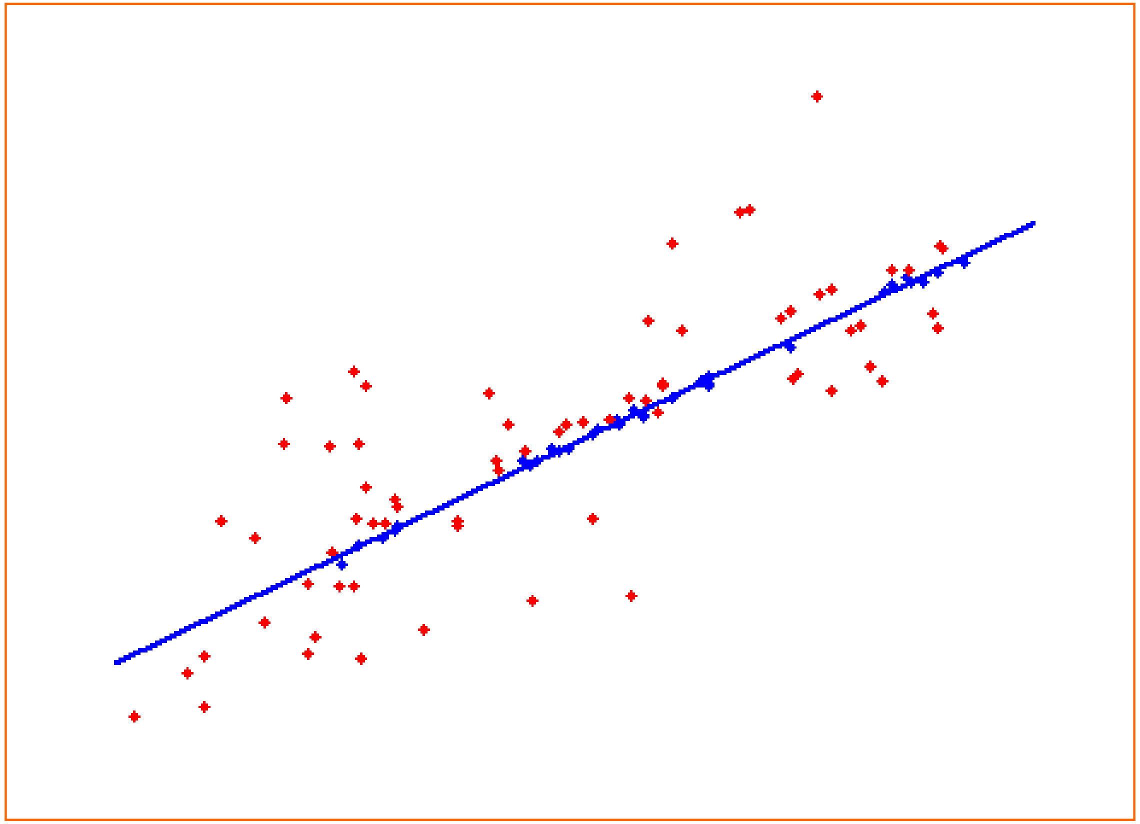
RANSAC – Robust Fitting



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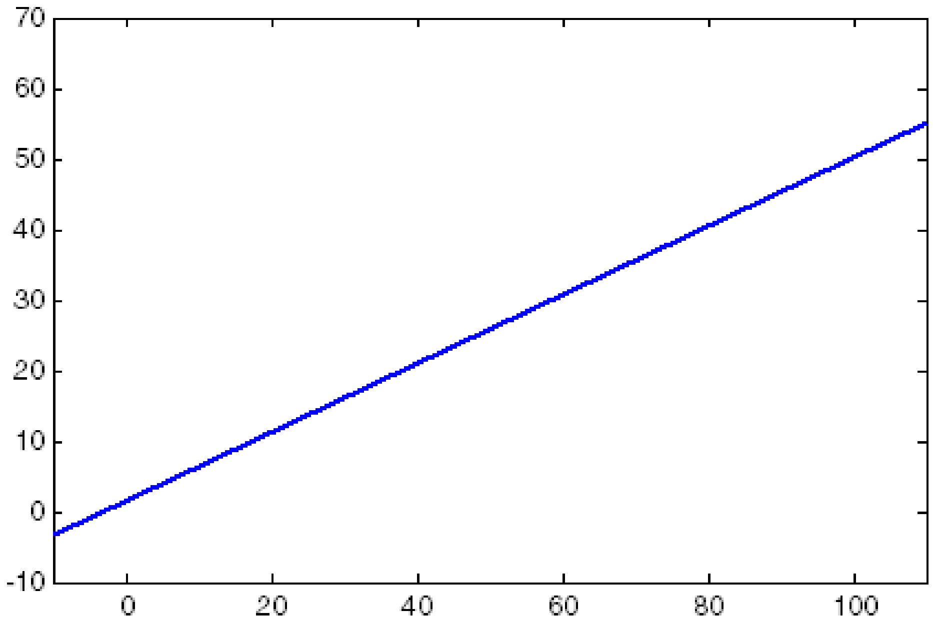
21 April 2007

