Cheirality in Epipolar Geometry

Tomáš Werner and Tomáš Pajdla

Center for Machine Perception, Czech Technical University in Prague

http://cmp.felk.cvut.cz

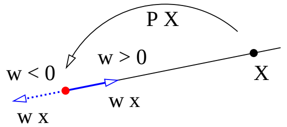
CENTER FOR MACHINE PERCEPTION

CONTRIBUTION

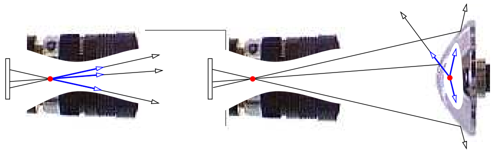
1. Theory of quasi-affine reconstruction for panoramic cameras.
2. Conditions for an image pair to correspond to a real scene.

CONCEPTS

1. $k = 1, \dots, K$ cameras with projection matrices: $\mathbf{P}^k \in \mathbb{R}^{3\times 4}$
2. $n = 1, \dots, N$ points $\mathbf{X}_n \in \mathbb{R}^4 \setminus \mathbf{0}$ in space
3. $K \times N$ image points $\mathbf{x}_n^k \in \mathbb{R}^3 \setminus \mathbf{0}$ represented by the direction vectors of camera rays
4. Projection equation: $w_n^k \mathbf{x}_n^k = \mathbf{P}^k \mathbf{X}_n$, $\quad w_n^k \in \mathbb{R} \setminus 0$
5. Camera rays are oriented



6. Conventional camera / Catadioptric camera



directional image / panoramic image

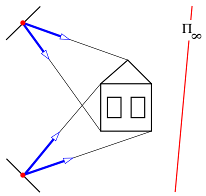
The image $\{\mathbf{x}_n\}$ is directional if there exists a vector $\mathbf{s}$ such that $\forall n$, $\mathbf{s}\mathbf{x}_n > 0$. Otherwise, the image is panoramic.

7. Oriented plane at infinity $\mathbf{\Pi}_\infty$.
8. Oriented camera center $C(\mathbf{P}^k) = \mathbf{p}_1 \wedge \mathbf{p}_2 \wedge \mathbf{p}_3$, where $\mathbf{p}_i$ are rows of $\mathbf{P}^k$.
9. $\mathbf{\Pi}_\infty \mathbf{X}_n > 0 \Leftrightarrow$ points lie in front of the plane at infinity
10. $\mathbf{\Pi}_\infty C(\mathbf{P}^k) > 0 \Leftrightarrow$ cameras preserve orientation of coordinate systems.

RECONSTRUCTIONS

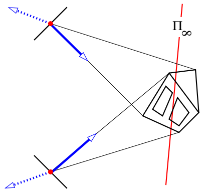
A reconstruction of image $\{\mathbf{x}_n^k\}$ is the triple $[\{\mathbf{P}^k\}, \{\mathbf{X}_n\}, \mathbf{\Pi}_\infty]$ of points, the corresponding camera matrices, and the plane at infinity such that the projection equation holds.

1. **Metric realization** (= real scene)



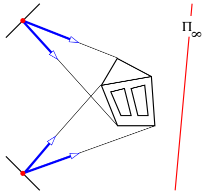
- is finite
- is in front of cameras
- $\mathbf{X}_n$ can be measured manually, e.g. by a (long) ruler, w.r.t. an orthonormal basis in which $\mathbf{\Pi}_\infty = (0,0,0,1)$

2. **Weak realization** (= projective reconstruction)



$$\exists w_n^k \neq 0: \quad \begin{aligned} w_n^k \mathbf{x}_n^k &= \mathbf{P}^k \mathbf{X}_n \\ \mathbf{\Pi}_\infty &\neq \mathbf{0} \end{aligned}$$

3. **Strong realization** (= quasi-affine reconstruction)



$$\exists w_n^k > 0: \quad \begin{aligned} w_n^k \mathbf{x}_n^k &= \mathbf{P}^k \mathbf{X}_n \\ \mathbf{\Pi}_\infty \mathbf{X}_n &> 0 \\ \mathbf{\Pi}_\infty C(\mathbf{P}^k) &> 0 \end{aligned}$$

Strong realizability is necessary and sufficient condition for the reconstruction $[\{\mathbf{P}^k\}, \{\mathbf{X}_n\}, \mathbf{\Pi}_\infty]$ to correspond to a real scene.

