Sequential Learned Linear Predictors for Object Tracking

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Object tracking

- **Tracking** - iterative estimation of an object pose in a sequence (e.g., 2D position of Basil's head).
- Trade-off among *accuracy, speed, robustness* is explicitly taken into account.

video: Basil head

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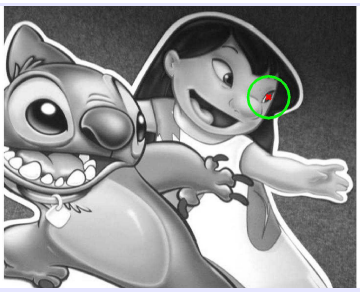
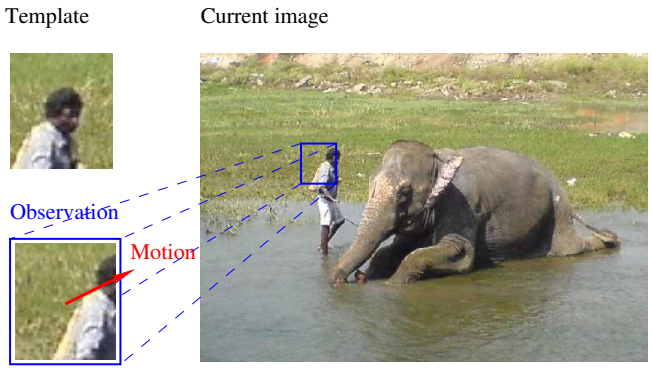


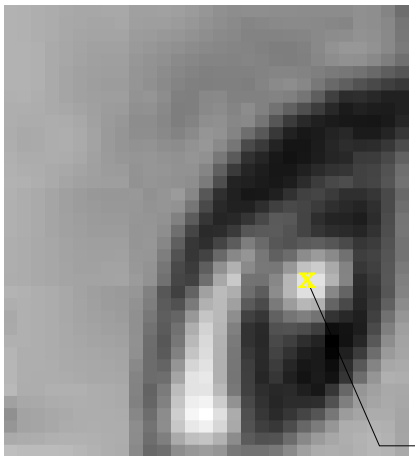
video: Cartoon eye

Tracking



Observation (image) **I** and template **J**

Tracking of a single point



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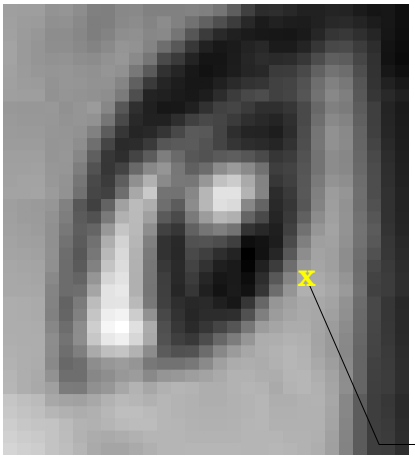

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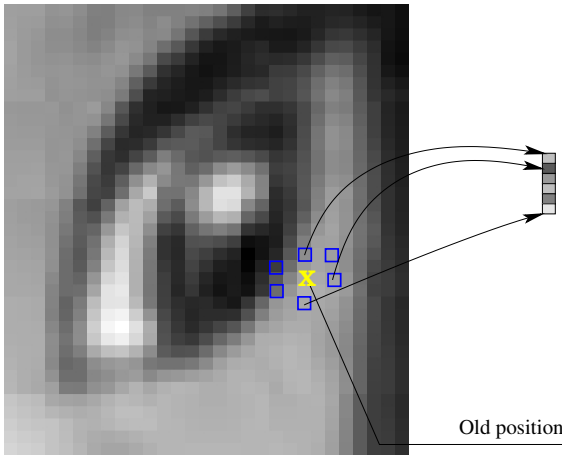
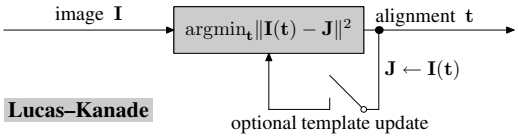
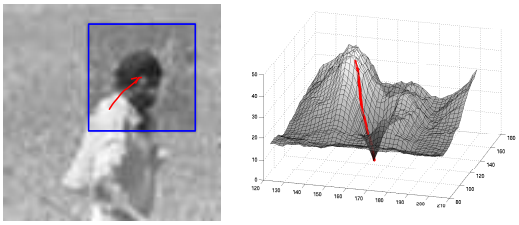


[Lucas-Kanade1981] - Iterative minimization





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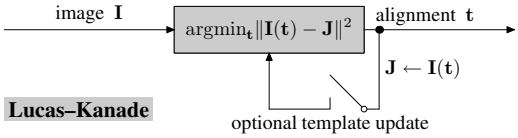
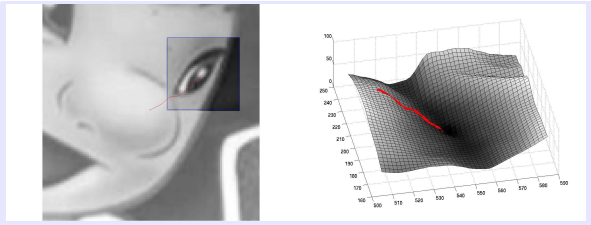
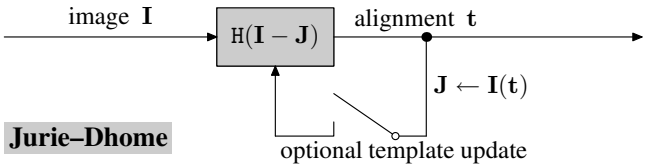