Example

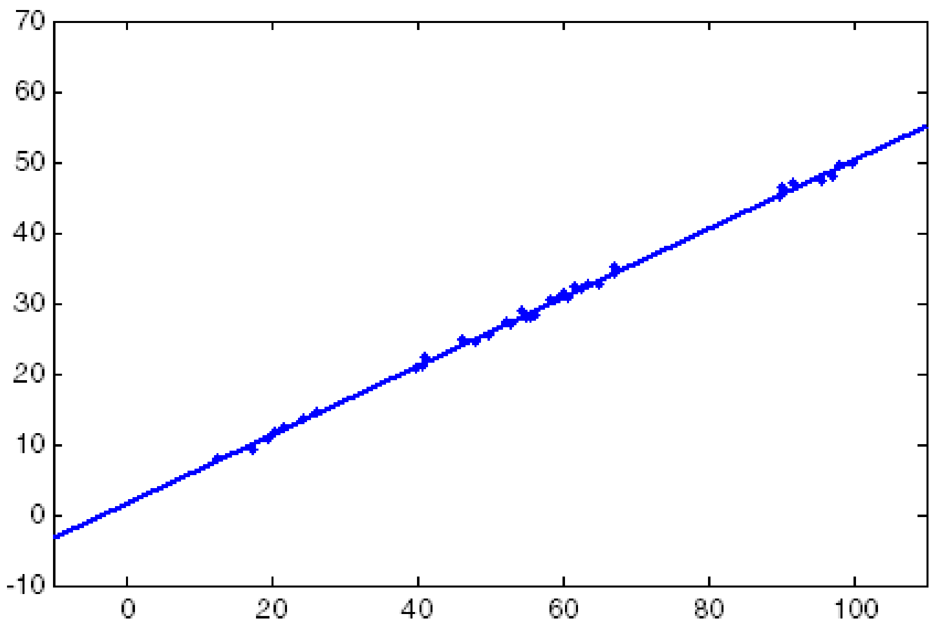
There is a line ...



Example

There is a line ...

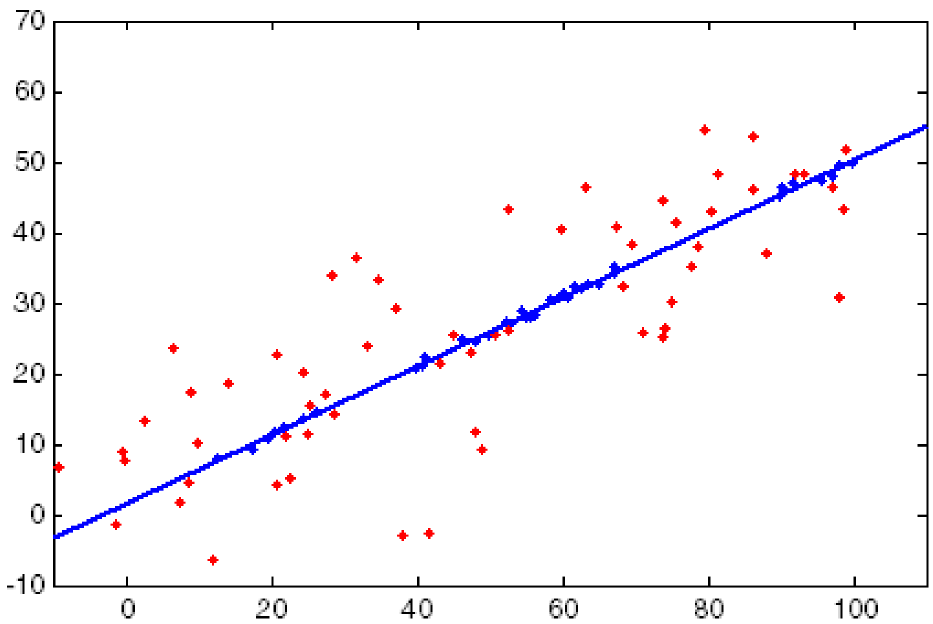
$Y = \{\mathbf{x}_i\}_{i=1}^{M}$ ... a set of points on the line l is measured with Gaussian noise $N(\mathbf{0}, \sigma)$



Example

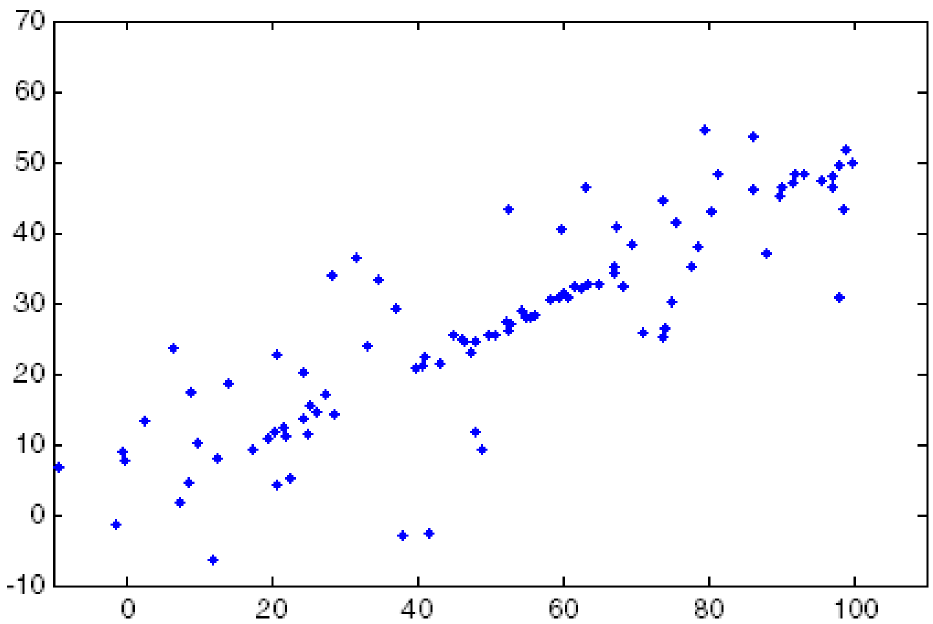
There is a line ...

