Rightarrow$ QARC$^+$ exists

SURVEY OF MATCHING CONSTRAINTS (new constraints in red)

In terms of...	PR	OPR	QAR/QARC$^+$
scene flats and cameras	$\exists\{\mathbf{P}_a^{A_k}, \mathbf{x}_n^a, \mathbf{x}_m^{ab}\}$: $\mathbf{x}_n^{A_k} \simeq \mathbf{P}_a^{A_k}\mathbf{x}_n^a$, $\mathbf{x}_n^{A_kB_k} \simeq \mathbf{P}_a^{A_k}\mathbf{P}_b^{B_k}\mathbf{x}_n^{ab}$	$\exists$ PR and all scales positive	$\exists$ OPR and $\exists\mathbf{\Omega}_a : \mathbf{\Omega}_a\mathbf{x}_n^a > 0, \mathbf{\Omega}_a\mathbf{e}_k^a > 0$
points in 2 images	$\mathbf{x}^{A_1}\mathbf{x}^{A_2}\mathbf{F}_{A_1A_2} = 0$	$\mathbf{e}_1^{A_2}\mathbf{x}^{B_2}\varepsilon_{A_2B_2C_2} \mathrel{\widehat{=}} \mathbf{x}^{C_1}\mathbf{F}_{C_1C_2}$	1+ camera directional: OPR implies QARC$^+$ for panoramic cameras: unknown
lines in 2 images	$\emptyset$	$\mathrm{sgn}(\mathbf{x}_{A_1}\mathbf{e}_2^{A_1}) + \mathrm{sgn}(\mathbf{x}_{A_2}\mathbf{e}_1^{A_2}) = 0$	QAR has no meaning QARC$^+$ unknown
points in 3 images	$\mathbf{x}^{A_1}\mathbf{x}^{A_2}\mathbf{x}^{A_3}\varepsilon_{A_1B_1C_1}\varepsilon_{A_2B_2C_2}\mathbf{T}_{A_3}^{B_1B_2} = \mathbf{0}_{C_1C_2}$	$\mathrm{sgn}(\mathbf{t}_{A_1}\mathbf{x}^{A_1}) = \mathrm{sgn}(\mathbf{t}_{A_2}\mathbf{x}^{A_2}) = \mathrm{sgn}(\mathbf{t}_{A_3}\mathbf{x}^{A_3})$	1+ camera directional: OPR implies QAR for panoramic cameras, QARC$^+$: unknown
lines in 3 images	$\mathbf{x}_{A_3} \simeq \mathbf{x}_{A_1}\mathbf{x}_{A_2}\mathbf{T}_{A_3}^{A_1A_2}$	$\mathbf{e}_1^{B_2}\mathbf{x}_{B_2}\mathbf{x}_{A_3} \mathrel{\widehat{=}} \mathbf{x}_{A_1}\mathbf{x}_{A_2}\mathbf{T}_{A_3}^{A_1A_2}$	QAR has no meaning QARC$^+$ unknown
joint image matrix	$\exists\rho_n^k \neq 0:\ \mathrm{rank}\,(\rho_n^k\mathbf{x}_n^{A_k}) \le 4$	$\exists\rho_n^k > 0:\ \mathrm{rank}\,(\rho_n^k\mathbf{x}_n^{A_k}) \le 4$	1+ camera directional: OPR implies QAR OPR and $\exists\mathbf{\Omega}_\alpha : \mathbf{\Omega}_\alpha\mathbf{x}^\alpha > 0$ imply QAR QARC$^+$: unknown