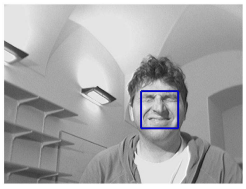
video: KLT iterations

Regression - learn the mapping in advanceo



- [Jurie-BMVC-2002] - learned motion and optional hard template update
- [Cootes-PAMI-2001] - learned regression during AAM iterations
- ...

Learning alignment for one predictor



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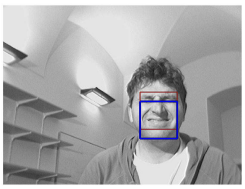

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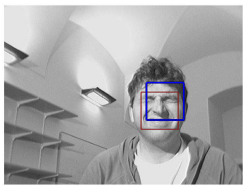

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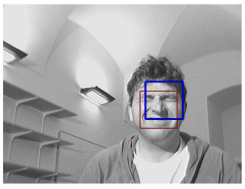

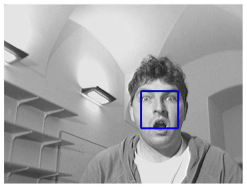

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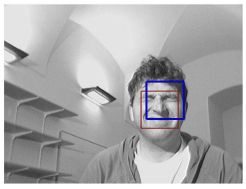

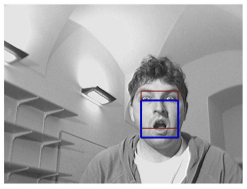

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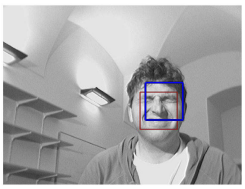

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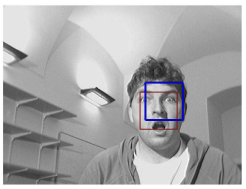

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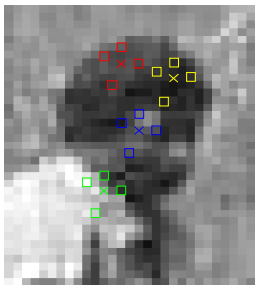
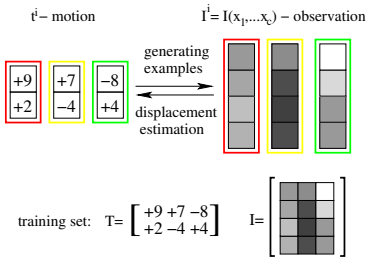
Generating training examples





Training set: $(\mathtt{I}, \mathtt{T})$
$\mathtt{I} = [\mathbf{I}^1 - \mathbf{J}, \mathbf{I}^2 - \mathbf{J}, \dots \mathbf{I}^d - \mathbf{J}]$ and $\mathtt{T} = [\mathbf{t}^1, \mathbf{t}^2, \dots \mathbf{t}^d]$.

LS learning

$$\varphi^* = \operatorname{argmin}_\varphi \sum_{\mathbf{t}} \|\varphi\Big(\mathbf{I}\big(\mathbf{t} \circ X\big)\Big) - \mathbf{t}\|^2.$$

Minimizes sum of square errors over all training set. Leads to matrix pseudoinverse computation.

Example for *linear mapping*:

$$\mathtt{H}^* = \operatorname*{argmin}_{\mathtt{H}} \sum_{i=1}^{d} \|\mathtt{H}(\mathbf{I}^i - \mathbf{J}) - \mathbf{t}^i\|_2^2 = \operatorname*{argmin}_{\mathtt{H}} \|\mathtt{HI} - \mathtt{T}\|_F^2$$

after some derivation

$$\mathtt{H}^* = \mathtt{T}\underbrace{\mathtt{I}^\top(\mathtt{I}\mathtt{I}^\top)^{-1}}_{\mathtt{I}^+} = \mathtt{T}\mathtt{I}^+.$$

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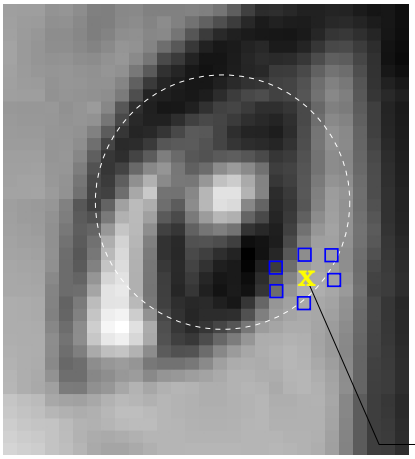
Min-max learning

$$\varphi^* = \mathrm{argmin}_\varphi \max_{\mathbf{t}} \|\varphi\Big(\mathbf{I}(\mathbf{t} \circ X)\Big) - \mathbf{t}\|_\infty.$$

Minimizes the *worst case* (the biggest estimation error) in the training set. Leads to linear programming.