$X = \{\mathbf{x}_i\}_{i=1}^{N}$ ... other points, unrelated to the line.



Task

Find a line and its points from data contaminated by mesurements unrelated to the line.



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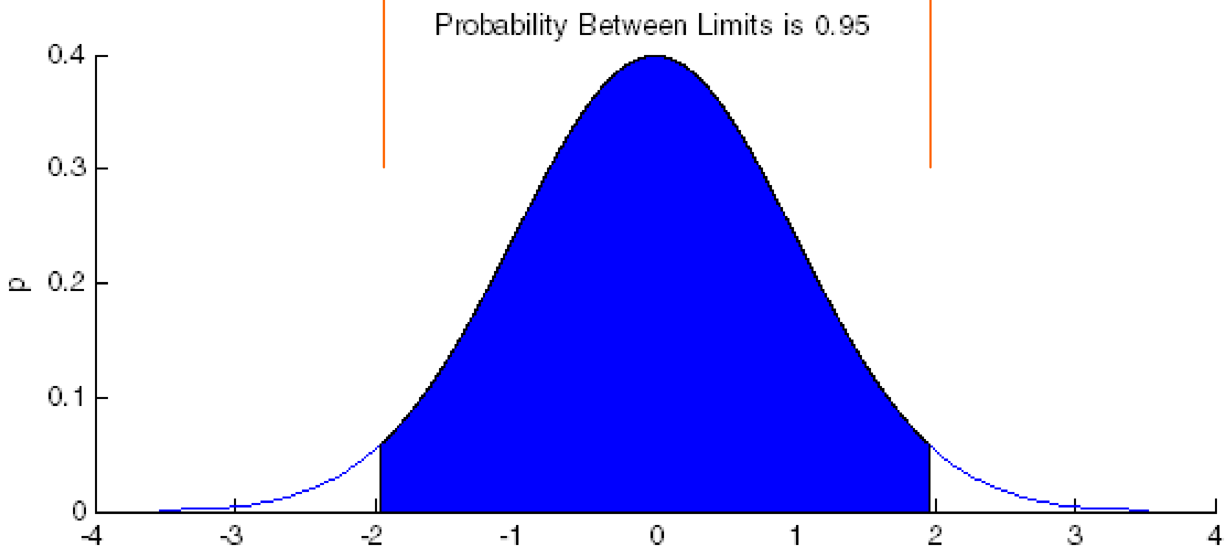
Assumptions

1. σ is known

2. The largest subset of X of points for which there is a line which is closer than 1.96 σ to all the points contains points that were measured on the line l.

$$\underbrace{\hspace{5cm}}_{}^{2 \times 1.96\,\sigma \sim 95\%}$$

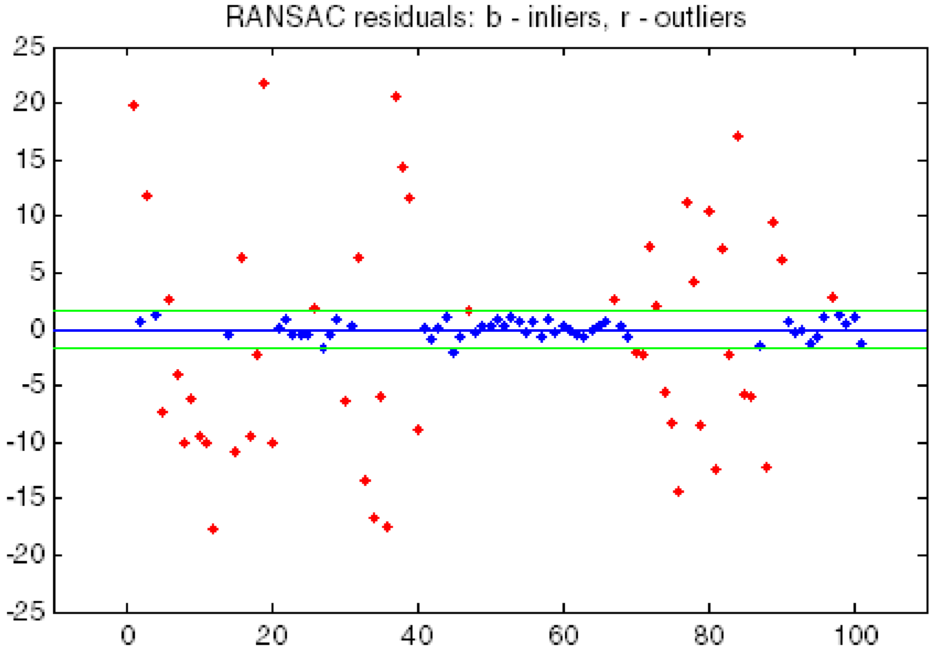


Problem formulation

Find the maximal subset I of the given set X

such that

there is a line which is closer than $1.96\,\sigma$ to all points in I.



