Welcome
What did I do?
What did I do?

Programming

#include "BIGUIKit.h"

/* Interpolation of a volume described by B-spline coefficients at arbitrary points.
   The function is designed to evaluate the B-spline coefficients of degree `degree'
   at `nxc` x `nyc` x `nzc` points given by `coord`, each point described by `ndc`
   coordinates. Uses mirror boundary conditions. Everything should be
   allocated in advance.

   In this routine, `x` is simply the index that changes fastest and
   `z` the index which changes slowest. C convention is applied for indexing,
   i.e. the first element of `input` is assumed to correspond to point (0,0,0). If `coord`
   gives multidimensional coordinates, they are laid consecutively, i.e., as the fastest changing (sub)index − even faster than `x`.

   #ifdef BIGSPLINES
   /* Takes an input matrix of size `nxi`*`nyi`*`nzi`, containing B-spline
   coefficients of degree `degree`. Samples the resulting function at
   `nxc`*`nyc`*`nzc` points given by `coord`, each point described by `ndc`
   coordinates. Uses mirror boundary conditions. Everything should be
   allocated in advance.

   #include "BIGsplines.h"
   #include "math.h"
   #ifdef BIGSPLINES
   extern splinterp
   #else
   extern int
   #endif

   #ifdef BIGSPLINES
   return
   #else
   #endif

   #ifdef BIGSPLINES
   extern int mfoldmirroronbound(int k, int n)
   #else
   extern int mfoldmirroronbound
   #endif

   extern double mfoldmirroronbound(double k, int n)
   #ifdef BIGSPLINES
   return
   #else
   #endif

   extern int mfoldmirroroffbound(int k, int n)
   #ifdef BIGSPLINES
   return
   #else
   #endif

   extern int mfoldmirroronbound(int k, int n)
   #ifdef BIGSPLINES
   return
   #else
   #endif

   extern int mfoldmirroroffbound(double k, int n)
   #ifdef BIGSPLINES
   return
   #else
   #endif

   extern int mfoldmirroroffbound(double k, int n)
   #ifdef BIGSPLINES
   return
   #else
   #endif

   extern int evalbspln(double *x, double *y, int n, int degree)
   #ifdef BIGSPLINES
   return
   #else
   #endif

   switch (bcond) {
   case MirrorOnBounds:
   case MirrorOffBounds:
   default:
   myErrMsg("Unsupported boundary conditions.");
   return 1;
   }

   extern int evalbspln(double *x, double *y, int n, int degree)
   #ifdef BIGSPLINES
   return
   #else
   #endif

   // mexPrintf("splninterp called with nxi=%d nyi=%d nzi=%d nxc=%d nyc=%d nzc=%d", nxi, nyi, nzi, nxc, nyc, nzc);
What did I do?

- Programming
- Mathematics

\[ B(f, g) = \int_{\mathbb{R}^2} f^T(x)V(x,y)g(y)\,dx\,dy \]

\[ B(f, g) = \frac{1}{(2\pi)^m} \int_{\mathbb{R}^m} \hat{f}^T(\omega) \hat{U}(\omega) \hat{g}^*(\omega) \,d\omega \]

\[ D^M f = \left[ \frac{\partial^M f}{\partial x_1^M}, \ldots, \frac{\partial^M f}{\partial x_k \ldots \partial x_M} \right] \]

Lemma 0 A function \( f_{\text{out}} \) from \( F \) satisfying \( \langle H, f_{\text{out}} \rangle = s \) solves the variational problem \( P \), if and only if there is a real vector \( \lambda \) such that for all \( g \in F \)

\[ B(f_{\text{out}}, g) = \lambda^T \langle H, g \rangle \]

\[ \frac{\partial E}{\partial c_{j,m}} = \sum_{i \in I} \frac{\partial e_i}{\partial f_{w}(i)} \frac{\partial f_{i,c}(x)}{\partial x_m} \bigg|_{x=g(i)} \beta_{nm}(i/h-j) \]
Unwarping of Unidirectionally Distorted EPI Images

Jan Kybic1, Philippe Thévenaz, Arto Nirkko and Michael Unser

Abstract — Echo-planar imaging (EPI) is a fast magnetic resonance imaging method. Unfortunately, local magnetic field inhomogeneities induced mainly by the subject’s presence cause a non-linear geometric distortion, predominantly along the phase-encoding direction. However, this effect has been too often neglected.