Tracking of a single point by a *sequence* of predictors



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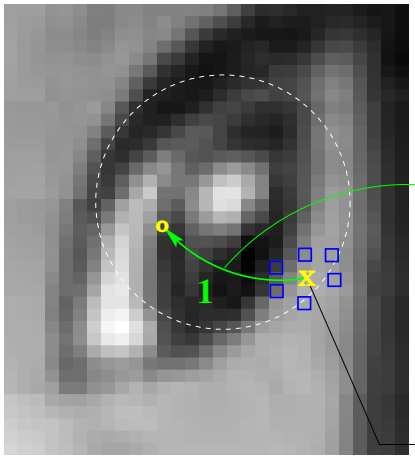

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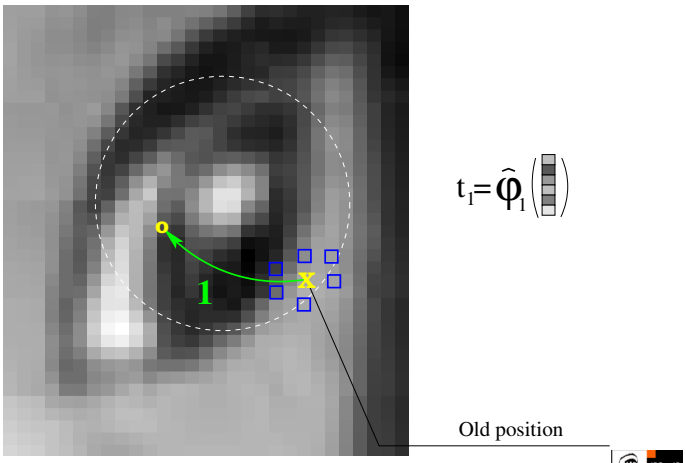

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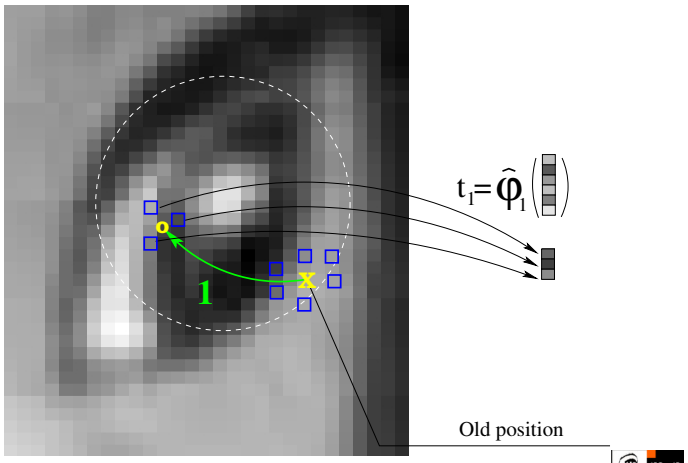

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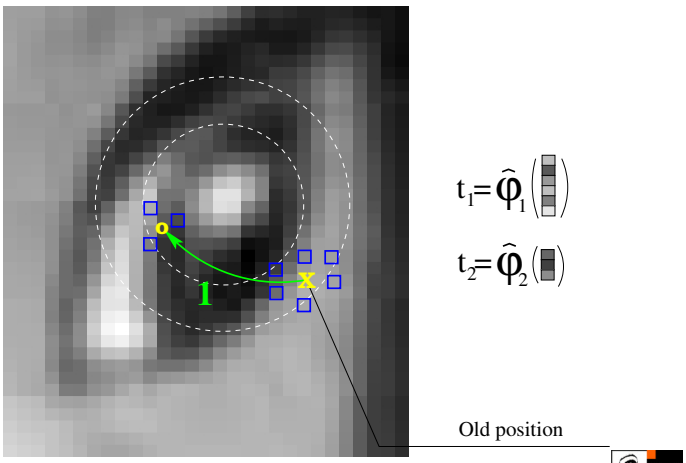

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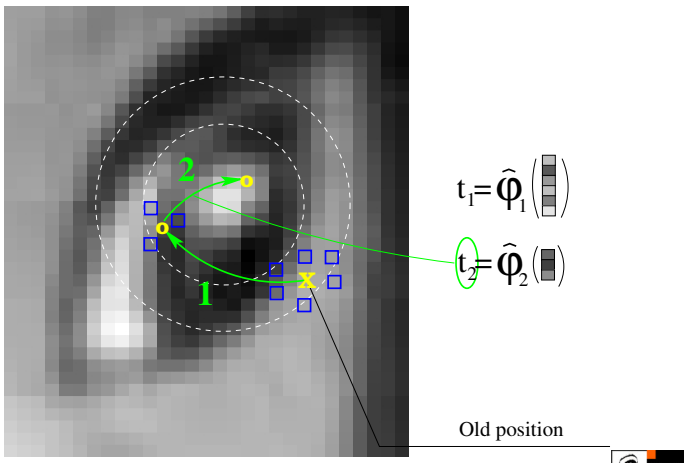