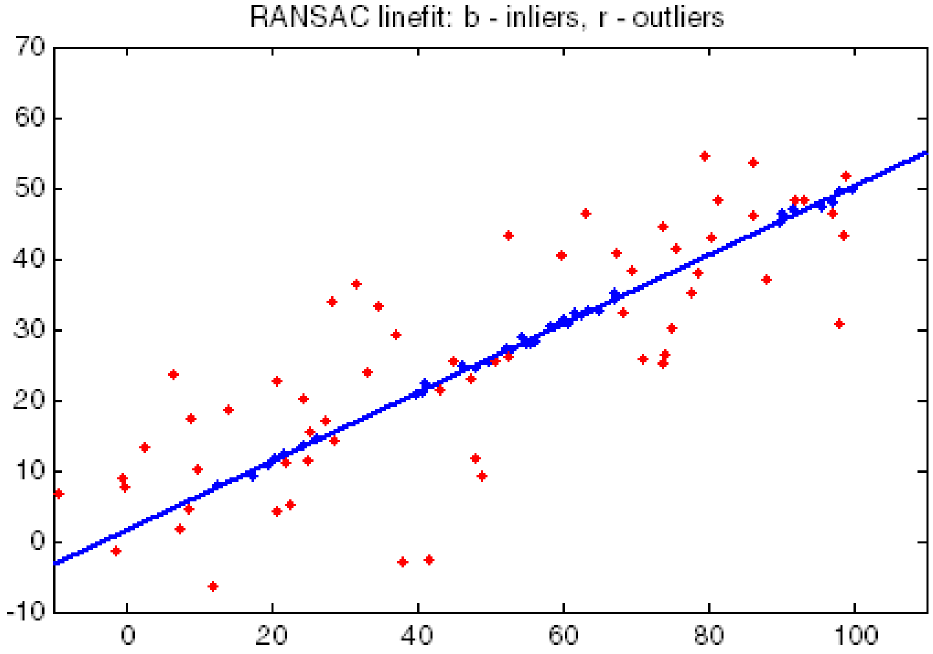
Notation

l . . . line

X . . . all data

I . . . inliers = points closer to l than a threshold τ

$O = X \setminus I$. . . outliers



Exhaustive Search

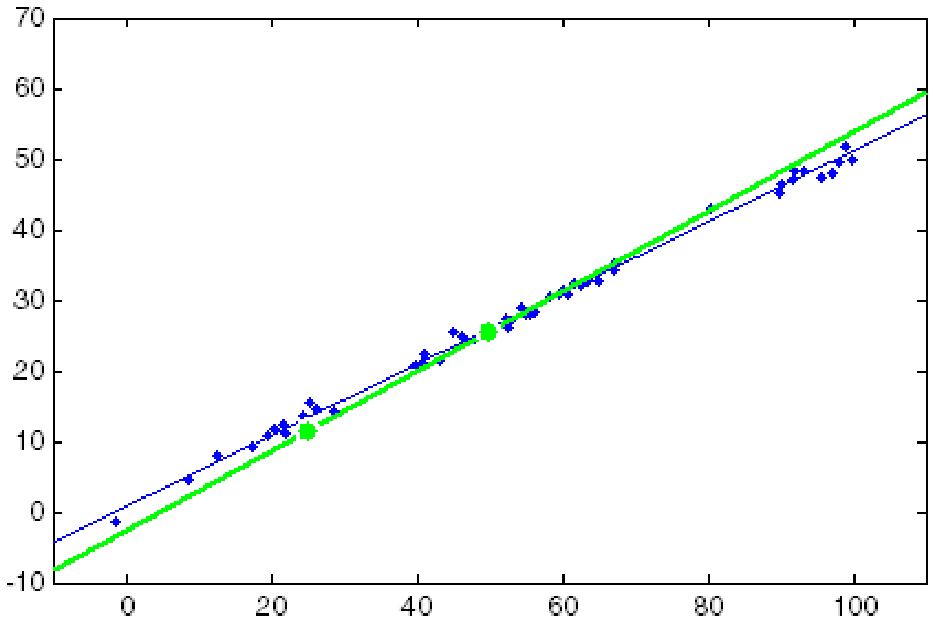
$$I = \arg\max_{S \subseteq X} (\text{the number of inliers in } X \text{ for the best line fit to } S)$$

does not work:

