

ADVANCED ROBOTICS



TOMÁŠ PAJDLA
2012



TOMÁŠ PAJDLA

Assistant Professor

Center for Machine Perception

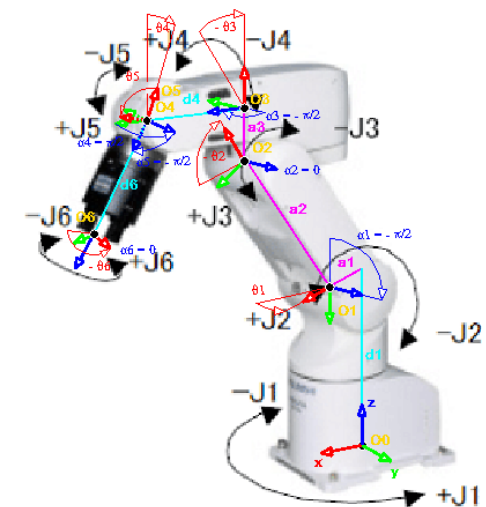
Department of Cybernetics

FEE CTU Prague, Karlovo náměstí 13



Research

Computer Vision & Robotics





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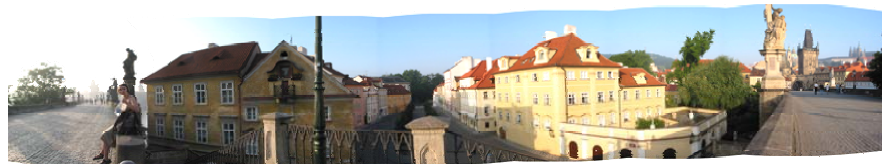
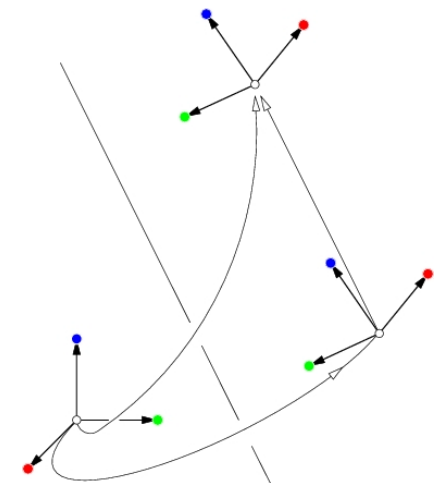
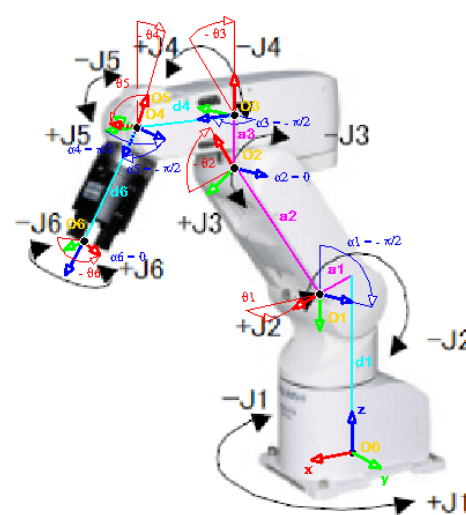
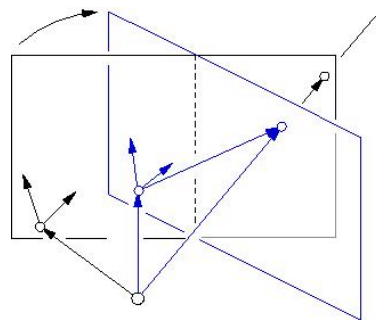
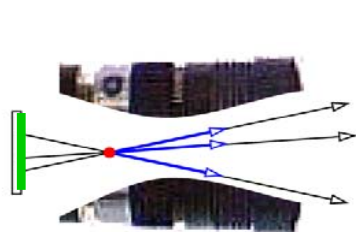
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Lecturer in

Geometry of Computer Vision & Robotics





T O M Á Š P A J D L A

Assistant Professor

Center for Machine Perception

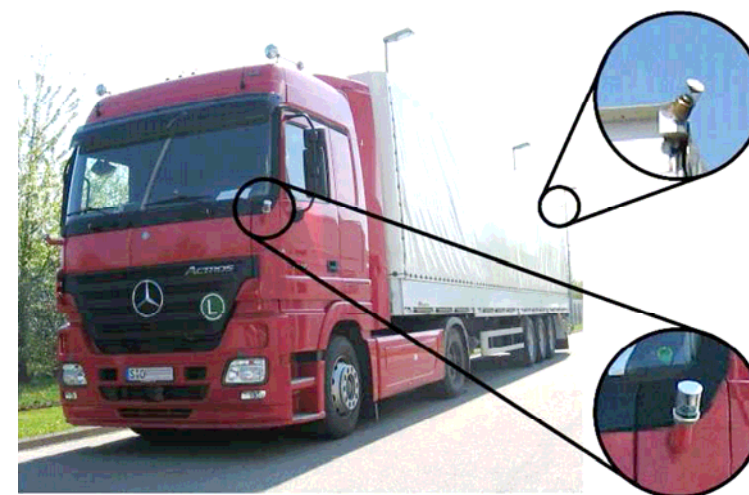
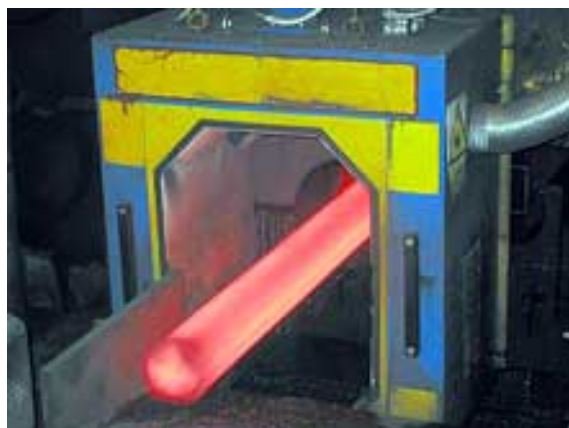
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Collaboration with industry

Neovision s.r.o., Olympus, Daimler



Advanced Robotics

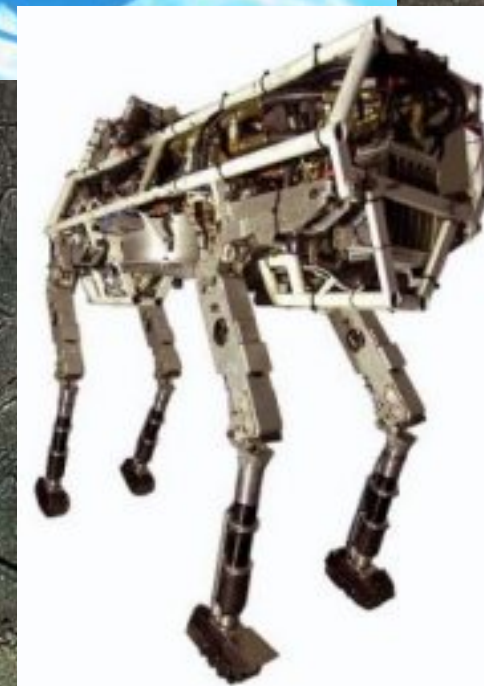
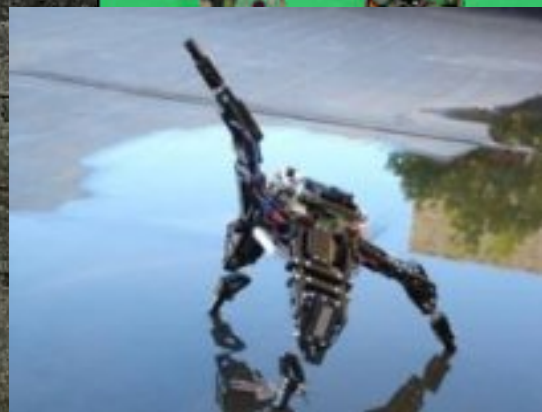
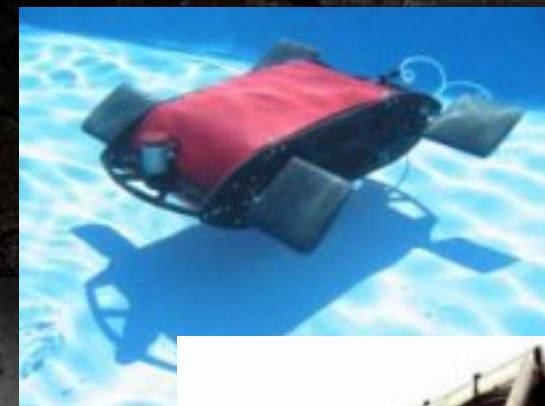
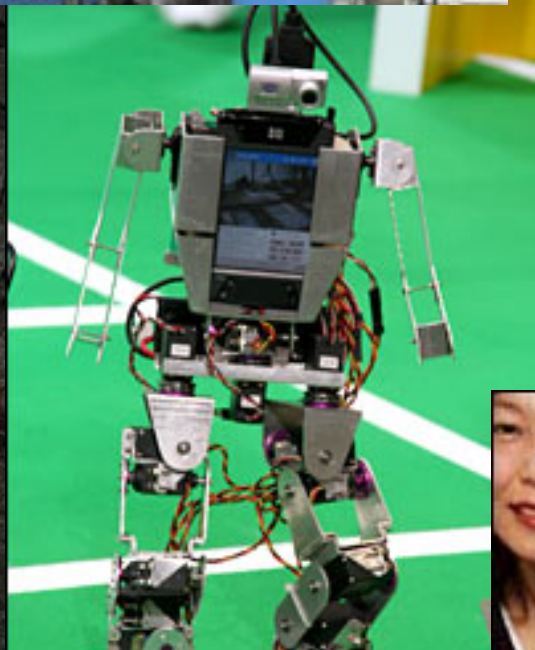
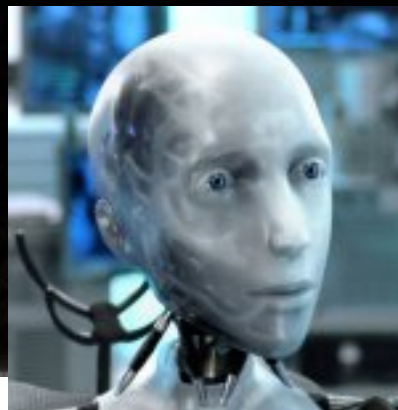
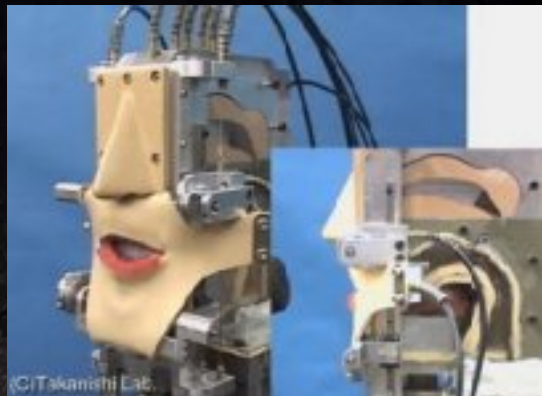
Lecture 1

We will build on Robotics by V. Smutny and study more advanced robot kinematics problems, e.g.,