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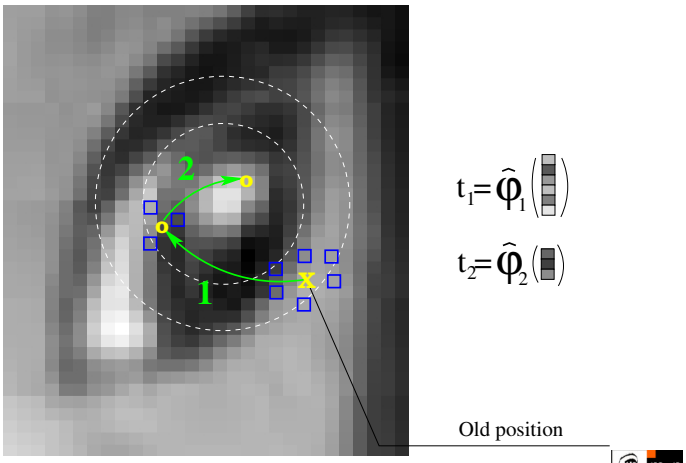

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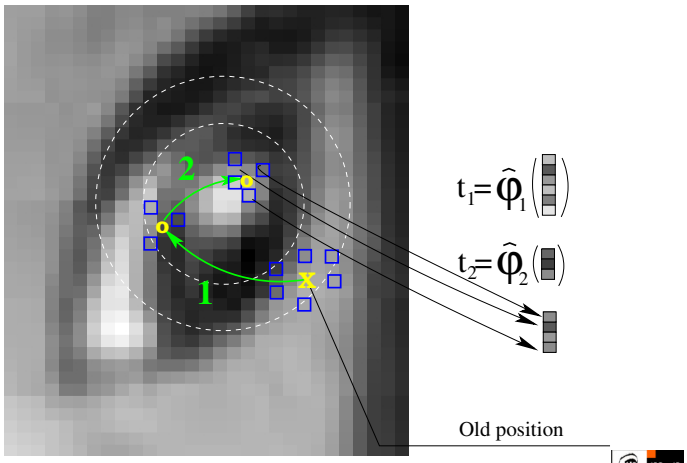

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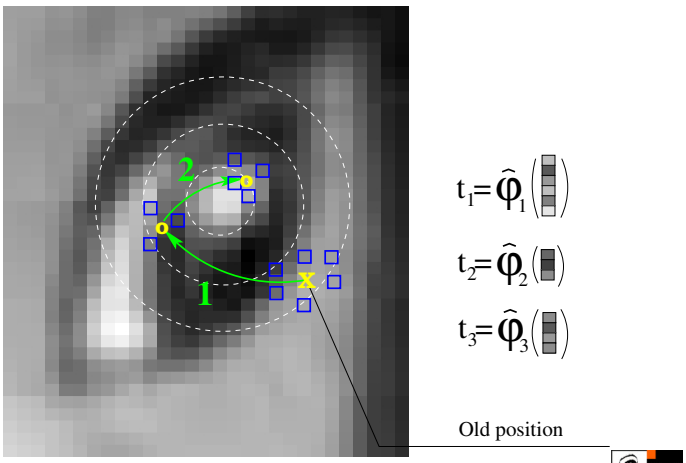

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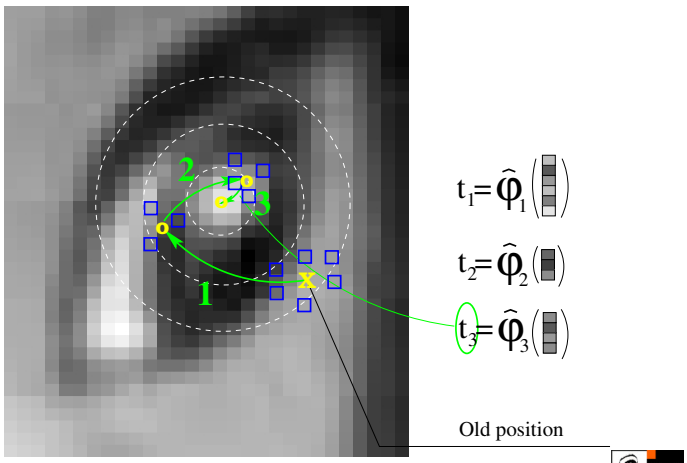

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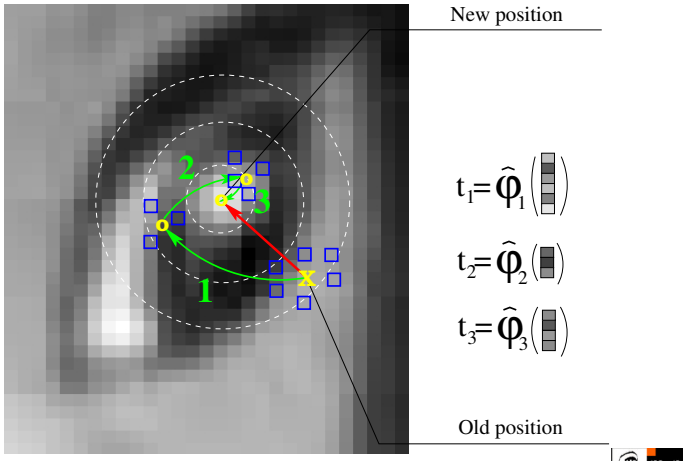