In this paper, we suggest a new approach using an algorithm specifically developed for the automatic registration of distorted EPI images with corresponding anatomically correct MRI images. We model the deformation field with splines, which gives us a lot of flexibility while computing the registration criterion. The criterion is least-squares. Interestingly, the complexity of its evaluation does not depend on the resolution of the central pixel. The spline model gives us good accuracy thanks to its high approximation order. The short support of splines leads to a fast algorithm. A multiresolution approach yields robustness and additional speed-up.

The algorithm was tested on real as well as synthetic data and is compared to other methods. A wavelet-based Sobolev-type random deformation generator was developed for testing purposes. A blind test indicates that the proposed automatic method is in better agreement with the manual warping.

Keywords — image registration, splines, geometrical distortion, unwarping

I. Introduction

A. EPI features

Echo-planar imaging (EPI) [1] is a fast magnetic resonance imaging (MRI) technique permitting an acquisition of a two-dimensional slice using a single excitation, which leads to very short scan times. It is used mainly for functional imaging (fMRI), where one can measure the temporal, spatial and behavioral dependencies of brain activations. The basis of fMRI lies in the fact that deoxygenated blood has a lower magnetic moment than oxygenated blood. The deoxygenated blood decreases the signal intensity as compared to oxygenated blood. A consequent increase in signal intensity is observed in the brain regions where the neuronal activity is high, which results in a measurable alteration of the magnetic field and an accordingly increased signal intensity in the appropriately weighted MRI images (blood oxygen-level dependent, BOLD). It is therefore difficult to compensate for the unwanted magnetic field inhomogeneities induced mainly by the spatially varying magnetic susceptibility of the subject [2]. In contrast to conventional MRI, where the number of excitations per slice is equal to the number of scan lines, in EPI the magnetic field gradients have to encode two co-ordinates simultaneously in one excitation. As one of the gradients (the so-called phase-encoding gradient) is several orders of magnitude weaker than the other, the inhomogeneous magnetic field will manifest itself mainly as a geometrical distortion of the 2D slice image along the direction of this gradient. This effect is clearly visible in Figure 1. Since the stronger gradient is less affected, the distortion is essentially unidirectional. Letting $g$ be the unknown warp function, we have

$$f^o(x,y) = f^o(x,y)$$

(1)

where $f^o$ is the observed EPI image and $f^u$ is the hypothetical ideal undistorted EPI image. We can consider each slice separately as the shift in the z-axis due to patient’s movement is insignificant because the head is attached. Should there be such a displacement, it can be readily corrected by existing algorithms [3].

B. The reason to study

The amplitudes of the deformation $g$ can be as large as 3–5 mm [4] (confirmed by our own observations), which typically amounts to several pixels. In some cases, as in Figure 1, specifically intended to illustrate EPI distortion, the deformation can be even more pronounced. Moreover, $g$ can vary significantly from slice to slice and from acquisition to acquisition. For localization applications like stereotactic surgery, this inaccuracy is much larger than the required limit of 1 mm and therefore EPI cannot currently be used to this end. It also severely hinders the performance of the statistical processing of sets of fMRI images used to obtain activation information. Since the task-induced signal changes represent typically only 5–10% of the mean signal intensity in fMRI [1,5], they will not stand out clearly unless the perturbations caused by the deformation $g$ are undone.

C. Existing distortion correction techniques

One approach consists in changing the acquisition procedure [2,4]. However, this is often not practicable due to technical or organizational limitations, for example lack of support or approval. Furthermore, while the alternative acquisition sequences reduce the distortion, the distortion is now present on the entire volume, and the methods usually sacrifice either sensitivity or acquisition speed.