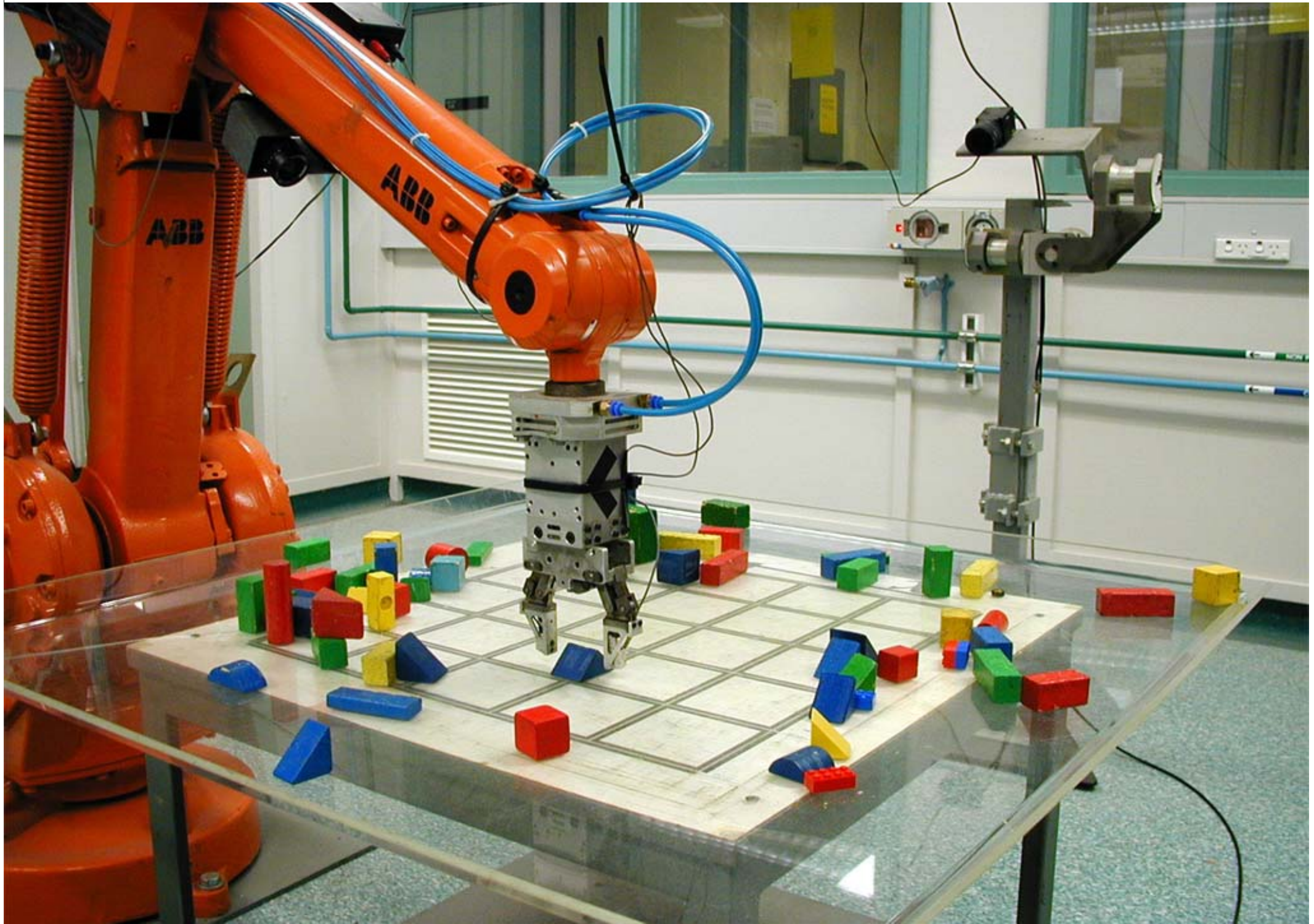
1. solving inverse kinematics of a general 6 DOF manipulator
2. identifying kinematic parameters of a manipulator
3. finding singular poses of a manipulator

with more advanced mathematical tools, such as

1. space rotation and motion and
2. solving algebraic equations



ROBOT = A GENERAL MANIPULATOR



Robotics

[Go to The ABB
Product Guide](#)

[Robotics startpage](#)

[+ Product range](#)

[- Application areas](#)

[Arc welding](#)

[Assembly](#)

[Foundry
applications](#)

[Gluing and
Sealing](#)

[+ Material
handling and
Machine
Tending](#)

[Packing](#)

[Palletizing](#)

[Picking](#)

[Painting and
coating](#)

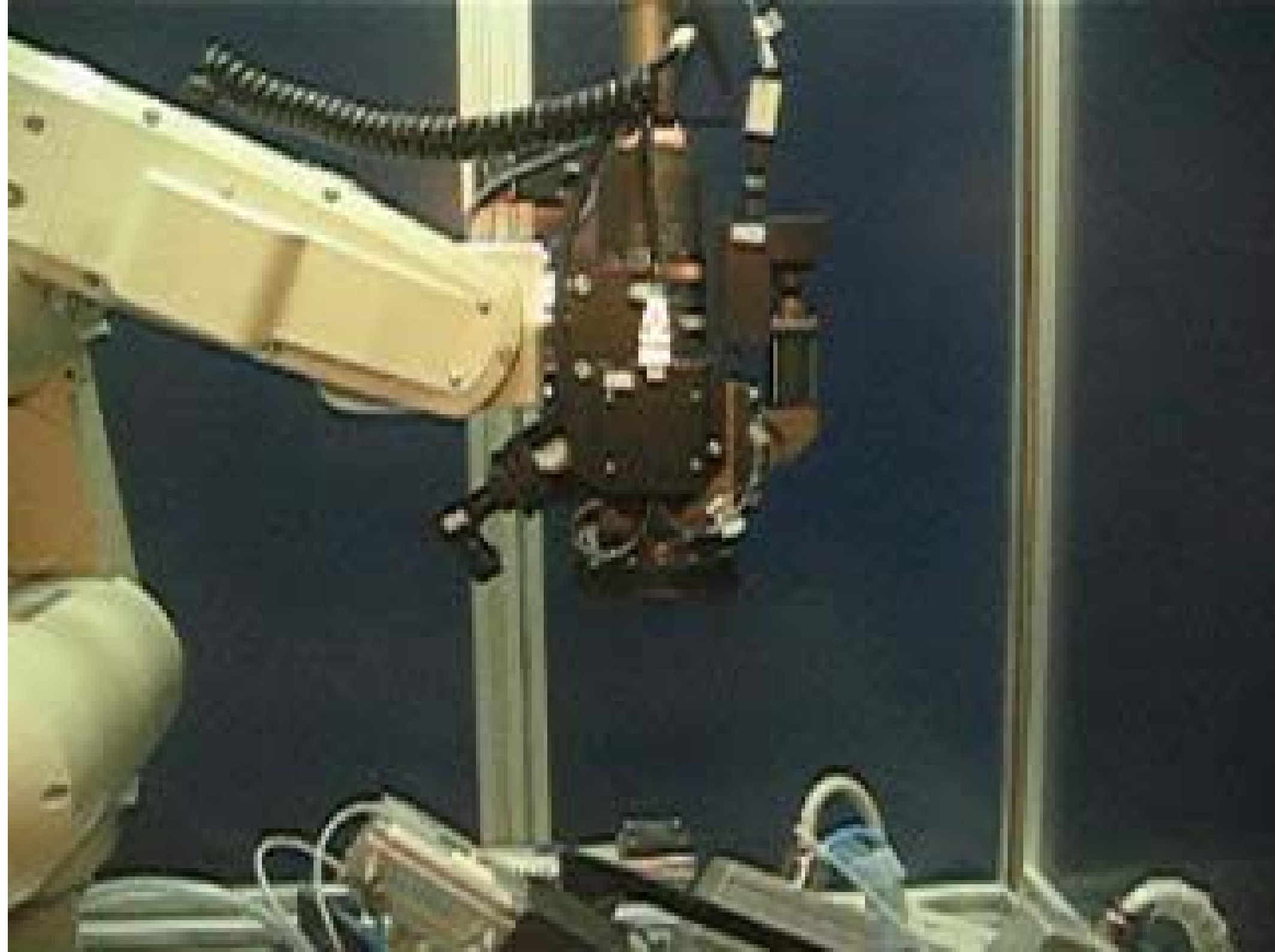
[Spot welding](#)

[Waterjet cutting](#)

Application areas

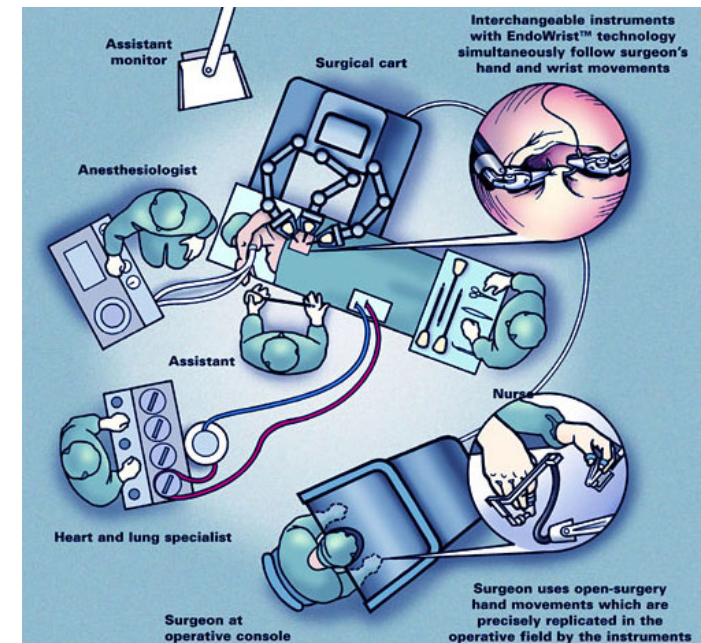
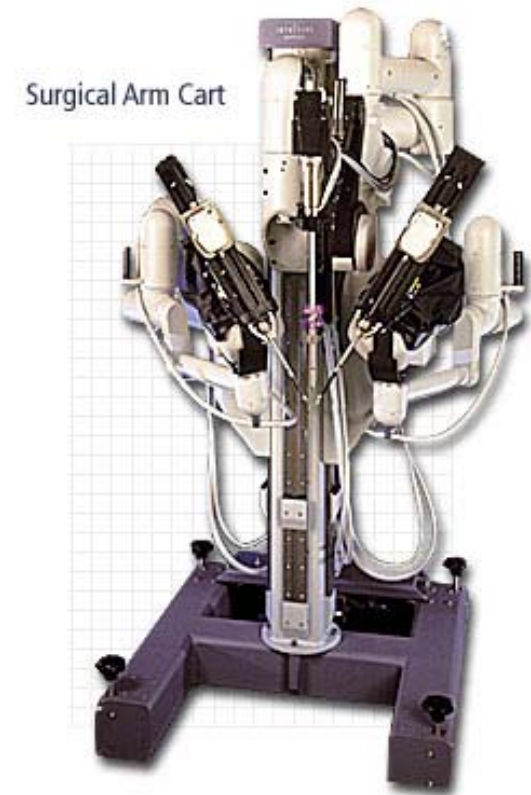
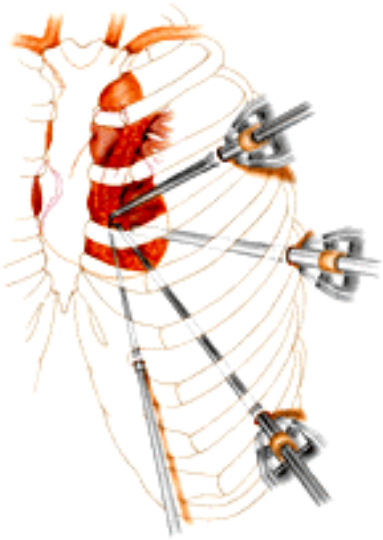




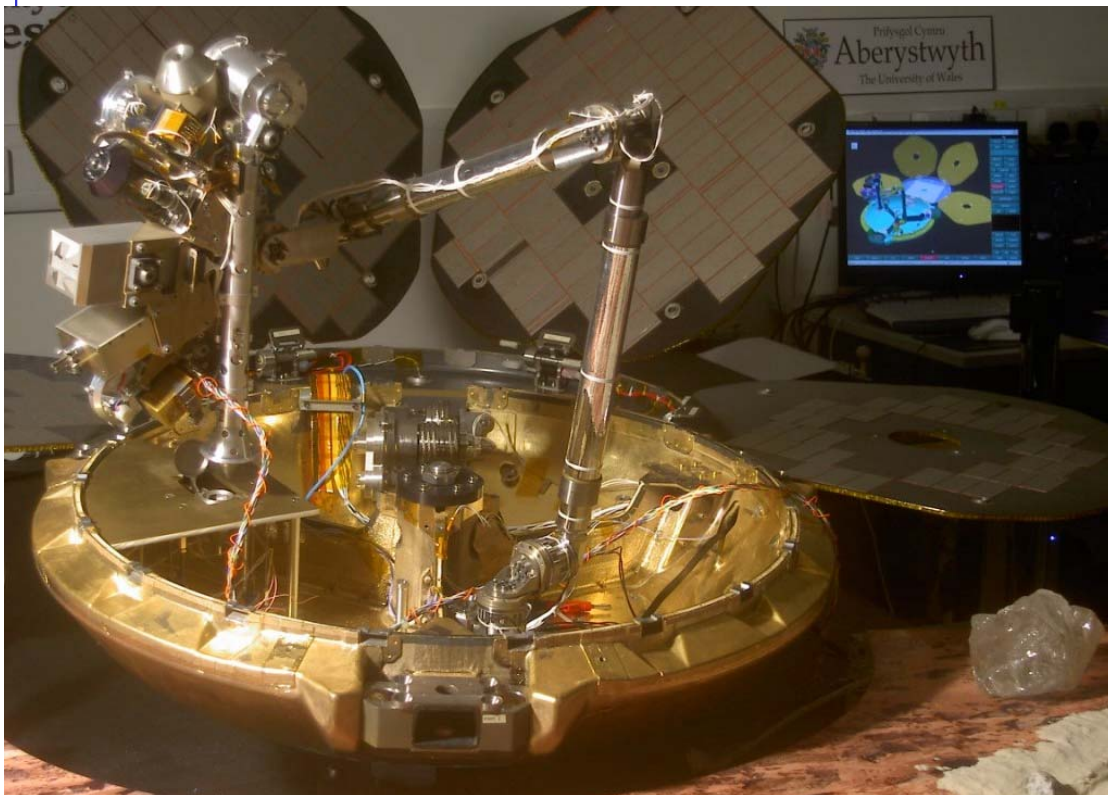




Precision for robotic surgery



<http://www.cts.usc.edu/rsi-article-robotputsuscatforefront.html>



Precision for industry

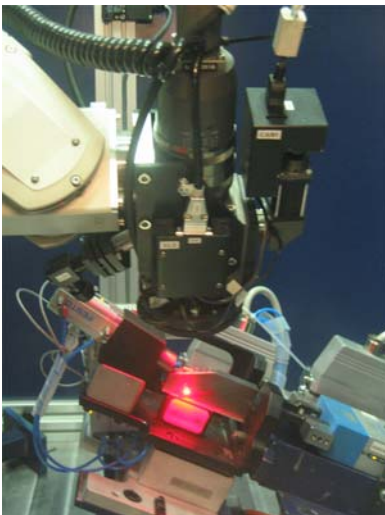


Low (e.g. manipulation)

