Homework from Fuzzy Logic. Choose the basic or advanced level.

## **Basic** level

Exercises from conversions of representations of fuzzy sets and generators of fuzzy negations.

**Exercise 2.1** Fuzzy set B is given by its collection of cuts:

$$\mathcal{R}_B(\alpha) = \begin{cases} \{A, B, C, D, E, F\}, & \alpha = 0, \\ \{A, B, C, D, E\}, & \alpha \in (0, 0.5), \\ \{A, B, C\}, & \alpha \in [0.5, 0.6), \\ \{A, B, C, D\}, & \alpha \in [0.6, 0.7), \\ \{A, B\}, & \alpha \in [0.7, 0.9), \\ \{A\}, & \alpha \in [0.9, 1]. \end{cases}$$

Find its vertical representation.

**Exercise 2.2** Fuzzy set C is given by its collection of cuts:

$$\mathcal{R}_C(\alpha) = \begin{cases} \mathbb{R}, & \alpha = 0, \\ (\alpha, 6 - 2 \alpha^2), & \alpha \in (0, 1]. \end{cases}$$

Find its vertical representation.

**Exercise 2.3** Fuzzy set D is given by its collection of cuts:

$$\mathcal{R}_D(\alpha) = \begin{cases} \mathbb{R}, & \alpha = 0, \\ [-2 + \alpha, 2 - \alpha], & \alpha \in (0, 1), \\ \{0\}, & \alpha = 1. \end{cases}$$

Find its vertical representation.

**Exercise 2.4** Find for which  $\lambda$  the formula

$$\sum_{\lambda} \alpha = \frac{1 - \alpha}{1 + \lambda \, \alpha}$$

defines a fuzzy negation  $\neg$ . Draw a typical graph.

**Exercise 2.5** Find the fuzzy negation with generator  $i(\alpha) = \alpha^2$ . Draw its graph.

Advanced level

Exercises from generators of negations.

**Exercise 2.6** Decide whether the following statement is true: For every two fuzzy negations  $\overline{1}, \overline{2}$ , there is an increasing bijection  $i: [0,1] \rightarrow [0,1]$  such that  $\overline{2} \alpha = i^{-1}(\overline{1}i(\alpha))$  for all  $\alpha \in [0,1]$ .

Exercise 2.7 Try to find a generator of the standard fuzzy negation, different from the identity.