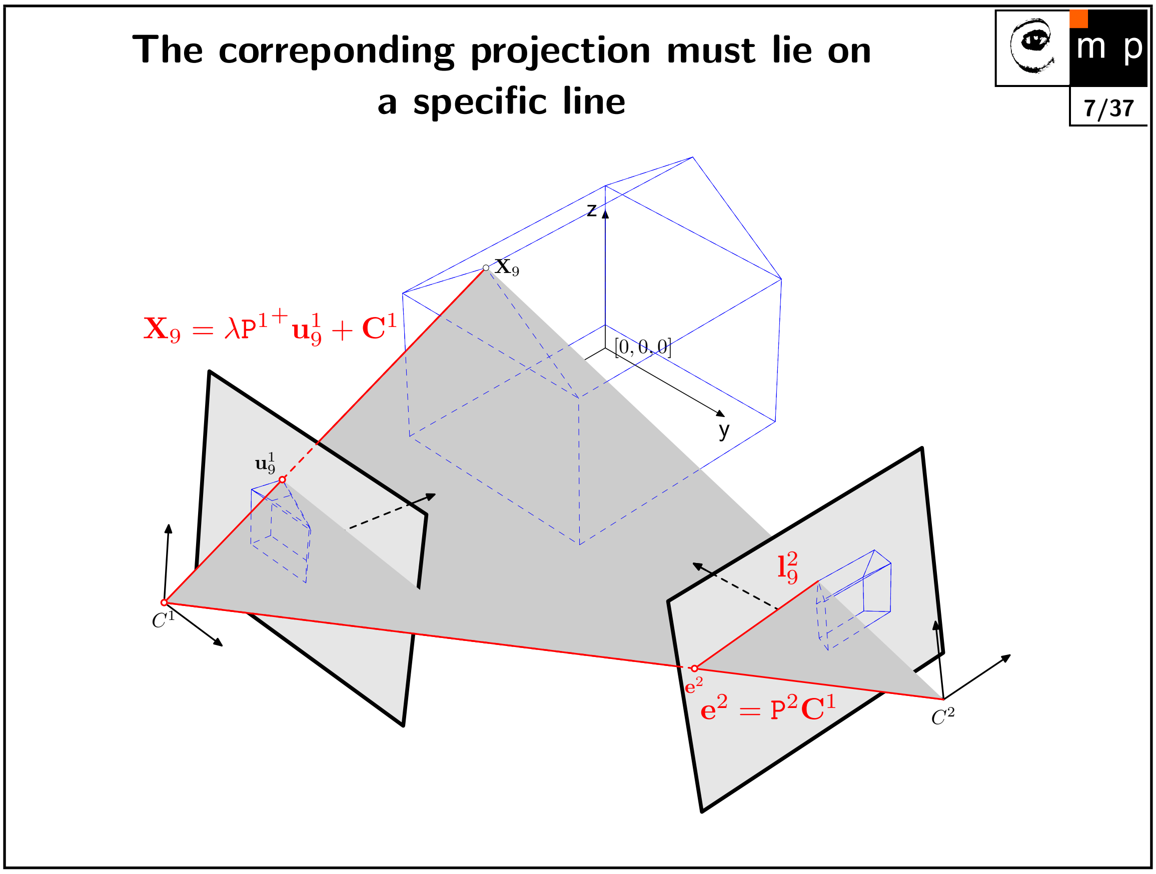

Pseudoinverse

Moore-Penrose Generalized Matrix Inverse

Given an $m \times n$ MATRIX B, the Moore-Penrose generalized MATRIX INVERSE (sometimes called the pseudoinverse) is a unique $n \times m$ MATRIX B^+ which satisfies

$$\mathsf{B}\mathsf{B}^+\mathsf{B} = \mathsf{B} \tag{1}$$

$$\mathsf{B}^+\mathsf{B}\mathsf{B}^+ = \mathsf{B}^+ \tag{2}$$

$$(\mathsf{B}\mathsf{B}^+)^{\mathrm{T}} = \mathsf{B}\mathsf{B}^+ \tag{3}$$

$$(\mathsf{B}^+\mathsf{B})^{\mathrm{T}} = \mathsf{B}^+\mathsf{B}. \tag{4}$$

It is also true that

$$\mathbf{z} = \mathsf{B}^+\mathbf{c} \tag{5}$$

is the shortest length LEAST SQUARES solution to the problem

$$\mathsf{B} = \mathbf{c}. \tag{6}$$

If the inverse of $(\mathsf{B}^{\mathrm{T}}\mathsf{B})$ exists, then

$$\mathsf{B}^+ = (\mathsf{B}^{\mathrm{T}}\mathsf{B})^{-1}\mathsf{B}^{\mathrm{T}}, \tag{7}$$

where B^{T} is the matrix TRANSPOSE, as can be seen by premultiplying both sides of (7) by B^{T} to create a SQUARE MATRIX which can then be inverted,

$$\mathsf{B}^{\mathrm{T}}\mathsf{B}_{\mathbf{z}} = \mathsf{B}^{\mathrm{T}}\mathbf{c}, \tag{8}$$

giving

$$\mathbf{z} = (\mathsf{B}^{\mathrm{T}}\mathsf{B})^{-1}\mathsf{B}^{\mathrm{T}}\mathbf{c} \equiv \mathsf{B}^+\mathbf{c}. \tag{9}$$

Just cropped from the CRC Encyclopedia of Mathematics (temporary solution).