± 5 mm in the whole working space

± 0.5 mm locally

... often available



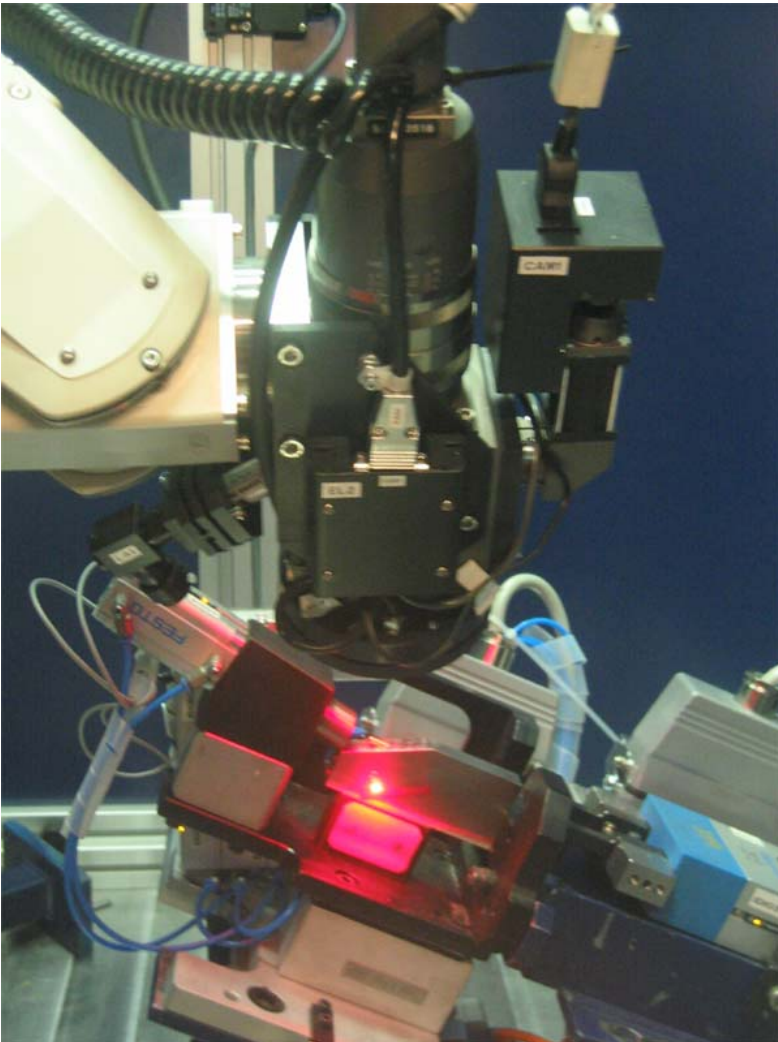
High (e.g. laser welding)

± 0.5 mm in the whole working space

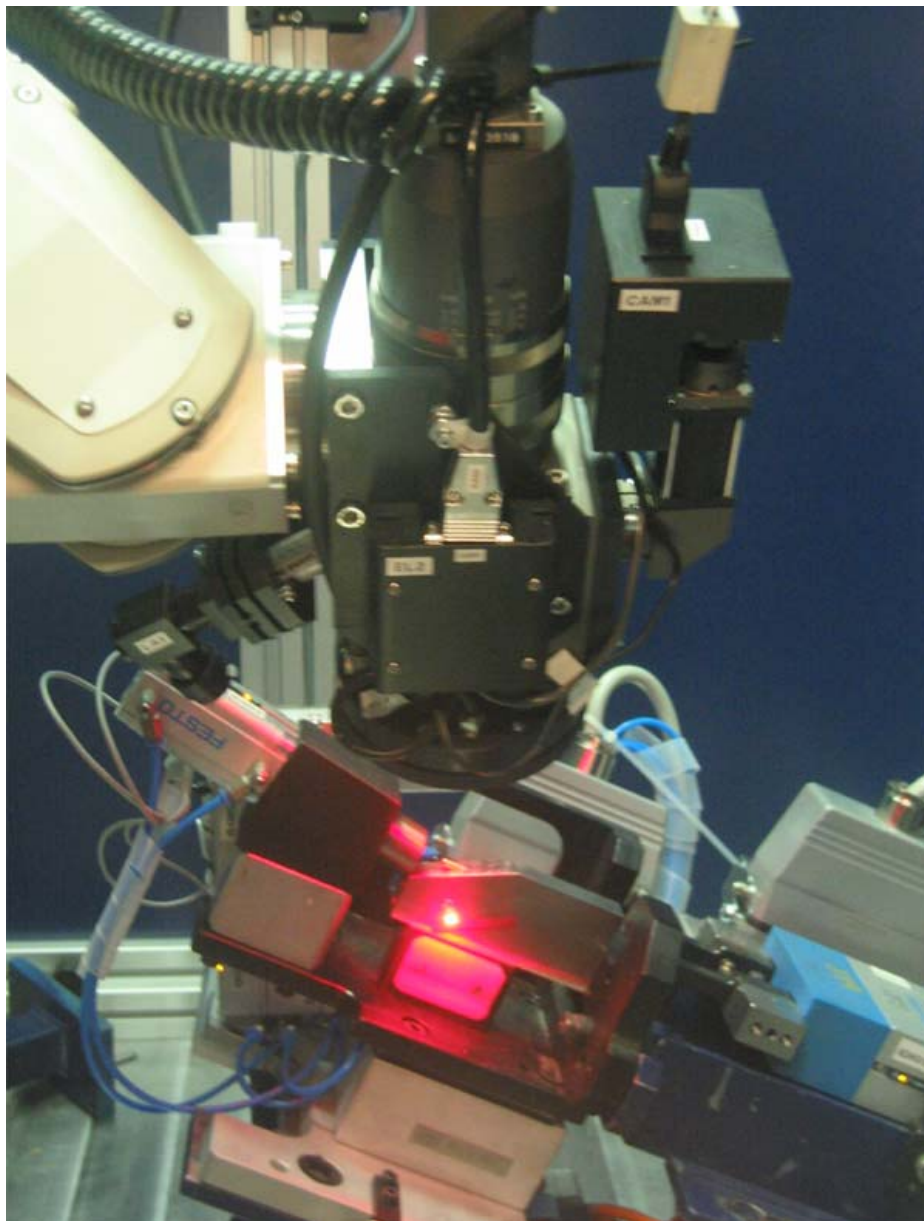
± 0.05 mm locally

... often not available

Error ± 0.5 mm



Modeling kinematics – calibration – absolute accuracy ± 0.05 mm



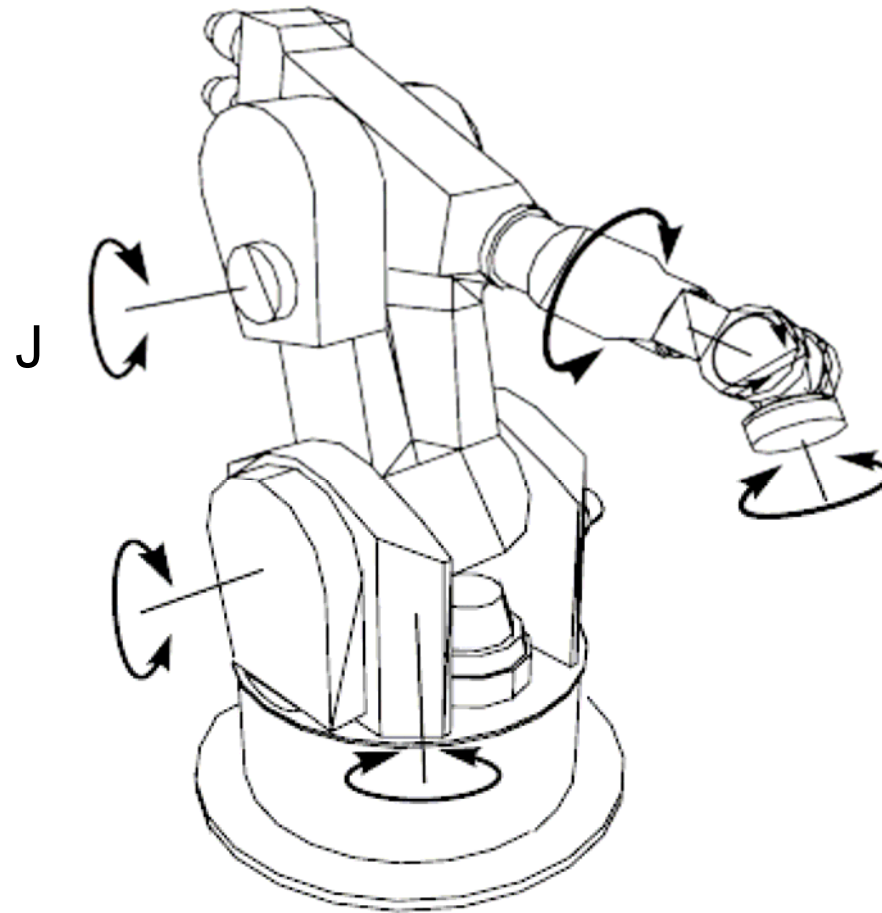
MITSUBISHI robot
TRUMPF welding laser
NEOVISION vision guiding

Robot-Vision calibration (courtesy Neovision s.r.o.)

Two kinds of manipulators

1. Serial manipulators
2. Parallel manipulators

Serial manipulators



KUKA manipulator

Serial manipulators



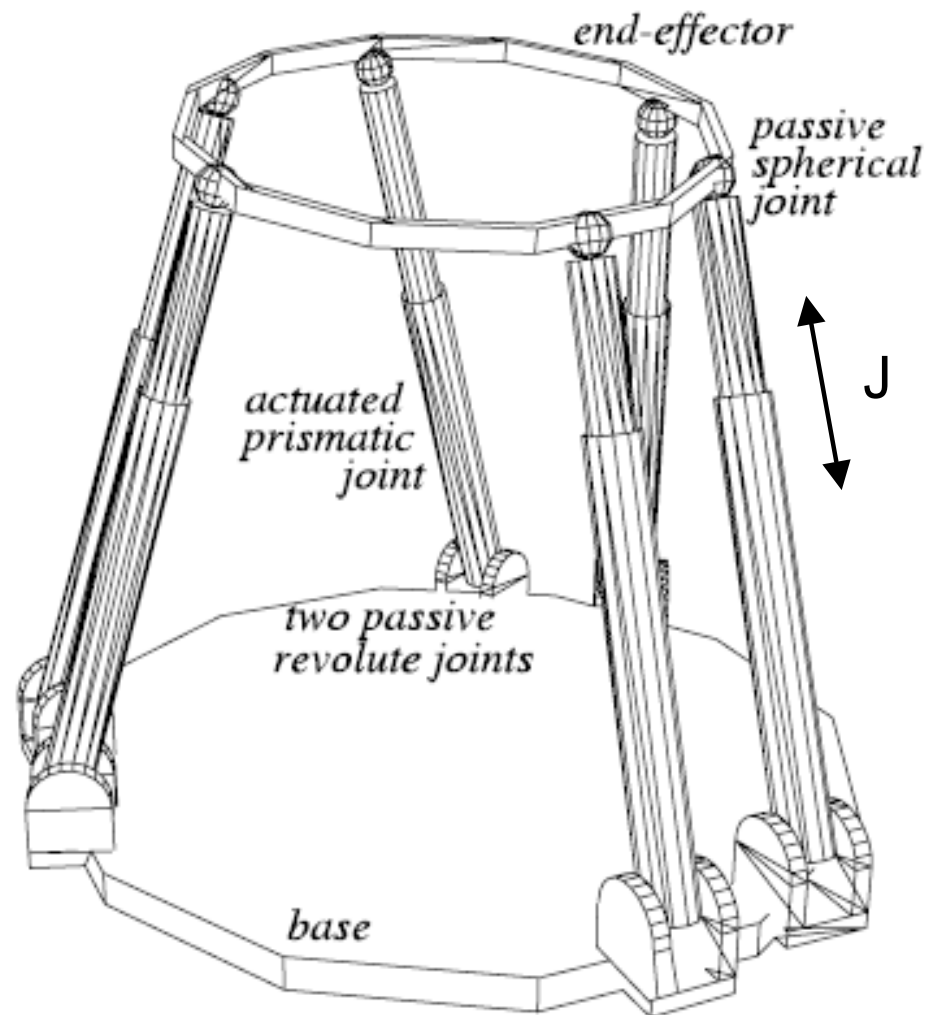
Stäubli (courtesy Neovision s.r.o.)



