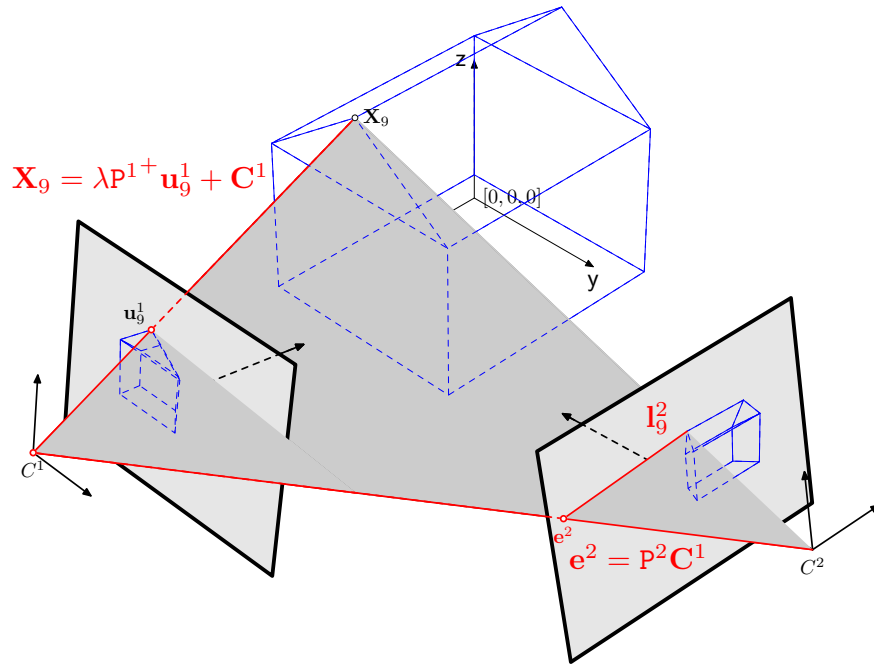


The corresponding projection must lie on a specific line



Pseudoinverse

Moore-Penrose Generalized Matrix Inverse

Given an $m \times n$ MATRIX B , the Moore-Penrose generalized MATRIX INVERSE (sometimes called the pseudoinverse) is a unique $n \times m$ MATRIX B^+ which satisfies

$$BB^+B = B \quad (1)$$

$$B^+BB^+ = B^+ \quad (2)$$

$$(BB^+)^T = BB^+ \quad (3)$$

$$(B^+B)^T = B^+B. \quad (4)$$

It is also true that

$$\mathbf{z} = B^+ \mathbf{c} \quad (5)$$

is the shortest length LEAST SQUARES solution to the problem

$$B\mathbf{z} = \mathbf{c}. \quad (6)$$

If the inverse of (B^TB) exists, then

$$B^+ = (B^TB)^{-1}B^T, \quad (7)$$

where B^T is the matrix TRANSPOSE, as can be seen by premultiplying both sides of (7) by B^T to create a SQUARE MATRIX which can then be inverted,

$$B^TB_z = B^T\mathbf{c}, \quad (8)$$

giving

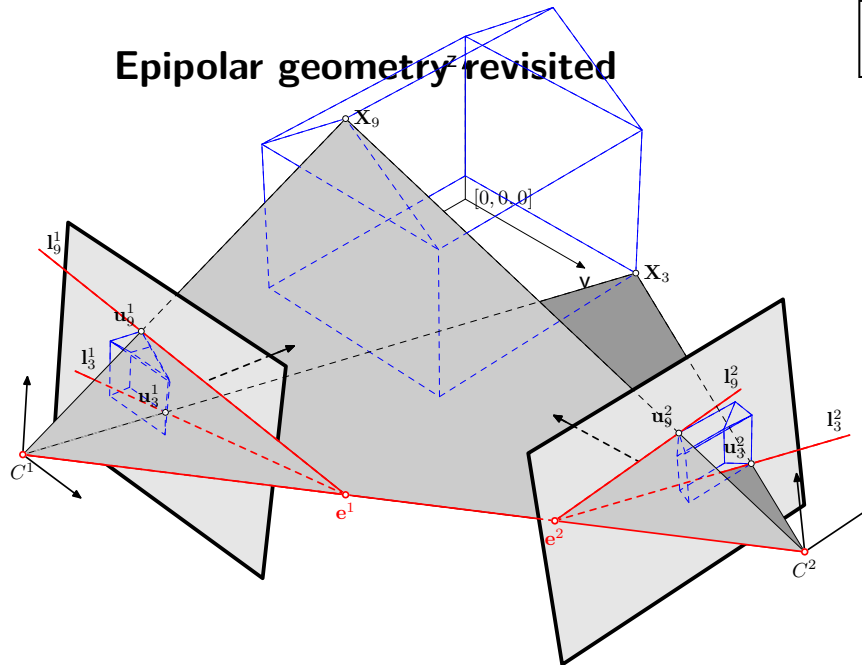
$$\mathbf{z} = (B^TB)^{-1}B^T\mathbf{c} \equiv B^+\mathbf{c}. \quad (9)$$

Just cropped from the CRC Encyclopedia of Mathematics (temporary solution).

Epipolar geometry revisited



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$\mathbf{u}_i^{2\top} \mathbf{F} \mathbf{u}_i^1 = 0$ holds for any **corresponding** pair $\mathbf{u}_i^1, \mathbf{u}_i^2$.

\mathbf{F} does not depend on the scene structure, only on cameras.

All **epipolar lines** intersect in **epipoles**.

1 Homework:

Assume that a Fundamental matrix \mathbf{F} is known. Derive the epipoles \mathbf{e}^1 and \mathbf{e}^2 . Shortly comment your derivations.

Hint: All epipolar lines intersect in epipoles. Hence, $\mathbf{e}^{2\top} \mathbf{F} \mathbf{u}_i^1$ holds for any i .

Essential matrix

For the Fundamental matrix we derived

$$\mathbf{u}_i^{1\top} \underbrace{\left([\mathbf{e}^2]_{\times} \mathbf{P}^2 \mathbf{P}^{1+} \right)}_{\mathbf{F}} \mathbf{u}_i^2 = 0$$

\mathbf{u} denote point coordinates in **pixels**.

$$\mathbf{u}^1 = \mathbf{K}^1 \begin{bmatrix} \mathbf{R}^1 & \mathbf{t}^1 \end{bmatrix} \mathbf{X} \quad \mathbf{u}^2 = \mathbf{K}^2 \begin{bmatrix} \mathbf{R}^2 & \mathbf{t}^2 \end{bmatrix} \mathbf{X}$$

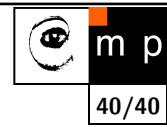
Remind the normalized image coordinates $\mathbf{x} = \mathbf{K}^{-1} \mathbf{u}$. We can define normalized cameras $\mathbf{x} = \hat{\mathbf{P}} \mathbf{X}$ and insert the equation above.

$$\mathbf{x}_i^{1\top} \underbrace{\left([\mathbf{x}_e^2]_{\times} \hat{\mathbf{P}}^2 (\hat{\mathbf{P}}^1)^+ \right)}_{\mathbf{E}} \mathbf{x}_i^2 = 0$$

where \mathbf{E} is the **Essential matrix**

Historically, the *Essential matrix* was introduced before the *Fundamental matrix* by Longuet-Higgins in his very seminal paper [5].

End



References

- [1] H.C. Longuet-Higgins. A computer algorithm for reconstruction a scene from two projections. *Nature*, 293:133–135, 1981.