There is 2^N candidate subsets S to be tested

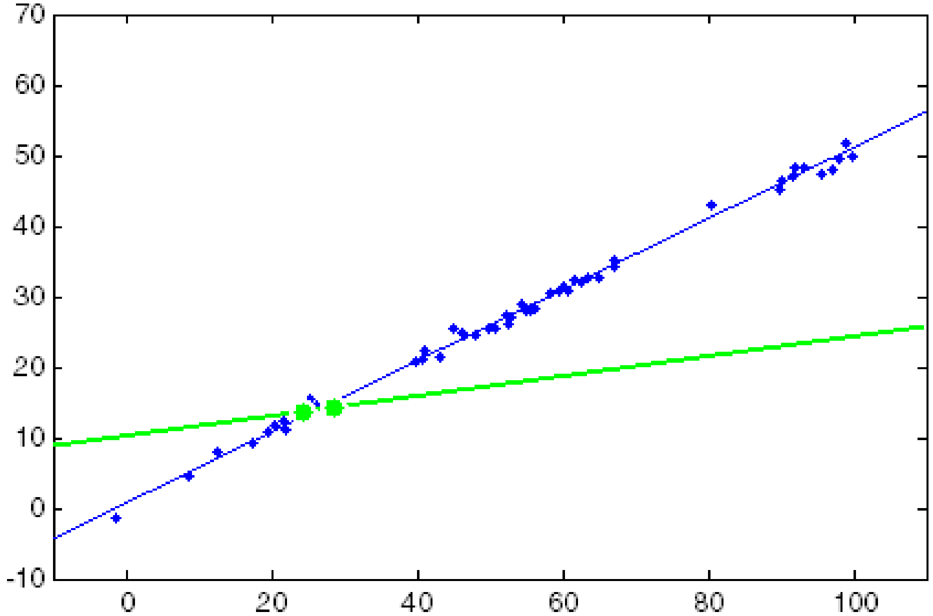
... infeasible for useful N's

Observation



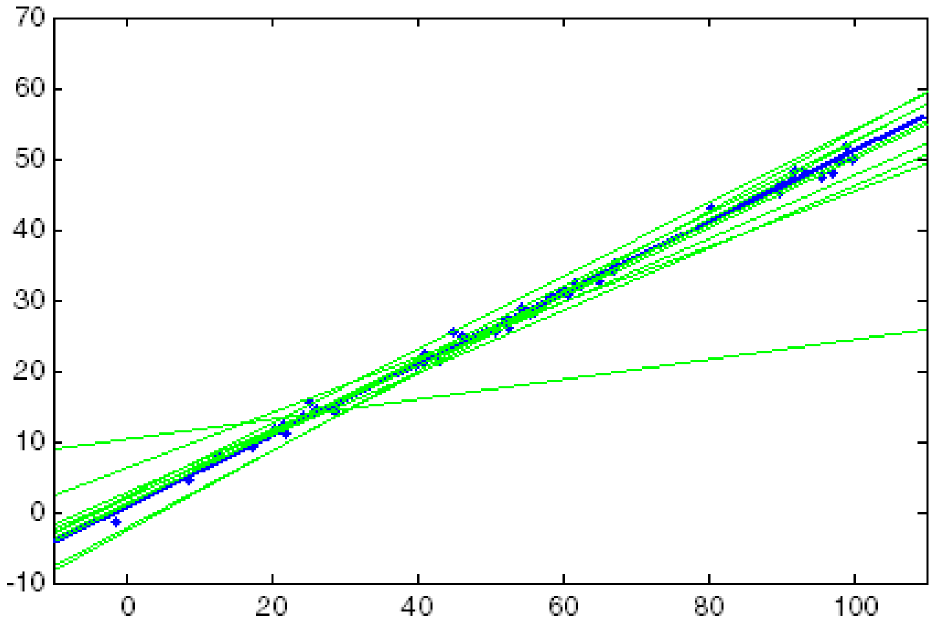
Two (a minimal sample of) *good* points (points measured on the line l) generate a line which is close to l.

Observation



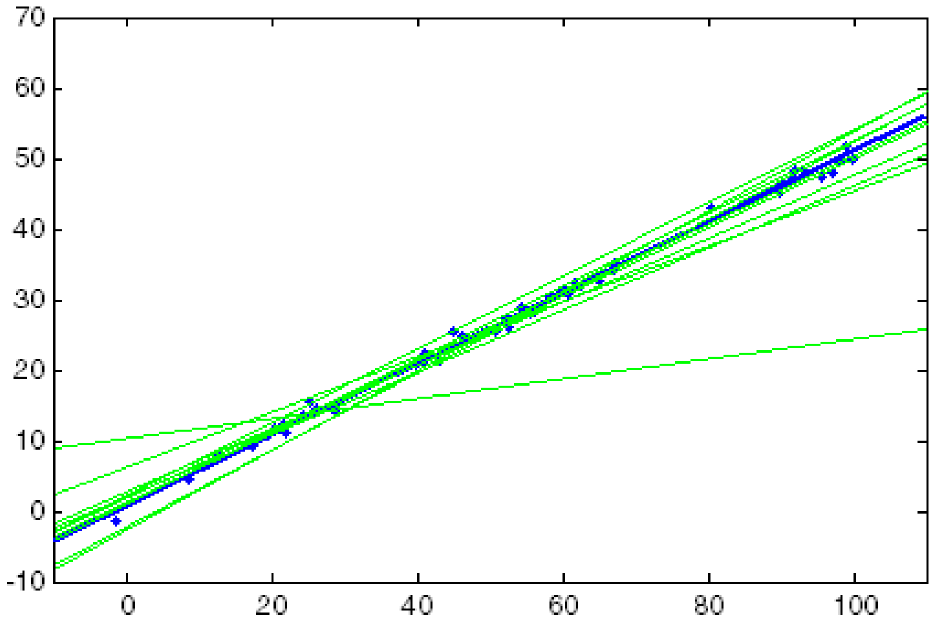
Not all pairs of "good" points are good due to noise ...