Mitsubishi (courtesy Neovision s.r.o.)

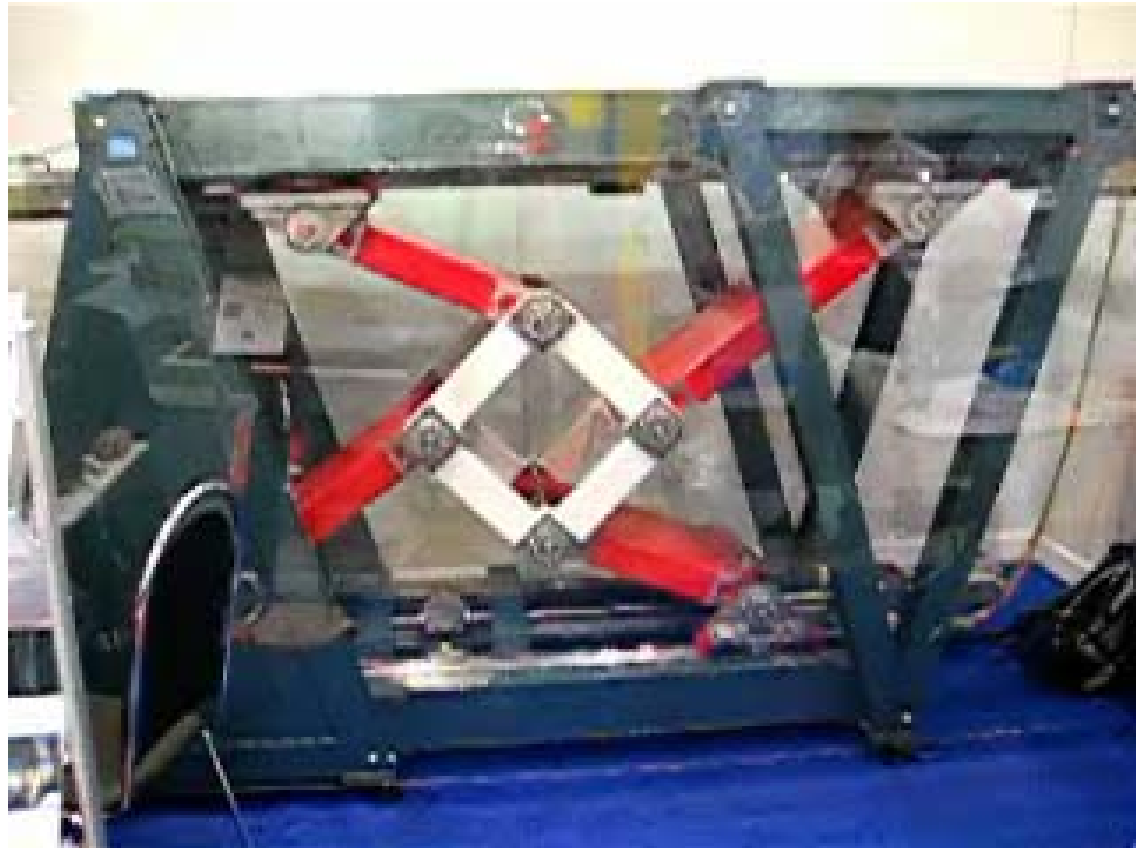
1. Direct kinematic task – easy
2. Inverse kinematic task – difficult

Parallel manipulators



Stewart-Gough Platform

Parallel manipulators



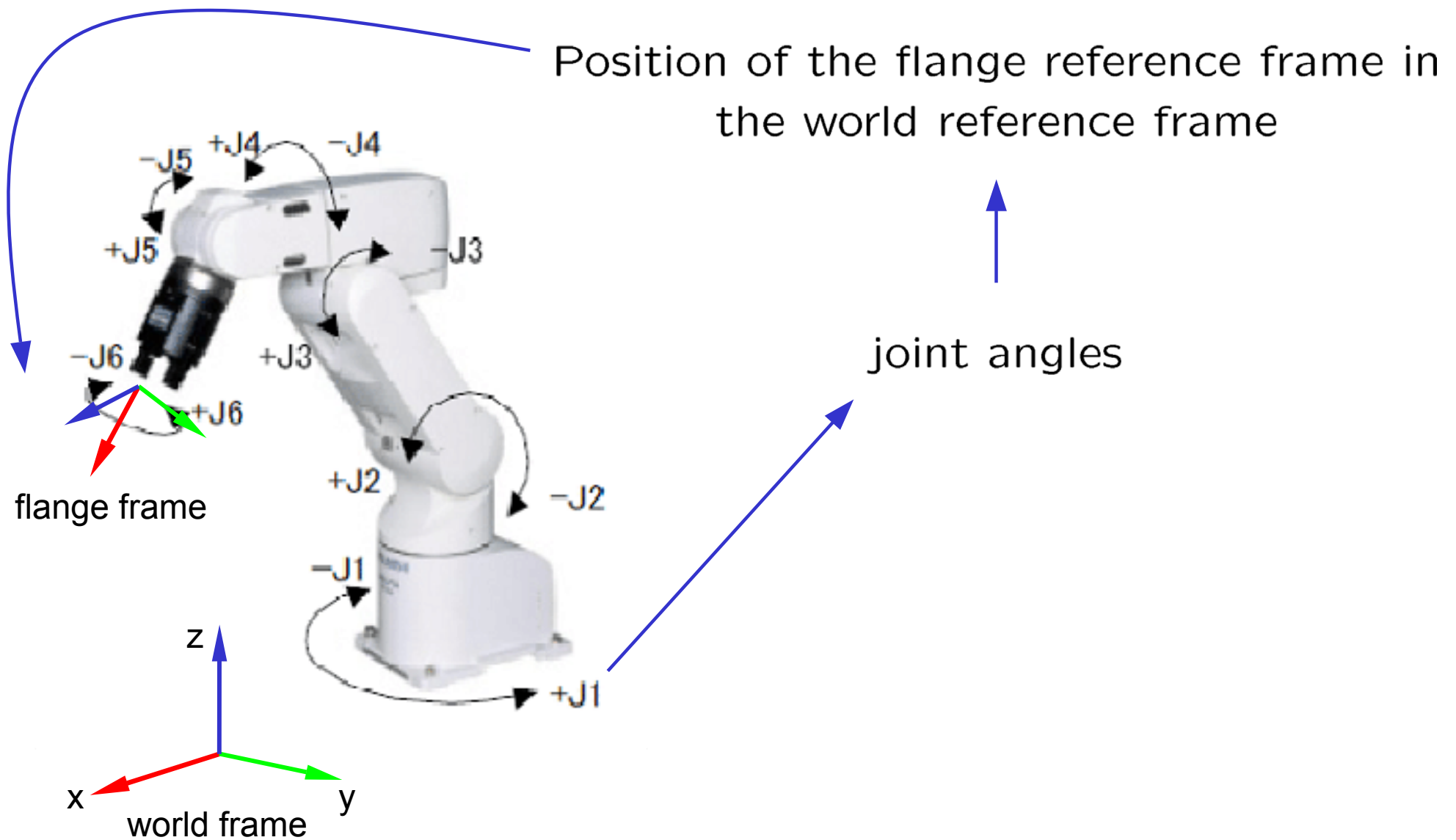
Sliding Star (courtesy of Prof. Valášek, CTU Prague)

1. Direct kinematic task – difficult
2. Inverse kinematic task – easy

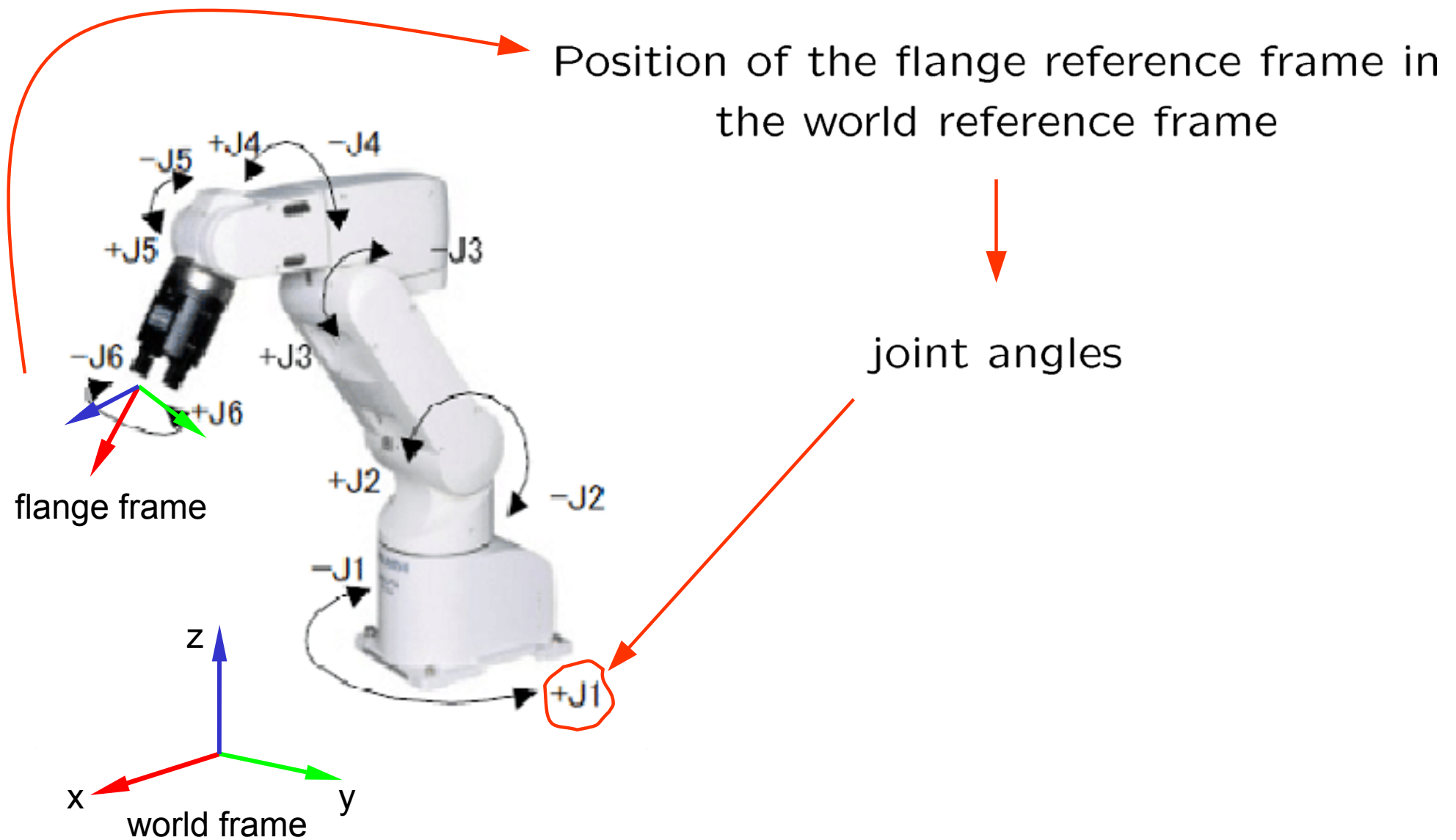
Three main problems

1. Direct kinematic task (přímá kinematická úloha)
2. Inverse kinematic task (inverzní kinematická úloha)
3. Kinematic calibration (kalibrace kinematiky)

