Improved sampling method for RANSAC and RHT

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The Randomised Hough Transform (RHT), RANSAC and MDL methods use randomised sampling as a first step towards detection of parametric curves. However, it can be shown that their performance degrades fast if either the noise level increases or the number of features in the image grows. We suggest a simple method designed to maintain a high likelihood of selecting a valid n-tuple of points in the randomised sampling of complex images. The method is based on the analysis of the number of points in windows whose size is controlled by local density of edge points.

1 Introduction

Most commonly used approaches for extraction of parametric curves - the Hough transform (HT), Randomised HT (RHT), RANSAC and methods based on the Minimum description length (MDL) principle can all be formulated in an optimisation framework [6, 10, 7]. It is therefore natural to analyse and compare their definitions of objective (cost) functions, of the search space (eg. the space of all partitions of the input data) and the search strategy. We observe that only the standard HT defines both the cost function and the search method. In particular (by definition [5]) the HT evaluates a cost function in the following form:

$$H(\Omega) = \sum_{i=1}^{n} p(X_i, \Omega), \tag{1}$$

where $X_i = [x_{1i}, \dots, x_{Ni}]$ are feature points defined in an N-dimensional feature space and $\Omega = [\omega_1, \dots, \omega_M]$ defines a point in the M-dimensional parameter space. The function p is the kernel of the HT.

Unlike HT, RANSAC [2] does not impose any constraints on the form of the cost (objective) function associated with a given parametric curve. The search for geometric primitives proceeds by evaluating the cost function at a point in the search space corresponding to a parametric curve passing through a randomly chosen n-tuple of feature points. This process is repeated for other n-tuples. The MDL principle provides a framework for selecting the cost function [7, 9], but is not concerned with the problem of minimising (maximising) it. Consequently, MDL-based algorithms may adopt an HT-like [7] or a RANSAC-like search technique. RHT evaluates the additive cost function of the form shown in eq. 1 at points selected as in RANSAC switching to a general, RANSAC-type cost function only if a threshold is reached[4].

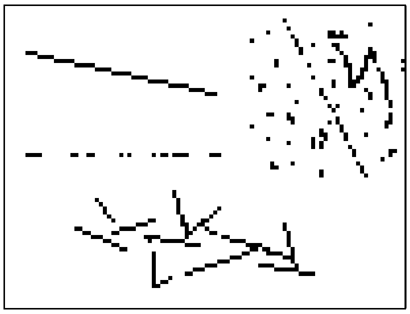

Figure 1: An example of a line detection problem.

Taking into account the similarities of structure of the above mentioned methods, it seems reasonable to identify, analyse and attempt to improve the *common components*. For instance, the RANSAC-like selection of n points for hypothesis generation has been used in RHT and MDL methods. If the selection scheme is improved than a number of algorithms will benefit. Similarly the issue of the cost function may be studied independently of the search method, eg. there is no reason why a cost function designed according to the MDL principle should not be invoked in the verification step of RHT.

In this short contribution we do not attempt to present a new method for the detection of parametric curves, neither do we discuss the other important issues related to the choice of the cost function. We focus on the RANSAC selection procedure, analyse its properties and propose an adaptive proximity based strategy that increases its robustness to noise and high density of features. The method attempts to maximise the distance among the selected n-tuples of points, thus making estimates of curve parameters precise while keeping the probability of selection of a 'valid' n-tuple high.

2 Random Sampling: the n-tuple selection problem

Both RANSAC and RHT start the process of extraction of parametric curves by selecting a random set of n points; n depends on the number of free parameters of the detected curves. It is assumed that if all selected n points lie on the same curve, then the estimated parameter point $p = f(n)$ is close to parameters of that curve and the precise value of p can be found by some form of local search.

A well-recognised limitation of the RANSAC (and RHT) selection scheme [3] in its pure form is linked to the probability of selecting n points that all belong to the same feature. If only a small fraction f of points belong to a feature, either because of high level of noise or because of presence of numerous features, the likelihood of selecting a 'valid' n-tuple is approximately f^N. For example, in an ellipse detection problem $N = 5$; for a moderate $f = 0.1$, ie. when 10% of all points in the image belong to the ellipse, on average 50000 ($f^N/2$) five point selections (and verifications in RANSAC, accumulations in RHT) are made before a first valid n-tuple is considered.

Standard solutions of this problem either use windowing techniques or connectivity (eg. [3]). Figure 1 shows a simple example that highlights problems with fixed sized windowing and connectivity. An algorithm that would select only a pair of connected pixels will work for only for the line in the top left corner if 8-connectivity is used. If a larger neighbourhood defined the adjacency relationship for the connected component analysis, than the points of the horizontal line will all be in the same component. At the same time, however, the points belonging to the line in the top left corner will be assigned to the same component as the many noise points in its vicinity. If windowing is used to control the selection process, we observe that the two long isolated lines would benefit from a large window size, allowing distant points on the same line to be selected (and hence good estimates of line parameters obtained). For the noisy part of the synthetic image and for the clustered small lines a smaller window size is needed to keep the fraction of points belonging to a single line high, ideally close to 1.

To overcome these problem, we propose an adaptive window based selection scheme based on the following observations: inside a window of width w centred at an edgel belonging to a curve

1. at most w edgels lie on the curve,
$$n_c(w) = k * w, \qquad 0 < k < 1 \tag{2}$$
2. the number of edgels belonging to other structures and noise, is proportional to the area of the window
$$n_o(w) = d * w^2 \tag{3}$$

From the two observations, we can see that the ratio of non-curve edgels and curve edgels $n_o(w)/n_c(w) = wd/k$ tends to zero if the window size w goes to zero. In other words, we obtain the intuitive result that the nearer the selected points to each other, the more likely will they belong to the same curve. Secondly, we see that the total number of edgels in a window of width w

$$n(w) = n_c(w) + n_c(w) = k * w + d * w^2 \tag{4}$$

will be for small w dominated by points on the curve and therefore linear; for large values of w the non-curve points dominate and the dependency becomes quadratic.

Since computation of curve parameters from nearby points is unstable, we attempt to find, for every edgel the largest w for which $n(w)$ is approximately linear. In practise it is sufficient to compute $n(w)$ for a relatively small number of window size, eg. selected on a logarithmic scale. Since it is known [8] that $n(w)$ can be computed in constant time, the computational cost of obtaining the 'ideal' window size is negligible compared with the cost of the rest of the curve detection process.

Having computed w_L, the window size at which the linear relationship breaks, at every edge point in a pre-processing step, we may modify the random selection in a number of ways:

1. Select the first point randomly. Select the subsequent point(s) randomly from within the w_L-sized window.
2. As before, but select the first point with likelihood which is a function of w_L.
3. As before, but include k of eq. 2 in the calculation of selection probability.

Efficient implementation of any of the strategies requires a fast method for selecting a point in a rectangular window. This can be accomplished in $2 * log(w_L)$ steps using the data structures necessary for evaluation of $n(w)$.

3 Conclusions

We have suggested a simple method designed to maintain a high likelihood of selection of a valid n-tuple of points in RANSAC and RHT in images with significant number of curves and/or amount of noise. This is a necessary prerequisite for achieving reasonable efficiency of the curve detection process. Under this constraint, the method attempts to select points which are as far apart as possible which increases the stability of the computation of curve (line) parameters. At present we are embedding the n-tuple selection scheme into a RHT, RANSAC and MDL algorithms to test its usefulness for realistic images.

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