The second group of methods uses a two-step procedure [4,7]. First, a field map or a deformation map is
What did I do?

- Programming
- Mathematics
- Papers
- Conferences
Elastic Image Registration using Parametric Deformation Models

Jan Kybic
Overview

- Registration and its applications
  - Manual registration
    - Interpolation
    - Variational reconstruction
  - Splines
- Automatic registration
  - Algorithm
  - Semi-automatic registration
  - Applications
- Conclusions
- Party
Overview

- Registration and its applications
  - Manual registration
    - Interpolation
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What did we do?
Overview

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What did we do?
What is image registration?
Find corresponding points

American Tux

Tux bordelais
Find corresponding points

American Tux

Tux bordelais
Find corresponding points

American Tux

Tux bordelais
Correspondence function

Reference image

Test image

Correspondence function

source

destination

\[ \begin{bmatrix} x' \\ y' \end{bmatrix}^T = \begin{bmatrix} x \\ y \end{bmatrix}^T \]
Correspondence function

\( \mathbf{g} \left( \begin{bmatrix} x & y \end{bmatrix}^T \right) = \begin{bmatrix} x' & y' \end{bmatrix}^T \)
Deformation field

0 % deformation
Deformation field

25 % deformation
Deformation field

50 % deformation
Deformation field

75 % deformation
Deformation field

100 % deformation
Image warping

0 % deformation
Image warping

25 % deformation
Image warping

50 % deformation
Image warping

75 % deformation
Image warping

100 % deformation
Image registration

- Reference image
- Test image
- Registration
- Deformation
Image registration

Reference image → registration → deformation → warped image

Test image → registration → warping → warped image
Is registration useful?
Is registration useful?

Yes!
(Biomedical) applications

... of image registration

- Comparing images
  - Different times
  - Different methods
  - Different subjects

- Analyzing sequences
  - Motion estimation
  - Segmentation

Qualitative and quantitative information.
Image alignment

reference \hspace{1cm} test

before
Image alignment

reference -> test

before -> warped
Registration types

- Manual
- Automatic
- Semi-automatic
Manual registration

- Landmark identification
Manual registration

- Landmark identification
- Landmark interpolation
What is interpolation?
Find a function
Find a function
Find a function
Rank functions

ugliest  . . .  prettiest

Variational criterion $J : (\mathbb{R}^m \to \mathbb{R}^n) \to \mathbb{R}_+ \geq 0$
Rank functions

ugliest \ldots prettiest

Variational criterion

\[ J : \left( \mathbb{R}^m \rightarrow \mathbb{R}^n \right) \rightarrow \mathbb{R}^+ \]

\[ J(f) \geq 0 \]
Variational reconstruction

Find the best function satisfying the constraints.
Tunable 1D interpolation

\[ J(f) = \left\| \frac{\partial^M f}{\partial x^M} \right\|_2^2 \]
Tunable 1D interpolation

\[ J(f) = \left\| \frac{\partial^M f}{\partial x^M} \right\|^2 \]
Tunable 2D interpolation
Tunable 2D interpolation
Tunable 2D interpolation

\[
\int \| \nabla^{0.5} g(x) \|^2 dx
\]
Tunable 2D interpolation

\[ \int \left\| \nabla^{0.9} g(x) \right\|^2 dx \]

alpha=0.9
Tunable 2D interpolation

\[ \int \| \nabla^{1.3} g(x) \|^2 \, dx \]
Tunable 2D interpolation

\[
\int \| \nabla^{2.5} g(x) \|^2 \, dx
\]
Tomographic reconstruction

The problem
Tomographic reconstruction

Solution
Tomographic reconstruction

Another solution
Radon transform

\[ (Rf)(\theta_i, u_j) = \int f \left( t \cos \theta_i - u_j \sin \theta_i, t \sin \theta_i + u_j \cos \theta_i \right) dt \]
Tomographic experiments

Phantom
Tomographic experiments

Backprojection

Variational reconstruction

8 angles (each $22^\circ$), 32 samples per angle
Overview (2)

- Registration and its applications
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  - Conclusions
  - Party
The splines

What are splines, anyway?
The splines

What are splines, anyway?