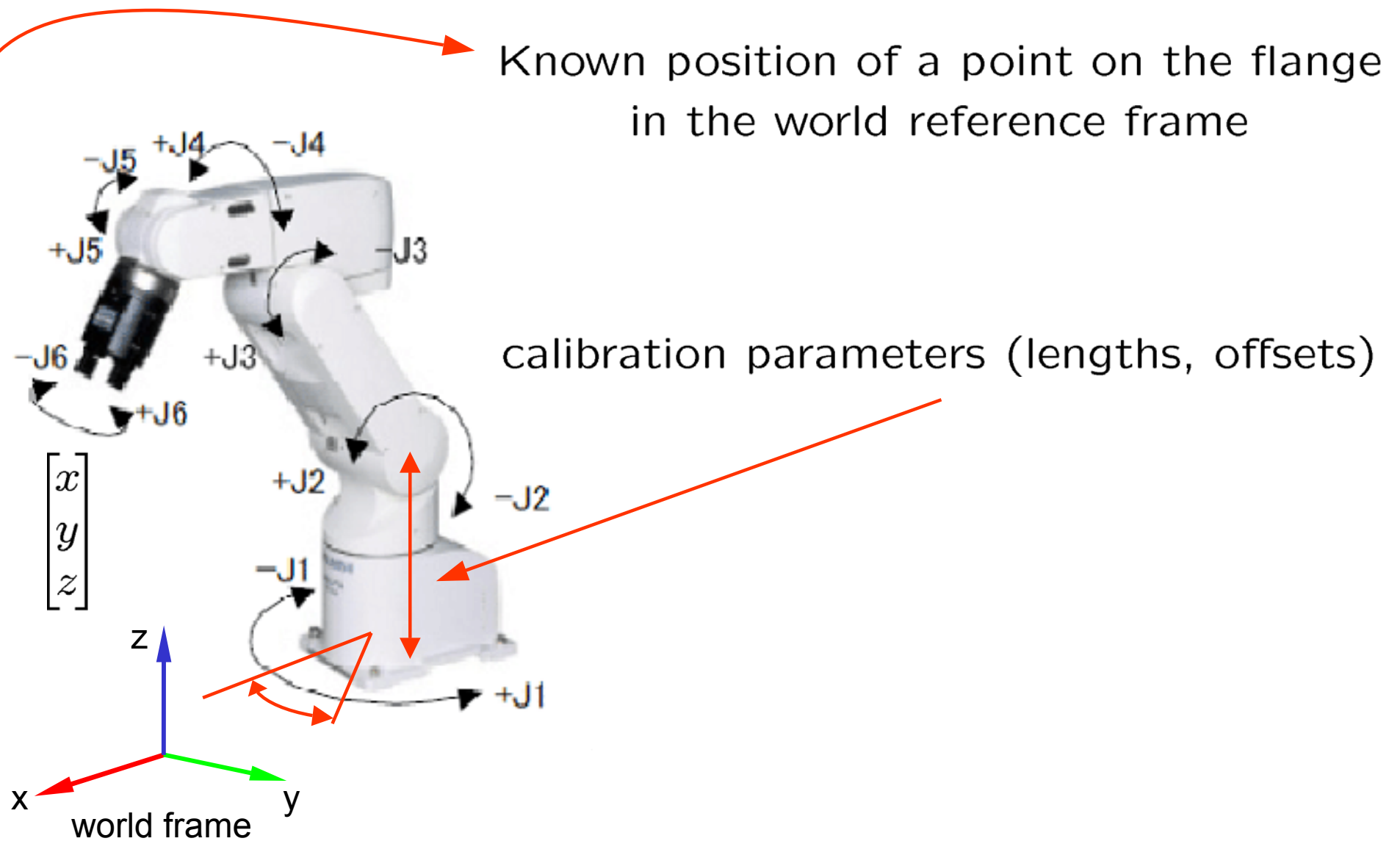
Direct kinematic task



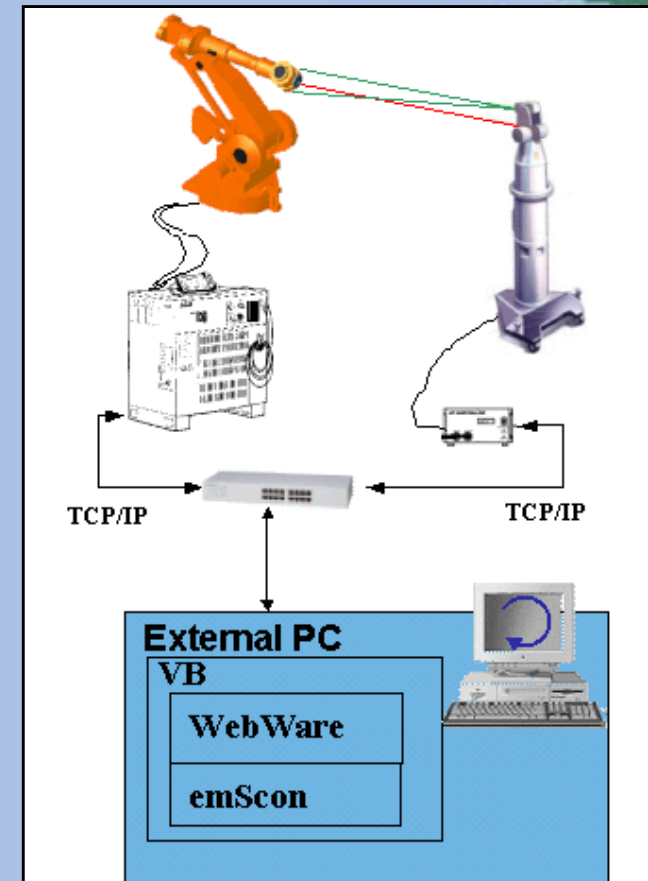
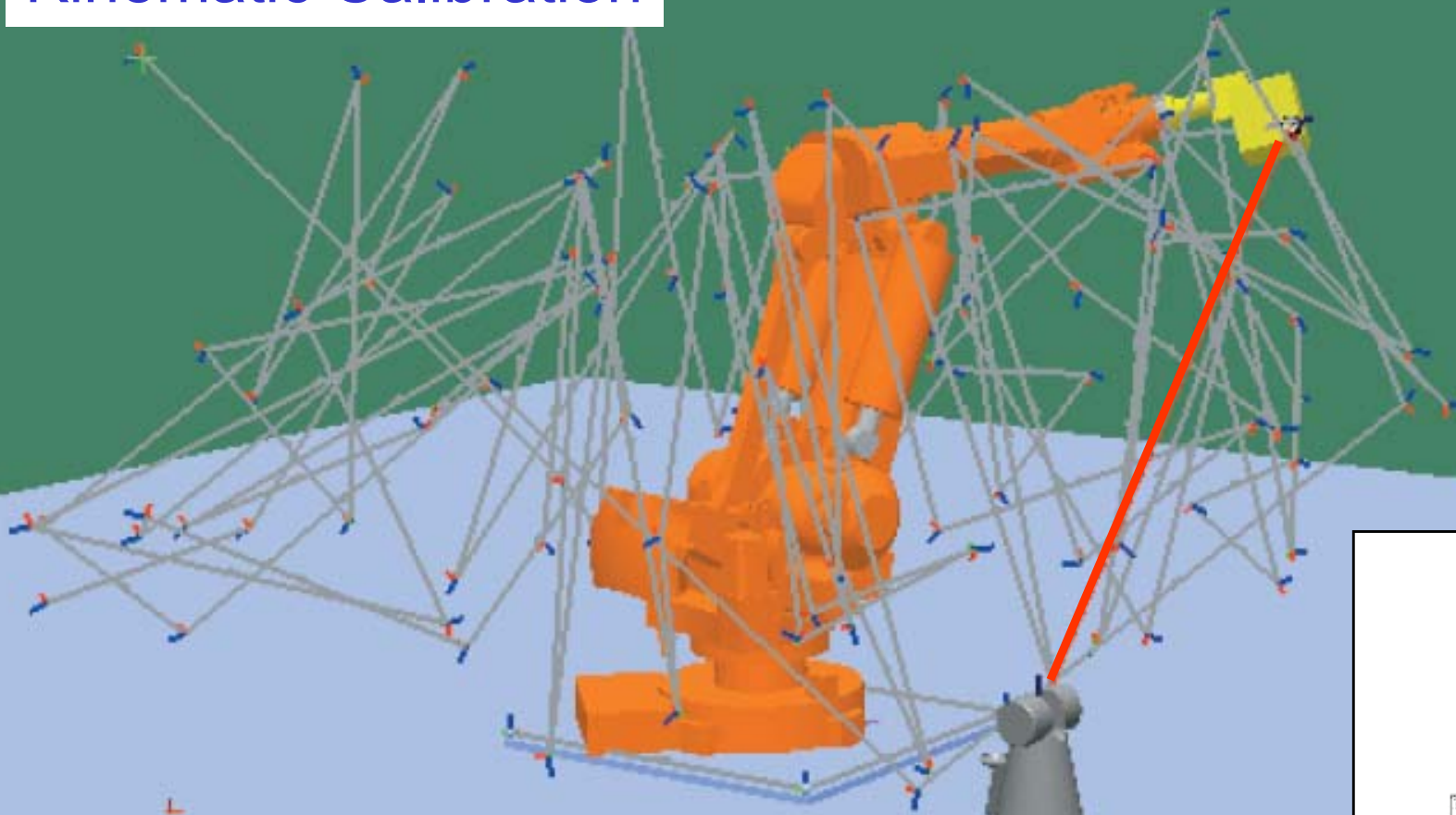
Inverse kinematic task



Kinematic calibration



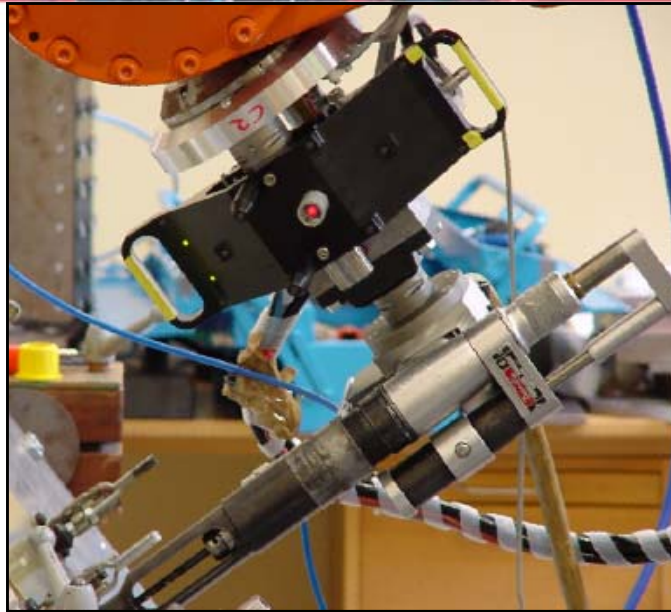
Kinematic Calibration



Robot Calibration



Kinematic Calibration

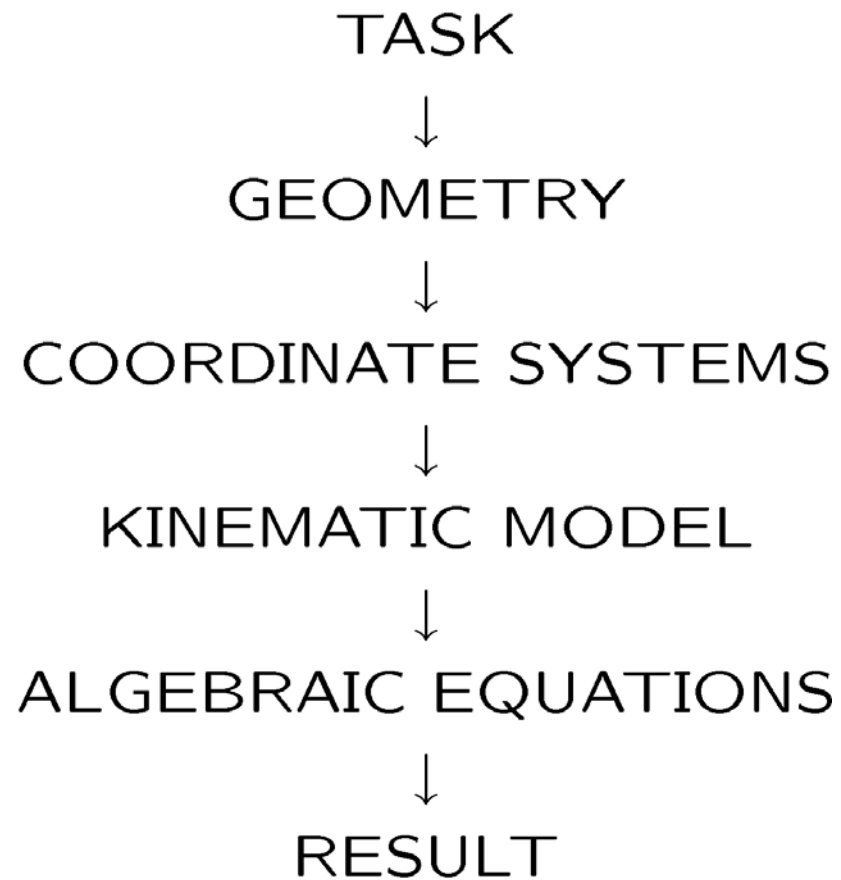








Solving kinematic tasks



Solving kinematic tasks

1968 Donald L. Pieper (Ph.D. thesis)

The inverse kinematics of any serial manipulator with six revolute joints, and with three consecutive joints intersecting, can be solved in closed-form, i.e., analytically.