Epipolar geometry revisited

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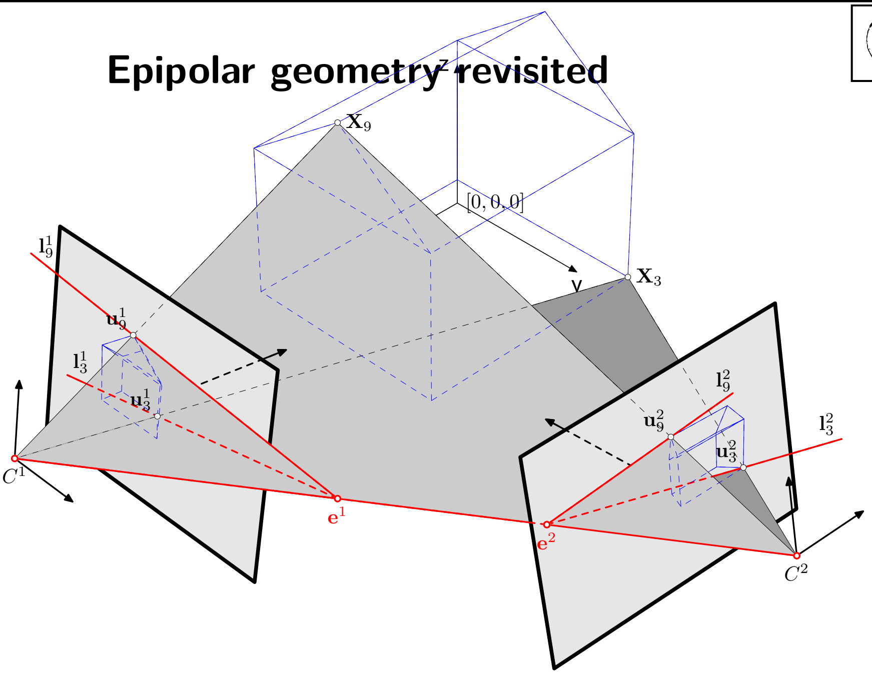

${\mathbf{u}_i^2}^\top \mathtt{F} \mathbf{u}_i^1 = 0$ holds for any corresponding pair $\mathbf{u}_i^1, \mathbf{u}_i^2$.

$\mathtt{F}$ does not depend on the scene structure, only on cameras.

All epipolar lines intersect in epipoles.

1 Homework:

Assume that a Fundamental matrix $\mathtt{F}$ is known. Derive the epipoles $\mathbf{e}^1$ and $\mathbf{e}^2$. Shortly comment your derivations.
Hint: All epipolar lines intersect in epipoles. Hence, ${\mathbf{e}^2}^\top \mathtt{F} \mathbf{u}_i^1$ holds for any i.

Essential matrix

For the Fundamental matrix we derived

$$\mathbf{u}_i^{1\top} \underbrace{\left([\mathbf{e}^2]_\times \mathtt{P}^2 \mathtt{P}^{1+}\right)^\top}_{\mathtt{F}} \mathbf{u}_i^2 = 0$$

$\mathbf{u}$ denote point coordinates in pixels. Let coincide the world system with the coordinate system of the first camera.

$$\mathbf{u}^1 = \mathtt{K}^1 \left[\ \mathtt{I} \quad \mathbf{0}\ \right] \mathbf{X} \qquad \mathbf{u}^2 = \mathtt{K}^2 \left[\ \mathtt{R} \quad \mathbf{t}\ \right] \mathbf{X}$$

Remind the normalized image coordinates $\mathbf{x} = \mathtt{K}^{-1}\mathbf{u}$. We can define normalized cameras $\mathbf{x} = \hat{\mathtt{P}}\mathbf{X}$ and insert the equation above.

$$\mathbf{x}_i^{1\top} \underbrace{\left([\mathbf{x}_\mathbf{e}^2]_\times \hat{\mathtt{P}}^2 (\hat{\mathtt{P}}^1)^+\right)^\top}_{\mathtt{E}} \mathbf{x}_i^2 = 0$$

where $\mathtt{E}$ is the Essential matrix

Historically, the *Essential matrix* was introduced before the *Fundamental matrix* by Longuet-Higgins in his very seminal paper [5].

End

References

[1] H.C. Longuett-Higgins. A computer algorithm for reconstruction a scene from two projections. *Nature*, 293:133–135, 1981.