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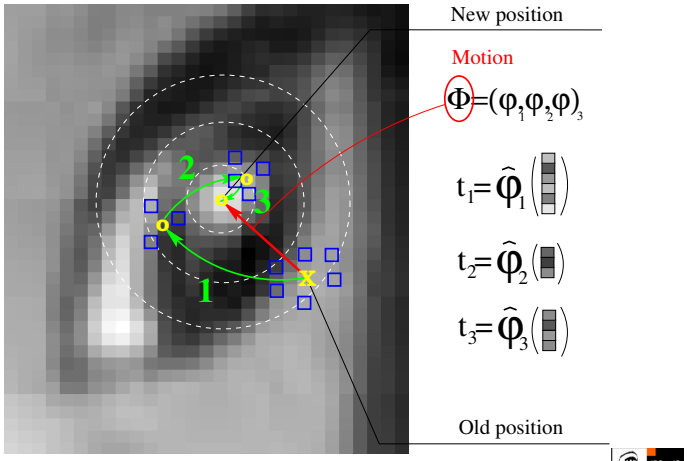

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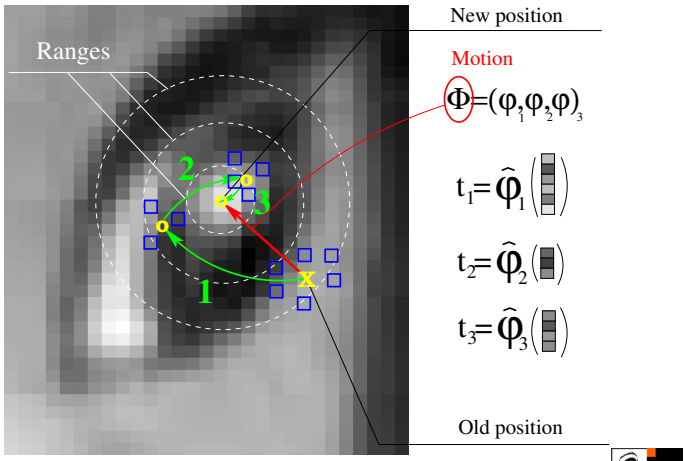

Learning of sequential predictor

- **Learning** - searching for the sequence with predefined *range*, *accuracy* and minimal *computational cost*.
  - [Zimmermann-PAMI-2009] - Dynamic programming estimates the optimal sequence of linear predictors.
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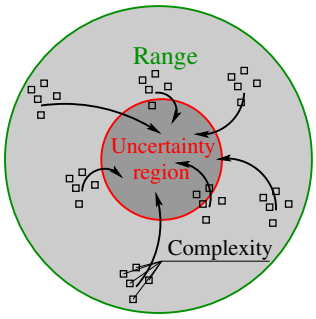
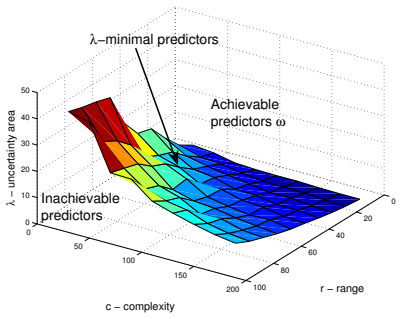
Learning of the optimal sequence of linear predictors



- **Range**: the set of admissible motions, r.
- **Complexity**: cardinality of support set, c.
- **Uncertainty region**: the region within which all predictions lie, λ. Small red circles show acceptable uncertainty.

Learning of the optimal sequence of linear predictors



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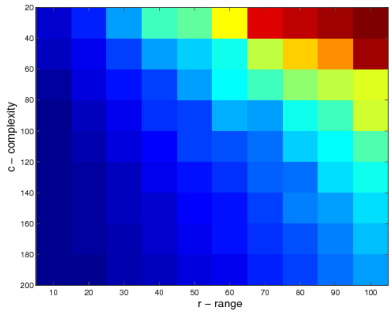

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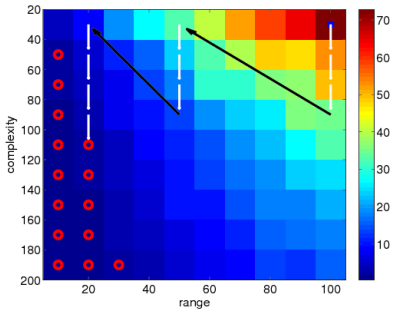

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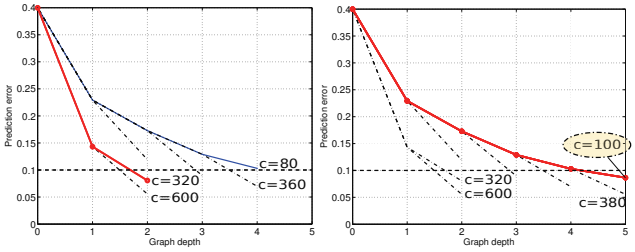
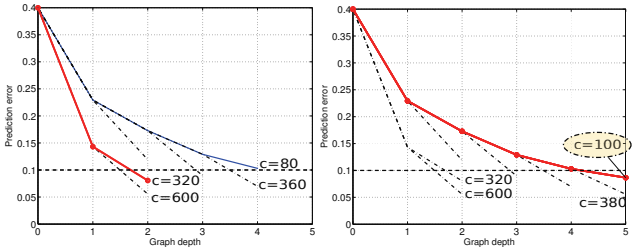
Branch and Bound