1989 M. Raghavan, B. Roth. *Kinematic Analysis of the 6R Manipulator of General Geometry*. Int. Symp. Robotics. Research. Pp. 314-320, Tokyo 1989/1990.

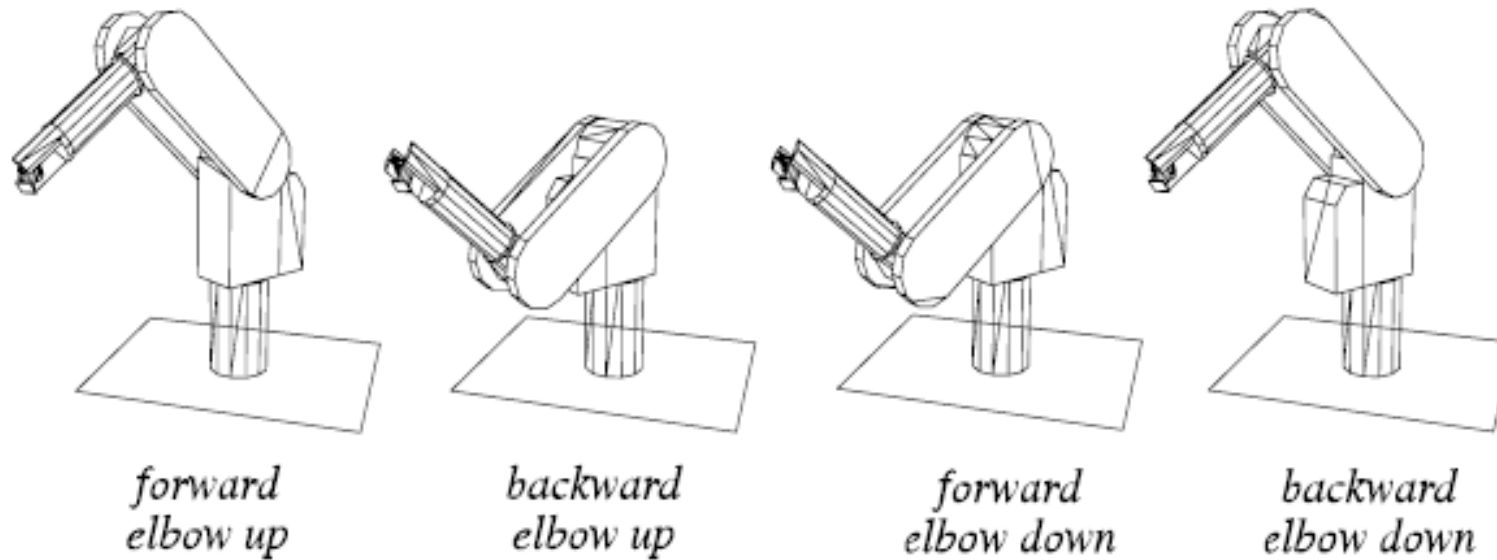
A general technique for computing inverse kinematics for any serial manipulator with six revolute joints.

... leads to solving an algebraic equation of degree 16.

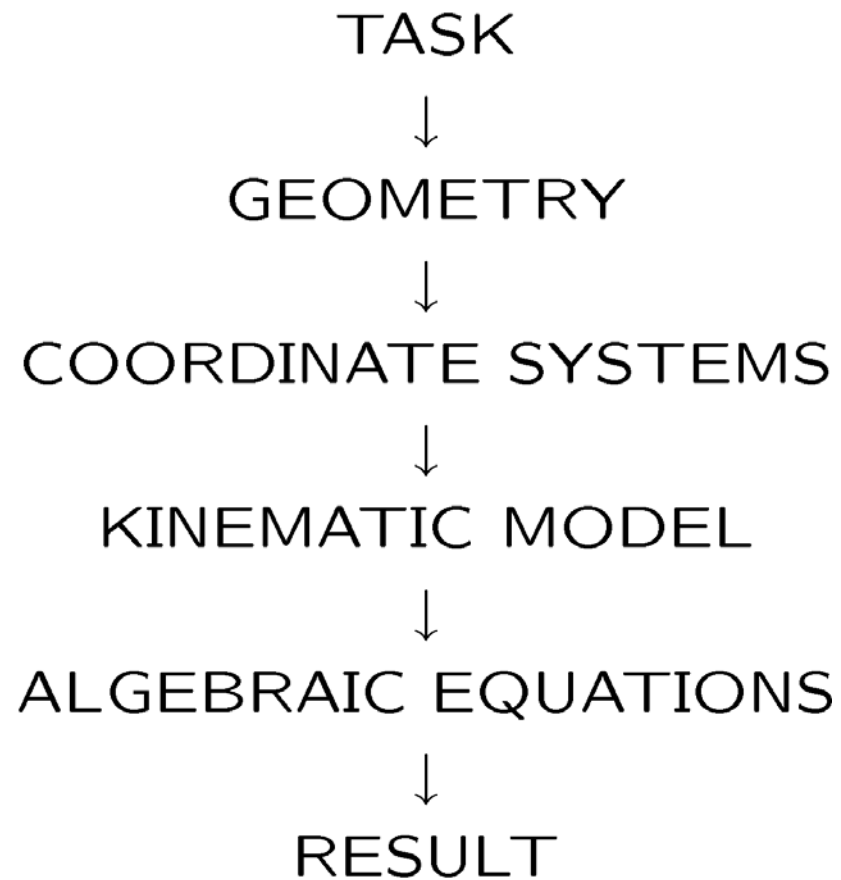
Solving kinematic tasks

Algebraic equation of degree 16 ... up to 16 solutions

4 typical solutions



Solving kinematic tasks



Stäubli TX-90 – Geometry

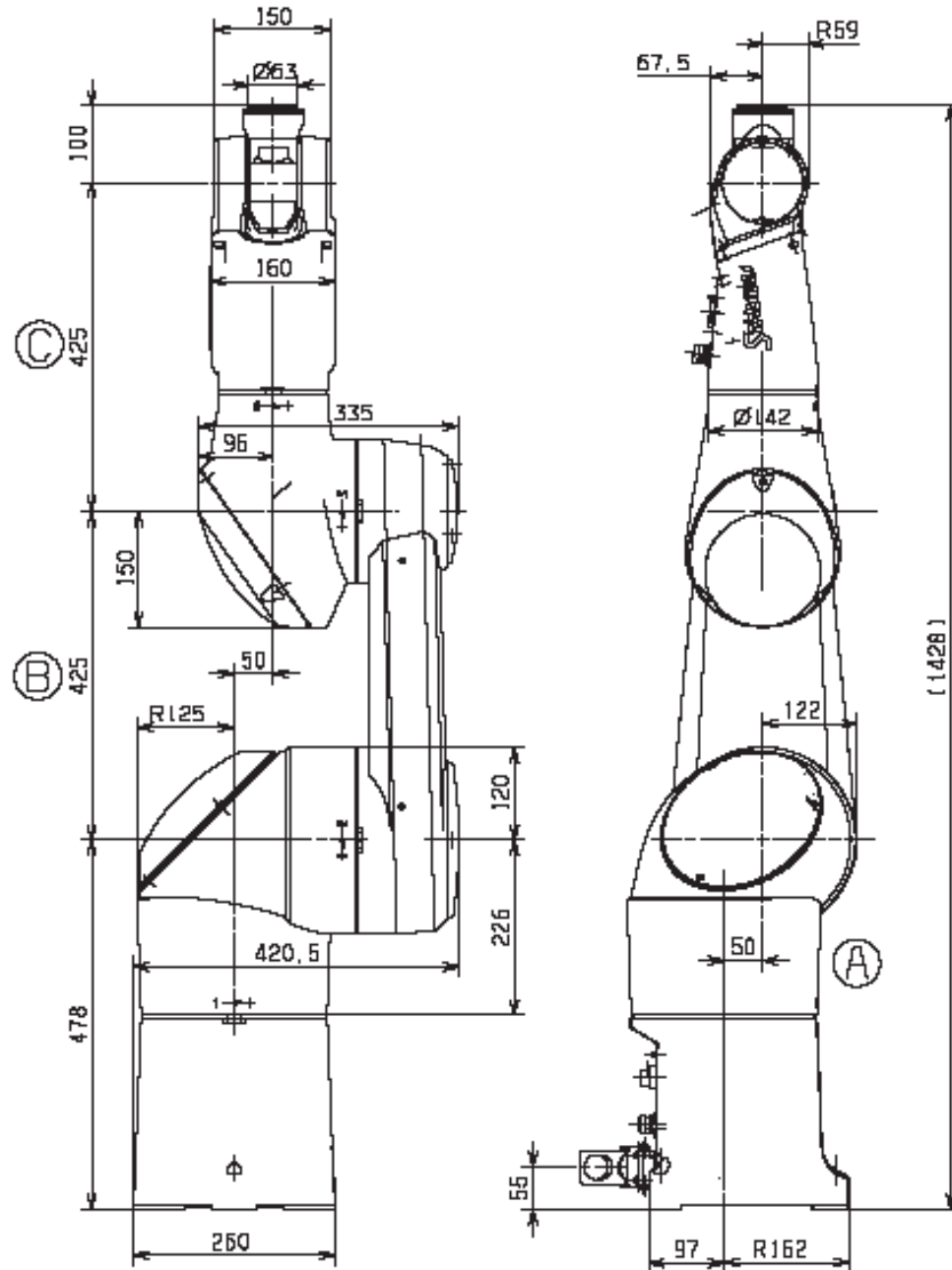
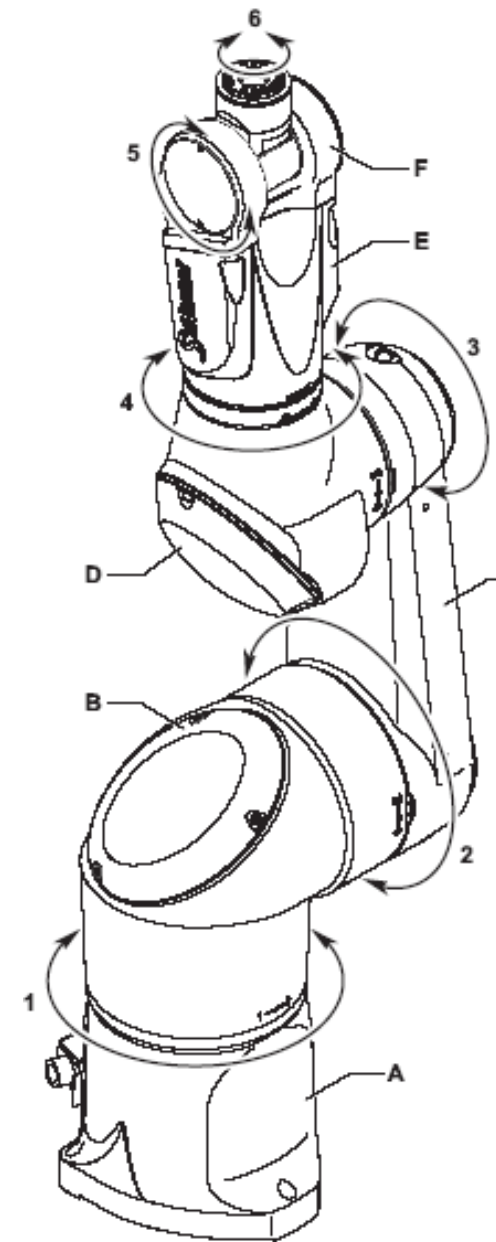
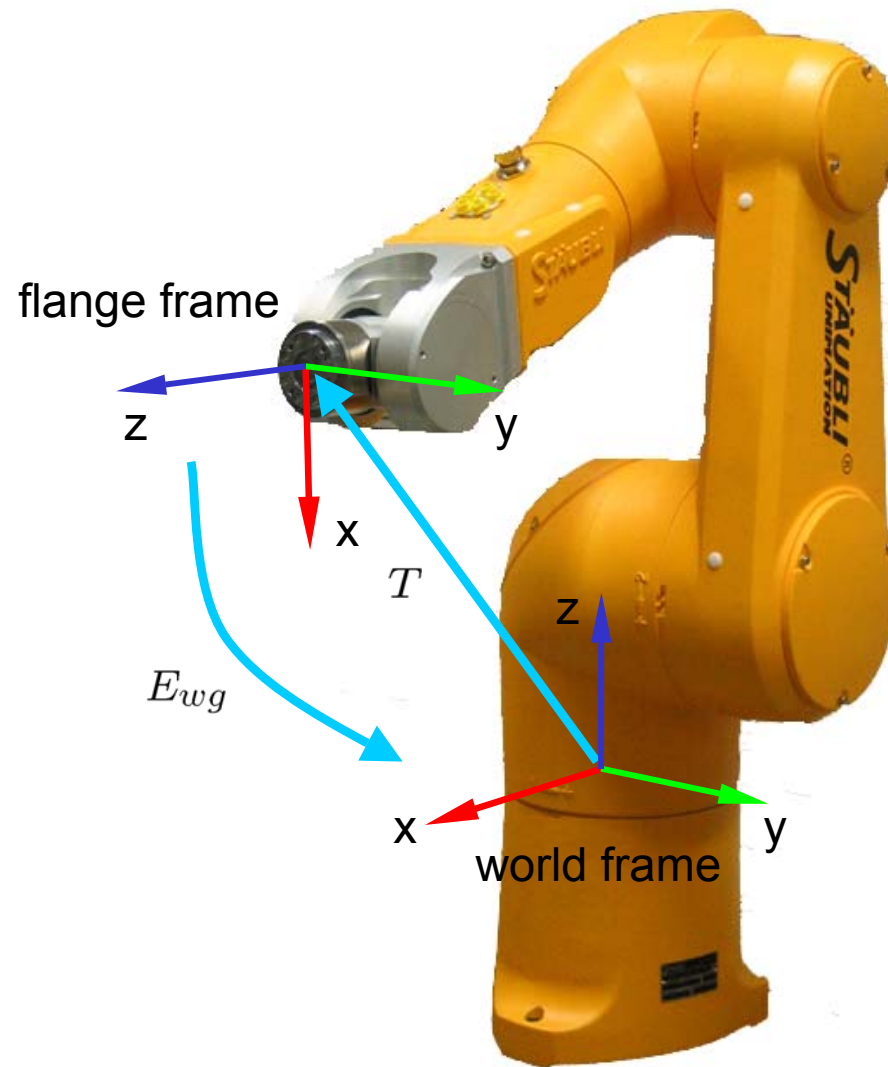


