LECTURE PLAN

1. Kinematics, what is?
2. Open, closed kinematic mechanisms.
3. Sequence of joint transformations (matrix multiplications)
4. Direct vs. inverse kinematic task.
KINEMATICS

KINEMATICS – the analytical study of the geometry of motion of a mechanism:

- with respect to a fixed reference co-ordinate system,
- without regard to the forces or moments that cause the motion.

In order to control and programme a robot we must have knowledge of both its spatial arrangement and a means of reference to the environment.
OPEN CHAIN MANIPULATOR KINEMATICS

- Mechanics of a manipulator can be represented as a kinematic chain of rigid bodies (links) connected by revolute or prismatic joints.

- One end of the chain is constrained to a base, while an end effector is mounted to the other end of the chain.

- The resulting motion is obtained by composition of the elementary motions of each link with respect to the previous one.
CLOSED KINEMATIC CHAIN

- Much more difficult.
- Even analysis has to take into account statics, constraints from other links, etc.
- Synthesis of closed kinematic mechanisms is very difficult.
**Kinematics** describes the analytical relationship between the joint positions and the end-effector position and orientation.

**Differential kinematics** describes the analytical relationship between the joint motion and the end-effector motion in terms of velocities.
There is a kinematic relationship between two frames, basically a translation and a rotation.

This relationship is represented by a $4 \times 4$ homogeneous transformation matrix.
Rotation matrix $R$ is orthogonal $\Leftrightarrow R^T R = I \Rightarrow 3$ independent entries, e.g., Euler angles.
TWO BASIC JOINTS

Revolute

Prismatic
OPEN KINEMATIC CHAIN
DIRECT vs. INVERSE KINEMATICS

In manipulator robotics, there are two kinematic tasks:

Direct (also forward) kinematics – Given are joint relations (rotations, translations) for the robot arm. Task: What is the orientation and position of the end effector?

Inverse kinematics – Given is desired end effector position and orientation. Task: What are the joint rotations and orientations to achieve this?
DIRECT KINEMATICS

- One joint: \( \mathbf{x}_i = A \mathbf{x}_{i-1} \).
- Chain of joints: \( \mathbf{x}_{n-1} = A_{n-1} A_{n-2} \ldots A_1 A_0 \mathbf{x}_0 \).
- Easy to compute (matrix multiplication).
- Unique solution.
INVERSE KINEMATICS

- For a kinematic mechanism, the inverse kinematic problem is difficult to solve.
- The robot controller must solve a set of non-linear simultaneous algebraic equations.

Source of problems:

- Non-linear equations ($\sin$, $\cos$ in rotation matrices).
- The existence of multiple solutions.
- The possible non-existence of a solution.
- Singularities.
INVERSE KINEMATICS, SIMPLIFICATIONS

- Divide and conquer strategy. Decouple the problem into independent subproblems.
- The spherical wrist. Positioning of the wrist + positioning within the wrist.
- Design conventions, e.g. Denavit-Hartenberg systematic frame assignment.
MANIPULATOR KINEMATIC (1)

Cartesian

Gantry
MANIPULATOR KINEMATIC (2)

Cylindrical

Sphere
MANIPULATOR KINEMATIC (3)

SCARA

Anthropomorphic
Kinematics is only the first step towards robot control!
CLOSED PARALLEL CHAIN

Hexamod
REAL HEXAMOD (1)
REAL HEXAMOD (2)