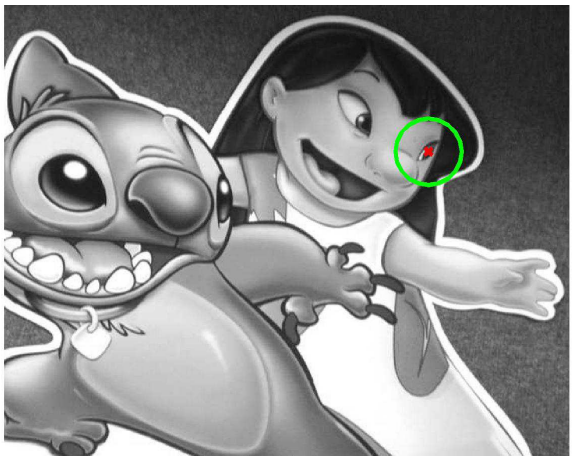
Don't forget to show the live demo!

Branch and Bound

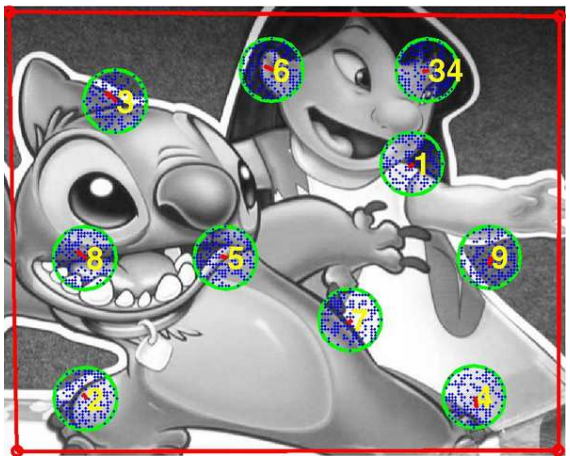


Don't forget to show the live demo!

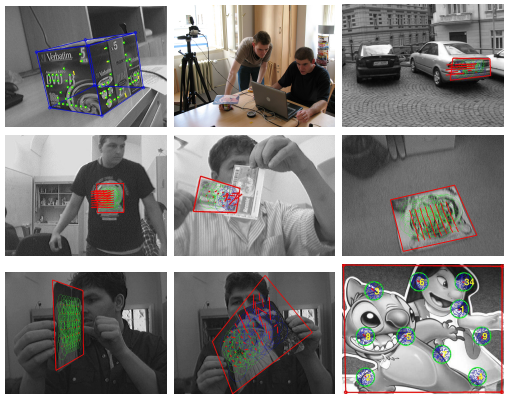
Tracking with one linear predictor.



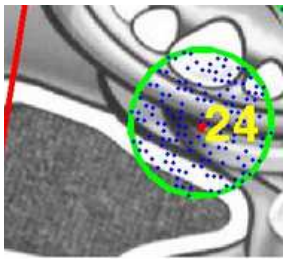
Modeling motion by number of linear predictors.

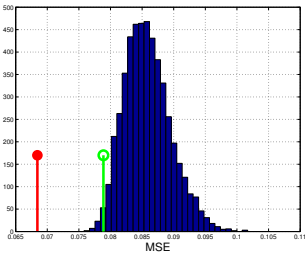


Motion blur, fast motion, views from acute angles and other image distortions.



Support set selection



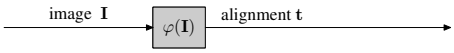


- Greedy LS selection (red) of an efficient support set.
- Much better than 1%-quantile (green) achieavable by randomized sampling

Tracking of objects with variable appearance

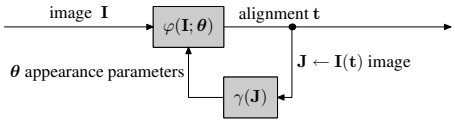
- **Variable appearance** - the way how the object looks like in the camera changes due to illumination, non-rigid deformation, out-of-plane rotation, ...

Simultaneous learning of motion and appearance



- Introduce feedback which encodes appearance in a low dimensional space and adjust the predictor.
- Appearance parameters learned in unsupervised way.
- Simultaneous learning of φ and $\gamma \Rightarrow$ appearance encoded in the low dimensional space, which is the most suitable for the motion estimation.

