Web-geometric approach to models of fuzzy logic

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Web geometry [1], which is usually understood as a branch of differential geometry, offers several concepts and tools which are known to characterize algebraic properties of *loops* in a surprisingly transparent geometric way. These are known as *closure conditions*; in particular, the associativity of loops is characterized by the *Reidemeister closure condition* [3]. In order to apply the web-geometric concepts in the framework of non-standard logics, we have to adapt the theory in a way which allows for dealing with mathematical structures more general than loops. Trying to keep the setup as general as possible, we mimic the standard machinery of web geometry in the case of totally ordered magmas.

Recall that a *totally ordered magma*, or a *togma* for short, is a structure $(M, \odot, \leq)$ where $(M, \leq)$ is a chain and $(M, \odot)$ is a magma [2] with operation isotone with respect to $\leq$. We introduce the notion *local togma* at the point $(u, v) \in M \times M$ and we explain how its associativity is characterized by an adopted version of the Reidemeister closure condition. As a result, we

give a visual characterization of associative togmas with a neutral element which are, actually, tomonoids. Using this tool we are able to discuss certain properties of MTL-chains geometrically.

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References

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