Three-dimensional distributions on Quantum logics and MV algebras

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It is well known that in the Kolmogorovian probability space

$$P(A|B,C) = P(A|C,B) = P(A|B \cap C).$$

It means that if we put two conditions on the event A, the resulting conditional probability is independent of the order of conditions. We show that this is not true in quantum logics (OML) and in MV-algebras. The conditional measure (probability) in OML can be defined by using the so-called *s-map*. Let L be an OML. The map $p_n : L^n \to [0,1]$ will be called an s_n*-map* if the following conditions hold:

(s1) $p_n(1, ..., 1) = 1$;

(s2) if there exist i, j, such that $a_i \perp a_j$, then $p_n(a_1, ..., a_n) = 0$;

(s3) if $a_i \perp b_i$ for some $i = 1, ..., n$, then

$$p_n(a_1, ..., a_i \vee b_i, ..., a_n) = p_n(a_1, ..., a_i, ..., a_n) + p_n(a_1, ..., b_i, ..., a_n).$$

We will discuss the properties of the s-map and of the corresponding conditional distributions. A different situation is in an MV-algebra $\mathcal{M}$. First we define the three-dimensional conditional distribution $\gamma(\cdot|\cdot,\cdot) : \mathcal{M}^3 \to [0,1]$ via the full probability theorem and then we define the joint distribution p by

$$p(f, g, h) = \gamma(f|g, h) \cdot \gamma(1, g|h) \cdot \gamma(h|1, 1).$$

The joint probability distribution is additive just in the first variable. We will show that the conditional distribution depends on the order of conditions.

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