Observation



… but many are.

Observation



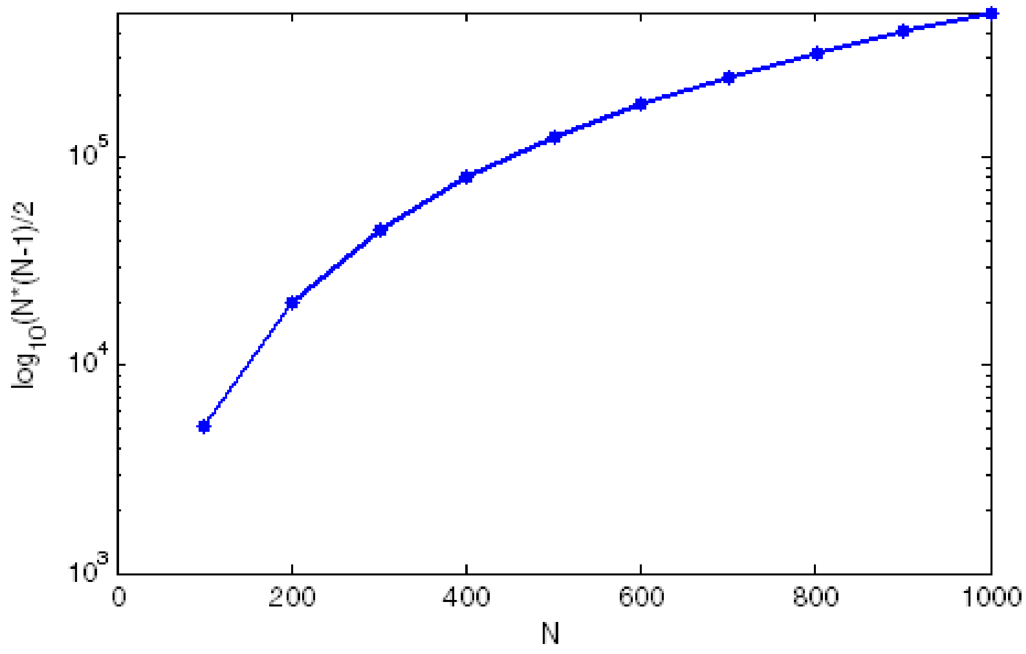
There is "only" $\binom{N}{2} = \frac{N(N-1)}{2}$ pairs of distinct points.

Exhaustive Minimal Sampling

$$I = \arg \max_{\{x_1, x_2\} \subseteq X} (\ \#\ \text{inliers in } X \text{ for the line through } \{x_1, x_2\})$$

needs to examine "only" $\frac{N(N-1)}{2} \ll 2^N$ pairs of distinct points.

Exhaustive Minimal Sampling



The number of samples: $\binom{N}{2} = \frac{N(N-1)}{2}$

... is often still too high.

Exhaustive Minimal Sampling with zero noise

Simplified analysis: assume no noise, i.e. $\sigma = 0$.

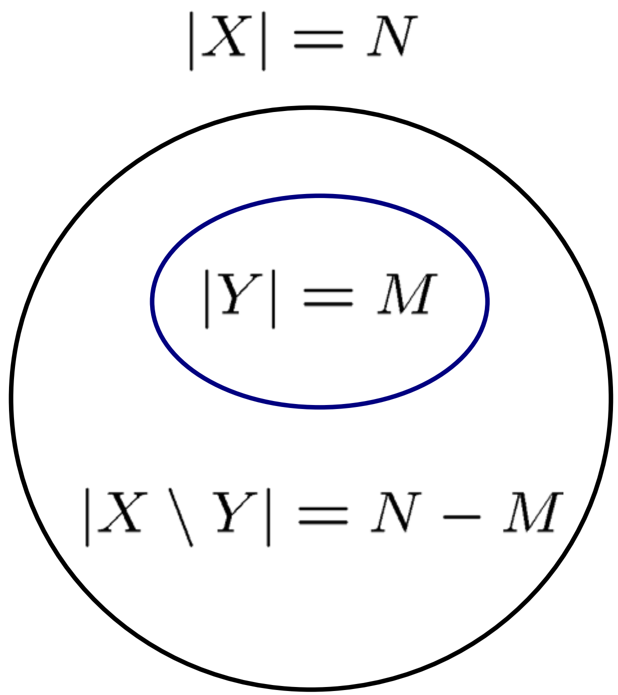
The goal is to make as many samples as to be sure not to miss the set Y of points on the line l.

Assume that there is M points in Y on the line l among the total number N of points in X.

One has to try more than

$$\binom{N}{2} - \binom{M}{2}$$

point pairs when drawing samples without repetition, which goest to ∞ for $N \to \infty$.