POSSIBLE APPLICATIONS

- Constraining search space in matching
- Ruling out spurious solutions for fundamental matrix estimated from 7 points.

CONDITIONS FOR REALIZABILITY OF TWO IMAGES

Assume the epipolar constraint in the plane+parallax form

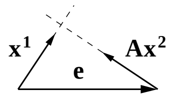
$$w_n \mathbf{x}_n^1 = \mathbf{A}\mathbf{x}_n^2 + \rho_n \mathbf{e}^1.$$

In two steps, the form $\mathbf{x}_n^{1\top} \mathbf{F} \mathbf{x}_n^2 = 0$ of the epipolar constraint is derived, but for the cost of loosing knowledge about w_n, ρ_n:

$$\begin{aligned} w_n \mathbf{x}_n^1 &= \mathbf{A}\mathbf{x}_n^2 + \rho_n \mathbf{e}^1 \\ w_n \mathbf{e}^1 \times \mathbf{x}_n^1 &= \mathbf{e}^1 \times \mathbf{A}\mathbf{x}_n^2 = \mathbf{F}\mathbf{x}_n^2 \\ \mathbf{x}_n^{1\top} \mathbf{F} \mathbf{x}_n^2 &= 0 \end{aligned}$$

Constraint on w_n, ρ_n / **Theorem**

1.

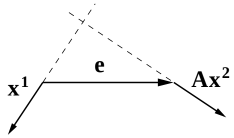


$w_n > 0$, $\rho_n > 0$

Image pair $\{\mathbf{x}_n^1, \mathbf{x}_n^2\}$ is strongly realizable iff

$$\exists \mathbf{A} \in \mathbb{R}^{3\times 3}, |\mathbf{A}| > 0, \exists \mathbf{e}^1 \in \mathbb{R}^3$$
$$\forall n\, \exists w_n > 0, \rho_n > 0: \; w_n \mathbf{x}_n^1 = \mathbf{A}\mathbf{x}_n^2 + \rho_n \mathbf{e}^1$$

2.

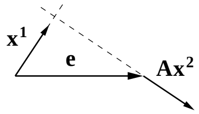


$w_n > 0$

Pair $\{\mathbf{x}_n^1, \mathbf{x}_n^2, x_{n3}^1 > 0, x_{n3}^2 > 0\}$ of directional images is strongly realizable iff

$$\exists \mathbf{F} \in \mathbb{R}^{3\times 3}, \text{rank}\,\mathbf{F} = 2, \exists \mathbf{e}^1 \in \mathbb{R}^3, \mathbf{e}^1 \neq \mathbf{0}, \mathbf{e}^{1\top}\mathbf{F} = \mathbf{0},$$
$$\forall n\, \exists w_n > 0: \; w_n \mathbf{e}^1 \times \mathbf{x}_n^1 = \mathbf{F}\mathbf{x}_n^2$$

3.



$w_n \neq 0$

Image pair $\{\mathbf{x}_n^1, \mathbf{x}_n^2\}$ is weakly realizable iff

$$\exists \mathbf{F} \in \mathbb{R}^{3\times 3}, \text{rank}\,\mathbf{F} = 2, \forall n:$$
$$[(\mathbf{x}_n^{1\top}\mathbf{F} \neq 0, \mathbf{F}\mathbf{x}_n^2 \neq 0) \vee (\mathbf{x}_n^{1\top}\mathbf{F} = 0, \mathbf{F}\mathbf{x}_n^2 = 0)]$$
$$\wedge (\mathbf{x}_n^{1\top}\mathbf{F}\mathbf{x}_n^2 = 0)$$

Main observation = Theorem 2: Condition $w_n > 0$ in the plane+parallax equation is sufficient for the strong realizability of directional images yet insufficient for panoramic images.

RELEVANT LITERATURE

[1] R.I.Hartley. Chirality. *IJCV*, 26(1): 41-61, 1998.

[3] B.Triggs. Matching Constraints and the Joint Image. *ICCV'95*: 338-343, 1995.

[2] J.Stolfi. *Oriented Projective Geometry*. Academic Press. 1991.

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