Simultaneous learning of motion and appearance



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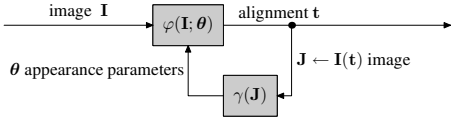

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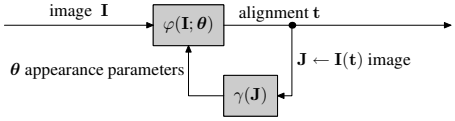

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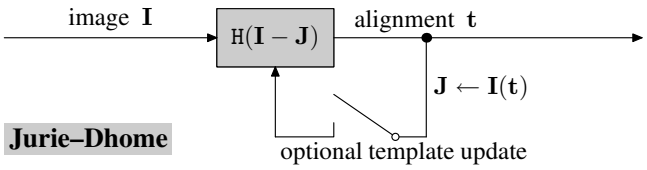
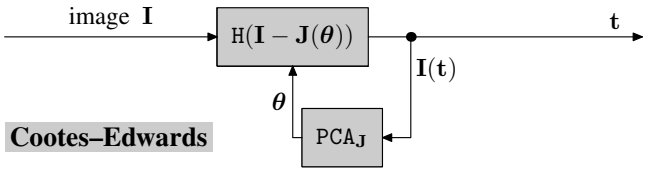
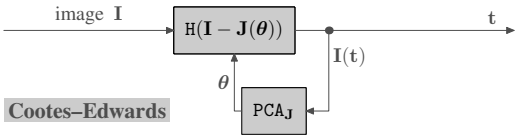
Learning appearance

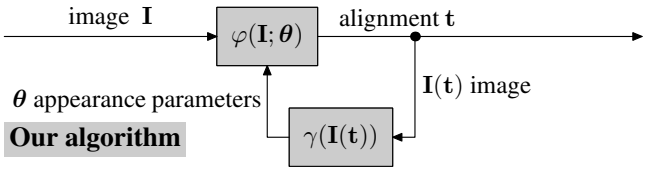


Learning appearance



Learning appearance – our approach





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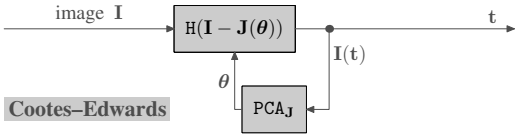
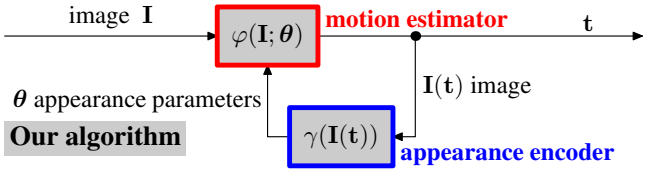
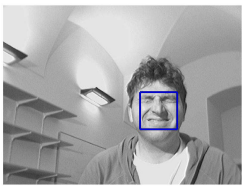
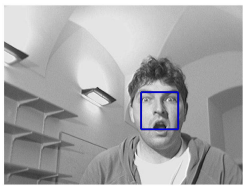




Learning the appearance encoder γ

- Current appearance encoded in low-dim parameters.





- $\gamma($ $) = \boldsymbol{\theta}_1$
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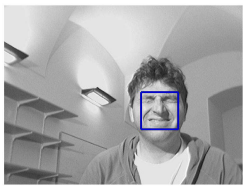

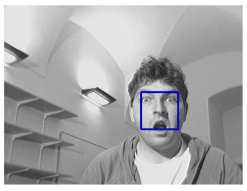

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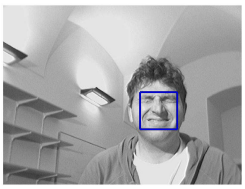
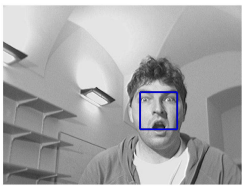
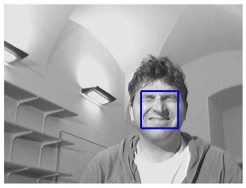


- $\gamma($  $) = \boldsymbol{\theta}_1$



- $\gamma($  $) = \boldsymbol{\theta}_2$

Learning the tracker $\varphi(\mathbf{I};\boldsymbol{\theta})$



- $\varphi($  $;\boldsymbol{\theta}_1) = (0,0)^\top$
- $\varphi($ $;\boldsymbol{\theta}_1) = (-25,0)^\top$
- $\varphi($ $;\boldsymbol{\theta}_1) = (25,-15)^\top$
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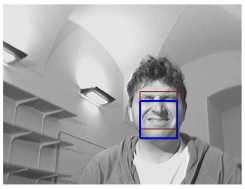

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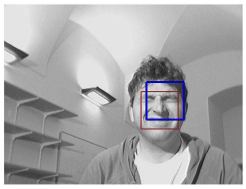

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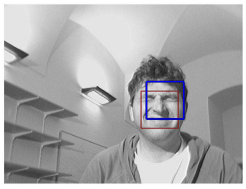

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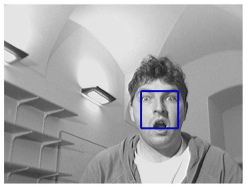

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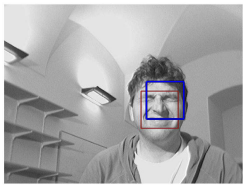

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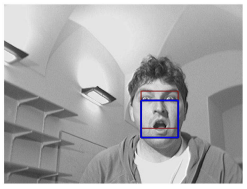

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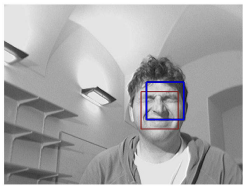