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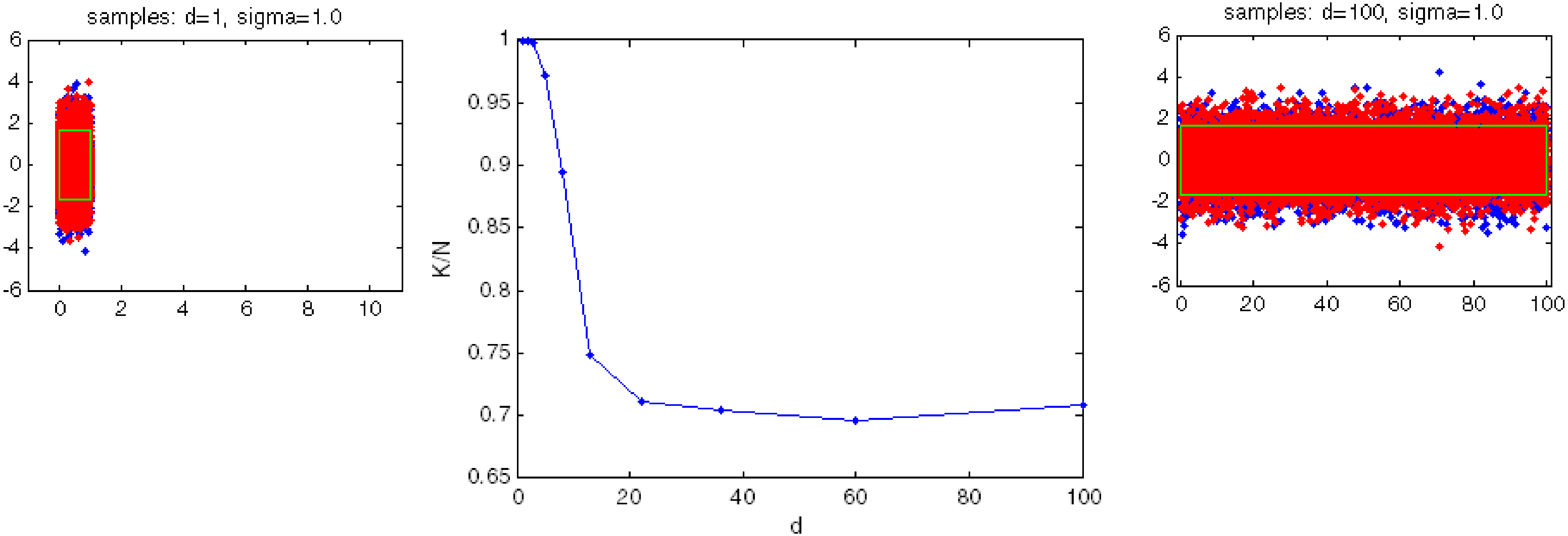
Observation



1. d ... the length of the line segment
2. K ... the number of pairs generating a line that has more that 95 % of points of X closer than 1.96 σ.
3. the shorter the line, the higher the chance to generate a good sample
4. for $d \gg \sigma$ ... $\frac{K}{N} \approx 0.7$

Relaxation

It is enough to have a high (95 %) chance of finding a good estimate of the true line segment l.

70 % of samples generate a line that is closer than $1.96\,\sigma$ to more than 95 % of points in X

Relaxation

Using sufficient number of randomly chosen pairs of points guarrantees average succeess.

Random Minimal Sampling

$$I = \arg \max_{\{x_1, x_2\} \subseteq R \subset X} (\# \text{ inliers in } X \text{ for the line through } \{x_1, x_2\})$$

needs to examine "only" $|R|$ pairs of distinct points in a randomly chosen subset R of X.

How many random samples should be tried?

Random Minimal Sampling for Robust Line Fitting

The goal is to make k samples to hit at leat one pair of points on the line l with probability larger than p.

Equivalently, we look for k such that the probability of not hitting any pair of points on l is smaller or equal to $1-p$

$$\left(1-\frac{M(M-1)}{N(N-1)}\right)^k \leq 1-p$$

$\frac{M}{N}$	...	the probability of drawing a good data point
$\frac{M(M-1)}{N(N-1)}$	...	the probability of drawing (without repetition) a good pair of data points
$1-\frac{M(M-1)}{N(N-1)}$	...	the probability of drawing a bad pair of data points
$\left(1-\frac{M(M-1)}{N(N-1)}\right)^k$	...	the probability of drawing (with repetitions) k bad pairs of data points in a row

Random Minimal Sampling for General Models

For $N \to \infty$ and fixed fraction $w = \frac{M}{N}$ of good points

$$\lim_{N\to\infty} \frac{M(M-1)}{N(N-1)} = \lim_{N\to\infty} \frac{wN(wN-1)}{N(N-1)} = \epsilon^2$$

which depends only on the fraction of good points w and for large N leads to

$$\left(1 - w^2\right)^k \leq 1 - p$$

or requivalently to the necessary number of samples

$$k \geq \frac{\log\left(1-p\right)}{\log\left(1 - w^2\right)}$$

Random Minimal Sampling for General Models

Generalization for the samples with m points:

$$\lim_{N \to \infty} \frac{M(M-1)\dots(M-m)}{N(N-1)\dots(N-m)} = \lim_{N \to \infty} \frac{wN(wN-1)\dots(wN-m)}{N(N-1)\dots(N-m)} = w^m$$

and thus for large N we get

$$(1-w^m)^k \leq 1-p$$

or equivalently

$$k \geq \frac{\log(1-p)}{\log(1-w^m)}$$

RANSAC - algorithm

Input: N m-tuples of data, threshold τ, probability p.

