Exercises from fuzzy propositional and set operations.

**Exercise 3.1**

1. *Verify that operation $\dot{\vee}$ defined by*

$$\alpha \mathbin{\dot{\vee}} \beta = \sqrt{\alpha^2 + \beta^2 - \alpha^2\,\beta^2}$$

   *is a fuzzy disjunction.*

2. *Find the fuzzy conjunction dual to $\dot{\vee}$ with respect to the fuzzy negation*

$$\mathbin{\dot{\neg}} \alpha = \sqrt{1 - \alpha^2}\,.\tag{1}$$

**Exercise 3.2** *Fuzzy subsets $A, B$ of $\mathbb{R}$ have membership functions*

$$\mu_A(x) = \begin{cases} x - 1\,, & 1 < x < 2\,, \\ 1\,, & 2 \le x \le 3\,, \\ \frac{5-x}{2}\,, & 3 < x < 5\,, \\ 0\,, & otherwise. \end{cases}$$

$$\mu_B(x) = \begin{cases} \frac{x}{3}\,, & 0 < x < 3\,, \\ 1\,, & 3 \le x \le 4\,, \\ \frac{6-x}{2}\,, & 4 < x < 6\,, \\ 0\,, & otherwise. \end{cases}$$

*Find*

1. $\overline{A}^{\,\dot{\neg}}$, *where $\dot{\neg}$ is the fuzzy negation from (1),*

2. $A \mathbin{\underset{\mathrm{P}}{\cap}} B$,

3. $A \mathbin{\overset{\mathrm{L}}{\cup}} B$.

*Describe the results by formulas and draw their graphs.*