The best functions in the world!
(Uniform) splines

Piecewise polynomial of degree \( n \) Continuous \((n-1)\)th derivative

Uniform knots
(Uniform) splines

- Piecewise polynomial of degree $n$
- Continuous $(n - 1)^{th}$ derivative
- (Uniform) knots
Uniform B-splines

Haar \( \beta_0 \)
linear \( \beta_1 \)
quadratic \( \beta_2 \)
cubic \( \beta_3 \)
Uniform B-splines

Haar $\beta_0$
linear $\beta_1$
quadratic $\beta_2$
cubic $\beta_3$

- Generation: $\beta_{n+1} = \beta_n \ast \beta_0$
- Basis for splines: $s(x) = \sum_i c_i \beta(x - i)$
Automatic registration

How does it work?
Spline based warping

\[ g(x) = x + \sum_{i \in \mathbb{Z}^2} c(i) \beta(x/h + d - i) \]
Registration as minimization

- reference image
- test image
- deformed image
- deformation
- criterion E
- deformation function $g(x)$
Registration as minimization

Reference image \rightarrow \text{difference} \rightarrow \text{optimization} \rightarrow \text{deformation function } g(x) \rightarrow \text{deformation} \rightarrow \text{deformed image}

Test image \rightarrow \text{difference} \rightarrow \text{optimization} \rightarrow \text{deformation function } g(x) \rightarrow \text{deformation} \rightarrow \text{deformed image}
Image warping (2)

0 % deformation
Image warping (2)

100% deformation
Registration as minimization (2)

reference image \rightarrow \text{difference} \rightarrow \text{optimization} \rightarrow \text{criterion } E \rightarrow \text{deformation function } g(x) \rightarrow \text{deformation} \rightarrow \text{test image} \rightarrow \text{deformed image}
Evaluating the difference
Evaluating the difference
Evaluating the difference
Evaluating the difference

\[ E = 435.7 \]
Evaluating the difference

\[ E = \frac{1}{N} \sum_i \left( f_t^c(g(i)) - f_r(i) \right)^2 \]
Registration as minimization (3)

Reference image \rightarrow \text{difference} \rightarrow \text{criterion } E \rightarrow \text{optimization} \rightarrow \text{deformation function } g(x) \rightarrow \text{deformation} \rightarrow \text{test image} \rightarrow \text{deformed image}

Elastic Image Registration using Parametric Deformation Models – p.35/45
Optimization

Minimize criterion $E$ with respect to parameters $c$. 
Optimization

Find the lowest point
Optimization

Frog search in the mist
Optimization

Next move
Acceleration

It works. Now let’s make it run fast.
Multiresolution

$32 \times 32$
Multiresolution

$64 \times 64$
Multiresolution

$128 \times 128$
Multiresolution

$256 \times 256$
Overview (3)

- Registration and its applications
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    - Algorithm
    - Semi-automatic registration
  - Applications
  - Conclusions
  - Party
Movie

click here
Applications

- EPI distortion

Before (with Arto Nirkko)
Applications

- EPI distortion

After

Elastic Image Registration using Parametric Deformation Models – p.41/45
Applications

- EPI distortion
- MRI atlas
Applications

- EPI distortion
- MRI atlas

Aligned
Applications

- EPI distortion
- MRI atlas
- CT alignment

Before
Applications

- EPI distortion
- MRI atlas
- CT alignment

After
Applications

- EPI distortion
- MRI atlas
- CT alignment
- SPECT atlas

(with University Hospital in Geneva)
Applications

- EPI distortion
- MRI atlas
- CT alignment
- SPECT atlas
- Ultrasound

velocity (with María J. Ledesma-Carbayo)
Conclusions

- Image registration is useful
- Our registration algorithm works
- Interpolation is interesting
- Variational is elegant
- Splines are great
The End
Díky!
Merci!
Thank you!
¡Gracias!
Grazie!
Danke!
Bâtiment de microtechnique, 4th floor