$K := \infty,\ k := 0$

$p^* := [],\ M^* := 0$

while $K > k$ **do**

Choose a sample S of size m

Compute the model parameter p from the sample S

Count the number M of inliers using p and τ

if $M > M^*$ **then**

$M^* := M$

$p^* := p$

$w := M/N$

$$K := \text{ceil}\left(\frac{\log(1-p)}{\log(1-w^m)}\right)$$

$k := k + 1$

end if

end while

Output: parameters p^*

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Example – Homography fitting

1. Model: $\alpha_i \mathbf{y}_i = \mathtt{H} \mathbf{x}_i, \qquad i = 1, \dots M$

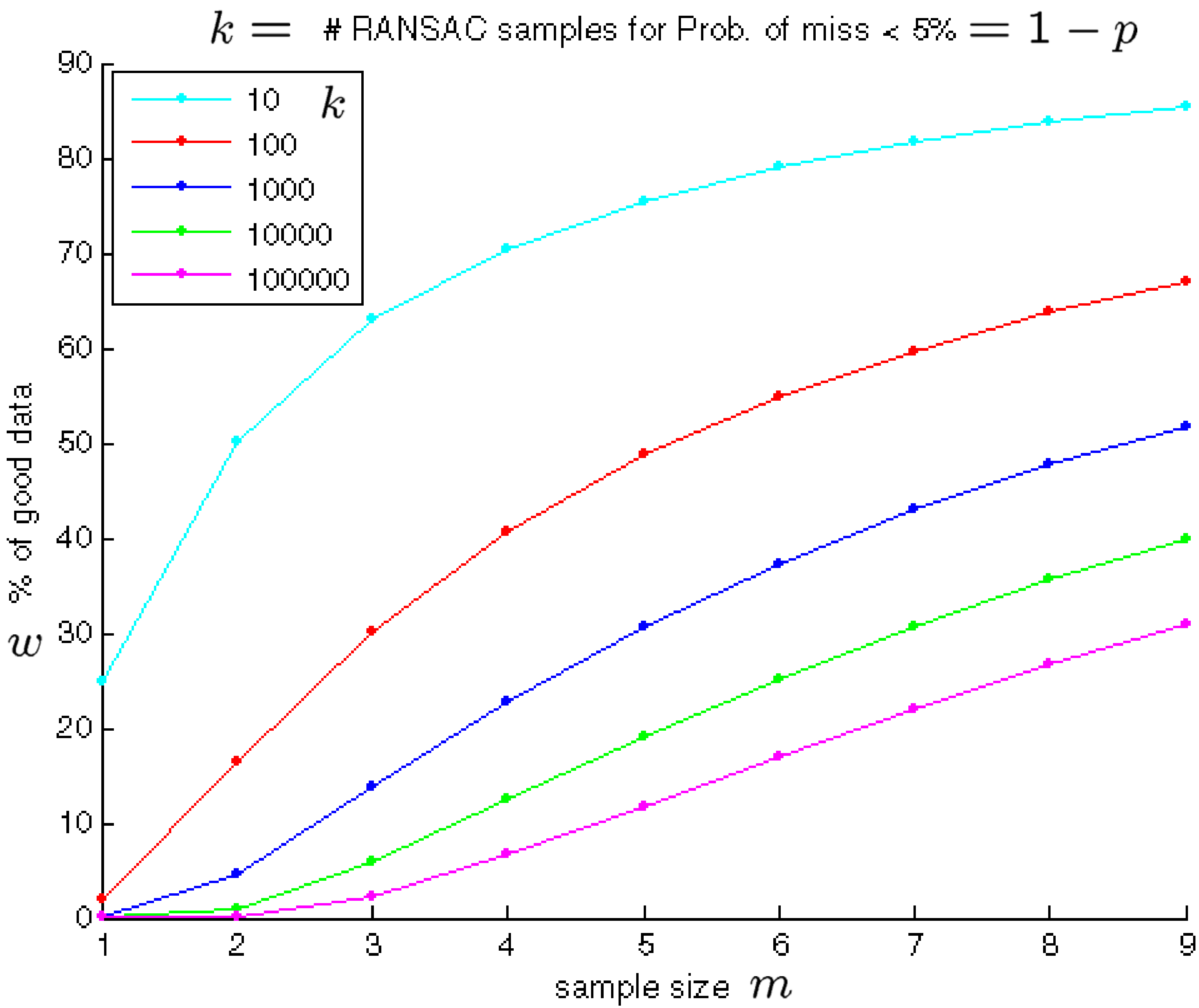
2. Data: 1000 points, 350 points related by the model, 750 points random

3. Minimal sample size $m = 4$

4. No measurement noise, i.e. $\sigma = 0$

5. Assume large N. How many random samples must be drawn with repetitions to hit at least one four-tupple o points related by the homography $\mathtt{H}$ with probability 0.93?

How many samples?



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