Figure 1.3 - Standard arm



Kinematic model



$$\alpha_i \mid a_i \mid \theta_i \mid d_i$$

$$A_i^{i-1} = \begin{bmatrix} \cos \theta_i & -\sin \theta_i \cos \alpha_i & \sin \theta_i \sin \alpha_i & a_i \cos \theta_i \\ \sin \theta_i & \cos \theta_i \cos \alpha_i & -\cos \theta_i \sin \alpha_i & a_i \sin \theta_i \\ 0 & \sin \alpha_i & \cos \alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\alpha_1}{-\frac{\pi}{2}} \mid \frac{a_1}{a_1} \mid \frac{\theta_1}{\theta_1} \mid \frac{d_1}{0}$$

$$G = A_1^0 A_2^1 A_3^2 A_4^3 A_5^4 A_6^5$$

$$A_1^0 = \begin{bmatrix} \cos \theta_1 & 0 & -\sin \theta_1 & a_1 \cos \theta_1 \\ \sin \theta_1 & 0 & \cos \theta_1 & a_1 \sin \theta_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\alpha_2}{0} \mid \frac{a_2}{a_2} \mid \frac{\theta_2}{\theta_2} \mid \frac{d_2}{d_2}$$

$$A_2^1 = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 & a_2 \cos \theta_2 \\ \sin \theta_2 & \cos \theta_2 & 0 & a_2 \sin \theta_2 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\alpha_3}{-\frac{\pi}{2}} \mid \frac{a_3}{0} \mid \frac{\theta_3}{\theta_3} \mid \frac{d_3}{0}$$

$$A_3^2 = \begin{bmatrix} \cos \theta_3 & 0 & -\sin \theta_3 & 0 \\ \sin \theta_3 & 0 & \cos \theta_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\alpha_4}{\frac{\pi}{2}} \mid \frac{a_4}{0} \mid \frac{\theta_4}{\theta_4} \mid \frac{d_4}{d_4}$$

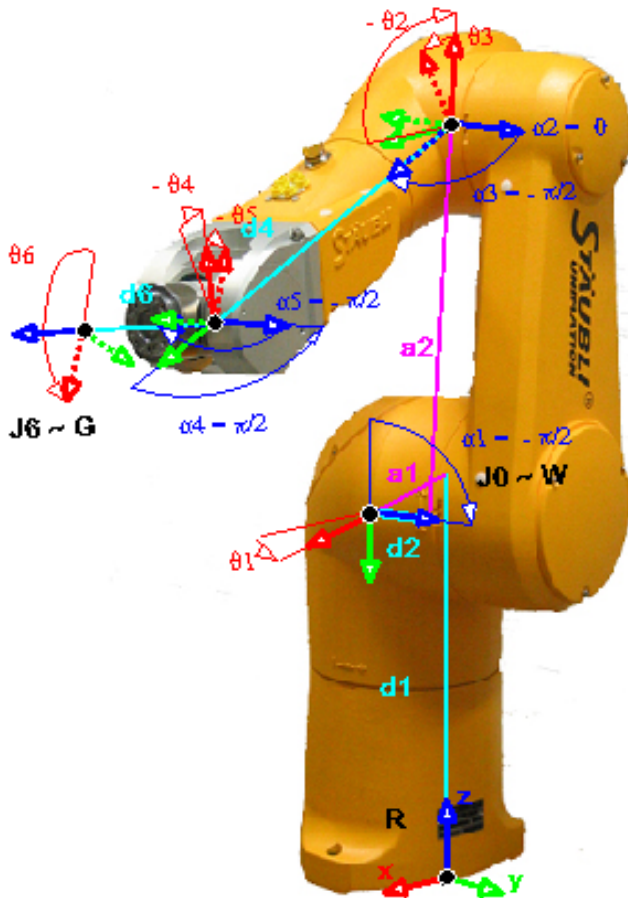
$$A_4^3 = \begin{bmatrix} \cos \theta_4 & 0 & \sin \theta_4 & 0 \\ \sin \theta_4 & 0 & -\cos \theta_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\alpha_5}{-\frac{\pi}{2}} \mid \frac{a_5}{0} \mid \frac{\theta_5}{\theta_5} \mid \frac{d_5}{0}$$

$$A_5^4 = \begin{bmatrix} \cos \theta_5 & 0 & -\sin \theta_5 & 0 \\ \sin \theta_5 & 0 & \cos \theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\alpha_6}{0} \mid \frac{a_6}{0} \mid \frac{\theta_6}{\theta_6} \mid \frac{d_6}{d_6}$$

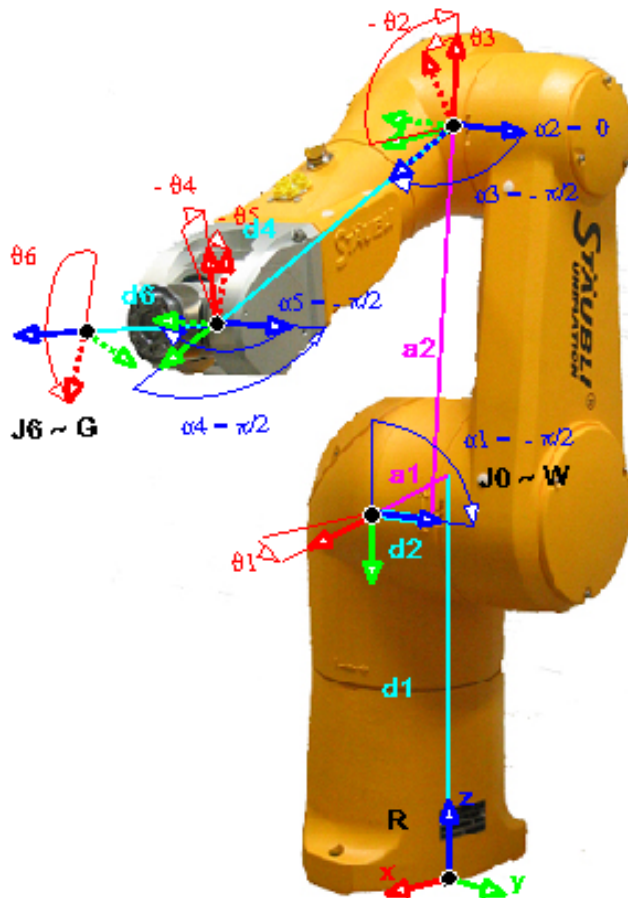
$$A_6^5 = \begin{bmatrix} \cos \theta_6 & -\sin \theta_6 & 0 & 0 \\ \sin \theta_6 & \cos \theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$\text{offset} = [0, -\frac{\pi}{2}, -\frac{\pi}{2}, 0, 0, -\pi]$$

The Standard Kinematic model in Denavit-Hartenberg Convention

Stäubli TX 90



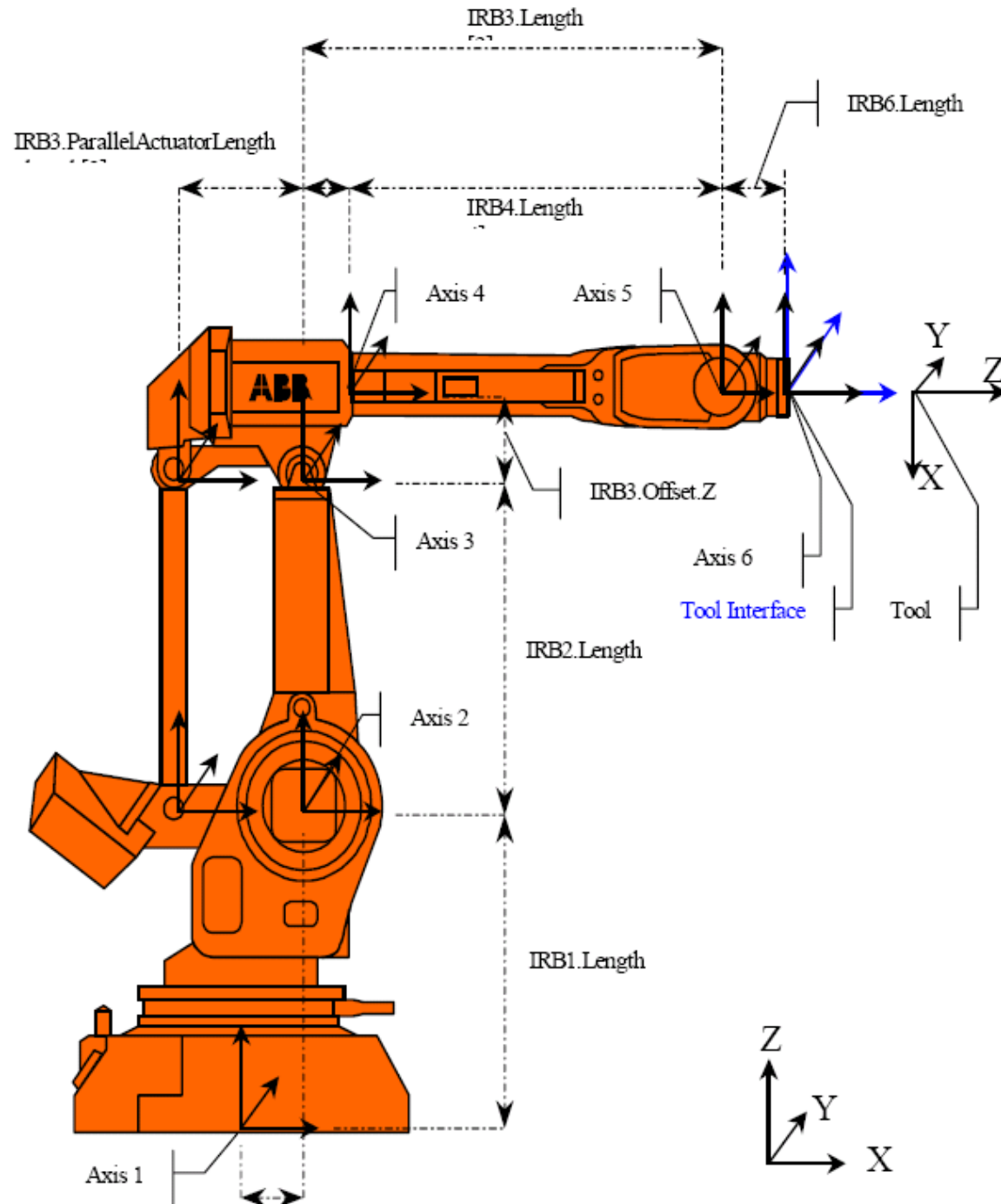
TX-90 (6 axis, RRRRRR) [Staubli]

