Three-dimensional distributions on Quantum logics and MV algebras

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It is well known that in the Kolmogorovian probability space

\[ P(A|B, C) = P(A|C, B) = P(A|B \cap C). \]

It means that if we put two conditions on the event \( A \), the resulting conditional probability is independent of the order of conditions. We show that this is not true in quantum logics (OML) and in MV-algebras. The conditional measure (probability) in OML can be defined by using the so-called \( s \)-map. Let \( L \) be an OML. The map \( p_n : L^n \to [0, 1] \) will be called an \( s_n \)-map if the following conditions hold:

(s1) \( p_n(1, ..., 1) = 1; \)
(s2) if there exist \( i, j \), such that \( a_i \perp a_j \), then \( p_n(a_1, ..., a_n) = 0; \)
(s3) if \( a_i \perp b_i \) for some \( i = 1, ..., n \), then

\[ p_n(a_1, ..., a_i \lor b_i, ..., a_n) = p_n(a_1, ..., a_i, ..., a_n) + p_n(a_1, ..., b_i, ..., a_n). \]

We will discuss the properties of the \( s \)-map and of the corresponding conditional distributions. A different situation is in an MV-algebra \( \mathcal{M} \). First we define the three-dimensional conditional distribution \( \gamma(\cdot|\cdot, \cdot) : \mathcal{M}^3 \to [0, 1] \) via the full probability theorem and then we define the joint distribution \( p \) by

\[ p(f, g, h) = \gamma(f|g, h) \cdot \gamma(1, g|h) \cdot \gamma(h|1, 1). \]

The joint probability distribution is additive just in the first variable. We will show that the conditional distribution depends on the order of conditions.

References


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