Image Registration – Introduction

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Outline

- What is image registration
- Classification of registration methods
- Manual landmark registration
- ITK Registration Toolkit
Find corresponding points

American Tux

Tux bordelais
Find corresponding points

American Tux

Tux bordelais
Find corresponding points

American Tux

Tux bordelais
Registration example 2

Michael Unser

Philippe Thévenaz
Registration example 3

James Bond

Lupe
Registration example 4

EPI MRI

anatomical MRI
Correspondence function

Reference image

Test image

source

destination
Correspondence function

\[ g([x \ y]^T) = [x' \ y']^T \]
Deformation field

0 % deformation
Deformation field

25 % deformation
Deformation field

50 % deformation
Deformation field

75 % deformation
Deformation field

100 % deformation
Image warping

0 % deformation
Image warping

25 % deformation
Image warping

50 % deformation
Image warping

75 % deformation
Image warping

100 % deformation
Image registration
Image registration

Reference image $\xrightarrow{\text{registration}}$ Test image

Deformation $\xrightarrow{\text{warping}}$ Warped image
(Biomedical) applications

...of image registration

- Comparing images
  - Different times
  - Different methods
  - Different subjects

- Analyzing sequences
  - Motion estimation
  - Segmentation

Qualitative and quantitative information.
Other applications of image registration

- video stabilization
- video compression
- image mosaicking
- stereo matching
- structure from motion
Image alignment

reference

before

test
Image alignment

reference

before

warped

test
Classification of registration methods

- **Feature space** — intermediate data extracted from image
- **Search space** — representation of the deformation
- **Similarity metric** — measuring the dissimilarity
- **Search strategy** — how to find the minimum
- **User interaction level**
Registration methods – Feature space

- Feature space
  - pixels
  - transforms
    - Fourier wavelet
  - features
    - landmarks
    - curves, surfaces, templates
      - intrinsic
      - extrinsic
Registration methods – Search space

- Local
  - Variational
  - PDE
- Semi-local
  - (Quad)tree
  - B-splines
  - Wavelets
- Global
  - linear
  - polynomial
  - harmonic
- RBF, krigging
- Image dependent models (e.g. adaptive quadtrees)
Similarity metrics

- **Data term for pixel based criteria**
  - $l_2$ norm (SSD)
  - $l_1$ norm
  - correlation, normalized correlation
  - mutual information, normalized mutual information

- **Other data terms**
  - image interpolation — important
  - feature-based methods — distance
  - template-based methods — windowed pixel-based criteria
  - transform-based methods — norm in the transform domain
  - *preprocessing* — filtering, histogram equalization, . . .

- **Regularization**
  - Norm ($l_p$) of the derivatives
  - Implicit regularization (constrained model)
  - Smoothing
Search strategy

- Direct solution
- Exhaustive search
- Dynamic programming
- PDE evolution
- Multidimensional optimisation
  - gradient descent
  - Newton-like methods, exact/estimated Hessian, Marquardt-Levenberg, conjugated gradients, BFGS, ...
- Multiresolution
User interaction level

- Manual
- Automatic
- Semi-automatic
Manual registration

- Landmark identification
Manual registration

- Landmark identification
- Landmark interpolation
Landmark interpolation (2)

- **Constraints**
  - **Hard constraints**
    \[ \mathbf{g}(\mathbf{x}_i) = \begin{bmatrix} g_x(x_i) \\ g_y(x_i) \end{bmatrix} = \mathbf{z}_i \quad \text{for all } i \in \{1, \ldots, N\} \]
  - **Soft constraints**
    \[ \sum_{i=1}^{N} \| \mathbf{g}(\mathbf{x}_i) - \mathbf{z}_i \|^2 \leq \varepsilon \]

- **Properties**
  - invariance to scale, shifts, rotations
  - representability of linear transforms
Thin-plate splines

- Minimize an energy

\[
J(g) = \int \left( \frac{\partial^2 g}{\partial x^2} \right)^2 + 2 \left( \frac{\partial^2 g}{\partial x \partial y} \right)^2 + \left( \frac{\partial^2 g}{\partial y^2} \right)^2 \, dx \, dy
\]

\[
J(g) = J(g_x) + J(g_y)
\]

under constraints

\[
g(x_i, y_i) = z_i
\]
Thin-plate splines

Minimize an energy

\[
J(g) = \int \left( \frac{\partial^2 g}{\partial x^2} \right)^2 + 2 \left( \frac{\partial^2 g}{\partial x \partial y} \right)^2 + \left( \frac{\partial^2 g}{\partial y^2} \right)^2 \, dx \, dy
\]

\[
J(g) = J(g_x) + J(g_y)
\]

under constraints

\[g(x_i, y_i) = z_i\]

Solution

\[
g(x, y) = \sum_{i=1}^{N} \lambda_i \varphi(\|x - x_i\|) + a_0 x + a_1 y + a_2
\]

with

\[
\|x - x_i\| = \sqrt{(x - x_i)^2 + (y - y_i)^2} = r
\]

where \(\varphi(r)\) is a radial basis function and \(\varphi(r) = r^2 \log r\)
Automatic Rigid registration

- Look for rigid (euclidean or affine) transformation
- To compensate different position, scale
- ... or to simplify a more complicated problem