α	a	θ	d
-1.5708	50.0	0.0	350.0
0.0	425.0	0.0	50.0
-1.5708	0.0	0.0	0.0
1.5708	0.0	0.0	425.0
-1.5708	0.0	0.0	0.0
0.0	0.0	0.0	100.0

6 non-trivial parameteres

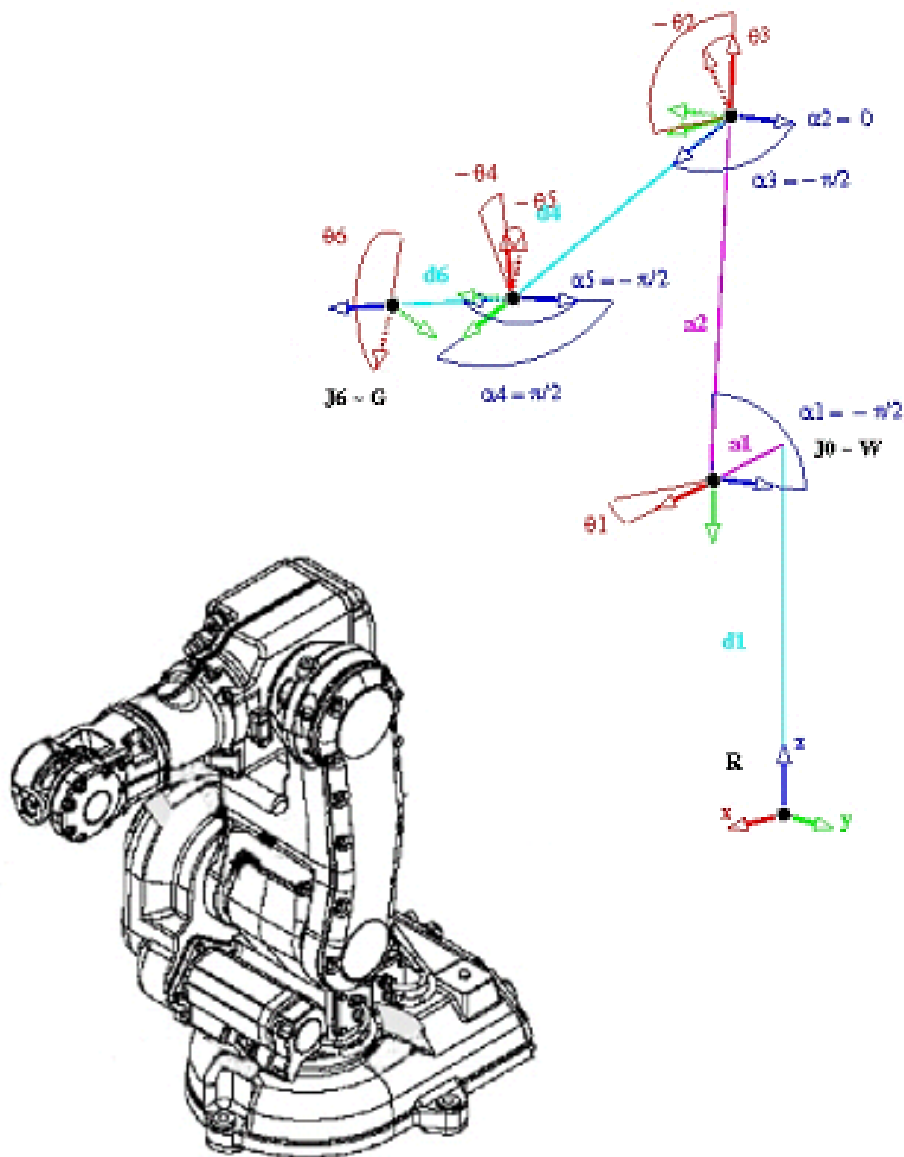
The Standard Kinematic model in Denavit-Hartenberg Convention

ABB IRB 140



The Standard Kinematic model in Denavit-Hartenberg Convention

ABB IRB 140



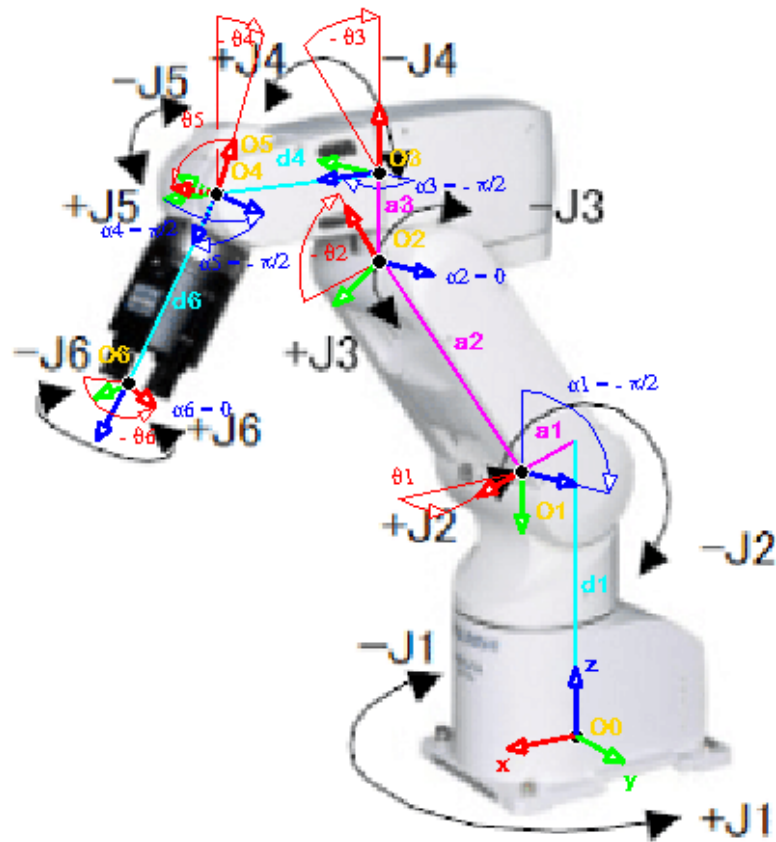
IBR-140 (6 axis) [ABB]

α	a	θ	d
-1.5708	70.0	0.0	352.0
0.0	360.0	0.0	0.0
-1.5708	0.0	0.0	0.0
1.5708	0.0	0.0	380.0
-1.5708	0.0	0.0	0.0
0.0	0.0	0.0	65.0

5 non-trivial parameteres

The Standard Kinematic model in Denavit-Hartenberg Convention

Stäubli TX 90



RV-6S (6 axis, RRRRRR) [Mitsubishi]

α	a	θ	d
-1.5708	85.0	0.0	350.0
0.0	280.0	0.0	0.0
-1.5708	100.0	0.0	0.0
1.5708	0.0	0.0	315.0
-1.5708	0.0	0.0	0.0
0.0	0.0	0.0	85.0

6 non-trivial parameteres

Special versus General Mechanisms

Special

×

General

simple & tractable

complicated & hard

α	a	θ	d
-1.5708	70.0	-	352.0
0.0	360.0	-	0.0
-1.5708	0.0	-	0.0
1.5708	0.0	-	380.0
-1.5708	0.0	-	0.0
0.0	0.0	-	65.0

α	a	θ	d
-1.42	70.1	- (+0.2)	352.0
0.10	360.0	- (+0.1)	0.2
-1.57	0.2	- (- 0.3)	0.3
1.58	0.1	- (+0.1)	380.2
-1.59	0.4	- (- 0.1)	0.1
0.07	0.2	- (- 0.2)	65.1

6 non-trivial parameters

×

18 (+6) non-trivial parameters

High precision → Small misalignments important → General mechanisms

Literature

Linear algebra

P. Pták. *Introduction to Linear Algebra*. Vydavatelství ČVUT, Praha, 2006.

Numerical linear algebra

E. Krajník. *Maticový počet*. Vydavatelství ČVUT, Praha, 2000.

The solution

M. Raghavan, B. Roth. *Kinematic Analysis of the 6R Manipulator of General Geometry*. Int. Symp. Robotics. Research. Pp. 314-320, Tokyo 1989/1990.

The numerical solution

D. Manocha, J. Canny. *Efficient Inverse Kinematics for General 6R Manipulators*. Robotics and Automation 1994.

The pedagogical solution will be developed using

D. Cox, J. Little, D. O'Shea. *Ideals, Varieties, and Algorithms*. Springer 1998.

Software

Matlab: www.matworks.com

Maple: www.maple.com

One algebraic equation in one variable

SOLVING 1 ALGEBRAIC EQUATION

1 equation, 1 variable $\rightarrow$ companion matrix $\rightarrow$ eigenvalues

$$f(x) = x^3 + 4x^2 + x - 6 = -6 + 1x + 4x^2 + 1x^3$$

$$M_x = \begin{bmatrix} 0 & 0 & 6 \\ 1 & 0 & -1 \\ 0 & 1 & -4 \end{bmatrix}$$

... a simple rule

```
>> e=eig(M_x)
```

$$e = \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}$$

$$x_1 = 1, x_2 = -2, x_3 = -3$$

It works when eig works, i.e. order 100 in Matlab is often OK.

SOLVING 1 ALGEBRAIC EQUATION

Linear mapping $M \in \mathbb{R}^{n \times n}$

Eigenvalues $M\mathbf{x} = \lambda \mathbf{x}$

$\Leftrightarrow$

$$M\mathbf{x} - \lambda \mathbf{x} = 0$$

$\Leftrightarrow$

$$M\mathbf{x} - \lambda I \mathbf{x} = 0$$

$\Leftrightarrow$

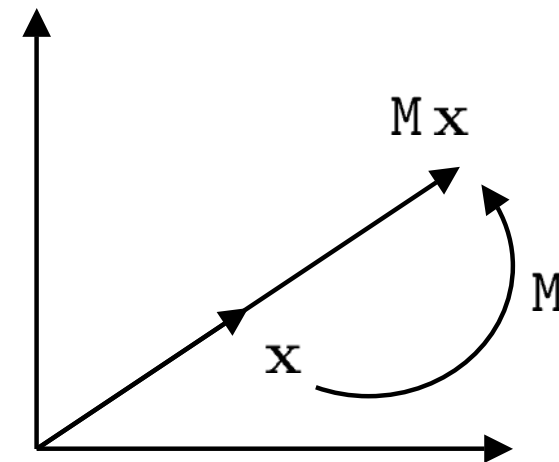
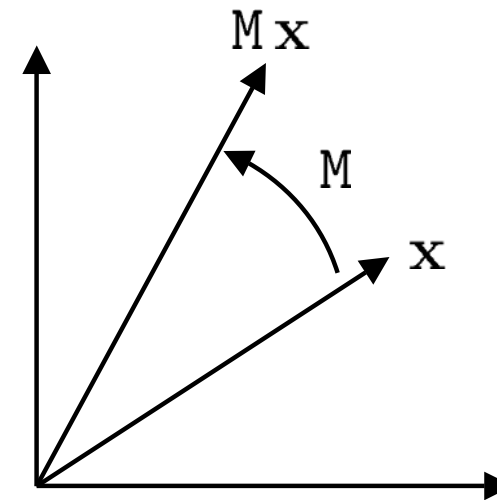
$$(M - \lambda I) \mathbf{x} = 0$$

$$\mathbf{x} \neq 0 \Rightarrow \Leftrightarrow$$

$$\text{rank}(M - \lambda I) < n$$

$\Leftrightarrow$

$$\det(M - \lambda I) = 0$$



SOLVING 1 ALGEBRAIC EQUATION

algebraic equation

$$f(x) = x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 = \det(-M + x I)$$

$$-M + x I = \begin{bmatrix} x & & & a_0 \\ -1 & x & & a_1 \\ & -1 & x & a_2 \\ & & -1 & x + a_3 \end{bmatrix}$$

$$f(x) = x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

Numerical solution to $f(x)$ is obtained by

```
>> x = eig(M);
```