2D Discrete Fourier Transform

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Abstract

This is assistant text for Signal and Image Processing subject. It reminds some properties of 2-D Discrete Fourier Transform and discrete convolution. It probably helps to solve problem of motion blur removing.

1 2D DFT

The discrete Fourier transform pair is

$$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \exp[-j2\pi(ux/M + vy/N)]. \tag{1}$$

remind that

$$\exp[-j2\pi(ux/M+vy/N)] = \cos(2\pi(ux/M+vy/N)) - j\sin(2\pi(ux/M+vy/N)). \tag{2}$$

2 Discrete Fourier transformation of discrete step function

Suppose discrete function f defined as

$$f[0,1,2,\ldots,A] = 1, \text{ and } f[A+1,\ldots,N-1] = 0. \tag{3}$$

DFT is computed using

$$F(u) = \frac{1}{N} \sum_{x=0}^{N-1} f(x) \exp(-j2\pi ux/N). \tag{4}$$

Note discrete nature of $u = 0 \ldots N-1$ and $x = 0 \ldots N-1$. Using (3) the equation (4) can be simplified to

$$F(u) = \frac{1}{N} \sum_{x=0}^{A} \exp(-j2\pi ux/N). \tag{5}$$

This summation gives the same value for each $u = kN/A$. The Fourier transform of the step function (3) is a periodic function with the period

$$T = \frac{N}{A} \tag{6}$$

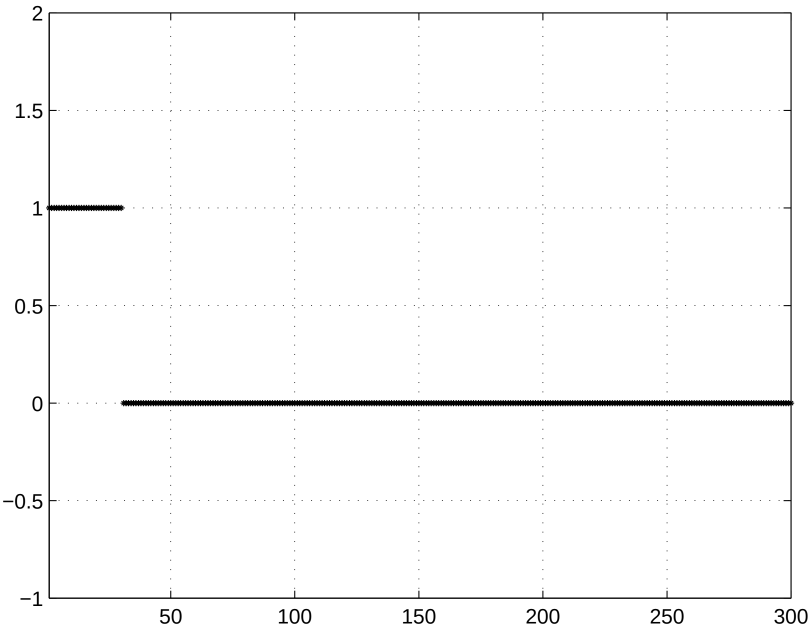

Figure 1: 1D discrete function.

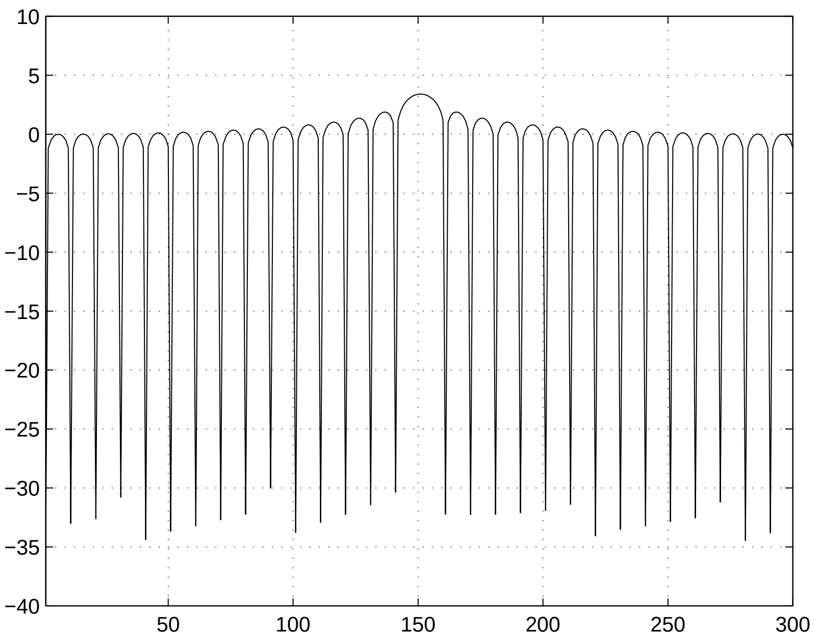

Figure 2: Shifted amplitude of DFT of the function from the figure above.