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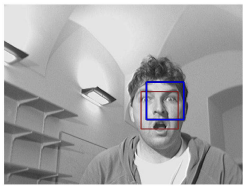

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Simultaneous learning of φ and γ

- **Learning = minimization of the least-squares error**

$$(\varphi^*,\gamma^*)=\arg\min_{\varphi,\gamma}\ \left[\varphi(\ \text{  }\ ;\gamma(\ \text{  }\ ))\ -(0,0)^\top\right]^2+$$
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$$\left[\varphi(\quad;\gamma(\quad))\ -(25,-15)^\top\right]^2+$$
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Linear mapping

- $\gamma(\mathbf{J}) : \boldsymbol{\theta} = \mathtt{G}\mathbf{J}$
- $\varphi(\mathbf{I}, \boldsymbol{\theta}) : \mathbf{t} = (\mathtt{H}_0 + \theta_1\mathtt{H}_1 + \cdots + \theta_n\mathtt{H}_n)\mathbf{I}$
- Criterion is sum of squares of bilinear functions.

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Algorithm: iterative minimization of criterion $e(\varphi, \gamma)$

φ – motion (geometry mapping), γ – appearance mapping

color encodes criterion value $e(\varphi, \gamma)$

- Iterative minimization:
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  - $\gamma^3 = \arg\min_{\gamma} e(\varphi^3, \gamma)$
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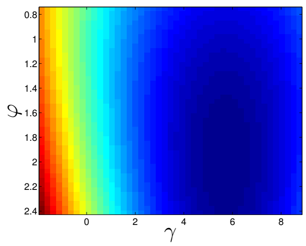

- Global optimality for linear φ, γ experimentally shown.

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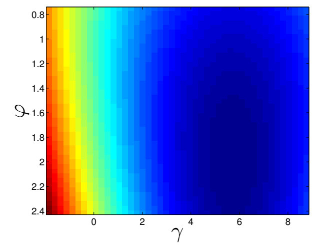

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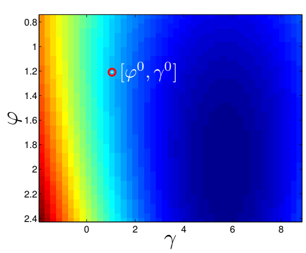

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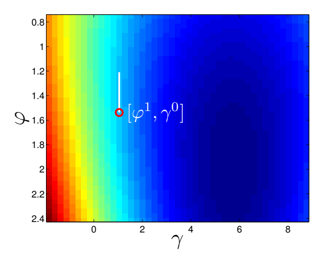

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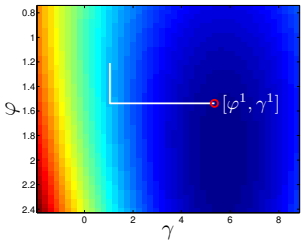

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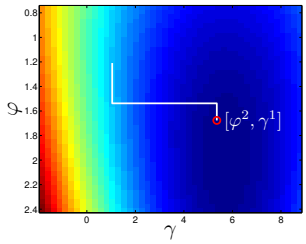
- Global optimality for linear φ, γ experimentally shown.



Algorithm: iterative minimization of criterion $e(\varphi, \gamma)$

φ – motion (geometry mapping), γ – appearance mapping

color encodes criterion value $e(\varphi, \gamma)$

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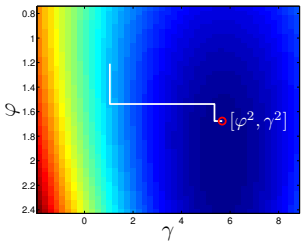

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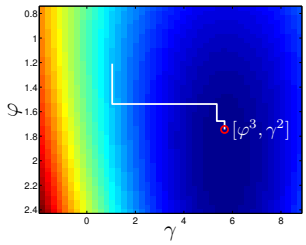

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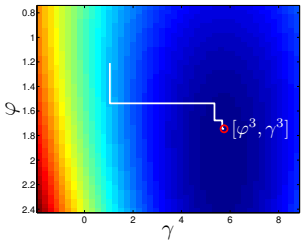

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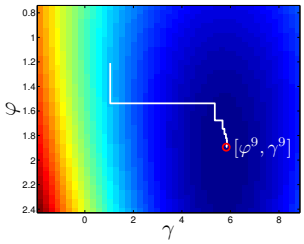

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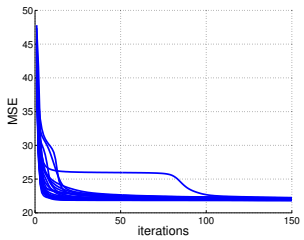

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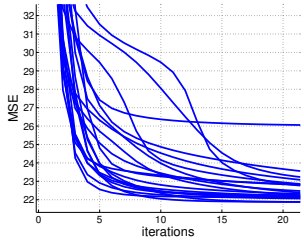

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Simultaneous learning of motion and appearance

Experiments - videos II

Conclusions

- Learnable and very efficient tracking of objects with variable appearance.
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Limitations

- Small, thin objects intractable.

Data, papers, various implemenations freely available at
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