On the forcing semantics for MTL logic

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Monoidal t-norm based logic (MTL logic) is the logic of left-continuous t-norms and was introduced by Esteva and Godo [1]. We define the weak forcing value $|\varphi|_{\mathcal{X}}$ of an MTL formula φ in a complete MTL-algebra $\mathcal{X}$. Besides the truth value $\|\varphi\|_{\mathcal{X}}$, $|\varphi|_{\mathcal{X}}$ constitutes an alternative way to evaluate the formula φ in $\mathcal{X}$. The weak forcing value is a refinement of the notion of validity in a Kripke model (in the sense of [2] and [3]).

It is of great interest to compare the two kinds of semantics: the truth value and the weak forcing. We conclude the the axioms (A2), (A6), (A7), (A9) and (A10) are not valid in the new semantics.

In [2] and [3] was proved that the r-forcing is a more adequate notion for reflecting the logical structure of MTL. Arising from r-forcing, we shall define the $\mathcal{X}$-valued forcing property and the forcing value $[\varphi]_{\mathcal{X}}$ of a formula of MTL in a complete MTL-algebra $\mathcal{X}$. The first notion is obtained from an $\mathcal{X}$-valued weak forcing property f by adding a condition that homogenizes the action of f w.r.t. the elements of X. Then one can define in a natural way the forcing value $[\varphi]_{\mathcal{X}}$, resulting a semantic $[\cdot]_{\mathcal{X}}$ distinct from $|\cdot|_{\mathcal{X}}$.

One of the main results of the above quoted papers asserts that the Kripke completeness (defined by means of r-forcing) coincides with the usual algebraic completeness of MTL. We shall extend this result by proving that $[\cdot]_{\mathcal{X}}$ and $\|\cdot\|_{\mathcal{X}}$ coincides.

We also give an algebraic proof for the fact that the forcing value coincides with the truth value by using the notion of forcing operator.

References

[1] Esteva, F., Godo, L.: Monoidal *t*-norm based logic: towards a logic for left-continuous *t*-norms, *Fuzzy Sets and Systems*, (123(3)), 2001, 271–288.

[2] Montagna, F., Ono, H.: Kripke semantics, undecidability and standard completeness for Esteva and Godo's logic $MTL\forall$, *Studia Logica*, (71(2)), 2002, 227–245.

[3] Montagna, F., Sacchetti, L.: Kripke-style semantics for many-valued logics, *Math. Logic Quart.*, 2003, 629–641.